

This is same calc as previously posted, except that we have added geometric analysis (CF1 and CF2) which prove the complex arithmetic is correct.

Although it may intuitively seem like there should be more difference between the vector and algebraic solutions than computed below, I believe that intuition is incorrect

Use transformer MVA as base

Items highlighted in purple are results which will be compared to Matlab

$$\text{IrunFullVoltage} = 0.1 \cdot \left( 0.8 - i \cdot \sqrt{1 - 0.8^2} \right)$$

$$\text{IrunFullVoltage} = 0.08 - 0.06 \cdot i$$

$$6 \cdot \text{IrunFullVoltage} = 0.48 - 0.36 \cdot i$$

$$\text{IstartFullVoltage} = 0.6 \cdot \left( 0.2 - i \cdot \sqrt{1 - 0.2^2} \right)$$

$$\text{IstartFullVoltage} = 0.12 - 0.5879 \cdot i$$

$$X := \frac{(1 + 6 \cdot i)}{\sqrt{1^2 + 6^2}} \cdot 0.06$$

$$X = 0.0099 + 0.0592 \cdot i$$

Huge simplifying assumption... currents don't change with voltage

"(otherwise we need load flow or a little more work)

First solution is using magnitudes

$$V2_{\text{algebraic}} = 1 - |X| \cdot (6 \cdot |\text{IrunFullVoltage}| + |\text{IstartFullVoltage}|)$$

$$V2_{\text{algebraic}} = 0.928$$

More exact solution using complex numbers

$$V2_{\text{vector}} = 1 - X \cdot (6 \cdot \text{IrunFullVoltage} + \text{IstartFullVoltage})$$

$$V2_{\text{vector}} = 0.938 - 0.0262 \cdot i$$

$$|V2_{\text{vector}}| = 0.9383$$

Compare the two solutions above in terms of correction factors CF1 and CF2

Correction Factor CF1 accounts for the fact that we added currents algebraically

Solve CF1 numerically:

$$\text{CF1} := \frac{|6 \cdot \text{IrunFullVoltage} + \text{IstartFullVoltage}|}{|6 \cdot \text{IrunFullVoltage}| + |\text{IstartFullVoltage}|}$$

$$\text{CF1} = 0.9348$$

Confirm CF1 by geometry - using law of cosines

$$A := 6 \cdot \text{IrunFullVoltage}$$

$$A = 0.48 - 0.36 \cdot i$$

$$|A| = 0.6$$

$$B := \text{IstartFullVoltage}$$

$$B = 0.12 - 0.5879 \cdot i$$

$$|B| = 0.6$$

$$\theta := \pi - (\arg(A) - \arg(B))$$

$$\theta = 2.4157$$

$$C_{\text{mag}} := \sqrt{|A|^2 + |B|^2 - 2 \cdot |A| \cdot |B| \cdot \cos(\theta)}$$

$$C_{\text{mag}} = 1.1218$$

$$CF1 := \frac{C_{\text{mag}}}{|A| + |B|}$$

$$CF1 = 0.9348$$

Correction Factor CF2 accounts for the fact that algebraic solution assumed the voltage drop was in-phase with the source voltage. i.e. we calculated  $V = 1 - (IZ)$  as if  $(IZ)$  was in phase with 1.

We will also include the effects of CF1 within the calculation of CF2 so that CF2 is the final exact correction from the algebraic method to the vector method

$$A := 1$$

$$A = 1$$

$$|A| = 1$$

$$B := -X \cdot (6 \cdot \text{IrunFullVoltage} + \text{IstartFullVoltage})$$

$$|B| = 0.0673$$

$$\theta := \pi - (\arg(A) - \arg(B))$$

$$\theta = 0.3992$$

$$C_{\text{mag}} := \sqrt{|A|^2 + |B|^2 - 2 \cdot |A| \cdot |B| \cdot \cos(\theta)}$$

$$C_{\text{mag}} = 0.9383$$

$C_{\text{mag}}$  we can see matches the length of the V2vector, as intended ( $C_{\text{mag}}$  is vector sum  $1 - IZ$ )

To develop final correction CF2, we need to compare the above geometrically-derived estimate of V2vectormag =  $C_{\text{mag}}$  to the algebraic estimate of V2, which itself includes the geometrically-derived error of CF1

$$CF2 := \frac{C_{mag}}{1 - \frac{|B|}{CF1}}$$

$$CF2 = 1.0112$$

Now check whether correction factor CF2, which is derived completely from geometry (2 applications of law of cosines), explains the difference between V2algebraic and V2vector

$$V2_{algebraic} = 0.928$$

$$CF2 \cdot V2_{algebraic} = 0.9383$$

$$V2_{vector} = 0.938 - 0.0262 \cdot i$$

$$|V2_{vector}| = 0.9383$$

The two items highlighted in yellow above match.

Therefore, the correction factors developed by geometry confirm the results of the complex