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[ STUDENT > # Rev 3 - corrected wde to negate the stuff inside sqrt
[ STUDENT > # Solve the system using mechanical parameters and using
[           electrical analogy
[ STUDENT >
[ STUDENT > # Contents
[ STUDENT > # Part 1 = Symbols
[ STUDENT > # Part 1a = Initialization
[ STUDENT > # Part 2 = Solve the system mechanically to determine
[           Hmech(s)
[ STUDENT > # Part 3 = Solve the system using electrical analogy to
[           determine Helec(s)
[ STUDENT > # Part 3a - verify Hmech(s) = Helec(s)
[ STUDENT > # Part 3b - determine roots from electrical solution
[ STUDENT > # Part 3c - analyse max power transfer using electrical
[           solution
[ STUDENT > # Part 4 - Conclusion - use Rmax
[ STUDENT > # Part 5 - Compare part 4 strategy to adjustment of R to
[           obtain rsonance
[ STUDENT >
[ STUDENT > #===== PART 1 - SYMBOLS=====
[ STUDENT > # capital letters R, L, C for electrical parameters
[ STUDENT > # small letters m, k, c for mechanical parameters
[ STUDENT >
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[ STUDENT > # Mechanical solution variables
[ STUDENT > # X(s) = displacement of the mass
[ STUDENT > # Xwave(s) = displacement of the wave
[ STUDENT > # Hmech(s) = X/Xwave - transfer functio
[ STUDENT > # wdm = wd in terms of mechanical parameters
[ STUDENT >
[ STUDENT > # Electrical solution variables
[ STUDENT > # Vmid(s) = voltage at midpoint node between L and R/C
[ STUDENT > # Vin(s) = voltage of the wave
[ STUDENT > # Helec(s) = Vmid / Vin - transfer function
[ STUDENT > # P = power
[ STUDENT > # Rmax = R chosen to maximize power
[ STUDENT > # dPdR = dP/dR
[ STUDENT > # dP2dR2 = d^2P/dR^2
[ STUDENT > # wde = wd electrical = damped radian freq
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[ STUDENT > # ====Part 1a === Initialize=====
[ STUDENT > restart;

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STUDENT > # Correspondence between the mech and elec parameters is
           given in the following, in a forma suitable for use with
           the subs substitution function:
STUDENT > subsM2E:={k=1/L,c=1/R,m=C};

           
$$\text{subsM2E} := \{m = C, c = \frac{1}{R}, k = \frac{1}{L}\}$$

STUDENT > subsE2M:={L=1/k,R=1/c,C=m};

           
$$\text{subsE2M} := \{R = \frac{1}{c}, L = \frac{1}{k}, C = m\}$$

STUDENT > #====Part 2 -Solve the mechanical system =====
STUDENT > # Sum of forces acting upward on the node at location of
           mass is acceleration
STUDENT > eq1:=(Xwave-X)*k -c*s*X = m*s^2*X;

           
$$\text{eq1} := (X\text{wave} - X)k - c s X = m s^2 X$$

STUDENT > eq1:=simplify(lhs(eq1)-k*Xwave-m*s^2*X =
           rhs(eq1)-k*Xwave-m*s^2*X);

           
$$\text{eq1} := -k X - c s X - m s^2 X = -k X\text{wave}$$

STUDENT > eq1:=simplify(-1*eq1);

           
$$\text{eq1} := k X + c s X + m s^2 X = k X\text{wave}$$

STUDENT > # Let H(s) = X/Xwave
STUDENT > H(s):=k/(m*s^2+c*s+k);

           
$$H(s) := \frac{k}{m s^2 + c s + k}$$

STUDENT > # =====Part 3 - Solve Electrical
           Analogy=====
STUDENT > # Now solve the same system using electrical analogy:
STUDENT > ZC:=1/(s*C);

           
$$ZC := \frac{1}{s C}$$

STUDENT > ZR:=R;

           
$$ZR := R$$

STUDENT > ZL:=s*L;

           
$$ZL := s L$$

STUDENT > Zpara:=ZC*ZR/(ZC+ZR);

           
$$Z\text{para} := \frac{R}{s C \left( \frac{1}{s C} + R \right)}$$

STUDENT > Zpara:=simplify(Zpara);

           
$$Z\text{para} := \frac{R}{\text{Page 2} + R s C}$$


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STUDENT > $V_{mid} := Vin * Z_{para} / (Z_L + Z_{para});$ # voltage at the midpoint of the ckt between L and R/C

$$V_{mid} := \frac{Vin R}{(1 + R s C) \left(s L + \frac{R}{1 + R s C} \right)}$$

STUDENT > $V_{mid} := \text{simplify}(V_{mid});$

$$V_{mid} := \frac{R Vin}{s L + s^2 L R C + R}$$

STUDENT > $Helec := V_{mid} / Vin;$

$$Helec := \frac{R}{s L + s^2 L R C + R}$$

STUDENT > #===== Part 3A - Compare Hmech to Helec=====

STUDENT > # Compare to mechanical system by substituting in the equivalence defined above subE2M

STUDENT > $\text{simplify}(\text{subs}(\text{subE2M}, Helec));$

$$\frac{k}{m s^2 + c s + k}$$

STUDENT > # This matches Hmech(s) as expected

STUDENT > # Therefore we get the SAME transfer function when we solve using electrical analogy as when we solved using mechanical parameters directly. (actually mechanical transfer function was ratio of displacements where electrical voltage ratio corresponded to ratio of velocities, but ratio of velocities is same as ratio of displacements)

STUDENT >

STUDENT > # ====Part 3B-Find Poles=Roots =====

STUDENT >

STUDENT > $TheElecRoots := \text{solve}(\text{denom}(V_{mid}) = 0, s);$

$$TheElecRoots := \frac{1}{2} \frac{-L + \sqrt{L^2 - 4 L R^2 C}}{L R C}, \frac{1}{2} \frac{-L - \sqrt{L^2 - 4 L R^2 C}}{L R C}$$

STUDENT > $wde := \text{sqrt}(4 * L * R^2 * C - L^2) / (2 * L * R * C);$

$$wde := \frac{1}{2} \frac{\sqrt{4 L R^2 C - L^2}}{L R C}$$

STUDENT > $wdm := \text{simplify}(\text{subs}(\text{subE2M}, wde));$

$$wdm := \frac{1}{2} \frac{\sqrt{\frac{4 m k - c^2}{k^2 c^2}} k c}{m}$$

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[ STUDENT >
[ STUDENT > #=====Part 3c-Analyse Max Power Transfer=====
[ STUDENT > # The objective is to maximixe the Power
[ STUDENT > # Subsitute s=I*w for sinusoidal waveform
[ STUDENT > Vmid:=subs(s=I*w,Vmid);


$$V_{mid} := \frac{R V_{in}}{I w L - w^2 L R C + R}$$

[ STUDENT > Vmidmag:=simplify(evalc(abs(Vmid))); # magnitude of Vmid


$$V_{midmag} := \sqrt{\frac{R^2 V_{in}^2}{w^4 L^2 R^2 C^2 - 2 w^2 L R^2 C + R^2 + w^2 L^2}}$$

[ STUDENT > P:=(1/2)*Vmidmag^2/R; #power


$$P := \frac{1}{2} \frac{R V_{in}^2}{w^4 L^2 R^2 C^2 - 2 w^2 L R^2 C + R^2 + w^2 L^2}$$

[ STUDENT > dPdR:=diff(P,R);


$$dPdR := \frac{1}{2} \frac{V_{in}^2}{w^4 L^2 R^2 C^2 - 2 w^2 L R^2 C + R^2 + w^2 L^2} - \frac{1}{2} \frac{R V_{in}^2 (2 w^4 L^2 R C^2 - 4 w^2 L R C + 2 R)}{(w^4 L^2 R^2 C^2 - 2 w^2 L R^2 C + R^2 + w^2 L^2)^2}$$

[ STUDENT > Rmax:=solve(dPdR=0,R);


$$R_{max} := \frac{w L}{w^2 L C - 1}, -\frac{w L}{w^2 L C - 1}$$

[ STUDENT > # Which one is positive depends on which side of the
undamed natural freq 1/sqrt(LC) is w
[ STUDENT >
[ STUDENT > Rmax:=Rmax[2]; # choose the root which is positive
assuming w^2LC-1 > 0 (above undamed nat freq)


$$R_{max} := -\frac{w L}{w^2 L C - 1}$$

[ STUDENT > # Rewrite in more usef-friendly form
[ STUDENT > Rmax:=w*L/(1-w^2*L*C);


$$R_{max} := \frac{w L}{1 - w^2 L C}$$

[ STUDENT > dP2dR2:=diff(dPdR,R);


$$dP2dR2 := -\frac{V_{in}^2 (2 w^4 L^2 R C^2 - 4 w^2 L R C + 2 R)}{\%1^2}$$


$$+ \frac{R V_{in}^2 (2 w^4 L^2 R C^2 - 4 w^2 L R C + 2 R)^2}{\%1^3} - \frac{1}{2} \frac{R V_{in}^2 (2 w^4 L^2 C^2 - 4 w^2 L C + 2)}{\%1^2}$$


$$\%1 := w^4 L^2 R^2 C^2 - 2 w^2 L R^2 C + R^2 + w^2 L^2$$

[ STUDENT > simplify(subs(R=Rmax,dP2dR2));

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$$\frac{1}{4} \frac{(w^2 L C - 1) V_{in}^2}{L^3 w^3}$$

STUDENT > # The 2nd derivative is negative (under same assumption $w^2 LC - 1$ is + as above), so this is a relative maximum of power

STUDENT > `cmax:=simplify(subs(subsE2M,Rmax));`

$$cmax := -\frac{w}{-k + w^2 m}$$

STUDENT > # rewrite in more user friendly form

STUDENT > `Cmax:=w/(k-w^2*m);`

$$Cmax := \frac{w}{k - w^2 m}$$

STUDENT >

STUDENT > # =====Part 4 = Conclusion=====

STUDENT > # Conclusion - maximize power by sleecting

`R=Rmax=w*L/(w^2*L*C-1)`

STUDENT > # equivalently selecting $c = cmax = w/(w^2*m-k)$

STUDENT >

STUDENT > # Note this gives a solution only when excitation freq w is below undamped resonant frequency

`wu=sqrt(k/m)=1/sqrt(LC)`

STUDENT > # For above undamped resonant freq, need some more thought

STUDENT >

STUDENT > #=== Part 5 - Compare setting R to $Rmax$ (for max power transfer) vs setting R to Resonant (for resonance)to - see if they are the same ===

STUDENT > # If w is excitatio frequency, part 4 tells us to set `R=Rmax(w)`

STUDENT >

STUDENT > # Find the $R=Resonance$ to make $wde = w$

STUDENT > `Rresonant:=solve(wde=w,R);`

$$Rresonant := 2 \frac{\sqrt{-C(w^2 L C - 1) L}}{-4 C + 4 w^2 L C^2}, -2 \frac{\sqrt{-C(w^2 L C - 1) L}}{-4 C + 4 w^2 L C^2}$$

STUDENT > `simplify(Rresonant[1]);`

$$\frac{1}{2} \frac{\sqrt{-C(w^2 L C - 1) L}}{C(w^2 L C - 1)}$$

STUDENT > # Compare to $Rmax$

STUDENT > `Rmax;`

$$\frac{w L}{1 - w^2 L C}$$

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[ STUDENT > # Rresonant does not appear to be the same as Rmax when  
excitation freq w is below undamped resonant frequency  
wu=sqrt(k/m)=1/sqrt(LC). Not sure about when above  
[ STUDENT > # Recommend using the Rmax formulation of part 4, rather  
than adjusting R to resonance  
[ STUDENT >  
[ STUDENT > #  
[ STUDENT >
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