

## SECTION A

### DERIVATION OF EQUATIONS FOR CODE REQUIRED INTERMEDIATE RAILING SPHERE PASS-THROUGH RESISTANCE

#### A.1—EFFECTIVE MODULUS OF ELASTICITY FOR WIRE ROPE CABLE

The modulus of elasticity for Type-316 Stainless Steel is:

$$E := 28000 \cdot \text{ksi}$$

However, the effective modulus of elasticity for wire rope is less than the nominal material modulus of elasticity because of the twist in the wire rope strands. With the rope twist, the individual wire strands are longer than the actual cable length, thus for a given stress, strain will be greater in the cable than in a solid round bar of equal area. This increase in strain for a given load appears as a lower "effective" modulus of elasticity. For ease of use, this effective modulus is calculated based on the nominal cable area, rather than actual steel area, to make calculations involving cable diameter simpler.

#### Symbols and Notations

$\epsilon$	Strain, in/in.
$\sigma$	Stress, ksi (ksi = 1000 psi).
$A$	Cross-sectional area, in <sup>2</sup> .
$D$	Diameter of wire rope cable, in.
$E$	Modulus of Elasticity, ksi.
$E_{\text{eff}}$	Effective Modulus of Elasticity, ksi.
$G$	Cable stretch factor, determined by wire rope manufacturer.
$W$	Load in cable, lbf.

#### Objective

Given the value for  $G$ , determine the effective modulus of elasticity,  $E_{\text{eff}}$

#### Stress and Strain

The basic stress-strain relationship for a material, throughout the linear-elastic range, is given by:

$$\epsilon = \frac{\sigma}{E} \quad (1)$$

Stress is defined as the force (load) divided by the area:

$$\sigma = \frac{W}{A} \quad (2)$$

Area for a circular cross-section of diameter,  $D$ , is given by:

$$A = \frac{\pi \cdot D^2}{4} \quad (3)$$

Combining Eqns (1), (2) and (3) produces this equation for strain, given a force,  $P$ :

$$\epsilon = \frac{4 \cdot W}{\pi \cdot D^2 \cdot E} \quad (4)$$

**Stretch in a Wire Rope**

The stretch in a wire rope under load has been quantified empirically, and follows the equation:

$$\varepsilon = \frac{W}{D^2} \cdot \frac{G}{100 \cdot \frac{\text{in}^2}{\text{lbf}}} \quad (5)$$

The value of  $G$  varies according to strand geometry and cable material. For stainless steel 1x19 wire rope:

$$G := 7.79 \cdot 10^{-6}$$

**Effective Modulus of Elasticity**

Since the strain from each of Eqns. (4) and (5) must be the same, we can set the two equations equal to each other and solve for the effective modulus of elasticity:

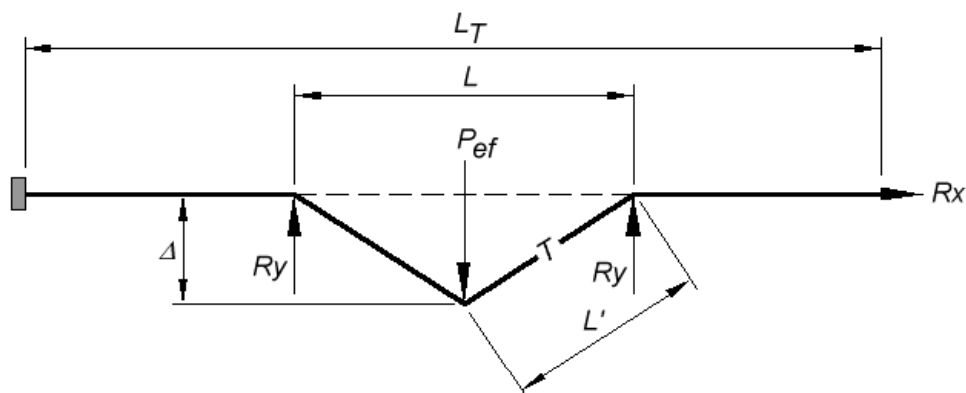
$$\frac{4 \cdot P}{\pi \cdot D^2 \cdot E_{\text{eff}}} = \frac{P}{D^2} \cdot \frac{G}{100 \cdot \frac{\text{in}^2}{\text{lbf}}}$$

$$E_{\text{eff}} := \frac{400 \cdot \text{lbf}}{\pi \cdot G \cdot \text{in}^2} \quad E_{\text{eff}} = 16345 \text{ ksi}$$

**Effective Modulus of  
Elasticity for  
316 Stainless Steel  
1x19 Wire Rope:**

$$E_{\text{eff}} := 16300 \cdot \text{ksi}$$

## A.2—LOAD-DEFLECTION RELATIONSHIP FOR AN EXTENSIBLE, FLEXIBLE CABLE



### Symbols and Notations

- $\Delta$  Deflection of cable under mid-span point load,  $P_{ef}$
- $L$  Spacing between intermediate supports, in.
- $L'$  Length of deflected cable from mid-span point load to support. In unloaded case,  $L' = L / 2$ , in.
- $L_T$  Length of cable between anchor points, ft.
- $P_{ef}$  Mid-span point load required to produce deflection  $\Delta$ , in.
- $R_x$  Axial end reaction at intermediate support, due to deflection of cable, lbs.
- $R_y$  Out-of-plane end reaction at intermediate support, due to deflection of cable, lbs.
- $T$  Tension load in cable between intermediate supports, due to deflection of cable, lbs.

### Objective

Given a mid-span deflection,  $\Delta$ , determine the point load required to produce that deflection, including the effects of stretch over the length of the cable between anchor points.

### Determination of Reactions

Out-of-plane end reactions,  $R_y$ , can be calculated by taking the sum of moments about one of the support points:

$$\begin{aligned}\sum M = 0: \quad P_{ef} \cdot \frac{L}{2} - R_y \cdot L &= 0 \\ R_y \cdot L &= \frac{P_{ef} \cdot L}{2} \\ R_y &= \frac{P_{ef}}{2} \quad (6)\end{aligned}$$

The axial end reaction,  $R_x$ , can be calculated by taking the sum of the moments about the point where the load is applied (mid-span) using forces to the right of the applied load:

$$\begin{aligned}\sum M = 0: \quad R_y \cdot \frac{L}{2} - R_x \cdot \Delta &= 0 \\ \frac{P_{ef}}{2} \cdot \frac{L}{2} &= R_x \cdot \Delta\end{aligned}$$

$$R_x = \frac{P_{ef} \cdot L}{4 \cdot \Delta} \quad (7)$$

Knowing the two end reactions, the tension in the cable may be resolved using the Pythagorean Theorem:

$$\begin{aligned} T &= \sqrt{R_y^2 + R_x^2} \\ T &= \sqrt{\left(\frac{P_{ef}}{2}\right)^2 + \left(\frac{P_{ef} \cdot L}{4 \cdot \Delta}\right)^2} \\ T &= \frac{1}{\Delta} \cdot \sqrt{\frac{P_{ef}^2}{4} \cdot \Delta^2 + \frac{P_{ef}^2}{4} \cdot \frac{L^2}{4 \cdot \Delta^2} \cdot \Delta^2} \\ T &= \frac{P_{ef}}{2 \cdot \Delta} \cdot \sqrt{\Delta^2 + \frac{L^2}{4}} \quad (8) \end{aligned}$$

### Strain Compatibility

The elongated (stretched) length of the cable under load can be found directly from the geometry, using the Pythagorean Theorem:

$$L' = \sqrt{\Delta^2 + \left(\frac{L}{2}\right)^2} = \sqrt{\Delta^2 + \frac{L^2}{4}}$$

where  $L'$  (the hypotenuse) represents one-half of the total elongated length.

The elongation (change in length) in the cable is given by:

$$\begin{aligned} \delta &= 2 \cdot L' - L \\ \delta &= 2 \cdot \sqrt{\Delta^2 + \frac{L^2}{4}} - L \quad (9) \end{aligned}$$

This total elongation is a result of cable stretch over two distinct regions: the region between the intermediate supports where the cable is loaded and where deflection in the cable is occurring, and the region beyond the intermediate supports where the cable is being stretched due to the tension from the axial reaction.

$$\delta = \frac{T \cdot L}{E \cdot A} + \frac{R_x \cdot (L_T - L)}{E \cdot A}$$

Substituting Eqns. (7) and (8) and simplifying yields:

$$\begin{aligned} \delta &= \frac{\left(\frac{P_{ef}}{2 \cdot \Delta} \cdot \sqrt{\Delta^2 + \frac{L^2}{4}}\right) \cdot L}{E \cdot A} + \frac{\left(\frac{P_{ef} \cdot L}{4 \cdot \Delta}\right) \cdot (L_T - L)}{E \cdot A} \\ \delta &= \frac{P_{ef} \cdot L}{2 \cdot \Delta \cdot E \cdot A} \cdot \sqrt{\Delta^2 + \frac{L^2}{4}} + \frac{P_{ef} \cdot L}{2 \cdot \Delta \cdot E \cdot A} \cdot \left(\frac{L_T - L}{2}\right) \\ \delta &= \frac{P_{ef} \cdot L}{2 \cdot \Delta \cdot E \cdot A} \cdot \left(\sqrt{\Delta^2 + \frac{L^2}{4}} + \frac{L_T - L}{2}\right) \quad (10) \end{aligned}$$

### Simultaneous Equations

Since the total elongation given by both Equations (9) and (10) must be the same, we now have two equations that can be solved for one variable in terms of the other.

Substituting Eqn. (9) for  $\delta$  in Eqn. (10) and solving for  $P_{ef}$  gives us:

$$2 \cdot \sqrt{\Delta^2 + \frac{L^2}{4}} - L = \frac{P_{ef} \cdot L}{2 \cdot \Delta \cdot E \cdot A} \cdot \left( \sqrt{\Delta^2 + \frac{L^2}{4}} + \frac{L_T - L}{2} \right)$$

$$P_{ef} = \frac{2 \cdot \Delta \cdot E \cdot A}{L} \cdot \frac{2 \cdot \sqrt{\Delta^2 + \frac{L^2}{4}} - L}{\sqrt{\Delta^2 + \frac{L^2}{4}} + \frac{L_T - L}{2}}$$

$$P_{ef} = \frac{4 \cdot \Delta \cdot E \cdot A}{L} \cdot \frac{\sqrt{4 \cdot \Delta^2 + L^2} - L}{\sqrt{4 \cdot \Delta^2 + L^2} + L_T - L}$$

**Mathcad Function:**

$$P_{ef}(\Delta, D, L, L_T) := \left| \begin{array}{l} A \leftarrow \frac{\pi \cdot D^2}{4} \\ \frac{4 \cdot \Delta \cdot E_{eff} \cdot A}{L} \cdot \frac{\sqrt{4 \cdot \Delta^2 + L^2} - L}{\sqrt{4 \cdot \Delta^2 + L^2} + L_T - L} \end{array} \right|$$

**Example 1:** Given a 3/8" diameter 1x19 wire rope supported and anchored at 42", calculate the point load required to cause a mid-span deflection of 1":

$$D := 0.375 \cdot \text{in}$$

$$L := 42 \cdot \text{in}$$

$$L_T := 42 \cdot \text{in}$$

$$\Delta := 1 \cdot \text{in}$$

$$P_{ef}(\Delta, D, L, L_T) = 194.1 \text{ lbf}$$

**Example 2:** Given a 3/8" diameter 1x19 wire rope supported every 42" and with a length of 42 feet between anchors, calculate the point load required to cause a mid-span deflection of 1":

$$D := 0.375 \cdot \text{in}$$

$$L := 42 \cdot \text{in}$$

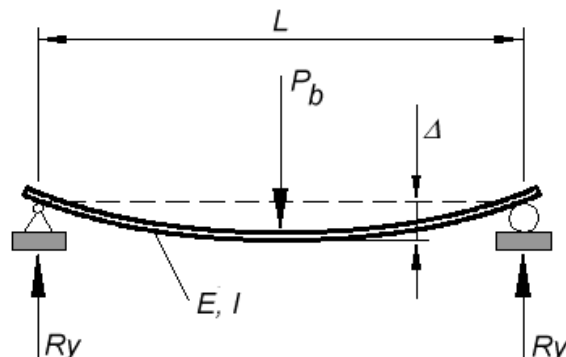
$$L_T := 42 \cdot \text{ft}$$

$$\Delta := 1 \cdot \text{in}$$

$$P_{ef}(\Delta, D, L, L_T) = 16.2 \text{ lbf}$$

### A.3—LOAD-DEFLECTION RELATIONSHIP IN FLEXURAL BENDING

The overall cable load-deflection relationship includes a component due to the flexural stiffness of the wire rope. Flexural stiffness is directly related to the effective modulus of elasticity and to the cable's moment of inertia.



#### Symbols and Notations

$\Delta$	Deflection of cable under mid-span point load, $P_b$ , in.
$A_s$	Area of individual wire rope strand, in <sup>2</sup> .
$C_o$	Circumference of cable outer strand centerline, in.
$d$	Distance of centroid of individual wire rope strand to centroid of wire rope, in.
$E$	Modulus of Elasticity, ksi.
$E_{eff}$	Effective Modulus of Elasticity for wire rope cable, ksi.
$I$	Moment of Inertia, in <sup>4</sup> .
$I_{1 \times 19}$	Moment of inertia of 1x19 wire rope, in <sup>4</sup> .
$I_s$	Moment of inertia of individual wire rope strand, in <sup>4</sup> .
$L$	Spacing between intermediate supports, in.
$P_b$	Mid-span point load required to produce deflection $\Delta$ , lbs.
$r_c$	Radius of cable, in.
$r_s$	Radius of individual wire rope strand, in.
$R_y$	Out-of-plane end reaction at intermediate support, due to deflection of cable, lbs.

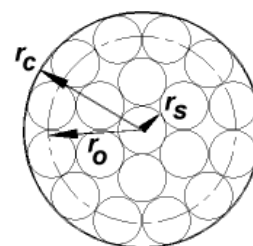
#### Objective

Given a mid-span deflection,  $\Delta$ , determine the point load required to produce that deflection due to flexural bending.

#### Strand Radius

Knowing strand geometry within the wire rope, we can determine the radius of individual strands,  $r_s$ , given the wire rope diameter,  $r_c$ .

In the 19-strand geometry, the outer layer contains 12 strands that fit snugly around their centerline, given by  $r_o$ . The circumference of this centerline circle,  $C_o$ , is given by the following two equations:



$$C_o = 12 \cdot \left[ 2 \cdot r_s + \left( r_c - r_s \cdot \cos\left(\frac{360 \cdot \text{deg}}{12}\right) \right) \right] \quad (11)$$

$$C_o = 2 \cdot \pi \cdot (r_c - r_s) \quad (12)$$

Setting Eqns. (11) and (12) equal to each other, and solving for  $r_s$ , we get:

$$12 \cdot \left[ 2 \cdot r_s + \left( r_c - r_s \cdot \cos(15 \cdot \text{deg}) \right) \right] = 2 \cdot \pi \cdot (r_c - r_s)$$

$$r_s = \frac{\pi \cdot r_c}{18 - (6 \cdot \cos(15 \cdot \text{deg}) + \pi)}$$

$$r_s = 0.2047 \cdot r_c \quad (13)$$

### Moment of Inertia

For a circular cross section, Area and Moment of Inertia are given by:

$$A_s = \pi \cdot r_s^2 \quad I_s = \frac{\pi \cdot r_s^4}{4}$$

Because the 1x19 wire rope is composed of individual strands with voids between them, the moment of inertia for the wire rope will be less than the moment of inertia of a solid circular area of the same diameter.

Moment of Inertia of a composite body is given by the equation:

$$I = \sum_j \left( I_j + A_j \cdot d_j^2 \right) \quad (14)$$

where  $d$  is the distance of the centroid of the individual part to the centroid of the composite body.

Expanding Eqns. (13) and (14) for the 19 strands yields:

$$I_{1 \times 19} = \left[ I_s + 8 \cdot \left( I_s + A_s \cdot r_s^2 \right) + 2 \cdot \left[ I_s + A_s \cdot (2 \cdot r_s)^2 \right] + 4 \cdot \left[ I_s + A_s \cdot \left( \frac{r_c - r_s}{\sqrt{2}} \right)^2 \right] \right] \dots$$

$$+ 4 \cdot \left[ I_s + A_s \cdot \left( \frac{r_s}{\tan(15 \cdot \text{deg})} \right)^2 \right]$$

$$I_{1 \times 19} = 19 \cdot I_s + 16 \cdot A_s \cdot r_s^2 + 2 \cdot A_s \cdot (r_c - r_s)^2 + 4 \cdot A_s \cdot \frac{r_s^2}{\tan(15 \cdot \text{deg})^2}$$

### Mathcad Function:

$$I_{1 \times 19}(D) := \left| \begin{array}{l} r_s \leftarrow 0.2047 \cdot \frac{D}{2} \\ A_s \leftarrow \pi \cdot r_s^2 \\ I_s \leftarrow \frac{\pi \cdot r_s^4}{4} \\ 19 \cdot I_s + 16 \cdot A_s \cdot r_s^2 + 2 \cdot A_s \cdot \left( \frac{D}{2} - r_s \right)^2 + 4 \cdot A_s \cdot \frac{r_s^2}{\tan(15 \cdot \text{deg})^2} \end{array} \right|$$

**Flexural Bending**

The deflection of a simply supported beam under a point load located at the mid-span is given by:

$$\Delta = \frac{P_b \cdot L^3}{48 \cdot E \cdot I}$$

The equation can also be rearranged to calculate the load necessary to cause a mid-span deflection of  $\Delta$ :

$$P_b = \frac{48 \cdot E \cdot I \cdot \Delta}{L^3}$$

**Mathcad Function:**

$$P_b(\Delta, D, L) := \frac{48 \cdot E_{\text{eff}} \cdot I_{1 \times 19}(D) \cdot \Delta}{L^3}$$

**Example:** Given a 3/8" diameter 1x19 wire rope supported every 42", calculate the point load required to cause a mid-span deflection of 1" due to pure bending:

$$D := 0.375 \cdot \text{in}$$

$$L := 42 \cdot \text{in}$$

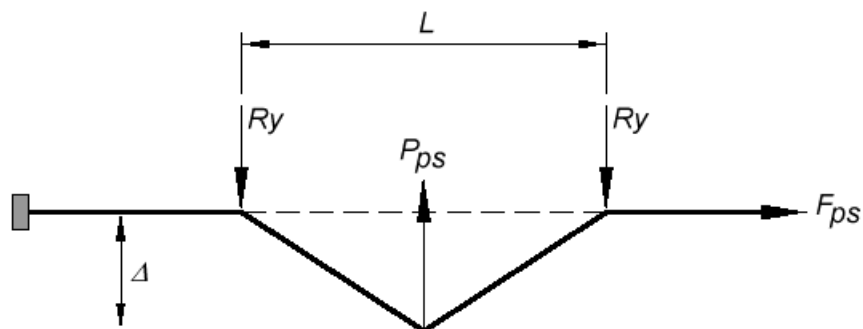
$$\Delta := 1 \cdot \text{in}$$

$$P_b(\Delta, D, L) = 7.7 \text{ lbf}$$



## A.4—EFFECTS OF CABLE PRESTRESSING

The effect of cable prestressing is to provide a force to balance an applied load. This balancing force is directly related to the geometry of the cable and the prestress force.



### Symbols and Notations

- $\Delta$  Deflection of cable at mid-span, in.
- $L$  Spacing between intermediate supports, in.
- $F_{ps}$  Applied prestressing force, lbs.
- $P_{ps}$  Mid-span balance point load due to prestressing, lbs.
- $R_y$  Out-of-plane end reaction at intermediate support, lbs.

### Objective

Given an initial prestress force,  $F_{ps}$ , and a mid-span deflection,  $\Delta$ , determine the resulting balancing force,  $P_{ps}$ .

### Balancing Force

The end reaction  $R_y$  is found by taking the sum of moments about the other end point:

$$\sum M = 0: \quad P_{ps} \cdot \frac{L}{2} - R_y \cdot L = 0$$

$$R_y \cdot L = \frac{P_{ps} \cdot L}{2}$$

$$R_y = \frac{P_{ps}}{2}$$

Taking the sum of the moments about the point where the load is applied (mid-span) using forces to the right of the applied load, we can compute the balance force,  $P_{ps}$ :

$$\sum M = 0: \quad R_y \cdot \frac{L}{2} + F_{ps} \cdot \Delta = 0$$

$$\frac{P_{ps}}{2} \cdot \frac{L}{2} = -F_{ps} \cdot \Delta$$

$$P_{ps} = -\frac{4 \cdot F_{ps} \cdot \Delta}{L}$$

Our applied load is equal to the magnitude of  $P_{ps}$ , but opposite in sign. Therefore, in the context of our applied load, the equation for  $P_{ps}$  becomes:

$$P_{ps} = \frac{4 \cdot F_{ps} \cdot \Delta}{L}$$

**Mathcad Function:**

$$P_{ps}(F_{ps}, \Delta, L) := \frac{4 \cdot F_{ps} \cdot \Delta}{L}$$

**Example:** Given a wire rope supported at 42", calculate the mid-span balancing load with a 400 lbs prestress load and a mid-span deflection of 1":

$$F_{ps} := 400 \cdot \text{lbf}$$

$$L := 42 \cdot \text{in}$$

$$\Delta := 1 \cdot \text{in}$$

$$P_{ps}(F_{ps}, \Delta, L) = 38.1 \text{ lbf}$$

## A.5—PUTTING IT ALL TOGETHER

### Symbols and Notations

$\Delta$	Deflection of cable under mid-span point load, $P$ .
$\Delta F_{psT}$	Change in prestress force due to temperature change, lbs.
$D$	Diameter of wire rope cable, in.
$F_{ps}$	Applied prestressing force, lbs.
$L$	Spacing between intermediate supports, in.
$L_T$	Length of cable between anchor points, ft.
$P$	Mid-span point load required to produce deflection $\Delta$ , lbs.
$P_b$	Component of mid-span point load, $P$ , resisted by flexural bending, lbs.
$P_{ef}$	Component of mid-span point load, $P$ , resisted by stretching of cable, lbs.
$P_{ps}$	Component of mid-span point load, $P$ , resisted by cable prestressing, lbs.

### Combined Load-Deflection Relationship

The effects of cable stretch, flexural beam action, and prestressing force combine to create a composite relationship between the applied load and the deflection of the cable. That is, for a given point load, applied at the mid-span of the cable, the cable will deflect until the load is balanced by the sum of the reactions due to cable stretch, flexure, and prestressing force.

Recall the load-deflection relationships previously derived:

Extensible,  
Flexible Cable:

$$P_{ef} = \frac{4 \cdot \Delta \cdot E \cdot A}{L} \cdot \frac{\sqrt{4 \cdot \Delta^2 + L^2} - L}{\sqrt{4 \cdot \Delta^2 + L^2} + L_T - L}$$

$$\text{Flexural Bending:} \quad P_b = \frac{48 \cdot E \cdot I \cdot \Delta}{L^3}$$

$$\text{Presstressing:} \quad P_{ps} = \frac{4 \cdot F_{ps} \cdot \Delta}{L}$$

Strain compatibility laws tell us that when a load is applied to the cable, the deflection in each of the above cases must be the same. Therefore, for a given deflection, the applied load required to cause that deflection,  $P$ , is the sum of the three components:

$$P = P_{ef} + P_b + P_{ps}$$

**Mathcad Function:**  $P(\Delta, D, L, L_T, F_{ps}) := P_{ef}(\Delta, D, L, L_T) + P_b(\Delta, D, L) + P_{ps}(F_{ps}, \Delta, L)$

**Example:** Given a 3/8" diameter 1x19 wire rope supported every 42", with an anchorage distance of 24'-6", and with a prestress load of 400 lbs., calculate the point load required to cause a mid-span deflection of 1":

$$D := 0.375 \cdot \text{in}$$

$$L := 42 \cdot \text{in}$$

$$L_T := 24.5 \cdot \text{ft}$$

$$F_{ps} := 400 \cdot \text{lbf}$$

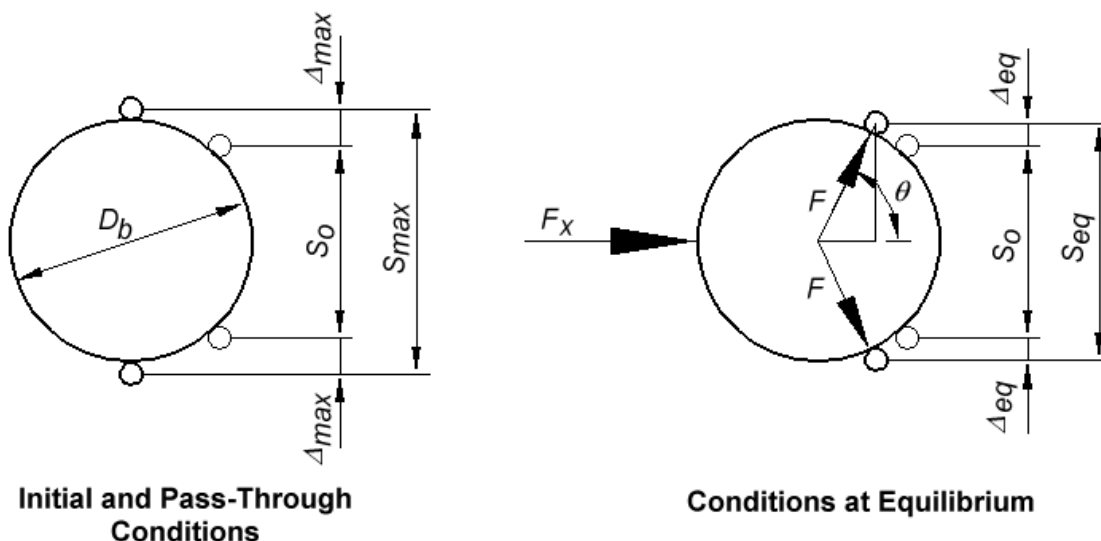
$$\Delta := 1 \cdot \text{in}$$

$$P(\Delta, D, L, L_T, F_{ps}) = 73.5 \text{ lbf}$$

## A.6—SPHERE PASS-THROUGH RESISTANCE

Building codes require that intermediate railing components for guardrail systems be spaced such that a sphere of 4" diameter shall not pass between the railings.

When the sphere is in contact with two of the cables and loaded perpendicular to the plane of the cables, the load will cause the cables to deflect and spread apart. If the load is great enough, the cables will spread far enough apart to allow the sphere to pass through the cables; if the load is less, the system will reach equilibrium, and the sphere will be prevented from passing through the cables.



### Symbols and Notations

- $\Delta$  Deflection of cable under force vector,  $F$ , in.
- $\Delta_{eq}$  Deflection of cable at equilibrium, in.
- $\Delta_{max}$  Deflection of cable at pass-through, in.
- $\theta$  Angle between applied load,  $F_x$ , and the resulting force vector,  $F$ , applied to the cable.
- $D$  Diameter of wire rope cable, in.
- $D_b$  Diameter of sphere, in.
- $F$  Force applied to cable, lbs; magnitude is a function of  $\alpha$ .
- $F_x$  Applied force on sphere, perpendicular to plane of cable railing infill, lbs.
- $S_{eq}$  Spacing (spread) between cables at equilibrium, in.
- $S_{max}$  Spacing (spread) between cables at pass-through, in.
- $S_0$  Initial spacing between cables, in.

### Objective

Derive the equations that govern the system, and provide a method for determining the adequacy of a cable intermediate railing system in resisting pass-through of the required sphere.

### Assumptions

1. The sphere and the cables are frictionless.
2. Curvature of the cable in contact with the sphere is negligible and force,  $F$ , may be assumed to be a point load.

3. The sphere is located at the mid-span of the cables, between the cable supports.

### Force Applied to the Sphere

The 2006 *International Building Code* (IBC) and 2007 *California Building Code* (CBC) require that guardrail intermediate railings be spaced so as to prevent a 4" diameter sphere from passing between the rails (IBC/CBC 1013.3). However, the code does not state what load is to be applied to the 4" diameter sphere. While such an oversight is not critical to solid railing members, it is of utmost importance in flexible railing systems such as wire-rope cables. In the absence of Code guidelines, a rational load must be developed.

The 2006 IBC and 2007 CBC also require a load of 50 psf, applied over a one square foot area to be applied horizontally to the intermediate railings (IBC/CBC 1607.7.1.2).

$$w := 50 \cdot \text{psf}$$

The projected horizontal area of the 4" sphere is a circle of 4" diameter:

$$d := 4 \cdot \text{in}$$

$$A := \frac{\pi \cdot d^2}{4}$$

$$A = 12.566 \text{ in}^2$$

Because objects that the guardrail infill system must resist may be moving, we will also conservatively include an impact factor of 2.0. Therefore, the force on the sphere under the IBC base load, and considering impact is:

$$F_x := 2 \cdot w \cdot A$$

$$F_x = 8.7 \text{ lbf}$$

### Cable Load Angle

When a force,  $F_x$ , is applied to the sphere in contact with two cables, the force is transmitted to the cables in proportion to the spread (spacing) of the cables. The transmitted force,  $F$ , is applied at an angle,  $\theta$ , which can be determined from the trigonometric relationship between the center of the sphere and the center of the cable:

$$\text{Hypotenuse} = \frac{D_b + D}{2}$$

$$\text{Opposite} = \frac{S_o}{2} + \Delta$$

$$\theta = \text{asin}\left(\frac{\text{Opposite}}{\text{Hypotenuse}}\right) = \text{asin}\left(\frac{\frac{S_o}{2} + \Delta}{\frac{D_b + D}{2}}\right)$$

$$\theta = \text{asin}\left(\frac{S_o + 2 \cdot \Delta}{D_b + D}\right)$$

### Mathcad Function:

$$\theta(\Delta, D, D_b, S_o) := \text{asin}\left(\frac{S_o + 2 \cdot \Delta}{D_b + D}\right)$$

**Example:** Given two 3/8" diameter wire rope cables with an initial spacing of 3" supporting a 4" diameter sphere. Calculate the angle at which the load is applied to each of the two cables when the sphere first contacts the cables (i.e.,  $\Delta = 0$ ).

$$\begin{aligned} D_b &:= 4 \cdot \text{in} \\ D &:= 0.375 \cdot \text{in} \\ S_o &:= 3 \cdot \text{in} \\ \Delta &:= 0 \cdot \text{in} \\ \theta(\Delta, D, D_b, S_o) &= 43.292 \text{ deg} \end{aligned}$$

### Force Applied to Cable

Knowing the angle  $\theta$ , the force applied to the cable can be easily calculated:

$$F = \frac{\frac{F_x}{2}}{\cos(\theta)}$$

$$F = \frac{F_x}{2 \cdot \cos(\theta)}$$

Note that as the cables spread farther apart, angle  $\theta$  increases. As  $\theta$  increases,  $\cos(\theta)$  decreases. Since the cosine term is in the denominator, as  $\cos(\theta)$  decreases, the force,  $F$ , applied to the cable increases. As  $\theta$  approaches  $90^\circ$ ,  $F$  approaches infinity.

**Mathcad Function:**

$$F(F_x, \Delta, D, D_b, S_o) := \frac{F_x}{2 \cdot \cos(\theta(\Delta, D, D_b, S_o))}$$

**Example:** Given two 3/8" diameter wire rope cables with an initial spacing of 3" supporting a 4" diameter sphere with a 10 lb load. Calculate the load applied to each of the two cables if the final cable spacing is 3.75".

$$\begin{aligned} F_x &:= 10 \cdot \text{lbf} \\ D_b &:= 4 \cdot \text{in} \\ D &:= 0.375 \cdot \text{in} \\ S_o &:= 3 \cdot \text{in} \\ S_{\text{final}} &:= 3.75 \cdot \text{in} \\ \Delta &:= \frac{S_{\text{final}} - S_o}{2} & \Delta = 0.375 \text{ in} \\ F(F_x, \Delta, D, D_b, S_o) &= 9.707 \text{ lbf} \end{aligned}$$

### Maximum Cable Deflection

The cable spread at pass-through is equal to the sum of the sphere diameter and the cable diameter:

$$S_{\text{max}} = D_b + D$$

The maximum cable deflection occurs at pass-through, and is equal to one-half of the difference between the maximum cable spread and the initial cable spacing:

$$\Delta_{\text{max}} = \frac{S_{\text{max}} - S_o}{2}$$

$$\Delta_{\max} = \frac{D_b + D - S_o}{2}$$

**Mathcad Function:**

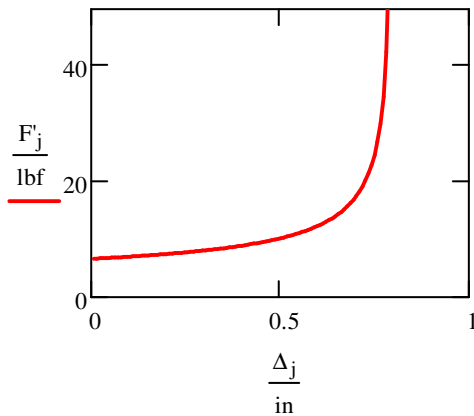
$$\Delta_{\max}(D, D_b, S_o) := \frac{D_b + D - S_o}{2}$$

### Equilibrium Deflection

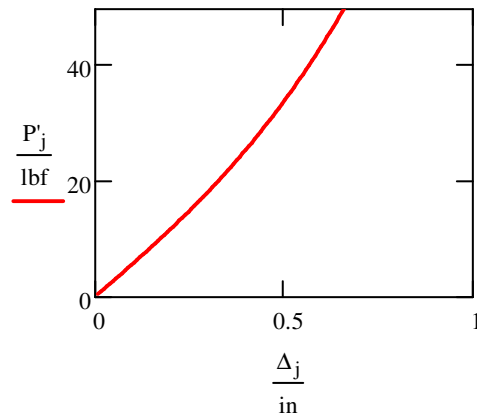
If the load on the sphere is not sufficient to cause a deflection in the cables at least equal to  $\Delta_{\max}$ , the system will reach equilibrium and the sphere will not pass between the cables. When this condition exists, the force,  $F$ , transmitted to each of the cables is equal to the resisting force of the cables,  $P$ .

Recall that the magnitude of the resisting force,  $P$ , is directly related to the cable deflection,  $\Delta$ . Likewise, the magnitude of the force applied to the cable by the loaded sphere is also related to the cable deflection,  $\Delta$  (because  $\Delta$  controls the angle  $\theta$ ).

To determine whether equilibrium occurs, we must find the deflection that results in the same magnitude of applied force,  $F$ , as the magnitude of the resisting force,  $P$ . The deflection at which this occurs is known as the *equilibrium deflection*,  $\Delta_{eq}$ . If the equilibrium deflection is less than the maximum deflection,  $\Delta_{\max}$ , the cables successfully resist the sphere, which does not pass through the cables.



**(A) Transmitted Force,  $F$ ,  
vs. Deflection,  $\Delta$ .**



**(B) Cable Resisting Force,  $P$ ,  
vs. Deflection,  $\Delta$ .**

The Figures above show typical force versus deflection relationships for the force transmitted by the sphere and the cable resisting force. Determining the equilibrium deflection is an iterative procedure to find the point at which the two curves cross.

### Mathcad Function:

The Mathcad **root()** function can be used to automate the iterative process of determining a value for  $\Delta$  where the transmitted force  $F$  is equal to the cable resistance force,  $P$ :

$$\Delta_{eq}(F_x, D_b, D, S_o, L, L_T, F_{ps}) := \begin{cases} \Delta \leftarrow 0 \cdot \text{in} \\ \text{root} \left( \begin{array}{l} F(F_x, \Delta, D, D_b, S_o) \dots, \Delta \\ + P(\Delta, D, L, L_T, F_{ps}) \end{array} \right) \end{cases}$$

(The first line assigns an initial value to the solver.)

This routine, however, is ill behaved—if the system does not reach equilibrium, i.e., the sphere passes through the cables, there is no root to the equation and the Mathcad **root()** function returns an error instead of a value. To make this routine better behaved, an error-trapping function can be included:

$$\Delta_{eq}(F_x, D_b, D, S_o, \Delta_{max}, L, L_T, F_{ps}) := \begin{cases} \Delta \leftarrow 0 \cdot \text{in} \\ \Delta_{max} \text{ on error root} \left( F(F_x, \Delta, D, D_b, S_o) \dots, \Delta \right) \\ \quad + -P(\Delta, D, L, L_T, F_{ps}) \end{cases}$$

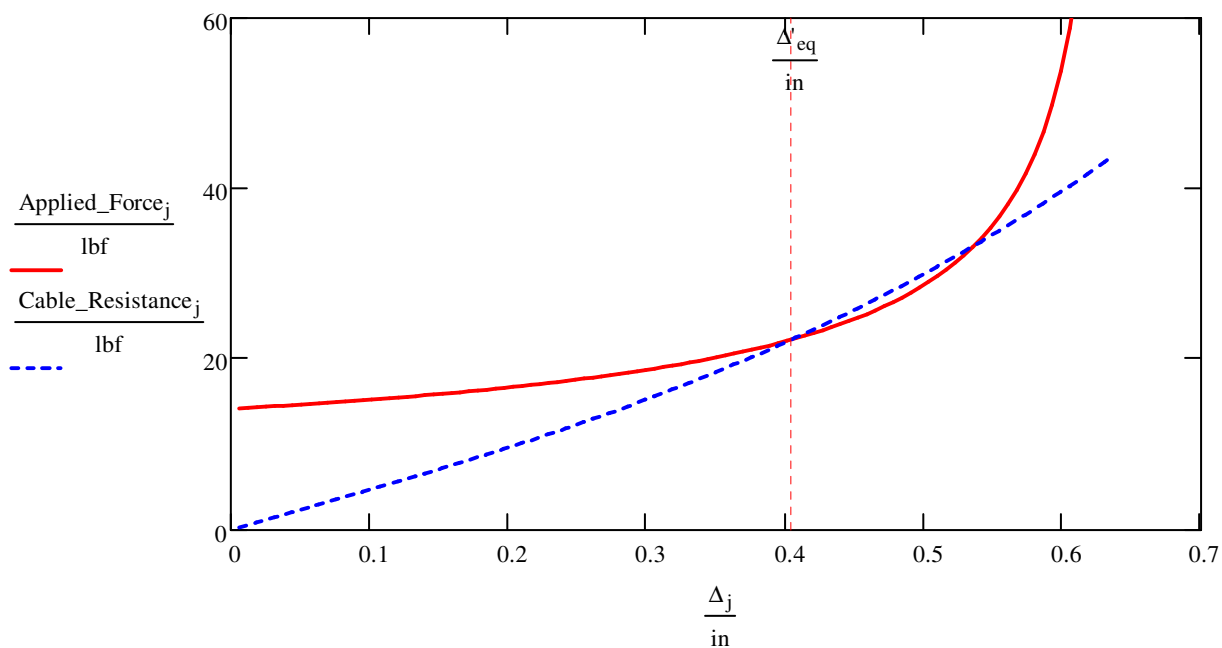
If equilibrium cannot be reached, this routine returns  $\Delta_{max}$ , indicating that the sphere passed-through the cables.

**Example:** Given the following parameters,

Diameter of Cable:	$D := 0.375 \cdot \text{in}$	Cable Spacing:	$S_o := 3.1 \cdot \text{in}$
Support Spacing:	$L := 42 \cdot \text{in}$	Anchor Spacing:	$L_T := 12 \cdot \text{ft}$
Prestress Force:	$F_{ps} := 400 \cdot \text{lbf}$		
Sphere Diameter:	$D_b := 4 \cdot \text{in}$	Load on Sphere	$F_x := 20 \cdot \text{lbf}$

Determine the equilibrium deflection, and plot the load vs. deflection curves.

Equilibrium Deflection:  $\Delta_{eq}(F_x, D_b, D, S_o, \Delta_{max}(D, D_b, S_o), L, L_T, F_{ps}) = 0.404 \text{ in}$



— END OF SECTION A —