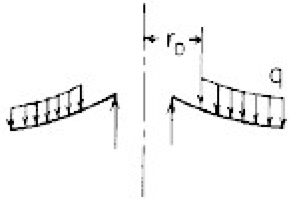
2k. Outer edge free, inner edge simply supported



$y_b = 0, \quad M_{rb} = 0, \quad M_{ra} = 0, \quad Q_a = 0$

$$\theta_b = \frac{-qa^3}{DC_7}\left[\frac{C_9}{2ab}(a^2 - r_o^2) - L_{17}\right]$$

$$Q_b = \frac{q}{2b}(a^2 - r_o^2)$$

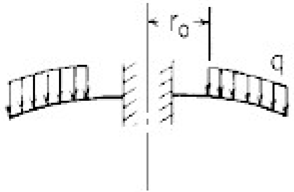
$$y_a = \theta_b a C_1 + Q_b \frac{a^3}{D} C_3 - \frac{qa^4}{D} L_{11}$$

$$\theta_a = \theta_b C_4 + Q_b \frac{a^2}{D} C_6 - \frac{qa^3}{D} L_{14}$$

If $r_o = b$ (uniform load over entire plate),

b/a	0.1	0.3	0.5	0.7	0.9
K_{y_a}	−0.1115	−0.1158	−0.0826	−0.0378	−0.0051
K_{θ_b}	−0.1400	−0.2026	−0.1876	−0.1340	−0.0515
K_{θ_a}	−0.1082	−0.1404	−0.1479	−0.1188	−0.0498
$K_{M_{tb}}$	−1.2734	−0.6146	−0.3414	−0.1742	−0.0521

2l. Outer edge free, inner edge fixed



$y_b = 0, \quad \theta_b = 0, \quad M_{ra} = 0, \quad Q_a = 0$

$$M_{rb} = \frac{-qa^2}{C_8}\left[\frac{C_9}{2ab}(a^2 - r_o^2) - L_{17}\right]$$

$$Q_b = \frac{q}{2b}(a^2 - r_o^2)$$

$$y_a = M_{rb} \frac{a^2}{D} C_2 + Q_b \frac{a^3}{D} C_3 - \frac{qa^4}{D} L_{11}$$

$$\theta_a = M_{rb} \frac{a}{D} C_5 + Q_b \frac{a^2}{D} C_6 - \frac{qa^3}{D} L_{14}$$

If $r_o = b$ (uniform load over entire plate),

b/a	0.1	0.3	0.5	0.7	0.9
K_{y_a}	−0.0757	−0.0318	−0.0086	−0.0011	
K_{θ_a}	−0.0868	−0.0512	−0.0207	−0.0046	−0.00017
$K_{M_{rb}}$	−0.9646	−0.4103	−0.1736	−0.0541	−0.00530

TABLE 11.1 Numerical values for functions used in Table 11.2 (*Continued*)

r_o/r	0.400	$\frac{1}{3}$	0.300	0.250	0.200	0.125	0.100	0.050
G_1	0.605736	0.704699	0.765608	0.881523	1.049227	1.547080	1.882168	3.588611
G_2	0.136697	0.161188	0.173321	0.191053	0.207811	0.229848	0.235987	0.245630
G_3	0.022290	0.027649	0.030175	0.033465	0.035691	0.035236	0.033390	0.025072
G_4	1.135000	1.266667	1.361667	1.562500	1.880000	2.881250	3.565000	7.032500
G_5	0.420000	0.444444	0.455000	0.468750	0.480000	0.492187	0.495000	0.498750
G_6	0.099258	0.109028	0.112346	0.114693	0.112944	0.099203	0.090379	0.062425
G_7	0.955500	1.213333	1.380167	1.706250	2.184000	3.583125	4.504500	9.077250
G_8	0.706000	0.688889	0.681500	0.671875	0.664000	0.655469	0.653500	0.650875
G_9	0.297036	0.289885	0.282550	0.266288	0.242827	0.190488	0.166993	0.106089
G_{11}	0.003833	0.005499	0.006463	0.008057	0.009792	0.012489	0.013350	0.014843
G_{12}	0.000827	0.001208	0.001435	0.001822	0.002266	0.003027	0.003302	0.003872
G_{13}	0.000289	0.000427	0.000510	0.000654	0.000822	0.001121	0.001233	0.001474
G_{14}	0.024248	0.031211	0.034904	0.040595	0.046306	0.054362	0.056737	0.060627
G_{15}	0.006691	0.008790	0.009945	0.011798	0.013777	0.016917	0.017991	0.020139
G_{16}	0.002840	0.003770	0.004290	0.005138	0.006065	0.007589	0.008130	0.009252
G_{17}	0.119723	0.139340	0.148888	0.162637	0.175397	0.191795	0.196271	0.203191
G_{18}	0.044939	0.053402	0.057723	0.064263	0.070816	0.080511	0.083666	0.089788
G_{19}	0.023971	0.028769	0.031261	0.035098	0.039031	0.045057	0.047086	0.051154
$C_1C_6 - C_3C_4$	0.034825	0.041810	0.044925	0.048816	0.051405	0.051951	0.051073	0.047702
$C_1C_9 - C_3C_7$	0.158627	0.170734	0.174676	0.177640	0.176832	0.168444	0.163902	0.153133
$C_2C_6 - C_3C_5$	0.004207	0.005285	0.005742	0.006226	0.006339	0.005459	0.004800	0.002829
$C_2C_9 - C_3C_8$	0.024867	0.027679	0.028408	0.028391	0.026763	0.020687	0.017588	0.009740
$C_4C_9 - C_6C_7$	0.242294	0.234900	0.229682	0.220381	0.209845	0.193385	0.188217	0.179431

† To obtain a value of either C_i, L_i, or F_i for a corresponding value of either b/a, r_o/a, or b/r, respectively, use the tabulated value of G_i for the corresponding value of r_o/r.

TABLE 11.2 Formulas for flat circular plates of constant thickness

NOTATION: W = total applied load (force); w = unit line load (force per unit of circumferential length); q = load per unit area; M_o = unit applied line moment loading (force-length per unit of circumferential length); θ_o = externally applied change in radial slope (radians); y_o = externally applied radial step in the vertical deflection (length); y = vertical deflection of plate (length); θ = radial slope of plate; M_r = unit radial bending moment; M_t = unit tangential bending moment; Q = unit shear force (force per unit of circumferential length); E = modulus of elasticity (force per unit area); ν = Poisson's ratio; γ = temperature coefficient of expansion (unit strain per degree); a = outer radius; b = inner radius for annular plate; t = plate thickness; r = radial location of quantity being evaluated; r_o = radial location of unit line loading or start of a distributed load. F_1 to F_9 and G_1 to G_{19} are the several functions of the radial location r. C_1 to C_9 are plate constants dependent upon the ratio a/b. L_1 to L_{19} are loading constants dependent upon the ratio a/r_o. When used as subscripts, r and t refer to radial and tangential directions, respectively. When used as subscripts, a, b, and o refer to an evaluation of the quantity subscripted at the outer edge, inner edge, and the position of the loading or start of distributed loading, respectively. When used as a subscript, c refers to an evaluation of the quantity subscripted at the center of the plate.

Positive signs are associated with the several quantities in the following manner: Deflections y and y_o are positive upward; slopes θ and θ_o are positive when the deflection y increases positively as r increases; moments M_r, M_t, and M_o are positive when creating compression on the top surface; and the shear force Q is positive when acting upward on the inner edge of a given annular section

Bending stresses can be found from the moments M_r and M_t by the expression $\sigma = 6M/t^2$. The plate constant $D = Et^3/12(1-\nu^2)$. The singularity function brackets $\langle\ \rangle$ indicate that the expression contained within the brackets must be equated to zero unless $r > r_o$, after which they are treated as any other brackets. Note that Q_b, Q_a, M_{rb}, and M_{ra} are reactions, not loads. They exist only when necessary edge restraints are provided.

General Plate Functions and Constants for Solid and Annular Circular Plates

$$F_1 = \frac{1+\nu}{2}\frac{b}{r}\ln\frac{r}{b} + \frac{1-\nu}{4}\left(\frac{r}{b} - \frac{b}{r}\right)$$

$$F_2 = \frac{1}{4}\left[1 - \left(\frac{b}{r}\right)^2\left(1 + 2\ln\frac{r}{b}\right)\right]$$

$$F_3 = \frac{b}{4r}\left\{\left[\left(\frac{b}{r}\right)^2 + 1\right]\ln\frac{r}{b} + \left(\frac{b}{r}\right)^2 - 1\right\}$$

$$F_4 = \frac{1}{2}\left[(1+\nu)\frac{b}{r} + (1-\nu)\frac{r}{b}\right]$$

$$F_5 = \frac{1}{2}\left[1 - \left(\frac{b}{r}\right)^2\right]$$

$$F_6 = \frac{b}{4r}\left[\left(\frac{b}{r}\right)^2 - 1 + 2\ln\frac{r}{b}\right]$$

$$F_7 = \frac{1}{2}(1-\nu^2)\left(\frac{r}{b} - \frac{b}{r}\right)$$

$$F_8 = \frac{1}{2}\left[1 + \nu + (1-\nu)\left(\frac{b}{r}\right)^2\right]$$

$$F_9 = \frac{b}{r}\left\{\frac{1+\nu}{2}\ln\frac{r}{b} + \frac{1-\nu}{4}\left[1 - \left(\frac{b}{r}\right)^2\right]\right\}$$

$$C_1 = \frac{1+\nu}{2}\frac{b}{a}\ln\frac{a}{b} + \frac{1-\nu}{4}\left(\frac{a}{b} - \frac{b}{a}\right)$$

$$C_2 = \frac{1}{4}\left[1 - \left(\frac{b}{a}\right)^2\left(1 + 2\ln\frac{a}{b}\right)\right]$$

$$C_3 = \frac{b}{4a}\left\{\left[\left(\frac{b}{a}\right)^2 + 1\right]\ln\frac{a}{b} + \left(\frac{b}{a}\right)^2 - 1\right\}$$

$$C_4 = \frac{1}{2}\left[(1+\nu)\frac{b}{a} + (1-\nu)\frac{a}{b}\right]$$

$$C_5 = \frac{1}{2}\left[1 - \left(\frac{b}{a}\right)^2\right]$$

$$C_6 = \frac{b}{4a}\left[\left(\frac{b}{a}\right)^2 - 1 + 2\ln\frac{a}{b}\right]$$

$$C_7 = \frac{1}{2}(1-\nu^2)\left(\frac{a}{b} - \frac{b}{a}\right)$$

$$C_8 = \frac{1}{2}\left[1 + \nu + (1-\nu)\left(\frac{b}{a}\right)^2\right]$$

$$C_9 = \frac{b}{a}\left\{\frac{1+\nu}{2}\ln\frac{a}{b} + \frac{1-\nu}{4}\left[1 - \left(\frac{b}{a}\right)^2\right]\right\}$$

TABLE 11.2 Formulas for flat circular plates of constant thickness (*Continued*)

$$L_1 = \frac{1+\nu}{2}\frac{r_o}{a}\ln\frac{r_0}{a} + \frac{1-\nu}{4}\left(\frac{a}{r_o} - \frac{r_o}{a}\right)$$

$$L_2 = \frac{1}{4}\left[1 - \left(\frac{r_o}{a}\right)^2\left(1 + 2\ln\frac{a}{r_o}\right)\right]$$

$$L_3 = \frac{r_o}{4a}\left\{\left[\left(\frac{r_o}{a}\right)^2 + 1\right]\ln\frac{a}{r_o} + \left(\frac{r_o}{a}\right)^2 - 1\right\}$$

$$L_4 = \frac{1}{2}\left[(1+\nu)\frac{r_o}{a} + (1-\nu)\frac{a}{r_o}\right]$$

$$L_5 = \frac{1}{2}\left[1 - \left(\frac{r_o}{a}\right)^2\right]$$

$$L_6 = \frac{r_o}{4a}\left[\left(\frac{r_o}{a}\right)^2 - 1 + 2\ln\frac{a}{r_o}\right]$$

$$L_7 = \frac{1}{2}(1-\nu^2)\left(\frac{a}{r_o} - \frac{r_o}{a}\right)$$

$$L_8 = \frac{1}{2}\left[1 - \nu + (1-\nu)\left(\frac{r_o}{a}\right)^2\right]$$

$$L_9 = \frac{r_o}{a}\left\{\frac{1+\nu}{2}\ln\frac{a}{r_o} + \frac{1-\nu}{4}\left[1 - \left(\frac{r_o}{a}\right)^2\right]\right\}$$

$$L_{11} = \frac{1}{64}\left\{1 + 4\left(\frac{r_o}{a}\right)^2 - 5\left(\frac{r_o}{a}\right)^4 - 4\left(\frac{r_o}{a}\right)^2\left[2 + \left(\frac{r_o}{a}\right)^2\right]\ln\frac{a}{r_o}\right\}$$

$$L_{12} = \frac{a}{14.400(a - r_o)}\left\{64 - 225\frac{r_o}{a} - 100\left(\frac{r_o}{a}\right)^3 + 261\left(\frac{r_o}{a}\right)^5 - 60\left(\frac{r_o}{a}\right)^3\left[3\left(\frac{r_o}{a}\right)^2 - 10\right]\ln\frac{a}{r_o}\right\}$$

$$L_{13} = \frac{a^2}{14.400(a - r_o)^2}\left\{25 - 128\frac{r_o}{a} + 225\left(\frac{r_o}{a}\right)^2 - 25\left(\frac{r_o}{a}\right)^4 - 97\left(\frac{r_o}{a}\right)^6 - 60\left(\frac{r_o}{a}\right)^4\left[5 + \left(\frac{r_o}{a}\right)^2\right]\ln\frac{a}{r_o}\right\}$$

$$L_{14} = \frac{1}{16}\left[1 - \left(\frac{r_o}{a}\right)^4 - 4\left(\frac{r_o}{a}\right)^2\ln\frac{a}{r_o}\right]$$

$$L_{15} = \frac{a}{720(a - r_o)}\left[16 - 45\frac{r_o}{a} + 9\left(\frac{r_o}{a}\right)^5 + 20\left(\frac{r_o}{a}\right)^3\left(1 + 3\ln\frac{a}{r_o}\right)\right]$$

$$L_{16} = \frac{a^2}{1440(a - r_o)^2}\left[15 - 64\frac{r_o}{a} + 90\left(\frac{r_o}{a}\right)^2 - 6\left(\frac{r_o}{a}\right)^6 - 5\left(\frac{r_o}{a}\right)^4\left(7 + 12\ln\frac{a}{r_o}\right)\right]$$

$$L_{17} = \frac{1}{4}\left\{1 - \frac{1-\nu}{4}\left[1 - \left(\frac{r_o}{a}\right)^4\right] - \left(\frac{r_o}{a}\right)^2\left[1 + (1+\nu)\ln\frac{a}{r_o}\right]\right\}$$

$$G_1 = \left[\frac{1+\nu}{2}\frac{r_o}{r}\ln\frac{r}{r_0} + \frac{1-\nu}{4}\left(\frac{r}{r_o} - \frac{r_o}{r}\right)\right]\langle r - r_o\rangle^0$$

$$G_2 = \frac{1}{4}\left[1 - \left(\frac{r_o}{r}\right)^2\left(1 + 2\ln\frac{r}{r_o}\right)\right]\langle r - r_o\rangle^0$$

$$G_3 = \frac{r_o}{4r}\left\{\left[\left(\frac{r_o}{r}\right)^2 + 1\right]\ln\frac{r}{r_o} + \left(\frac{r_o}{r}\right)^2 - 1\right\}\langle r - r_o\rangle^0$$

$$G_4 = \frac{1}{2}\left[(1+\nu)\frac{r_o}{r} + (1-\nu)\frac{r}{r_o}\right]\langle r - r_o\rangle^0$$

$$G_5 = \frac{1}{2}\left[1 - \left(\frac{r_o}{r}\right)^2\right]\langle r - r_o\rangle^0$$

$$G_6 = \frac{r_o}{4r}\left[\left(\frac{r_o}{r}\right)^2 - 1 + 2\ln\frac{r}{r_o}\right]\langle r - r_o\rangle^0$$

$$G_7 = \frac{1}{2}(1-\nu^2)\left(\frac{r}{r_o} - \frac{r_o}{r}\right)\langle r - r_o\rangle^0$$

$$G_8 = \frac{1}{2}\left[1 + \nu + (1-\nu)\left(\frac{r_o}{r}\right)^2\right]\langle r - r_o\rangle^0$$

$$G_9 = \frac{r_o}{r}\left\{\frac{1+\nu}{2}\ln\frac{r}{r_o} + \frac{1-\nu}{4}\left[1 - \left(\frac{r_o}{r}\right)^2\right]\right\}\langle r - r_o\rangle^0$$

$$G_{11} = \frac{1}{64}\left\{1 + 4\left(\frac{r_o}{r}\right)^2 - 5\left(\frac{r_o}{r}\right)^4 - 4\left(\frac{r_o}{r}\right)^2\left[2 + \left(\frac{r_o}{r}\right)^2\right]\ln\frac{r}{r_o}\right\}\langle r - r_o\rangle^0$$

$$G_{12} = \frac{r\langle r - r_o\rangle^0}{14.400(r - r_o)}\left\{64 - 225\frac{r_o}{r} - 100\left(\frac{r_o}{r}\right)^3 + 261\left(\frac{r_o}{r}\right)^5 - 60\left(\frac{r_o}{r}\right)^3\left[3\left(\frac{r_o}{r}\right)^2 + 10\right]\ln\frac{r}{r_o}\right\}$$

$$G_{13} = \frac{r^2\langle r - r_o\rangle^0}{14.400(r - r_o)^2}\left\{25 - 128\frac{r_o}{r} + 225\left(\frac{r_o}{r}\right)^2 - 25\left(\frac{r_o}{r}\right)^4 - 97\left(\frac{r_o}{r}\right)^6 - 60\left(\frac{r_o}{r}\right)^4\left[5 - \left(\frac{r_o}{r}\right)^2\right]\ln\frac{r}{r_o}\right\}$$

$$G_{14} = \frac{1}{16}\left[1 - \left(\frac{r_o}{r}\right)^4 - 4\left(\frac{r_o}{r}\right)^2\ln\frac{r}{r_o}\right]\langle r - r_o\rangle^0$$

$$G_{15} = \frac{r\langle r - r_o\rangle^0}{720(r - r_o)}\left[16 - 45\frac{r_o}{r} + 9\left(\frac{r_o}{r}\right)^5 + 20\left(\frac{r_o}{r}\right)^3\left(1 + 3\ln\frac{r}{r_o}\right)\right]$$

$$G_{16} = \frac{r^2\langle r - r_o\rangle^0}{1440(r - r_o)^2}\left[15 - 64\frac{r_o}{a} + 90\left(\frac{r_o}{r}\right)^2 - 6\left(\frac{r_o}{r}\right)^6 - 5\left(\frac{r_o}{r}\right)^4\left(7 + 12\ln\frac{r}{r_o}\right)\right]$$

$$G_{17} = \frac{1}{4}\left\{1 - \frac{1-\nu}{4}\left[1 - \left(\frac{r_o}{r}\right)^4\right] - \left(\frac{r_o}{r}\right)^2\left[1 + (1+\nu)\ln\frac{r}{r_o}\right]\right\}\langle r - r_o\rangle^0$$

TABLE 11.2 Formulas for flat circular plates of constant thickness (*Continued*)

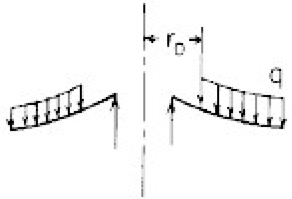
$$L_{18} = \frac{a}{720(a - r_o)}\left\{\left[20\left(\frac{r_o}{a}\right)^3 + 16\right](4 + \nu) - 45\frac{r_o}{a}(3 + \nu) - 9\left(\frac{r_o}{a}\right)^5(1 - \nu) + 60\left(\frac{r_o}{a}\right)^3(1 + \nu)\ln\frac{a}{r_o}\right\}$$

$$L_{19} = \frac{a^2}{1440(a - r_o)^2}\left[15(5 - \nu) - 64\frac{r_o}{a}(4 + \nu) - 90\left(\frac{r_o}{a}\right)^2(3 + \nu) - 5\left(\frac{r_o}{a}\right)^4(19 + 7\nu) + 6\left(\frac{r_o}{a}\right)^6(1 - \nu) - 60\left(\frac{r_o}{a}\right)^4(1 + \nu)\ln\frac{a}{r_o}\right]$$

$$G_{18} = \frac{r\langle r - r_o\rangle^0}{720(r - r_o)}\left\{\left[20\left(\frac{r_o}{r}\right)^3 + 16\right](4 + \nu) - 45\frac{r_o}{r}(3 + \nu) - 9\left(\frac{r_o}{r}\right)^5(1 - \nu) + 60\left(\frac{r_o}{r}\right)^3(1 + \nu)\ln\frac{r}{r_o}\right\}$$

$$G_{19} = \frac{r^2\langle r - r_o\rangle^0}{1440(r - r_o)^2}\left[15(5 + \nu) - 64\frac{r_o}{r}(4 - \nu) + 90\left(\frac{r_o}{r}\right)^2(3 + \nu) - 5\left(\frac{r_o}{r}\right)^4(19 - 7\nu) + 6\left(\frac{r_o}{r}\right)^6(1 - \nu) - 60\left(\frac{r_o}{r}\right)^4(1 + \nu)\ln\frac{r}{r_o}\right]$$

2k. Outer edge free, inner edge simply supported



$y_b = 0, \quad M_{rb} = 0, \quad M_{ra} = 0, \quad Q_a = 0$

$$\theta_b = \frac{-qa^3}{DC_7}\left[\frac{C_9}{2ab}(a^2 - r_o^2) - L_{17}\right]$$

$$Q_b = \frac{q}{2b}(a^2 - r_o^2)$$

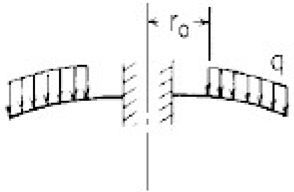
$$y_a = \theta_b a C_1 + Q_b \frac{a^3}{D} C_3 - \frac{qa^4}{D} L_{11}$$

$$\theta_a = \theta_b C_4 + Q_b \frac{a^2}{D} C_6 - \frac{qa^3}{D} L_{14}$$

If $r_o = b$ (uniform load over entire plate),

b/a	0.1	0.3	0.5	0.7	0.9
K_{y_a}	−0.1115	−0.1158	−0.0826	−0.0378	−0.0051
K_{θ_b}	−0.1400	−0.2026	−0.1876	−0.1340	−0.0515
K_{θ_a}	−0.1082	−0.1404	−0.1479	−0.1188	−0.0498
$K_{M_{tb}}$	−1.2734	−0.6146	−0.3414	−0.1742	−0.0521

2l. Outer edge free, inner edge fixed



$y_b = 0, \quad \theta_b = 0, \quad M_{ra} = 0, \quad Q_a = 0$

$$M_{rb} = \frac{-qa^2}{C_8}\left[\frac{C_9}{2ab}(a^2 - r_o^2) - L_{17}\right]$$

$$Q_b = \frac{q}{2b}(a^2 - r_o^2)$$

$$y_a = M_{rb}\frac{a^2}{D} C_2 + Q_b \frac{a^3}{D} C_3 - \frac{qa^4}{D} L_{11}$$

$$\theta_a = M_{rb}\frac{a}{D} C_5 + Q_b \frac{a^2}{D} C_6 - \frac{qa^3}{D} L_{14}$$

If $r_o = b$ (uniform load over entire plate),

b/a	0.1	0.3	0.5	0.7	0.9
K_{y_a}	−0.0757	−0.0318	−0.0086	−0.0011	
K_{θ_a}	−0.0868	−0.0512	−0.0207	−0.0046	−0.00017
$K_{M_{rb}}$	−0.9646	−0.4103	−0.1736	−0.0541	−0.00530

11.14 Tables

TABLE 11.1 Numerical values for functions used in Table 11.2

Numerical values for the plate coefficients F, C, L, and G for values of b/r, b/a, r_o/a, and r_o/r, respectively, from 0.05 to 1.0. Poisson's ratio is 0.30. The table headings are given for G_1 to G_{19} for the various values of r_o/r.† Also listed in the last five lines are values for the most used denominators for the ratios b/a

r_o/r	1.000	0.900	0.800	0.750	0.700	$\frac{2}{3}$	0.600	0.500
G_1	0.000	0.098580346	0.19478465	0.2423283	0.2897871	0.3215349	0.3858887	0.487773
G_2	0.000	0.004828991	0.01859406	0.0284644	0.0401146	0.0487855	0.0680514	0.100857
G_3	0.000	0.000158070	0.00119108	0.0022506	0.0037530	0.0050194	0.0082084	0.014554
G_4	1.000	0.973888889	0.95750000	0.9541667	0.9550000	0.9583333	0.9733333	1.025000
G_5	0.000	0.095000000	0.18000000	0.2187500	0.2550000	0.2777778	0.3200000	0.375000
G_6	0.000	0.004662232	0.01725742	0.0258495	0.0355862	0.0425624	0.0572477	0.079537
G_7	0.000	0.096055556	0.20475000	0.2654167	0.3315000	0.3791667	0.4853333	0.682500
G_8	1.000	0.933500000	0.87400000	0.8468750	0.8215000	0.8055556	0.7760000	0.737500
G_9	0.000	0.091560902	0.16643465	0.1976669	0.2247621	0.2405164	0.2664220	0.290898
G_{11}	0.000	0.000003996	0.00006104	0.0001453	0.0002935	0.0004391	0.0008752	0.001999
G_{12}	0.000	0.000000805	0.00001240	0.0000297	0.0000603	0.0000905	0.0001820	0.000422
G_{13}	0.000	0.000000270	0.00000418	0.0000100	0.0000205	0.0000308	0.0000623	0.000146
G_{14}	0.000	0.000158246	0.00119703	0.0022693	0.0038011	0.0051026	0.0084257	0.015272
G_{15}	0.000	0.000039985	0.00030618	0.0005844	0.0009861	0.0013307	0.0022227	0.004111
G_{16}	0.000	0.000016107	0.00012431	0.0002383	0.0004039	0.0005468	0.0009196	0.001721
G_{17}	0.000	0.004718219	0.01775614	0.0268759	0.0374539	0.0452137	0.0621534	0.090166
G_{18}	0.000	0.001596148	0.00610470	0.0093209	0.0131094	0.0159275	0.0221962	0.032948
G_{19}	0.000	0.000805106	0.00310827	0.0047694	0.0067426	0.0082212	0.0115422	0.017341
$C_1C_6 - C_3C_4$	0.000	0.000305662	0.00222102	0.0041166	0.0067283	0.0088751	0.0141017	0.023878
$C_1C_9 - C_3C_7$	0.000	0.009010922	0.03217504	0.0473029	0.0638890	0.0754312	0.0988254	0.131959
$C_2C_6 - C_3C_5$	0.000	0.000007497	0.00010649	0.0002435	0.0004705	0.0006822	0.0012691	0.002564
$C_2C_9 - C_3C_8$	0.000	0.000294588	0.00205369	0.0037205	0.0059332	0.0076903	0.0117606	0.018605
$C_4C_9 - C_6C_7$	0.000	0.088722311	0.15582772	0.1817463	0.2028510	0.2143566	0.2315332	0.243886

TABLE 11.1 Numerical values for functions used in Table 11.2 (*Continued*)

r_o/r	0.400	$\frac{1}{3}$	0.300	0.250	0.200	0.125	0.100	0.050
G_1	0.605736	0.704699	0.765608	0.881523	1.049227	1.547080	1.882168	3.588611
G_2	0.136697	0.161188	0.173321	0.191053	0.207811	0.229848	0.235987	0.245630
G_3	0.022290	0.027649	0.030175	0.033465	0.035691	0.035236	0.033390	0.025072
G_4	1.135000	1.266667	1.361667	1.562500	1.880000	2.881250	3.565000	7.032500
G_5	0.420000	0.444444	0.455000	0.468750	0.480000	0.492187	0.495000	0.498750
G_6	0.099258	0.109028	0.112346	0.114693	0.112944	0.099203	0.090379	0.062425
G_7	0.955500	1.213333	1.380167	1.706250	2.184000	3.583125	4.504500	9.077250
G_8	0.706000	0.688889	0.681500	0.671875	0.664000	0.655469	0.653500	0.650875
G_9	0.297036	0.289885	0.282550	0.266288	0.242827	0.190488	0.166993	0.106089
G_{11}	0.003833	0.005499	0.006463	0.008057	0.009792	0.012489	0.013350	0.014843
G_{12}	0.000827	0.001208	0.001435	0.001822	0.002266	0.003027	0.003302	0.003872
G_{13}	0.000289	0.000427	0.000510	0.000654	0.000822	0.001121	0.001233	0.001474
G_{14}	0.024248	0.031211	0.034904	0.040595	0.046306	0.054362	0.056737	0.060627
G_{15}	0.006691	0.008790	0.009945	0.011798	0.013777	0.016917	0.017991	0.020139
G_{16}	0.002840	0.003770	0.004290	0.005138	0.006065	0.007589	0.008130	0.009252
G_{17}	0.119723	0.139340	0.148888	0.162637	0.175397	0.191795	0.196271	0.203191
G_{18}	0.044939	0.053402	0.057723	0.064263	0.070816	0.080511	0.083666	0.089788
G_{19}	0.023971	0.028769	0.031261	0.035098	0.039031	0.045057	0.047086	0.051154
$C_1C_6 - C_3C_4$	0.034825	0.041810	0.044925	0.048816	0.051405	0.051951	0.051073	0.047702
$C_1C_9 - C_3C_7$	0.158627	0.170734	0.174676	0.177640	0.176832	0.168444	0.163902	0.153133
$C_2C_6 - C_3C_5$	0.004207	0.005285	0.005742	0.006226	0.006339	0.005459	0.004800	0.002829
$C_2C_9 - C_3C_8$	0.024867	0.027679	0.028408	0.028391	0.026763	0.020687	0.017588	0.009740
$C_4C_9 - C_6C_7$	0.242294	0.234900	0.229682	0.220381	0.209845	0.193385	0.188217	0.179431

† To obtain a value of either C_i, L_i, or F_i for a corresponding value of either b/a, r_o/a, or b/r, respectively, use the tabulated value of G_i for the corresponding value of r_o/r.

TABLE 11.2 Formulas for flat circular plates of constant thickness

NOTATION: W = total applied load (force); w = unit line load (force per unit of circumferential length); q = load per unit area; M_o = unit applied line moment loading (force-length per unit of circumferential length); θ_o = externally applied change in radial slope (radians); y_o = externally applied radial step in the vertical deflection (length); y = vertical deflection of plate (length); θ = radial slope of plate; M_r = unit radial bending moment; M_t = unit tangential bending moment; Q = unit shear force (force per unit of circumferential length); E = modulus of elasticity (force per unit area); ν = Poisson's ratio; γ = temperature coefficient of expansion (unit strain per degree); a = outer radius; b = inner radius for annular plate; t = plate thickness; r = radial location of quantity being evaluated; r_o = radial location of unit line loading or start of a distributed load. F_1 to F_9 and G_1 to G_{19} are the several functions of the radial location r. C_1 to C_9 are plate constants dependent upon the ratio a/b. L_1 to L_{19} are loading constants dependent upon the ratio a/r_o. When used as subscripts, r and t refer to radial and tangential directions, respectively. When used as subscripts, a, b, and o refer to an evaluation of the quantity subscripted at the outer edge, inner edge, and the position of the loading or start of distributed loading, respectively. When used as a subscript, c refers to an evaluation of the quantity subscripted at the center of the plate.

Positive signs are associated with the several quantities in the following manner: Deflections y and y_o are positive upward; slopes θ and θ_o are positive when the deflection y increases positively as r increases; moments M_r, M_t, and M_o are positive when creating compression on the top surface; and the shear force Q is positive when acting upward on the inner edge of a given annular section

Bending stresses can be found from the moments M_r and M_t by the expression $\sigma = 6M/t^2$. The plate constant $D = Et^3/12(1-\nu^2)$. The singularity function brackets $\langle\ \rangle$ indicate that the expression contained within the brackets must be equated to zero unless $r > r_o$, after which they are treated as any other brackets. Note that Q_b, Q_a, M_{rb}, and M_{ra} are reactions, not loads. They exist only when necessary edge restraints are provided.

General Plate Functions and Constants for Solid and Annular Circular Plates

$$F_1 = \frac{1+\nu}{2}\frac{b}{r}\ln\frac{r}{b} + \frac{1-\nu}{4}\left(\frac{r}{b} - \frac{b}{r}\right)$$

$$F_2 = \frac{1}{4}\left[1 - \left(\frac{b}{r}\right)^2\left(1 + 2\ln\frac{r}{b}\right)\right]$$

$$F_3 = \frac{b}{4r}\left\{\left[\left(\frac{b}{r}\right)^2 + 1\right]\ln\frac{r}{b} - \left(\frac{b}{r}\right)^2 - 1\right\}$$

$$F_4 = \frac{1}{2}\left[(1+\nu)\frac{b}{r} + (1-\nu)\frac{r}{b}\right]$$

$$F_5 = \frac{1}{2}\left[1 - \left(\frac{b}{r}\right)^2\right]$$

$$F_6 = \frac{b}{4r}\left[\left(\frac{b}{r}\right)^2 - 1 + 2\ln\frac{r}{b}\right]$$

$$F_7 = \frac{1}{2}(1-\nu^2)\left(\frac{r}{b} - \frac{b}{r}\right)$$

$$F_8 = \frac{1}{2}\left[1 + \nu + (1-\nu)\left(\frac{b}{r}\right)^2\right]$$

$$F_9 = \frac{b}{r}\left\{\frac{1+\nu}{2}\ln\frac{r}{b} + \frac{1-\nu}{4}\left[1 - \left(\frac{b}{r}\right)^2\right]\right\}$$

$$C_1 = \frac{1+\nu}{2}\frac{b}{a}\ln\frac{a}{b} + \frac{1-\nu}{4}\left(\frac{a}{b} - \frac{b}{a}\right)$$

$$C_2 = \frac{1}{4}\left[1 - \left(\frac{b}{a}\right)^2\left(1 + 2\ln\frac{a}{b}\right)\right]$$

$$C_3 = \frac{b}{4a}\left\{\left[\left(\frac{b}{a}\right)^2 + 1\right]\ln\frac{a}{b} + \left(\frac{b}{a}\right)^2 - 1\right\}$$

$$C_4 = \frac{1}{2}\left[(1+\nu)\frac{b}{a} + (1-\nu)\frac{a}{b}\right]$$

$$C_5 = \frac{1}{2}\left[1 - \left(\frac{b}{a}\right)^2\right]$$

$$C_6 = \frac{b}{4a}\left[\left(\frac{b}{a}\right)^2 - 1 + 2\ln\frac{a}{b}\right]$$

$$C_7 = \frac{1}{2}(1-\nu^2)\left(\frac{a}{b} - \frac{b}{a}\right)$$

$$C_8 = \frac{1}{2}\left[1 + \nu + (1-\nu)\left(\frac{b}{a}\right)^2\right]$$

$$C_9 = \frac{b}{a}\left\{\frac{1+\nu}{2}\ln\frac{a}{b} + \frac{1-\nu}{4}\left[1 - \left(\frac{b}{a}\right)^2\right]\right\}$$

TABLE 11.2 Formulas for flat circular plates of constant thickness (*Continued*)

$$L_1 = \frac{1+\nu}{2}\frac{r_o}{a}\ln\frac{r_0}{a} + \frac{1-\nu}{4}\left(\frac{a}{r_o} - \frac{r_o}{a}\right)$$

$$L_2 = \frac{1}{4}\left[1 - \left(\frac{r_o}{a}\right)^2\left(1 + 2\ln\frac{a}{r_o}\right)\right]$$

$$L_3 = \frac{r_o}{4a}\left\{\left[\left(\frac{r_o}{a}\right)^2 + 1\right]\ln\frac{a}{r_o} + \left(\frac{r_o}{a}\right)^2 - 1\right\}$$

$$L_4 = \frac{1}{2}\left[(1+\nu)\frac{r_o}{a} + (1-\nu)\frac{a}{r_o}\right]$$

$$L_5 = \frac{1}{2}\left[1 - \left(\frac{r_o}{a}\right)^2\right]$$

$$L_6 = \frac{r_o}{4a}\left[\left(\frac{r_o}{a}\right)^2 - 1 + 2\ln\frac{a}{r_o}\right]$$

$$L_7 = \frac{1}{2}(1-\nu^2)\left(\frac{a}{r_o} - \frac{r_o}{a}\right)$$

$$L_8 = \frac{1}{2}\left[1 - \nu + (1-\nu)\left(\frac{r_o}{a}\right)^2\right]$$

$$L_9 = \frac{r_o}{a}\left\{\frac{1+\nu}{2}\ln\frac{a}{r_o} + \frac{1-\nu}{4}\left[1 - \left(\frac{r_o}{a}\right)^2\right]\right\}$$

$$L_{11} = \frac{1}{64}\left\{1 + 4\left(\frac{r_o}{a}\right)^2 - 5\left(\frac{r_o}{a}\right)^4 - 4\left(\frac{r_o}{a}\right)^2\left[2 + \left(\frac{r_o}{a}\right)^2\right]\ln\frac{a}{r_o}\right\}$$

$$L_{12} = \frac{a}{14.400(a-r_o)}\left\{64 - 225\frac{r_o}{a} - 100\left(\frac{r_o}{a}\right)^3 + 261\left(\frac{r_o}{a}\right)^5 - 60\left(\frac{r_o}{a}\right)^3\left[3\left(\frac{r_o}{a}\right)^2 - 10\right]\ln\frac{a}{r_o}\right\}$$

$$L_{13} = \frac{a^2}{14.400(a-r_o)^2}\left\{25 - 128\frac{r_o}{a} + 225\left(\frac{r_o}{a}\right)^2 - 25\left(\frac{r_o}{a}\right)^4 - 97\left(\frac{r_o}{a}\right)^6 - 60\left(\frac{r_o}{a}\right)^4\left[5 + \left(\frac{r_o}{a}\right)^2\right]\ln\frac{a}{r_o}\right\}$$

$$L_{14} = \frac{1}{16}\left[1 - \left(\frac{r_o}{a}\right)^4 - 4\left(\frac{r_o}{a}\right)^2\ln\frac{a}{r_o}\right]$$

$$L_{15} = \frac{a}{720(a-r_o)}\left[16 - 45\frac{r_o}{a} + 9\left(\frac{r_o}{a}\right)^5 + 20\left(\frac{r_o}{a}\right)^3\left(1 + 3\ln\frac{a}{r_o}\right)\right]$$

$$L_{16} = \frac{a^2}{1440(a-r_o)^2}\left[15 - 64\frac{r_o}{a} + 90\left(\frac{r_o}{a}\right)^2 - 6\left(\frac{r_o}{a}\right)^6 - 5\left(\frac{r_o}{a}\right)^4\left(7 + 12\ln\frac{a}{r_o}\right)\right]$$

$$L_{17} = \frac{1}{4}\left\{1 - \frac{1-\nu}{4}\left[1 - \left(\frac{r_o}{a}\right)^4\right] - \left(\frac{r_o}{a}\right)^2\left[1 + (1+\nu)\ln\frac{a}{r_o}\right]\right\}$$

$$G_1 = \left[\frac{1+\nu}{2}\frac{r_o}{r}\ln\frac{r}{r_0} + \frac{1-\nu}{4}\left(\frac{r}{r_o} - \frac{r_o}{r}\right)\right]\langle r - r_o\rangle^0$$

$$G_2 = \frac{1}{4}\left[1 - \left(\frac{r_o}{r}\right)^2\left(1 + 2\ln\frac{r}{r_o}\right)\right]\langle r - r_o\rangle^0$$

$$G_3 = \frac{r_o}{4r}\left\{\left[\left(\frac{r_o}{r}\right)^2 + 1\right]\ln\frac{r}{r_o} + \left(\frac{r_o}{r}\right)^2 - 1\right\}\langle r - r_o\rangle^0$$

$$G_4 = \frac{1}{2}\left[(1+\nu)\frac{r_o}{r} + (1-\nu)\frac{r}{r_o}\right]\langle r - r_o\rangle^0$$

$$G_5 = \frac{1}{2}\left[1 - \left(\frac{r_o}{r}\right)^2\right]\langle r - r_o\rangle^0$$

$$G_6 = \frac{r_o}{4r}\left[\left(\frac{r_o}{r}\right)^2 - 1 + 2\ln\frac{r}{r_o}\right]\langle r - r_o\rangle^0$$

$$G_7 = \frac{1}{2}(1-\nu^2)\left(\frac{r}{r_o} - \frac{r_o}{r}\right)\langle r - r_o\rangle^0$$

$$G_8 = \frac{1}{2}\left[1 + \nu + (1-\nu)\left(\frac{r_o}{r}\right)^2\right]\langle r - r_o\rangle^0$$

$$G_9 = \frac{r_o}{r}\left\{\frac{1+\nu}{2}\ln\frac{r}{r_o} + \frac{1-\nu}{4}\left[1 - \left(\frac{r_o}{r}\right)^2\right]\right\}\langle r - r_o\rangle^0$$

$$G_{11} = \frac{1}{64}\left\{1 + 4\left(\frac{r_o}{r}\right)^2 - 5\left(\frac{r_o}{r}\right)^4 - 4\left(\frac{r_o}{r}\right)^2\left[2 + \left(\frac{r_o}{r}\right)^2\right]\ln\frac{r}{r_o}\right\}\langle r - r_o\rangle^0$$

$$G_{12} = \frac{r\langle r - r_o\rangle^0}{14.400(r-r_o)}\left\{64 - 225\frac{r_o}{r} - 100\left(\frac{r_o}{r}\right)^3 + 261\left(\frac{r_o}{r}\right)^5 - 60\left(\frac{r_o}{r}\right)^3\left[3\left(\frac{r_o}{r}\right)^2 + 10\right]\ln\frac{r}{r_o}\right\}$$

$$G_{13} = \frac{r^2\langle r - r_o\rangle^0}{14.400(r-r_o)^2}\left\{25 - 128\frac{r_o}{r} + 225\left(\frac{r_o}{r}\right)^2 - 25\left(\frac{r_o}{r}\right)^4 - 97\left(\frac{r_o}{r}\right)^6 - 60\left(\frac{r_o}{r}\right)^4\left[5 - \left(\frac{r_o}{r}\right)^2\right]\ln\frac{r}{r_o}\right\}$$

$$G_{14} = \frac{1}{16}\left[1 - \left(\frac{r_o}{r}\right)^4 - 4\left(\frac{r_o}{r}\right)^2\ln\frac{r}{r_o}\right]\langle r - r_o\rangle^0$$

$$G_{15} = \frac{r\langle r - r_o\rangle^0}{720(r-r_o)}\left[16 - 45\frac{r_o}{r} + 9\left(\frac{r_o}{r}\right)^5 + 20\left(\frac{r_o}{r}\right)^3\left(1 + 3\ln\frac{r}{r_o}\right)\right]$$

$$G_{16} = \frac{r^2\langle r - r_o\rangle^0}{1440(r-r_o)^2}\left[15 - 64\frac{r_o}{a} + 90\left(\frac{r_o}{r}\right)^2 - 6\left(\frac{r_o}{r}\right)^6 - 5\left(\frac{r_o}{r}\right)^4\left(7 + 12\ln\frac{r}{r_o}\right)\right]$$

$$G_{17} = \frac{1}{4}\left\{1 - \frac{1-\nu}{4}\left[1 - \left(\frac{r_o}{r}\right)^4\right] - \left(\frac{r_o}{r}\right)^2\left[1 + (1+\nu)\ln\frac{r}{r_o}\right]\right\}\langle r - r_o\rangle^0$$

TABLE 11.2 Formulas for flat circular plates of constant thickness (*Continued*)

$$L_{18} = \frac{a}{720(a - r_o)}\left\{\left[20\left(\frac{r_o}{a}\right)^3 + 16\right](4+\nu) - 45\frac{r_o}{a}(3+\nu) - 9\left(\frac{r_o}{a}\right)^5(1-\nu) + 60\left(\frac{r_o}{a}\right)^3(1+\nu)\ln\frac{a}{r_o}\right\}$$

$$G_{18} = \frac{r\langle r - r_o\rangle^0}{720(r - r_o)}\left\{\left[20\left(\frac{r_o}{r}\right)^3 + 16\right](4+\nu) - 45\frac{r_o}{r}(3+\nu) - 9\left(\frac{r_o}{r}\right)^5(1-\nu) + 60\left(\frac{r_o}{r}\right)^3(1+\nu)\ln\frac{r}{r_o}\right\}$$

$$L_{19} = \frac{a^2}{1440(a - r_o)^2}\left[15(5-\nu) - 64\frac{r_o}{a}(4+\nu) - 90\left(\frac{r_o}{a}\right)^2(3+\nu) - 5\left(\frac{r_o}{a}\right)^4(19+7\nu) + 6\left(\frac{r_o}{a}\right)^6(1-\nu) - 60\left(\frac{r_o}{a}\right)^4(1+\nu)\ln\frac{a}{r_o}\right]$$

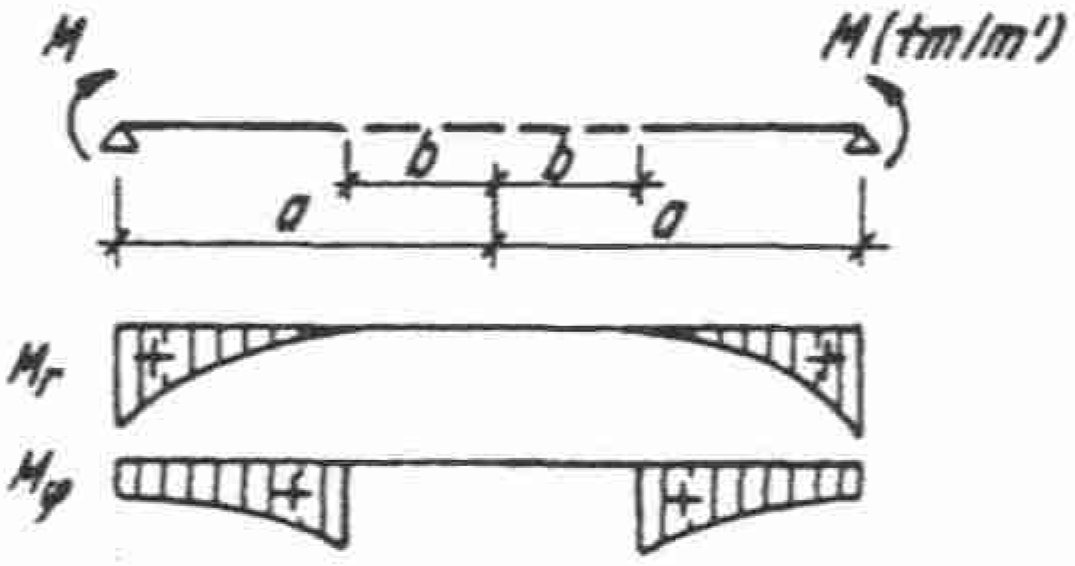
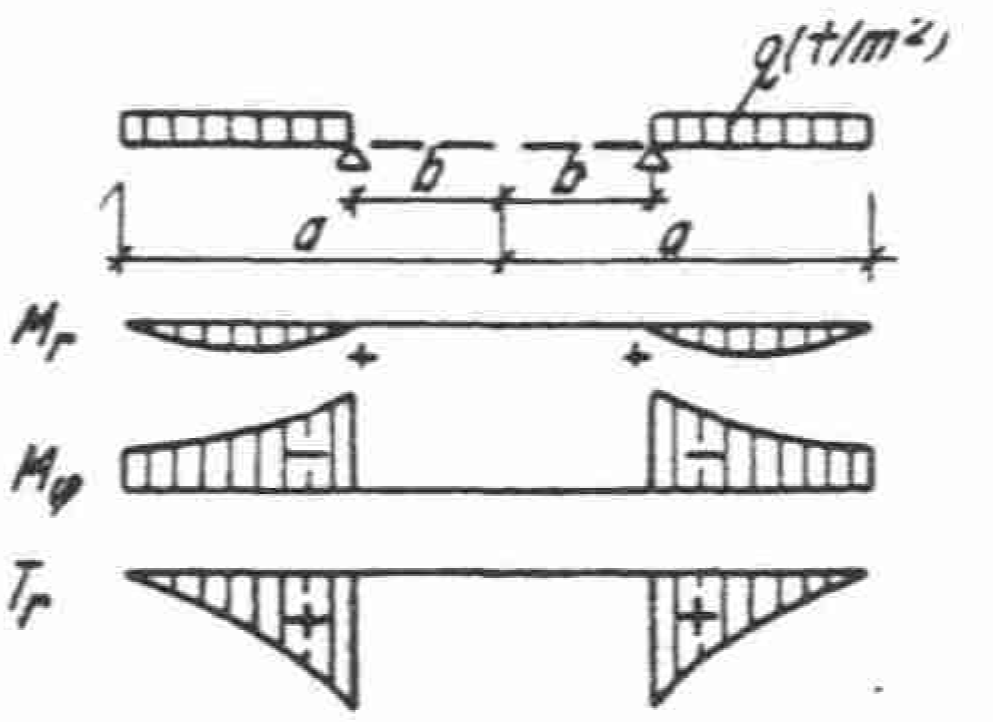
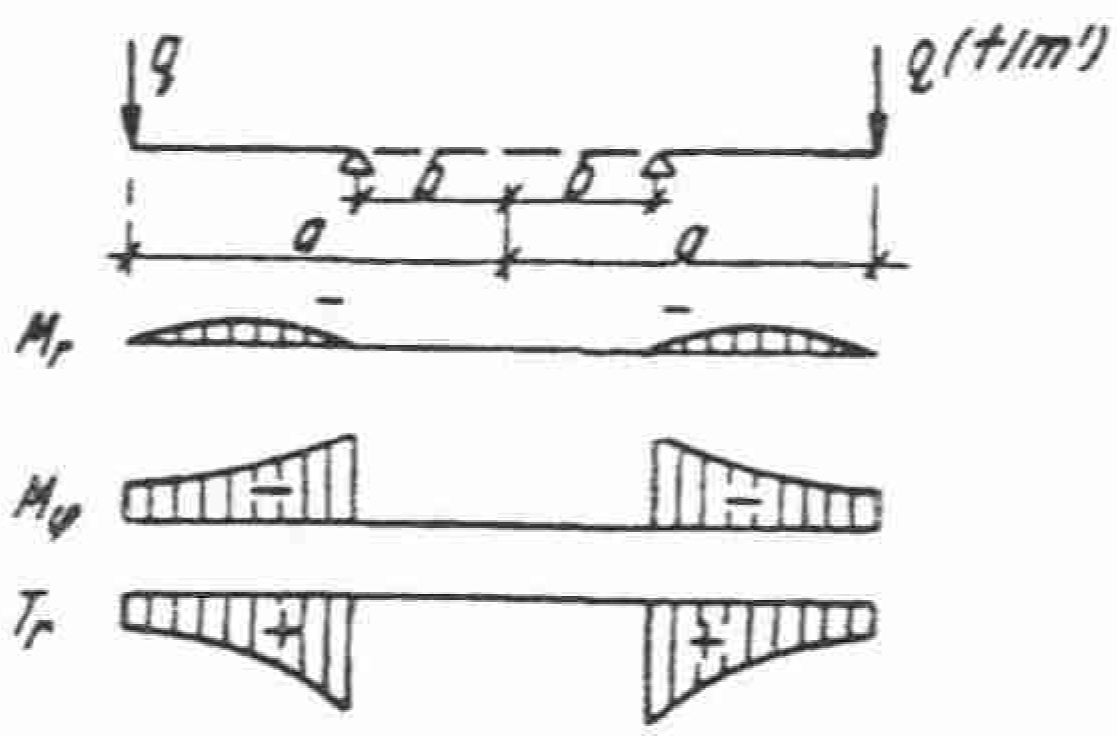
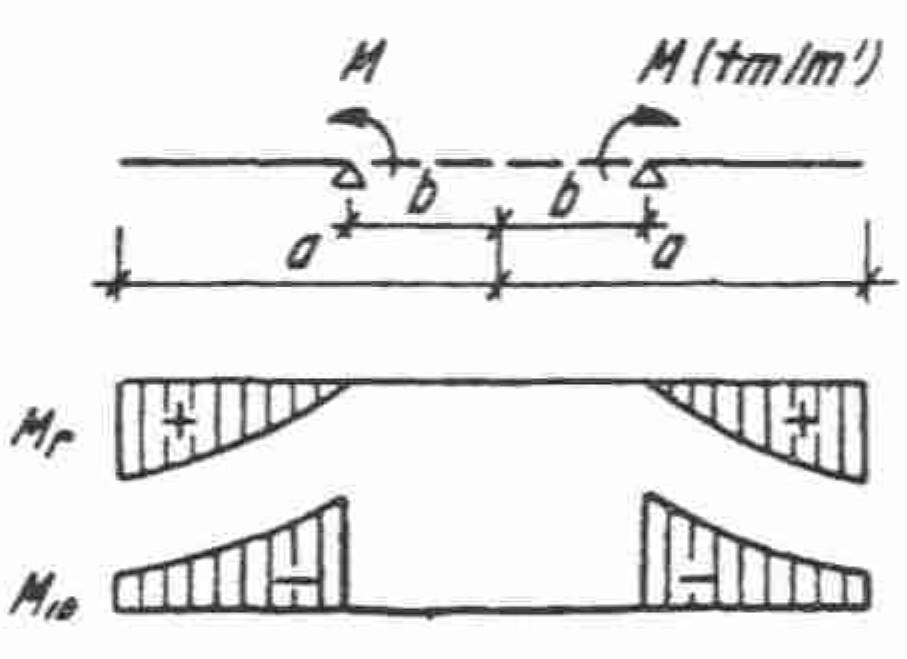
$$G_{19} = \frac{r^2\langle r - r_o\rangle^0}{1440(r - r_o)^2}\left[15(5+\nu) - 64\frac{r_o}{r}(4-\nu) + 90\left(\frac{r_o}{r}\right)^2(3+\nu) - 5\left(\frac{r_o}{r}\right)^4(19-7\nu) + 6\left(\frac{r_o}{r}\right)^6(1-\nu) - 60\left(\frac{r_o}{r}\right)^4(1+\nu)\ln\frac{r}{r_o}\right]$$

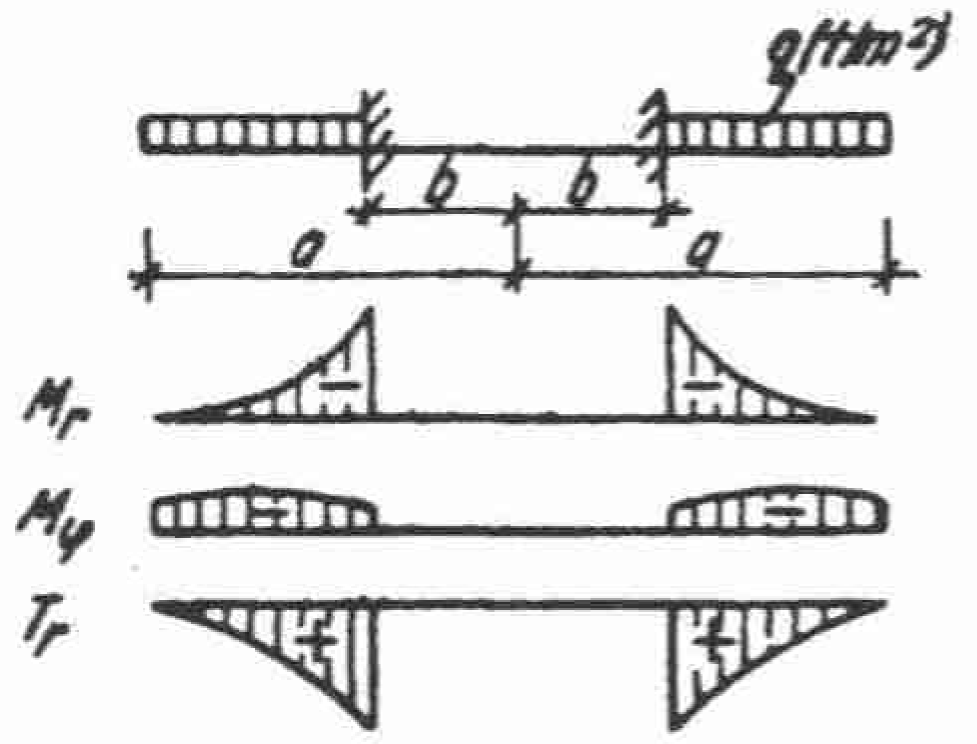
11.14 Tables

TABLE 11.1 Numerical values for functions used in Table 11.2

Numerical values for the plate coefficients F, C, L, and G for values of b/r, b/a, r_o/a, and r_o/r, respectively, from 0.05 to 1.0. Poisson's ratio is 0.30. The table headings are given for G_1 to G_{19} for the various values of r_o/r.† Also listed in the last five lines are values for the most used denominators for the ratios b/a

r_o/r	1.000	0.900	0.800	0.750	0.700	$\frac{2}{3}$	0.600	0.500
G_1	0.000	0.098580346	0.19478465	0.2423283	0.2897871	0.3215349	0.3858887	0.487773
G_2	0.000	0.004828991	0.01859406	0.0284644	0.0401146	0.0487855	0.0680514	0.100857
G_3	0.000	0.000158070	0.00119108	0.0022506	0.0037530	0.0050194	0.0082084	0.014554
G_4	1.000	0.973888889	0.95750000	0.9541667	0.9550000	0.9583333	0.9733333	1.025000
G_5	0.000	0.095000000	0.18000000	0.2187500	0.2550000	0.2777778	0.3200000	0.375000
G_6	0.000	0.004662232	0.01725742	0.0258495	0.0355862	0.0425624	0.0572477	0.079537
G_7	0.000	0.096055556	0.20475000	0.2654167	0.3315000	0.3791667	0.4853333	0.682500
G_8	1.000	0.933500000	0.87400000	0.8468750	0.8215000	0.8055556	0.7760000	0.737500
G_9	0.000	0.091560902	0.16643465	0.1976669	0.2247621	0.2405164	0.2664220	0.290898
G_{11}	0.000	0.000003996	0.00006104	0.0001453	0.0002935	0.0004391	0.0008752	0.001999
G_{12}	0.000	0.000000805	0.00001240	0.0000297	0.0000603	0.0000905	0.0001820	0.000422
G_{13}	0.000	0.000000270	0.00000418	0.0000100	0.0000205	0.0000308	0.0000623	0.000146
G_{14}	0.000	0.000158246	0.00119703	0.0022693	0.0038011	0.0051026	0.0084257	0.015272
G_{15}	0.000	0.000039985	0.00030618	0.0005844	0.0009861	0.0013307	0.0022227	0.004111
G_{16}	0.000	0.000016107	0.00012431	0.0002383	0.0004039	0.0005468	0.0009196	0.001721
G_{17}	0.000	0.004718219	0.01775614	0.0268759	0.0374539	0.0452137	0.0621534	0.090166
G_{18}	0.000	0.001596148	0.00610470	0.0093209	0.0131094	0.0159275	0.0221962	0.032948
G_{19}	0.000	0.000805106	0.00310827	0.0047694	0.0067426	0.0082212	0.0115422	0.017341
$C_1C_6 - C_3C_4$	0.000	0.000305662	0.00222102	0.0041166	0.0067283	0.0088751	0.0141017	0.023878
$C_1C_9 - C_3C_7$	0.000	0.009010922	0.03217504	0.0473029	0.0638890	0.0754312	0.0988254	0.131959
$C_2C_6 - C_3C_5$	0.000	0.000007497	0.00010649	0.0002435	0.0004705	0.0006822	0.0012691	0.002564
$C_2C_9 - C_3C_8$	0.000	0.000294588	0.00205369	0.0037205	0.0059332	0.0076903	0.0117606	0.018605
$C_4C_9 - C_6C_7$	0.000	0.088722311	0.15582772	0.1817463	0.2028510	0.2143566	0.2315332	0.243886

	$w = \frac{Ma^2}{2D} \frac{1}{(1-\beta^2)(1+\mu)} \left(1 - \varrho^2 - 2\frac{1+\mu}{1-\mu} \beta^2 \lg \varrho\right)$ $M_r = \frac{M\beta^2}{1-\beta^2}\left(\frac{1}{\beta^2} - \frac{1}{\varrho^2}\right)$ $M_\varphi = \frac{M\beta^2}{1-\beta^2}\left(\frac{1}{\beta^2} + \frac{1}{\varrho^2}\right)$ $T_r = 0$
	$w = \frac{qa^4}{64D}\left\{\frac{2}{1+\mu}[(3+\mu)(\beta^2-2)+k_5](\beta^2-\varrho^2) - (\beta^4-\varrho^4) - \right.$ $\left. - \frac{4\beta^2}{1-\mu} k_5 \lg\frac{\varrho}{\beta} - 8\varrho^2 \lg\frac{\varrho}{\beta}\right\}$ $M_r = \frac{qa^2}{16}\left[(3+\mu)(\beta^2-\varrho^2) + k_5\left(1 - \frac{\beta^2}{\varrho^2}\right) + 4(1+\mu)\lg\frac{\varrho}{\beta}\right]$ $M_\varphi = \frac{qa^2}{16}\left[2(1-\mu)(\beta^2-2) + (1+3\mu)(\beta^2-\varrho^2) + \right.$ $\left. + k_5\left(1+\frac{\beta^2}{\varrho^2}\right) + 4(1+\mu)\lg\frac{\varrho}{\beta}\right]$ $T_r = \frac{qa}{2}\left(\frac{1}{\varrho} - \varrho\right)$ $k_5 = 3 + \mu + 4(1+\mu)\frac{1}{1-\beta^2}\lg\beta$
	$w = -\frac{qa^3}{8D}\left[\frac{3+\mu-2k_6}{1+\mu}(\beta^2-\varrho^2) + \right.$ $\left. + \frac{4k_6}{1-\mu}\beta^2 \lg\frac{\varrho}{\beta} + 2\varrho^2\lg\frac{\varrho}{\beta}\right]$ $T_r = \frac{q}{\varrho}$ $M_r = -\frac{qa}{2}\left[k_6\left(\frac{\beta^2}{\varrho^2} - 1\right) - (1+\mu)\lg\frac{\varrho}{\beta}\right]$ $M_\varphi = -\frac{qa}{2}\left[1 - \mu - k_6\left(\frac{\beta^2}{\varrho^2} + 1\right) - (1+\mu)\lg\frac{\varrho}{\beta}\right]$ $k_6 = \frac{1+\mu}{1-\beta^2}\lg\beta$
	$w = \frac{Ma^2}{2D}\frac{\beta^2}{(1+\mu)(\beta^2-1)}\left(\beta^2 - \varrho^2 - 2\frac{1+\mu}{1-\mu}\lg\frac{\varrho}{\beta}\right)$ $T_r = 0$ $M_r = M\frac{\beta^2}{\beta^2-1}\left(1 - \frac{1}{\varrho^2}\right)$ $M_\varphi = M\frac{\beta^2}{\beta^2-1}\left(1 + \frac{1}{\varrho^2}\right)$



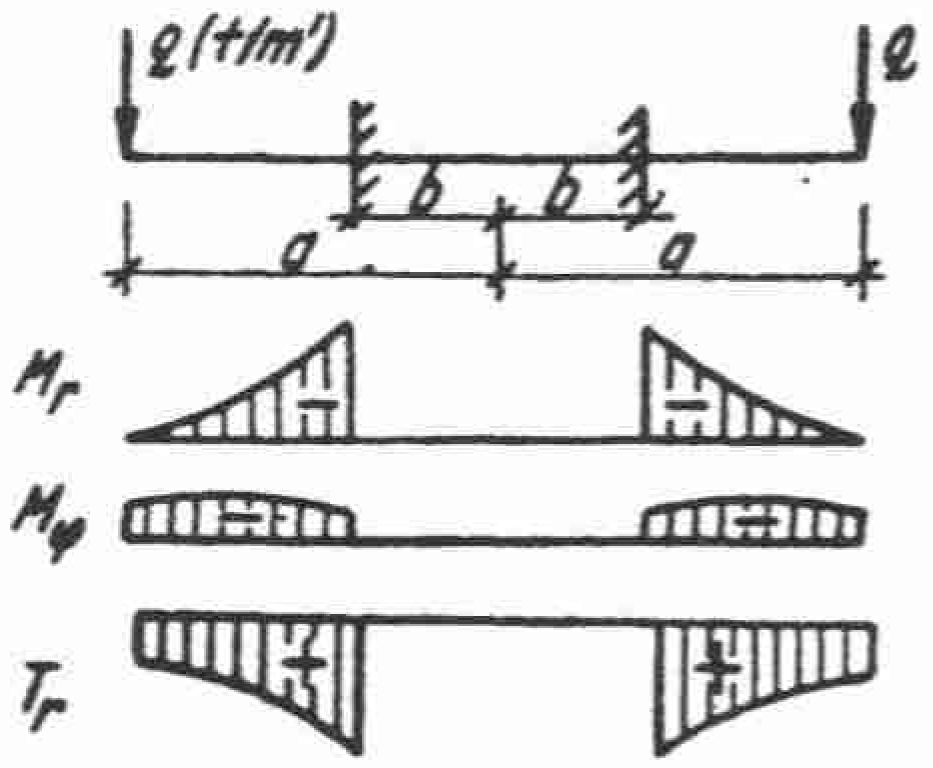
$$w = \frac{qa^4}{64D}\left[-\beta^4 + 2(\beta^2 - k_{10} - 2)(\beta^2 - \varrho^2) + \varrho^4 - 4k_{10}\beta^2 \lg\frac{\varrho}{\beta} - 8\varrho^2 \lg\frac{\varrho}{\beta}\right]$$

$$T_r = -\frac{qa}{2}\left(\varrho - \frac{1}{\varrho}\right)$$

$$M_r = \frac{qa^2}{16}\left[(1+\mu)(\beta^2 - k_{10}) + 4 - (3+\mu)\varrho^2 - (1-\mu)k_{10}\frac{\beta^2}{\varrho^2} + 4(1+\mu)\lg\frac{\varrho}{\beta}\right]$$

$$M_\varphi = \frac{qa^2}{16}\left[(1+\mu)(\beta^2 - k_{10}) + 4\mu - (1+3\mu)\varrho^2 + (1-\mu)k_{10}\frac{\beta^2}{\varrho^2} + 4(1+\mu)\lg\frac{\varrho}{\beta}\right]$$

$$k_{10} = \frac{1 - \mu + (1+\mu)(\beta^2 - 4\lg\beta)}{1 + \mu + (1-\mu)\beta^2}$$



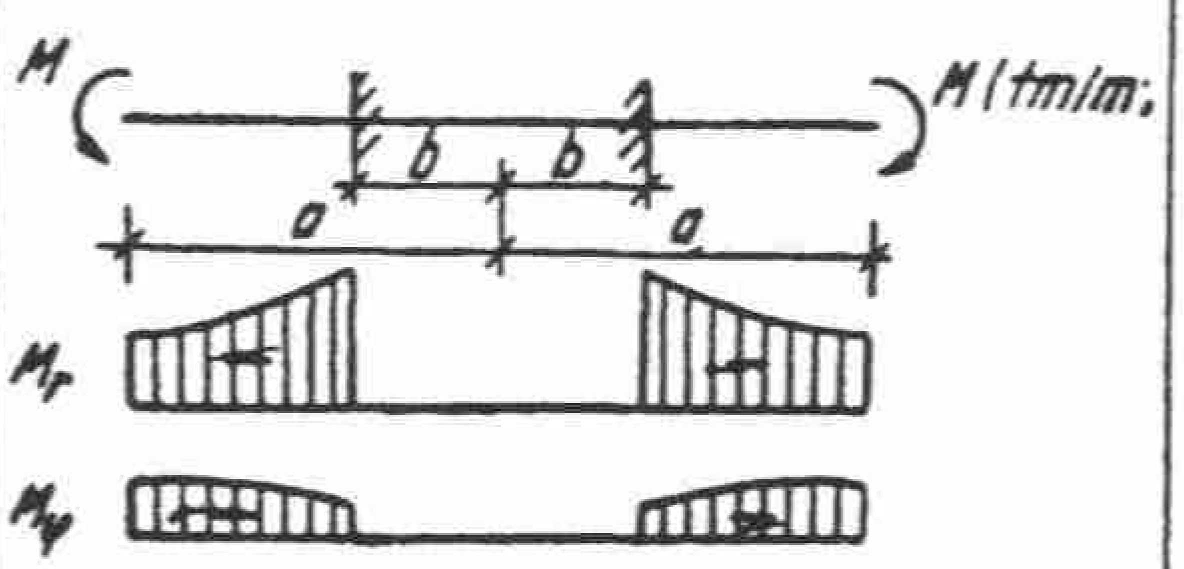
$$w = \frac{qa^3}{8D}\left[(1 + 2k_{11})(\beta^2 - \varrho^2) + 4k_{11}\beta^2\lg\frac{\varrho}{\beta} + 2\varrho^2\lg\frac{\varrho}{\beta}\right]$$

$$T_r = \frac{q}{\varrho}$$

$$M_r = -\frac{qa}{2}\left[k_{11}(1+\mu) - 1 + (1-\mu)k_{11}\frac{\beta^2}{\varrho^2} - (1+\mu)\lg\frac{\varrho}{\beta}\right]$$

$$M_\varphi = -\frac{qa}{2}\left[-\mu + (1+\mu)k_{11} - (1-\mu)k_{11}\frac{\beta^2}{\varrho^2} - (1+\mu)\lg\frac{\varrho}{\beta}\right]$$

$$k_{11} = \frac{1 - (1+\mu)\lg\beta}{1 + \mu + (1-\mu)\beta^2}$$



$$w = -\frac{Ma^2}{2D}k_{12}\left(-\beta^2 + \varrho^2 - 2\beta^2\lg\frac{\varrho}{\beta}\right)$$

$$T_r = 0$$

$$M_r = Mk_{12}\left[1 + \mu + (1-\mu)\frac{\beta^2}{\varrho^2}\right]$$

$$M_\varphi = Mk_{12}\left[1 + \mu - (1-\mu)\frac{\beta^2}{\varrho^2}\right]$$

$$k_{12} = \frac{1}{1 + \mu + (1-\mu)\beta^2}$$

Tab. 1.162

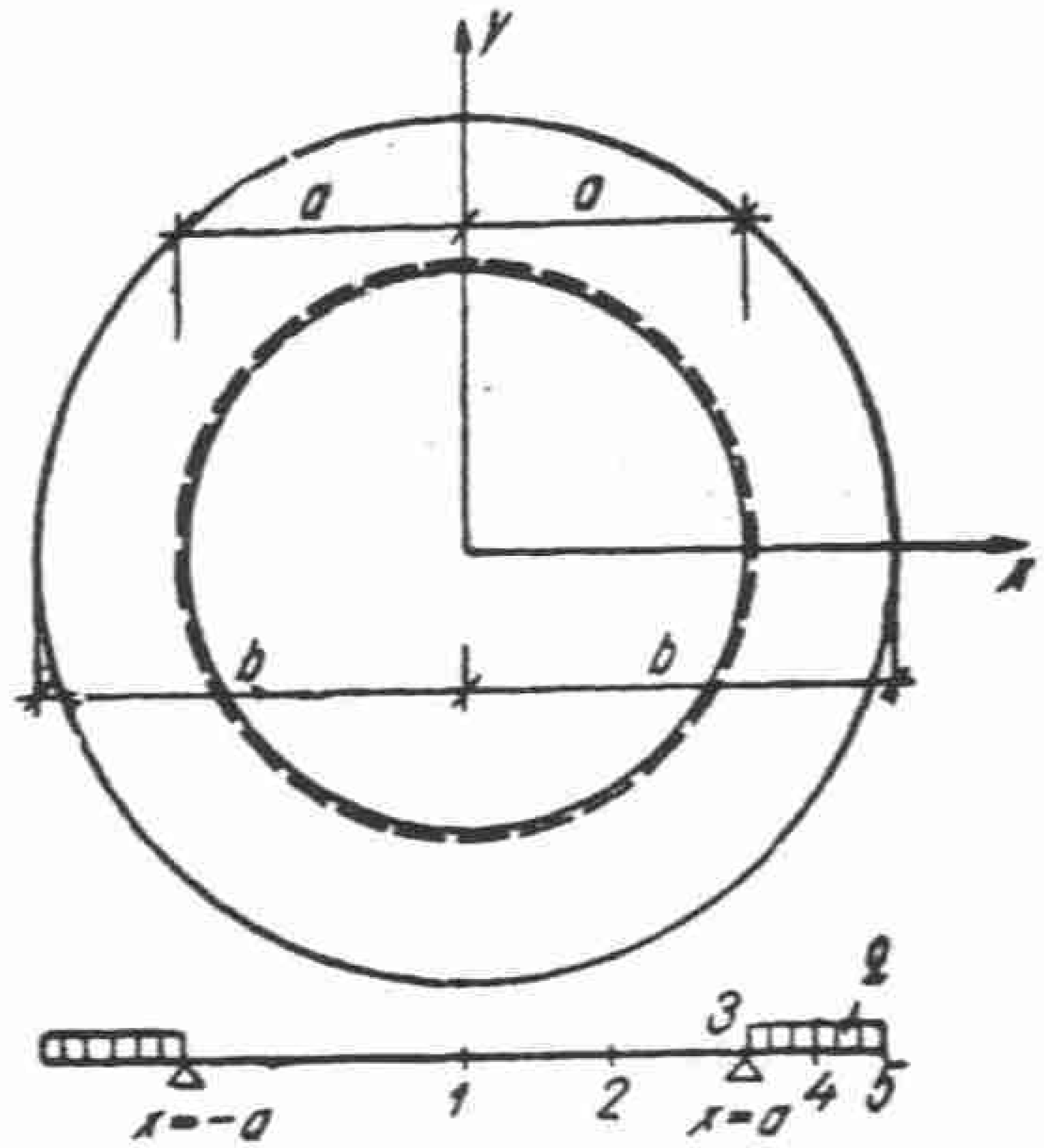
$$\mu = 0$$

$$\beta = \frac{b}{a}$$

$M_{r1} = M_{r2} = M_{r3} =$
$= M_{\varphi 1} = M_{\varphi 2} = M_{\varphi 3}$

$M_{r5} = 0$

$T_{r1} = T_{r2} = T_{r3}$ (interno)
$= T_{r5} = 0$



β	M_{r1}	M_{r4}	$M_{\varphi 4}$	$M_{\varphi 5}$	T_{r3} (esterno)	T_{r4}
1,1	−0,005	−0,002	−0,004	−0,004	0,105	0,050
1,2	−0,019	−0,004	−0,016	−0,014	0,220	0,105
1,3	−0,043	−0,008	−0,035	−0,029	0,345	0,160
1,4	−0,077	−0,014	−0,060	−0,049	0,480	0,215
1,5	−0,120	−0,020	−0,092	−0,072	0,625	0,275
1,6	−0,173	−0,027	−0,129	−0,099	0,780	0,335
1,7	−0,236	−0,034	−0,173	−0,128	0,945	0,395
1,8	−0,310	−0,042	−0,221	−0,161	1,120	0,460
1,9	−0,393	−0,044	−0,269	−0,197	1,305	0,520
2,0	−0,487	−0,059	−0,335	−0,234	1,500	0,580
2,1	−0,592	−0,079	−0,410	−0,274	1,705	0,645
2,2	−0,711	−0,081	−0,472	−0,317	1,920	0,710
f.m.	qa^2	qa^2	qa^2	qa^2	qa	qa