

If the pipe is horizontal, then $z_2 = z_1$ and

$$\frac{p_1 - p_2}{\rho} = \frac{\Delta p}{\rho} = h_f \quad (8.30)$$

Thus the major head loss can be expressed as the pressure loss for fully developed flow through a horizontal pipe of constant area.

Since head loss represents the energy converted by frictional effects from mechanical to thermal energy, head loss for fully developed flow in a constant-area duct depends only on the details of the flow through the duct. Head loss is independent of pipe orientation.

a. Laminar Flow

In laminar flow the pressure drop may be computed analytically for fully developed flow in a horizontal pipe. Thus, from Eq. 8.13c,

$$\Delta p = \frac{128 \mu L Q}{\pi D^4} = \frac{128 \mu L \bar{V} (\pi D^2/4)}{\pi D^4} = 32 \frac{L \mu \bar{V}}{D^3}$$

Substituting in Eq. 8.30 gives

$$h_f = 32 \frac{L \mu \bar{V}}{D^3} = \frac{L \mu \bar{V}^2}{D^3} \left(64 \frac{\mu}{\rho \bar{V} D} \right) = \left(\frac{64}{Re} \right) \frac{L \mu \bar{V}^2}{D^3} \quad (8.31)$$

(We shall see the reason for writing h_f in this form shortly.)

b. Turbulent Flow

In turbulent flow we cannot evaluate the pressure drop analytically, so we must resort to experimental results and use dimensional analysis to correlate the experimental data. In fully developed turbulent flow, the pressure drop, Δp , due to friction in a horizontal constant-area pipe is known to depend on the pipe diameter, D , the pipe length, L , the pipe roughness, e , the average flow velocity, \bar{V} , the fluid density, ρ , and the fluid viscosity, μ . In functional form

$$\Delta p = \Delta p(D, L, e, \bar{V}, \rho, \mu)$$

We applied dimensional analysis to this problem in Example Problem 7.2. The results were a correlation of the form

$$\frac{\Delta p}{\rho \bar{V}^2} = f \left(\frac{\mu}{\rho \bar{V} D}, \frac{L}{D}, \frac{e}{D} \right)$$

We recognize that $\mu/\rho \bar{V} D = 1/Re$, so we could just as well write

$$\frac{\Delta p}{\rho \bar{V}^2} = f \left(\frac{1}{Re}, \frac{L}{D}, \frac{e}{D} \right)$$

Substituting from Eq. 8.30, we see that

$$\frac{h_f}{\bar{V}^2} = \phi \left(Re, \frac{L}{D}, \frac{e}{D} \right)$$

Although dimensional analysis predicts the functional relationship, we must resort to experiment to obtain actual values.

Experiments show that the nondimensional head loss is directly proportional to L/D . Hence we can write

$$\frac{h_f}{\bar{V}^2} = \frac{L}{D} \phi_1 \left(Re, \frac{e}{D} \right)$$

Since the function ϕ_1 is still undetermined, it is permissible to introduce a constant into the left side of the above equation. The number $\frac{1}{2}$ is introduced into the denominator such that the head loss is nondimensionalized on the kinetic energy per unit mass of flow. Then

$$\frac{h_f}{\frac{1}{2} \bar{V}^2} = \frac{L}{D} \phi_2 \left(Re, \frac{e}{D} \right)$$

The unknown function $\phi_2(Re, e/D)$ is defined as the friction factor, f ,

$$f \equiv \phi_2 \left(Re, \frac{e}{D} \right)$$

and

$$h_f = f \frac{L \bar{V}^2}{D^5} \quad (8.32)$$

The friction factor, f , is determined experimentally. The results, published by L. F. Moody, are shown in Fig. 8.14.

To determine head loss for fully developed flow with known conditions, the flow Reynolds number is evaluated first. The value of relative roughness, e/D , for the flow is obtained from Fig. 8.15. Then the friction factor, f , is read from the appropriate curve in Fig. 8.14, at the known values of Re and e/D . Finally, the head loss is found using Eq. 8.32.

Several features of Fig. 8.14 require some discussion. The friction factor for laminar flow may be obtained by comparing Eqs. 8.31 and 8.32

$$h_f = \left(\frac{64}{Re} \right) \frac{L \bar{V}^2}{D^5} = f \frac{L \bar{V}^2}{D^5}$$

Consequently, for laminar flow

$$f_{\text{laminar}} = \frac{64}{Re} \quad (8.33)$$

Thus, in laminar flow, the friction factor is a function of Reynolds number only; it is