JOURNAL OF STRUCTURAL ENGINEERING
INFORMATION FOR AUTHORS

The *Journal of Structural Engineering* is the technical publication of the Structural Division of the ASCE. As stated in the *Official Register*, the purpose of the division is "to advance the science of structural design by increasing the knowledge of the physical properties of engineering materials, developing methods of analysis, studying the relative merits of various types of structures and methods of fabrication, and disseminating knowledge relating to this division of the civil engineering profession." Consistent with this purpose, the division encourages prospective authors to submit quality manuscript papers in all fields of structural engineering for consideration for publication in the journal. Papers must contain work that has not been published, and ASCE will not review a paper that is simultaneously submitted elsewhere.

The Structural Division is currently composed of 10 technical administrative committees, which are responsible for the review of papers for publication in the journal. When authors submit papers for publication consideration, they should indicate the committee most appropriate for review of their paper; however, all papers must be submitted directly to ASCE Headquarters. The technical committees and the areas of their responsibilities are described here briefly:

Analysis and Design of Structures—Technical problems in the analysis, design, safety, and serviceability of structures.

Bridges—Methods of analysis, design, erection, and materials used for bridges.

Concrete and Masonry Structures—The design and construction of structures of reinforced concrete or masonry.

Dynamic Effects—Dynamic forces and their effects upon the behavior of structures.

Electronic Computation—Applications of modern computing devices to the general field of structures.

Metals—Mechanical properties and structural applications of metals and the behavior of metal structures.

Nuclear Structures and Materials—Structures and structural materials of nuclear facilities.

Structural Safety and Reliability—Principles and methods of structural safety and reliability, and formulation of probability-based design.

Tall Buildings—Analysis and design of tall buildings and related systems.

Wood—Properties and structural applications of wood and the behavior of wood structures.

bridge deck modular roads and bridges." *Civ. Engrg.*, ASCE,

., K. F., W. pf, T. J., and Sanders, W. W., Jr. (1987).
:ning existing highway bridges." *NCHRP Report 293*.

:cess in New Jersey." (1985). *Pittsburgh. Engr.*, summer. Early
dermic deck (Insta-Dek) and the Driscoll Bridge.

y bridge deck ENR." (1985). *Public Works*, Oct.–Dec.

ened with prefabricated pavement modules." (1984). *Construc-*
–14.

(Insta-Dek) speeds bridge expansion job in New Jersey." (1985).
b.

d new bridge lane." (1984). *Civ. Engrg.*, ASCE, Sept. 20–22.

d new bridge lane." (1985). *Amer. Transp. Builder*, winter.

ing (Insta-Dek) construction." (1985). *Concr. Int.*, 3, June, 97.

985). "Modular deck simplifies, speeds highway bridge modifi-
vs. Apr. 8.

86). "Prefab components may offer speed, savings." *Roads and*
55.

LATERAL BUCKLING IN CURTAIN WALL SYSTEMS

By Charles D. Clift,[1] Member, ASCE, and Walter J. Austin,[2]
Fellow, ASCE

ABSTRACT: Recent demand for increasingly complex exterior facades of commercial buildings has driven a need for more efficient high-performance curtain wall designs. Current codes can be improved by formulations that more closely parallel required criteria. In this paper, formulas for the elastic lateral buckling strength of slender monosymmetrical beams that represent curtain wall mullions with practical loading and supporting conditions have been derived by approximate energy procedures for four different assumptions of possible glazing restraint. The equations have been constructed in a general manner so that a large variety of glazing (bracing) positions is accommodated. Analysis of a typical design situation indicates that the currently used allowable compressive stresses of the Aluminum Association Specifications for buildings are extremely conservative for this application. Also, a method to experimentally verify the assumed glazing restraint is presented via the Southwell plot method. Use of the more accurate analyses herein will result in significant design efficiencies on many curtain wall systems.

INTRODUCTION

Glass curtain walls consist of large glass panes supported by vertical mullions and horizontal members between mullions. Wind forces on the curtain walls are transmitted to the building frame at the floor levels by the vertical mullions. The glazing transmits the wind forces to the mullions and also acts to restrain lateral-torsional motion of the mullions. The horizontal members between mullions also may restrain lateral-torsional buckling of the mullions. This paper is concerned with the lateral buckling strength of the long slender aluminum mullions, and the design of such members. Since there is practically no literature on the design of curtain wall mullions, most designers make very conservative assumptions.

Approximate analyses are given herein for several different supporting conditions that approximate minimum and maximum bracing and a realistic approximation to the restraint afforded by the glazing. For a typical application, comparisons are made of the allowable compressive stress given by current design procedures with corresponding values derived from the different theoretical analyses. These comparisons show the great influence of the restraint that may be offered by the glazing. Large increases in the allowable compressive stresses are possible. However, further study including experimental verification is suggested.

The cross section of a typical curtain wall mullion system is shown in Fig. 1. The mullion is represented in this paper by a prismatic monosymmetrical cross section, as in Fig. 2. The horizontal principal axis is denoted as x, the vertical principal axis as y and the longitudinal centroidal axis as z, forming

[1] Prin., Curtain Wall Design and Consulting, Inc., 10450 Brockwood Road, Dallas, TX 75238.
[2] Prof. Emeritus, Civ. Engrg. Dept., Rice Univ., P.O. Box 1892, Houston, TX 77251.
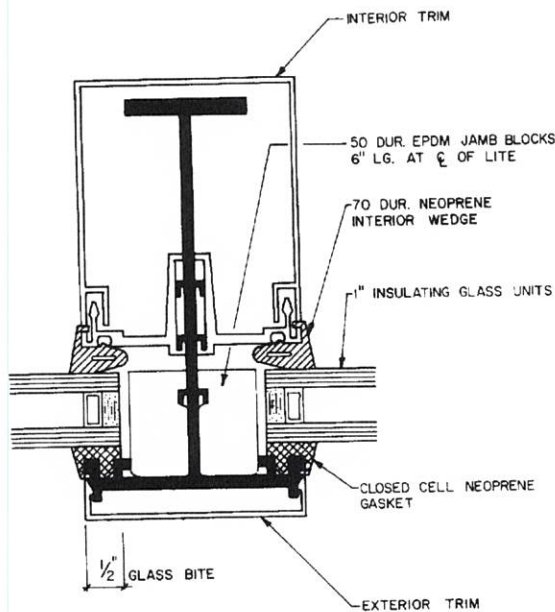
FIG. 1. Typical Curtain Wall Mullion

a right-hand system. The shear center has coordinates $(0, y_o)$. The horizontal displacement of the shear center is denoted as u and the rotation of the cross section as β. The uniform load w, positive in the positive y direction, is considered to be applied by the glazing a distance, $\bar{a}$, below the shear center, as shown in Fig. 2. The mullions span two equal floor heights, λ, and are
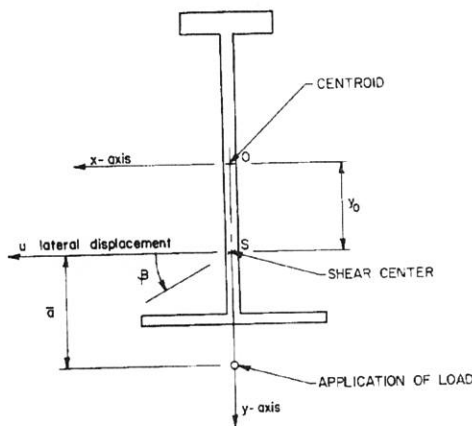
FIG. 2. Nomenclature of Mullion Cross Section and Axis System

2482
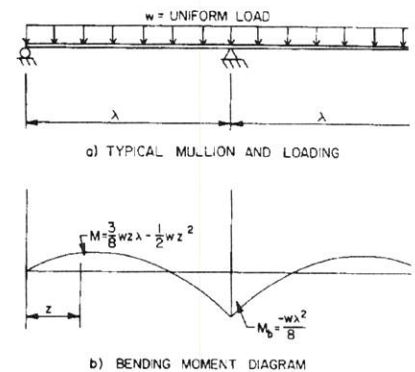
FIG. 3. Beam Analysis Model for Curtain Wall

considered as simply supported at the ends. Lateral wind to be static, uniformly distributed loads, as in Fig. 3(a). bending moment about a horizontal axis is denoted as M ment diagram is shown in Fig. 3(b). Note that the wind positive or negative on the glazing plane.

The current state of knowledge of lateral buckling of in Galambos (1988); this extensive presentation contains of references. Texts by Timoshenko and Gere (1961), Galambos (1968) give derivations of basic relationship solutions, mostly of rectangular or I-shaped cross secti Trahair (1972) give the first extensive presentation of dat elastic lateral buckling strength of simply supported an symmetrical beams subjected to uniform and concentra perimental study on cantilevers verifies the validity of tl

CASE I. MINIMUM BRACING RESTRAINT

Assume first that the mullion of Fig. 2 is not restrained but is freestanding. Each span of the two span beam w mode as for a simply supported beam subjected to the sponding moment diagram of Fig. 3. A Rayleigh-Ritz dure is used.

The general potential energy expression for lateral bu Masur and Milbradt (1957) is as follows:

$$V = \frac{1}{2} \int_o^\lambda [EI_y(u'')^2 + EC_w(\beta'')^2 + GJ(\beta')^2$$

$$+ 2M\beta u'' - 2K_oM(\beta')^2 + w\bar{a}\beta^2]dz \dots\dots\dots\dots$$

in which E = Young's modulus of elasticity; G = shear form torsion constant; C_w = warping torsion constant; I_y = about the y axis, each prime denoting one derivative wit K_o = section constant given by the following expression

2483

Typical Curtain Wall Mullion

hear center has coordinates $(0, y_o)$. The horizontal
:enter is denoted as u and the rotation of the cross
1 load w, positive in the positive y direction, is
, the glazing a distance, $\bar{a}$, below the shear center,
nullions span two equal floor heights, λ, and are

re of Mullion Cross Section and Axis System

a) TYPICAL MULLION AND LOADING

b) BENDING MOMENT DIAGRAM

FIG. 3. Beam Analysis Model for Curtain Wall Mullion

considered as simply supported at the ends. Lateral wind loads are assumed to be static, uniformly distributed loads, as in Fig. 3(a). The corresponding bending moment about a horizontal axis is denoted as M. The bending moment diagram is shown in Fig. 3(b). Note that the wind pressure may be positive or negative on the glazing plane.

The current state of knowledge of lateral buckling of beams is described in Galambos (1988); this extensive presentation contains a very complete list of references. Texts by Timoshenko and Gere (1961), Bleich (1952), and Galambos (1968) give derivations of basic relationships and many elastic solutions, mostly of rectangular or I-shaped cross sections. Anderson and Trahair (1972) give the first extensive presentation of data on the theoretical elastic lateral buckling strength of simply supported and cantilever monosymmetrical beams subjected to uniform and concentrated loads. An experimental study on cantilevers verifies the validity of the theory.
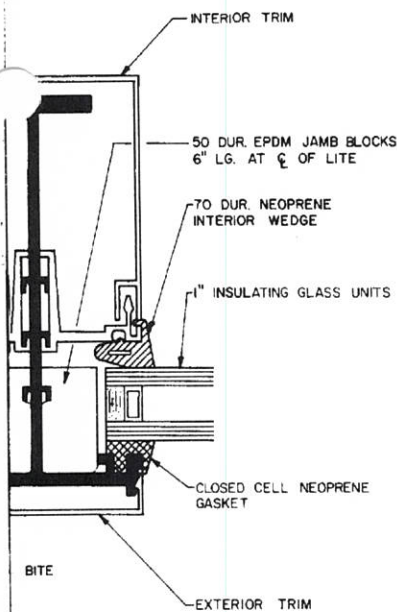
CASE I. MINIMUM BRACING RESTRAINT

Assume first that the mullion of Fig. 2 is not restrained at all by the glass, but is freestanding. Each span of the two span beam will have a buckling mode as for a simply supported beam subjected to the loading and corresponding moment diagram of Fig. 3. A Rayleigh-Ritz approximate procedure is used.

The general potential energy expression for lateral buckling, derived by Masur and Milbradt (1957) is as follows:

$$V = \frac{1}{2} \int_o^\lambda [EI_y(u'')^2 + EC_w(\beta'')^2 + GJ(\beta')^2$$

$$+ 2M\beta u'' - 2K_o M(\beta')^2 + w\bar{a}\beta^2]dz \dots\dots\dots\dots\dots (1a)$$

in which E = Young's modulus of elasticity; G = shear modulus; J = uniform torsion constant; C_w = warping torsion constant; I_y = moment of inertia about the y axis, each prime denoting one derivative with respect to z; and K_o = section constant given by the following expression:

$$K_o = y_o - \frac{1}{2I_x} \int_A y(x^2 + y^2)dA \dots \dots \dots \dots \dots \dots \dots (1b)$$

in which A = area of the cross section.

For this case of a beam subjected to no lateral forces and with both ends "simply supported," the lateral curvature in Eq. 1a can be eliminated by the following relation, $u'' = -\beta M/EI_y$, to give the following potential energy expression

$$V = \frac{1}{2} \int_o^\lambda [EC_w(\beta'')^2 + GJ(\beta')^2 - \frac{M^2}{EI_y}\beta^2 - 2K_oM(\beta')^2 + w\bar{a}\beta^2]dz \dots \dots (2)$$

Assume a one-term approximation to the buckling shape

$$\beta = B \sin \frac{\pi z}{\lambda} \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (3)$$

in which B = magnitude coordinate. Minimizing the potential energy with respect to a variation in B, $\partial V/\partial B = 0$, gives Eq. 4.

$$(w\lambda^2)_{cr} = \frac{166}{\lambda} \sqrt{EI_yGJ} \left[\pm \sqrt{\delta_1^2 + 0.1191\left(1 + \frac{\pi^2}{\lambda^2}\frac{EC_w}{GJ}\right)} + \delta_1 \right] \dots \dots (4)$$

in which

$$\delta_1 = \left(\frac{\bar{a}}{\lambda} + 0.08877 \frac{K_o}{\lambda}\right) \sqrt{\frac{EI_y}{GJ}} \dots \dots \dots \dots \dots \dots \dots \dots (5)$$

The plus sign in front of the second radical in Eq. 4 indicates that the direction of loading coincides with the positive direction of the y axis; the minus sign indicates the opposite direction of loading. Note that both K_o and $\bar{a}$ may be negative; hence δ_1 may be negative.

Eq. 4 gives an upper bound. To check the accuracy of Eq. 4, the solution for a symmetric section, $K_o = 0$, with loading applied at the shear center, $\bar{a} = 0$, and with a wide range of properties was found by finite differences with a large number of divisions so that the answers are accurate. For this case Eq. 4 reduces to the following:

$$(w\lambda^2)_{cr} = 18.2 \frac{\pi}{\lambda} \sqrt{EIGJ} \sqrt{1 + \frac{\pi^2}{\lambda^2}\frac{EC_w}{GJ}}$$

It was found that the numerical coefficient 18.2 is not constant but varies slightly with the parameter $GJ\lambda^2/EC_w$. The error varies from about 0.5 percent for very small values of the parameter to 2.5 percent for $(GJ\lambda^2/EC_w)$ = 1,000, a large value. Hence, it appears that Eq. 4 is sufficiently accurate for design purposes. In general, as noted by Galambos (1968) on p. 106, the Rayleigh-Ritz procedure applied to lateral buckling problems gives good results with relatively simple calculations, provided the assumed trial mode is well chosen.

CASE II. MIDSTORY BRACING

One may wish to consider the restraint afforded by intermediate wall elements in addition to the supports at the floor levels. Thus, assume the mul-

lion is supported at midstory so that it cannot deflec glazing restraints, it will then buckle into two half-w may obtain an approximate value for this problem by the mullion from outer support to midspan as "simply torsional buckling. This would be conservative. Thu: expression is given by Eq. 2, except the limits of int $\lambda/2$. Assume a trial buckled shape

$$\beta = B \sin \frac{2\pi z}{\lambda}$$

and minimize the potential energy. One obtains

$$(w\lambda^2)_{cr} = \frac{561}{\lambda} \sqrt{EI_yGJ} \left[\pm \sqrt{\delta_2^2 + 0.0352\left(1 + \frac{4\pi^2}{\lambda^2}\frac{EC}{GJ}\right)} \right]$$

in which

$$\delta_2 = \left(\frac{\bar{a}}{\lambda} - 0.903 \frac{K_o}{\lambda}\right) \sqrt{\frac{EI_y}{GJ}} \dots \dots \dots \dots \dots$$

CASE III. ELASTIC RESTRAINT

Next consider that the mullion is elastically restrair deflection and rotation at the glazing plane that is a shear center. Although a is usually equal to $\bar{a}$, there a load in which a is not equal to $\bar{a}$. Also, it may be note where the load is applied outside the flanges of the mu plane, two restraining forces act: (1) The restraining t $= K_\theta\beta$; and (2) the restraining lateral force resisting lat point of attachment of the glazing $= K_s\bar{u}$, in which $\bar{u} =$ of the point of attachment to the glazing. Hence, $\bar{u} =$

The potential energy expression is given by Eq. 8

$$V = \frac{1}{2} \int_o^\lambda [EI_y(u'')^2 + EC_w(\beta'')^2 + GJ(\beta')^2 + K_\theta\beta^2 + K_s\bar{u}^2$$

$$- 2K_oM(\beta')^2 + w\bar{a}\beta^2]dz \dots \dots \dots \dots \dots \dots$$

Eq. 8 is the general potential energy expression of Mas: 1a, supplemented by the strain energy of the rotational a springs $= \int_o^\lambda (K_\theta\beta^2 + K_s\bar{u}^2)dz/2$.

Assume

$$\beta = B \sin \frac{\pi z}{\lambda} \dots \dots \dots \dots \dots \dots \dots \dots$$

and

$$u'' = C\left[\sin \frac{\pi z}{\lambda}\left(\frac{3}{8}\frac{z}{\lambda} - \frac{1}{2}\left(\frac{z}{\lambda}\right)^2\right) \right] \dots \dots \dots \dots$$

and minimize with respect to B and C. The approxima

$+ y^2)dA$ .... (1b)

.he cross section.

beam subjected to no lateral forces and with both ends
the lateral curvature in Eq. 1a can be eliminated by the
$u'' = -\beta M/EI_y$, to give the following potential energy

$+ GJ(\beta')^2 - \dfrac{M^2}{EI_y}\beta^2 - 2K_oM(\beta')^2 + w\bar{a}\beta^2]dz$ .... (2)

m approximation to the buckling shape

.... (3)

litude coordinate. Minimizing the potential energy with
h in B, $\partial V/\partial B = 0$, gives Eq. 4.

$GJ\left[\pm\sqrt{\delta_1^2 + 0.1191\left(1 + \dfrac{\pi^2}{\lambda^2}\dfrac{EC_w}{GJ}\right)} + \delta_1 \right]$ .... (4)

$\dfrac{K_o}{\lambda}\right)\sqrt{\dfrac{EI_y}{GJ}}$ .... (5)

front of the second radical in Eq. 4 indicates that the
coincides with the positive direction of the y axis; the
the opposite direction of loading. Note that both K_o and
hence δ_1 may be negative.

er bound. To check the accuracy of Eq. 4, the solution
tion, $K_o = 0$, with loading applied at the shear center,
wide range of properties was found by finite differences
divisions so that the answers are accurate. For this
the following:

$\overline{EIGJ}\sqrt{1 + \dfrac{\pi^2}{\lambda^2}\dfrac{EC_w}{GJ}}$

le numerical coefficient 18.2 is not constant but varies
ameter $GJ\lambda^2/EC_w$. The error varies from about 0.5 per-
values of the parameter to 2.5 percent for $(GJ\lambda^2/EC_w)$
ue. Hence, it appears that Eq. 4 is sufficiently accurate
In general, as noted by Galambos (1968) on p. 106,
ocedure applied to lateral buckling problems gives good
y simple calculations, provided the assumed trial mode

BRACING

consider the restraint afforded by intermediate wall ele-
the supports at the floor levels. Thus, assume the mul-

---

lion is supported at midstory so that it cannot deflect laterally. Neglecting
glazing restraints, it will then buckle into two half-waves in the story. One
may obtain an approximate value for this problem by considering in Fig. 3
the mullion from outer support to midspan as "simply supported" for lateral-
torsional buckling. This would be conservative. Thus, the potential energy
expression is given by Eq. 2, except the limits of integration are from 0 to
$\lambda/2$. Assume a trial buckled shape

$$\beta = B \sin\frac{2\pi z}{\lambda}$$

and minimize the potential energy. One obtains

$$(w\lambda^2)_{cr} = \frac{561}{\lambda}\sqrt{EI_yGJ}\left[\pm\sqrt{\delta_2^2 + 0.0352\left(1 + \frac{4\pi^2}{\lambda^2}\frac{EC_w}{GJ}\right)} + \delta_2 \right] \dots\dots (6)$$

in which

$$\delta_2 = \left(\frac{\bar{a}}{\lambda} - 0.903\frac{K_o}{\lambda}\right)\sqrt{\frac{EI_y}{GJ}} \dots\dots\dots\dots\dots\dots\dots\dots (7)$$

CASE III. ELASTIC RESTRAINT

Next consider that the mullion is elastically restrained from both lateral
deflection and rotation at the glazing plane that is a distance a below the
shear center. Although a is usually equal to $\bar{a}$, there are ways to apply the
load in which a is not equal to $\bar{a}$. Also, it may be noted that there are cases
where the load is applied outside the flanges of the mullion. At the glazing
plane, two restraining forces act: (1) The restraining torque per unit length
$= K_\theta\beta$; and (2) the restraining lateral force resisting lateral deflection of the
point of attachment of the glazing $= K_s\bar{u}$, in which $\bar{u}$ = lateral displacement
of the point of attachment to the glazing. Hence, $\bar{u} = u - a\beta$.

The potential energy expression is given by Eq. 8

$$V = \frac{1}{2}\int_o^\lambda [EI_y(u'')^2 + EC_w(\beta'')^2 + GJ(\beta')^2 + K_\theta\beta^2 + K_s\bar{u}^2 + 2M\beta u''$$

$$- 2K_oM(\beta')^2 + w\bar{a}\beta^2]dz \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (8)$$

Eq. 8 is the general potential energy expression of Masur and Milbradt, Eq.
1a, supplemented by the strain energy of the rotational and lateral restraining
springs $= \int_o^\lambda (K_\theta\beta^2 + K_s\bar{u}^2)dz/2$.

Assume

$$\beta = B \sin\frac{\pi z}{\lambda} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (9a)$$

and

$$u'' = C\left[\sin\frac{\pi z}{\lambda}\left(\frac{3}{8}\frac{z}{\lambda} - \frac{1}{2}\left(\frac{z}{\lambda}\right)^2\right)\right] \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (9b)$$

and minimize with respect to B and C. The approximations of Eq. 9 were

chosen so that when K_s and K_θ vanish, the solution will agree with that of Case I, Eq. 4. It isn't possible to arrange the answer in a simple explicit form. One obtains the following quadratic equation in $w\lambda^2$

$$0 = 0.003016 \frac{(w\lambda^2)^2}{EI_y} - w\lambda^2 \left[\left(\frac{\bar{a}}{\lambda^2} + 0.08877 \frac{K_o}{\lambda^2} \right) C_1 - \frac{1}{106.9} \frac{K_s\lambda^4}{EI_y} \cdot \frac{a}{\lambda^2} \right]$$

$$- C_1 \left(EC_w \frac{\pi^4}{\lambda^4} + GJ \frac{\pi^2}{\lambda^2} + K_\theta + K_s a^2 \right) + \frac{1}{137.8} \frac{K_s^2\lambda^4 a^2}{EI_y} \quad \ldots \ldots \ldots \ldots \quad (10)$$

in which

$$C_1 = 1 + \frac{1}{137.8} \frac{K_s\lambda^4}{EI_y} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (11)$$

For the case where the glazing is attached at the shear center, $a = 0$ (often about correct in practical cases), Eq. 10 gives the following solution

$$(w\lambda^2)_{cr} = \frac{166}{\lambda} \sqrt{EI_y GJ} \left[\pm \sqrt{(C_1\delta_1)^2 + 0.119 C_1 \left(1 + \frac{\pi^2 EC_w}{\lambda^2 GJ} + \frac{K_\theta\lambda^2}{\pi^2 GJ} \right)} \right.$$

$$\left. + C_1\delta_1 \right] \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (12)$$

Note δ_1 is given by Eq. 5.

The very simple trial deflection functions assumed in Eq. 9 are most appropriate for relatively weak springs, K_s, for which Eq. 10 should give good results. However, the critical load may be off considerably for strong lateral restraining springs.

There is also the possibility that the mullion will buckle into several waves at a lower load. To investigate this possibility the mullion is assumed to be subjected to a uniform moment given by Eq. 13

$$M = \frac{9}{128} w\lambda^2 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (13)$$

The following mode shape is assumed with n half-waves in each span

$$\beta = B \sin \frac{n\pi z}{\lambda} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (14a)$$

and

$$u = C \sin \frac{n\pi z}{\lambda} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (14b)$$

The following critical load is obtained:

$$(w\lambda^2)_{cr} = \frac{128}{9} \frac{n\pi}{\lambda} \sqrt{EI_y GJ} \left\{ \pm \left[n^2\pi^2\delta_3^2 + C_2 \left(1 + \frac{n^2\pi^2}{\lambda^2} \frac{EC_w}{GJ} + \frac{K_\theta\lambda^2}{n^2\pi^2 GJ} \right. \right. \right.$$

$$\left. \left. \left. + \frac{K_s a^2\lambda^2}{n^2\pi^2 GJ} \right) - \left(\frac{K_s\lambda^4}{n^4\pi^4 EI_y} \right) \left(\frac{K_s a^2\lambda^2}{n^2\pi^2 GJ} \right) \right]^{0.50} - n\pi\delta_3 \right\} \quad \ldots \ldots \ldots \ldots \quad (15a)$$

2486

in which

$$C_2 = 1 + \frac{K_s\lambda^4}{n^4\pi^4 EI_y} \quad \ldots \ldots \ldots \ldots \ldots \ldots$$

and

$$\delta_3 = \left[\frac{K_o}{\lambda} C_2 + \left(\frac{K_s\lambda^4}{n^4\pi^4 EI_y} \right) \frac{a}{\lambda} \right] \sqrt{\frac{EI_y}{GJ}} \quad \ldots \ldots \ldots$$

This simplifies considerably for the case where the gl center, $a = 0$.

Case IV. Maximum Bracing Restraint

Finally, consider that the mullion is completely restra flection at the glazing plane and is elastically restrained $\to \infty$ in Eq. 10, the following equation for the critical

$$\omega\lambda^2(1.289a - 0.08877K_o - \bar{a})$$

$$= \pi^2 GJ \left(1 + \pi^2 \frac{EC_w}{\lambda^2 GJ} + \frac{K_\theta\lambda^2}{\pi^2 GJ} + \frac{137.8}{\pi^2} \frac{EI_y a^2}{\lambda^2 GJ} \right) \quad \ldots \ldots$$

However, this cannot be expected to be very accurate b previously, the trail deflection function for u is not appr Therefore, a separate solution was obtained as describe

The potential energy expression for this case is given 0. Obviously

$$u = a\beta \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

Using a one-term approximation, $\beta = B \sin \pi z/\lambda$, before, one obtains Eq. 18

$$w\lambda^2(0.9112a - \bar{a} - 0.08877K_o)$$

$$= \pi^2 GJ \left(1 + \frac{\pi^2 EI_y a^2}{GJ\lambda^2} + \frac{\pi^2 EC_w}{GJ\lambda^2} + \frac{K_\theta\lambda^2}{\pi^2 GJ} \right) \quad \ldots \ldots \ldots$$

Note that this is of the same form as Eq. 16.

If K_θ were of significant magnitude there is the possibil would buckle into several waves at the lowest critical will be shown, the buckling strength for practical mullio for $K_\theta = 0$), there is no need to consider this possibility

Roeder and Assadi (1982) give the exact solution for a wide flange beam ($K_o = 0$) with the lower flange prev displacement ($a = d/2$) by a thin membrane ($K_\theta = 0$) an form moment. They give

$$M_{cr} = \left[\frac{\pi^2}{d\lambda^2} \left(\frac{d^2}{4} EI_y + EC_w \right) + \frac{GJ}{d} \right] \quad \ldots \ldots \ldots \ldots$$

Eq. 18 gives for this beam with $\bar{a} = 0$

2487

and K_θ vanish, the solution will agree with that of
possible to arrange the answer in a simple explicit
lowing quadratic equation in $w\lambda^2$

$$w\lambda^2\left[\left(\frac{\bar{a}}{\lambda^2} + 0.08877\frac{K_o}{\lambda^2}\right)C_1 - \frac{1}{106.9}\frac{K_s\lambda^4}{EI_y}\cdot\frac{a}{\lambda^2}\right]$$
$$+ K_\theta + K_s a^2\Bigg) + \frac{1}{137.8}\frac{K_s^2\lambda^4 a^2}{EI_y} \quad\dots\dots\dots\dots \text{(10)}$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(11)}$$

glazing is attached at the shear center, $a = 0$ (often
al cases), Eq. 10 gives the following solution

$$\left[\pm\sqrt{(C_1\delta_1)^2 + 0.119C_1\left(1 + \frac{\pi^2 EC_w}{\lambda^2 GJ} + \frac{K_\theta\lambda^2}{\pi^2 GJ}\right)}\right.$$
$$\dots\dots\dots\dots\dots\dots \text{(12)}$$

5.
l deflection functions assumed in Eq. 9 are most ap-
weak springs, K_s, for which Eq. 10 should give good
ritical load may be off considerably for strong lateral

sibility that the mullion will buckle into several waves
restigate this possibility the mullion is assumed to be
moment given by Eq. 13

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(13)}$$

nape is assumed with n half-waves in each span

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(14a)}$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(14b)}$$

load is obtained:

$$\overline{GJ}\left\{\pm\left[n^2\pi^2\delta_3^2 + C_2\left(1 + \frac{n^2\pi^2}{\lambda^2}\frac{EC_w}{GJ} + \frac{K_\theta\lambda^2}{n^2\pi^2 GJ}\right)\right.\right.$$
$$\left.\left.\left(\frac{K_s a^2\lambda^2}{n^2\pi^2 GJ}\right)\right]^{0.50} - n\pi\delta_3\right\} \dots\dots\dots \text{(15a)}$$

2486

in which

$$C_2 = 1 + \frac{K_s\lambda^4}{n^4\pi^4 EI_y} \quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(15b)}$$

and

$$\delta_3 = \left[\frac{K_o}{\lambda}C_2 + \left(\frac{K_s\lambda^4}{n^4\pi^4 EI_y}\right)\frac{a}{\lambda}\right]\sqrt{\frac{EI_y}{GJ}} \quad\dots\dots \text{(15c)}$$

This simplifies considerably for the case where the glazing is at the shear
center, $a = 0$.

CASE IV. MAXIMUM BRACING RESTRAINT

Finally, consider that the mullion is completely restrained from lateral de-
flection at the glazing plane and is elastically restrained from rotation. If $K_s \to \infty$ in Eq. 10, the following equation for the critical load is obtained

$$w\lambda^2(1.289a - 0.08877K_o - \bar{a})$$
$$= \pi^2 GJ\left(1 + \pi^2\frac{EC_w}{\lambda^2 GJ} + \frac{K_\theta\lambda^2}{\pi^2 GJ} + \frac{137.8}{\pi^2}\frac{EI_y a^2}{\lambda^2 GJ}\right) \quad\dots\dots \text{(16)}$$

However, this cannot be expected to be very accurate because, as explained
previously, the trail deflection function for u is not appropriate for this case.
Therefore, a separate solution was obtained as described below.

The potential energy expression for this case is given by Eq. 8 with $\bar{u} = 0$. Obviously

$$u = a\beta \quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(17)}$$

Using a one-term approximation, $\beta = B\sin\pi z/\lambda$, and minimizing, as
before, one obtains Eq. 18

$$w\lambda^2(0.9112a - \bar{a} - 0.08877K_o)$$
$$= \pi^2 GJ\left(1 + \frac{\pi^2 EI_y a^2}{GJ\lambda^2} + \frac{\pi^2 EC_w}{GJ\lambda^2} + \frac{K_\theta\lambda^2}{\pi^2 GJ}\right) \quad\dots\dots\dots \text{(18)}$$

Note that this is of the same form as Eq. 16.

If K_θ were of significant magnitude there is the possibility that the mullion
would buckle into several waves at the lowest critical load. However, as
will be shown, the buckling strength for practical mullions is so high (even
for $K_\theta = 0$), there is no need to consider this possibility further.

Roeder and Assadi (1982) give the exact solution for a doubly symmetric
wide flange beam ($K_o = 0$) with the lower flange prevented from lateral
displacement ($a = d/2$) by a thin membrane ($K_\theta = 0$) and subjected to uni-
form moment. They give

$$M_{cr} = \left[\frac{\pi^2}{d\lambda^2}\left(\frac{d^2}{4}EI_y + EC_w\right) + \frac{GJ}{d}\right] \quad\dots\dots\dots\dots\dots \text{(19)}$$

Eq. 18 gives for this beam with $\bar{a} = 0$

2487

$$(w\lambda^2)_{cr} = \frac{2\pi^2}{0.9112}\left[\frac{\pi^2}{d\lambda^2}\left(\frac{d^2}{4}EI_y + EC_w\right) + \frac{GJ}{d}\right] \quad \ldots\ldots\ldots\ldots\ldots \quad (20)$$

The corresponding maximum positive moment is

$$(M+)_{cr} = \frac{9}{128}(w\lambda^2)_{cr} = 1.52\left[\frac{\pi^2}{d\lambda^2}\left(\frac{d^2}{4}EI_y + EC_w\right) + \frac{GJ}{d}\right] \quad \ldots\ldots\ldots \quad (21)$$

Thus, the form of this solution is the same as the exact solution of Roeder and Assadi for uniform moment and the coefficient is substantially larger, as would be expected.

Design Comparisons

The monosymmetric beam of Fig. 4 represents a simplified mullion that spans two stories with a story height, $\lambda = 150$ in. (381 cm). The glazing is roughly at the position of the shear center and hence it is assumed that the wind load is applied at the shear center. The mullion is assumed to be an extrusion composed of 6063-T6 aluminum.

Aluminum Association Specifications (1986)

The beam is fully supported against lateral buckling over the center support. Therefore, by Table 3.3.29 the allowable stress at this location is assumed to be 15 ksi (103.4 MPa) in either tension or compression. The limitations on compressive stress due to lateral buckling are assumed to apply only to the interior portions of the mullion, for which the maximum moment is $0.0703w\lambda^2$. For this example only downward forces will be considered.

Table 3.3.29 gives the following formula for allowable compressive stress in beams, in ksi, for $L_b/r_y^* \geq 94$
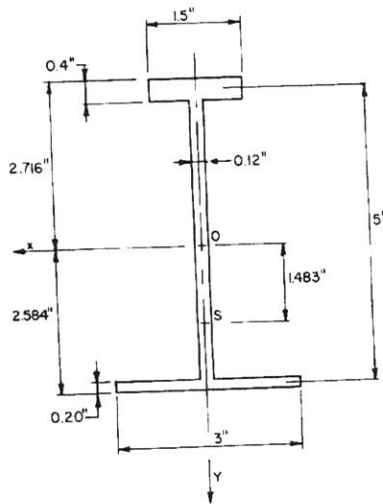

FIG. 4. Design Example of Mullion Cross Section Dimensions

$$F_b = \frac{87,000}{\left(\dfrac{L_b}{r_y^*}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots$$

in which L_b = unsupported length of compression flang of gyration about the y axis calculated as though both flan as the compressive flange. For the given mullion L_b = cm); $r_y^* = 0.360$ in. (0.914 cm); and $(L_b/r_y^*) = 416$. Fo beam, Eq. 22a gives

$$F_b = 0.502 \text{ ksi (3.46 MPa)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

The Specifications for Aluminum Structures also give formula for allowable compression stress in Section 4.9, 22 is replaced by an effective r_y^*, as follows for load app (tension) flange

$$\text{effective} \quad r_y^* = r_{ye}^* = \frac{k_b}{1.7}\sqrt{\frac{I_y^* d}{S^*}}\left[0.50 + \sqrt{1.25 + 0.15}\right]$$

in which k_b = coefficient = 1 for uniform transverse load I_y^* = moment of inertia about axis parallel to web; S^* = d = depth of section; and J^* = uniform torsion constan though both flanges were the same as the compression f 0.226 in.^{4} (9.41 cm^4); $S^* = 3.07$ in.^{3} (50.3 cm^3); $d = 5.$ and $J^* = 0.0666$ in.^{4} (2.77 cm^4). Then, by Eq. 23, r_{ye}^* cm); and by Eq. 22, $F_b = 3.45$ ksi (23.8 MPa).

Thus, the allowable compressive stress by this provision than that given by the provision of Section 3.4.11.

Case I. No Bracing Restraint

The critical value of $w\lambda^2$ is given by Eq. 4, in which fc of Fig. 4, $I_y = 0.563$ in.^{4} (23.4 cm^4); $G = 0.385E$; $E = 1$ MPa); $J = 0.0429$ in.^{4} (1.79 cm^4); $C_w = 2.250$ in.^{6} (60 1.450 in. (3.68 cm). One obtains $(w\lambda^2)_{cr} = 384$ in.-kip the corresponding maximum positive moment $= (M+)$ $= 27.0$ in.-kip (3.05 kN·m). The corresponding compress buckling is $f_c = M/S = 8.58$ ksi (59.2 MPa) in which S cm^3). If one applies a factor of safety of 1.65 as is used Association formulas for buildings, one obtains the fo compressive stress for the interior region

$$F_b = 5.20 \text{ ksi (35.8 MPa)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

This allowable compressive stress is 10 times larger than Eq. 22 and 1.5 times larger than the value given using Eq. 23. The Case I analysis neglects all restraint offered additional strength results partly from an accurate evaluation moment variation in Eq. 4 and partly from the approxim Aluminum Association specification procedure when app

$$\left[\frac{\pi^2}{} \left(\frac{d^2}{4} EI_y + EC_w \right) + \frac{GJ}{d} \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

ng maximum positive moment is

$$\lambda^2)_{cr} = 1.52 \left[\frac{\pi^2}{d\lambda^2} \left(\frac{d^2}{4} EI_y + EC_w \right) + \frac{GJ}{d} \right] \quad \ldots\ldots\ldots\ldots \quad (21)$$
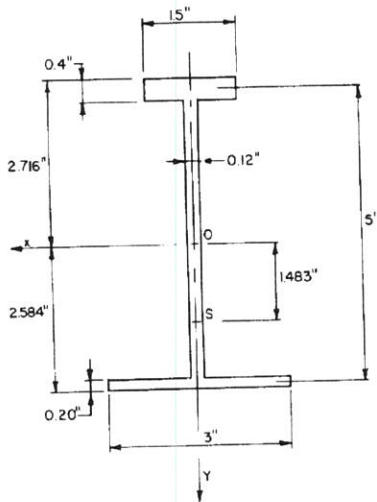
of this solution is the same as the exact solution of Roeder uniform moment and the coefficient is substantially larger. ected.

ISONS

metric beam of Fig. 4 represents a simplified mullion that s with a story height, $\lambda = 150$ in. (381 cm). The glazing e position of the shear center and hence it is assumed that s applied at the shear center. The mullion is assumed to be mposed of 6063-T6 aluminum.

ciation Specifications (1986)

fully supported against lateral buckling over the center sup- by Table 3.3.29 the allowable stress at this location is as- ksi (103.4 MPa) in either tension or compression. The lim- pressive stress due to lateral buckling are assumed to apply ior portions of the mullion, for which the maximum moment for this example only downward forces will be considered. gives the following formula for allowable compressive stress ii, for $L_b/r_y^* \geq 94$

Design Example of Mullion Cross Section Dimensions

$$F_b = \frac{87{,}000}{\left(\dfrac{L_b}{r_y^*} \right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (22a)$$

in which L_b = unsupported length of compression flange and r_y^* = radius of gyration about the y axis calculated as though both flanges were the same as the compressive flange. For the given mullion $L_b = \lambda = 150$ in. (381 cm); $r_y^* = 0.360$ in. (0.914 cm); and $(L_b/r_y^*) = 416$. For this very slender beam, Eq. 22a gives

$$F_b = 0.502 \text{ ksi (3.46 MPa)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (22b)$$

The Specifications for Aluminum Structures also give a more accurate formula for allowable compression stress in Section 4.9, in which r_y^* in Eq. 22 is replaced by an effective r_y^*, as follows for load applied to the bottom (tension) flange

$$\text{effective} \quad r_y^* = r_{ye}^* = \frac{k_b}{1.7} \sqrt{ \frac{I_y^* d}{S^*} \left[0.50 + \sqrt{ 1.25 + 0.152 \frac{J^*}{I_y^*} \left(\frac{L_b}{d} \right)^2 } \right] } \quad (23)$$

in which k_b = coefficient = 1 for uniform transverse load as in this example; I_y^* = moment of inertia about axis parallel to web; S^* = section modulus; d = depth of section; and J^* = uniform torsion constant, all computed as though both flanges were the same as the compression flange. Thus, $I_y^* = 0.226$ in.^{4} (9.41 cm^4); $S^* = 3.07$ in.^{3} (50.3 cm^3); $d = 5.30$ in. (13.46 cm); and $J^* = 0.0666$ in.^{4} (2.77 cm^4). Then, by Eq. 23, $r_{ve}^* = 0.944$ in. (2.40 cm); and by Eq. 22, $F_b = 3.45$ ksi (23.8 MPa).

Thus, the allowable compressive stress by this provision is 6.9 times larger than that given by the provision of Section 3.4.11.

Case I. No Bracing Restraint

The critical value of $w\lambda^2$ is given by Eq. 4, in which for the cross section of Fig. 4, $I_v = 0.563$ in.^{4} (23.4 cm^4); $G = 0.385E$; $E = 10{,}000$ ksi (68,950 MPa); $J = 0.0429$ in.^{4} (1.79 cm^4); $C_w = 2.250$ in.^{6} (604 cm^6); and $K_o = 1.450$ in. (3.68 cm). One obtains $(w\lambda^2)_{cr} = 384$ in.-kip (43.4 kN·m), and the corresponding maximum positive moment = $(M+)$ max = $0.0703w\lambda^2$ = 27.0 in.-kip (3.05 kN·m). The corresponding compressive stress at elastic buckling is $f_c = M/S = 8.58$ ksi (59.2 MPa) in which $S = 3.15$ in.^{3} (51.6 cm^3). If one applies a factor of safety of 1.65 as is used for the Alluminum Association formulas for buildings, one obtains the following allowable compressive stress for the interior region

$$F_b = 5.20 \text{ ksi (35.8 MPa)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$

This allowable compressive stress is 10 times larger than the value given by Eq. 22 and 1.5 times larger than the value given using the effective r_{ye}^* of Eq. 23. The Case I analysis neglects all restraint offered by the glazing; the additional strength results partly from an accurate evaluation of the true bending moment variation in Eq. 4 and partly from the approximate nature of the Aluminum Association specification procedure when applied to this case.
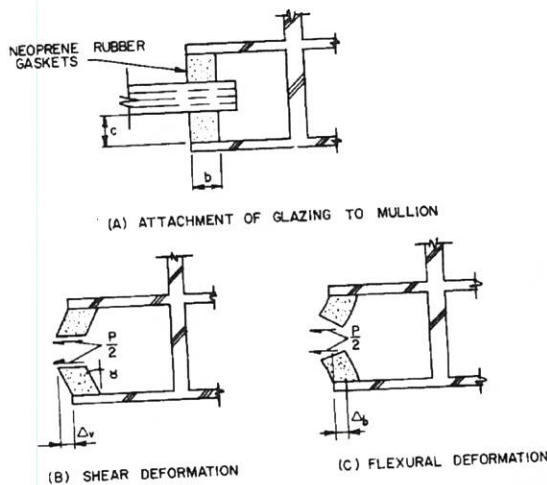
(A) ATTACHMENT OF GLAZING TO MULLION

(B) SHEAR DEFORMATION   (C) FLEXURAL DEFORMATION

FIG. 5. Glazing Pocket Detail for Modeling Elastic Restraint

Case II. Midspan Bracing

If an intermediate element prevents lateral displacement at midstory of the given mullion, the critical load is approximated by Eq. 6, giving $(w\lambda^2)_{cr} = 591$ in.-kip (66.8 kN·m). This gives a maximum positive moment = 41.6 in.-kip (4.70 kN·m), which corresponds to a critical compressive stress of 13.2 ksi (91.0 MPa). Again, assuming a factor of safety of 1.65, one obtains the following allowable stress

$$F_b = 8.00 \text{ ksi (55.2 MPa)} \dots \dots \tag{25}$$

Thus the intermediate supporting element increased the strength over the freestanding mullion, Case I, by a factor of 1.54.

Case III. Elastic Restraint

Small restraints can greatly increase the lateral-torsional buckling strength of the aluminum mullions. A reliable estimate of the restraint constants K_θ and K_s provided by the glazing is not available. As a very rough estimate of the minimum elastic restraint, assume that $K_\theta = 0$ and that K_s is provided by the elasticity of the soft Neoprene seals only as it seems reasonable to assume that the glass does not move horizontally in its plane.

The attachment of the glass to one side of the mullion is shown in Fig. 5(a). Although the glass is assumed fixed and the mullion moves, it is easier to visualize the mullion as fixed and the glass as moving, as in Figs. 5(b) and 5(c). The very complex stress-displacement state in the seals is roughly approximated herein as composed of pure shear and pure flexure. The total displacement of the glass Δ is the sum of the displacements due to shear, Δ_v, and flexure, Δ_b. The force per unit length of seal of each glass pane is denoted as P. Each seal resists one-half of this force.

First consider the displacement due to shear. Assume the load $P/2$ causes uniform shearing stress in the seal. Then

shearing stress $= \tau = \dfrac{P}{2b}$ ......

in which b is the width of the seal, as in Fig. 5(a). The corresponding strain is

shearing strain $= \gamma = \dfrac{\tau}{G} = \dfrac{3}{2}\dfrac{P}{bE}$ ......

as it is assumed for the seal material that $G = E/3$. The displacement due to shear is then

$$\Delta_v = \gamma c = \frac{3}{2}\frac{P}{E}\frac{c}{b} \dots \dots$$

in which c = height of the seal, Fig. 5(a).

As a conservative assumption assume simple flexure of a cantilever beam. Then the beam displacement is

$$\Delta_b = \frac{\left(\dfrac{P}{2}\right)c^3}{3EI_s} = \frac{\left(\dfrac{P}{2}\right)c^3}{3E\left(\dfrac{b^3}{12}\right)} = \frac{2Pc^3}{Eb^3} \dots \dots$$

in which I_s = moment of inertia of seal per unit width.
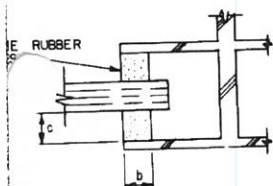The total displacement Δ is

$$\Delta = \Delta_v + \Delta_b = \frac{P}{E}\left(\frac{3}{2}\frac{c}{b} + 2\frac{c^3}{b^3}\right) \dots \dots$$

Assuming that a glass pane is restraining the mullion on total spring stiffness K_s is

$$K_s = \frac{2P}{\Delta} = \frac{2E}{\left(\dfrac{3}{2}\dfrac{c}{b} + 2\dfrac{c^3}{b^3}\right)} \dots \dots$$

Assume as typical dimensions $b = c = 0.250$ in. (0.635 Neoprene rubber under short time loading, $E = 2,000$ psi (1 $K_s = 1,143$ lb/in./in. (788 N/cm/cm). Obviously, this is mate but it should be low since consistently conservative a been made. In the following calculation $K_s = 1,000$ lb/in./ cm) has been assumed.

Assume that the central plane of the glazing is at the she $a = \bar{a} = 0$. For a story height $\lambda = 150$ in. (381 cm); as wave solution of Case III gives the lowest critical load as $= 965$ in.-kip (109 kN·m). This is 2.51 times larger than of Case I. The elastic critical compressive stress is 21.6 ksi compressive yield strength of 6063-T6 aluminum is 25 Therefore, this case should fail inelastically at about 17.7

(A) ATTACHMENT OF GLAZING TO MULLION
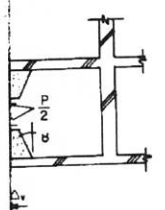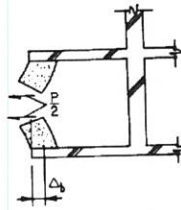
SHEAR DEFORMATION   (C) FLEXURAL DEFORMATION

Glazing Pocket Detail for Modeling Elastic Restraint

an Bracing

ate element prevents lateral displacement at midstory of the
e critical load is approximated by Eq. 6, giving $(w\lambda^2)_{cr} =$
kN·m). This gives a maximum positive moment = 41.6
m), which corresponds to a critical compressive stress of
Pa). Again, assuming a factor of safety of 1.65, one obtains
owable stress

$$2 \text{ MPa}) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (25)$$

diate supporting element increased the strength over the
ion, Case 1, by a factor of 1.54.

c Restraint

s can greatly increase the lateral-torsional buckling strength
mullions. A reliable estimate of the restraint constants K_θ
by the glazing is not available. As a very rough estimate
elastic restraint, assume that $K_\theta = 0$ and that K_s is provided
of the soft Neoprene seals only as it seems reasonable to
glass does not move horizontally in its plane.
t of the glass to one side of the mullion is shown in Fig.
ne glass is assumed fixed and the mullion moves, it is easier
mullion as fixed and the glass as moving, as in Figs. 5(b)
ry complex stress-displacement state in the seals is roughly
rein as composed of pure shear and pure flexure. The total
the glass Δ is the sum of the displacements due to shear,
Δ_b. The force per unit length of seal of each glass pane is
ach seal resists one-half of this force.
the displacement due to shear. Assume the load $P/2$ causes
g stress in the seal. Then

2490

shearing stress $= \tau = \dfrac{P}{2b} \dots\dots\dots\dots\dots\dots\dots\dots\dots (26)$

in which b is the width of the seal, as in Fig. 5(a). The corresponding shearing strain is

shearing strain $= \gamma = \dfrac{\tau}{G} = \dfrac{3}{2}\dfrac{P}{bE} \dots\dots\dots\dots\dots\dots (27)$

as it is assumed for the seal material that $G = E/3$. The displacement due to shear is then

$$\Delta_\tau = \gamma c = \dfrac{3}{2}\dfrac{P}{E}\dfrac{c}{b} \dots\dots\dots\dots\dots\dots\dots\dots (28)$$

in which $c =$ height of the seal, Fig. 5(a).

As a conservative assumption assume simple flexure of each seal as a cantilever beam. Then the beam displacement is

$$\Delta_b = \dfrac{\left(\dfrac{P}{2}\right)c^3}{3EI_s} = \dfrac{\left(\dfrac{P}{2}\right)c^3}{3E\left(\dfrac{b^3}{12}\right)} = \dfrac{2Pc^3}{Eb^3} \dots\dots\dots\dots (29)$$

in which $I_s =$ moment of inertia of seal per unit width.

The total displacement Δ is

$$\Delta = \Delta_\tau + \Delta_b = \dfrac{P}{E}\left(\dfrac{3}{2}\dfrac{c}{b} + 2\dfrac{c^3}{b^3}\right) \dots\dots\dots\dots (30)$$

Assuming that a glass pane is restraining the mullion on each side, the total spring stiffness K_s is

$$K_s = \dfrac{2P}{\Delta} = \dfrac{2E}{\left(\dfrac{3}{2}\dfrac{c}{b} + 2\dfrac{c^3}{b^3}\right)} \dots\dots\dots\dots (31)$$

Assume as typical dimensions $b = c = 0.250$ in. (0.635 cm) and for soft Neoprene rubber under short time loading, $E = 2,000$ psi (13.8 MPa). Then, $K_s = 1,143$ lb/in./in. (788 N/cm/cm). Obviously, this is a ballpark estimate but it should be low since consistently conservative assumptions have been made. In the following calculation $K_s = 1,000$ lb/in./in. (690 N/cm/cm) has been assumed.

Assume that the central plane of the glazing is at the shear center; hence, $a = \bar{a} = 0$. For a story height $\lambda = 150$ in. (381 cm); as before, the two-wave solution of Case III gives the lowest critical load as follows: $(w\lambda^2)_{cr} = 965$ in.-kip (109 kN·m). This is 2.51 times larger than the critical load of Case I. The elastic critical compressive stress is 21.6 ksi (149 MPa). The compressive yield strength of 6063-T6 aluminum is 25 ksi (172 MPa). Therefore, this case should fail inelastically at about 17.7 ksi (122 MPa).

2491

Or using a factor of safety of 1.65 gives the following allowable stress

$$F_b = 10.7 \text{ ksi (74 MPa)} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (32)$$

This is about three times the value allowed by the Specifications for Aluminum Structures and over twice as much as the suggested allowable stress for the freestanding beam, Case I.

Note that in this example a minimum glazing restraint is assumed as well as a uniform moment. The true critical load is probably much higher.

Case IV. Maximum Bracing Restraint

This case assumes that the mullion is completely restrained from lateral deflection at the glazing plane and elastically restrained from rotation. For this example assume the glazing is at the shear center and neglect entirely the rotational restraint. Then $a = \bar{a} = 0$ and $K_\theta = 0$. Eq. 18 gives the following result: $(w\lambda^2)_{cr} = -13,420$ in.-kip $(-1,516 \text{ kN} \cdot \text{m})$. Thus, the absolute value of $w\lambda^2$ is 35 times larger than the value of Case I. The negative sign means that the wind force would have to be reversed (suction). This result shows that if the given mullion is prevented from lateral displacements by the glazing located at the shear center there is really no lateral buckling problem and the allowable stresses do not have to be reduced.

Comment

The dramatic difference in results between the regular Aluminum Association allowable compressive stress equation, its more precise effective r, equation, and an allowable compressive stress based on the minimum possible critical load, Case I, is readily discerned. Case I gives an increased capacity of 51% compared to the effective radius equation and an increase of over 900% compared to the regular equation. The contrasts are even greater when comparisons to Cases II, III, and IV are made. In Case III an attempt is made to conservatively estimate the elastic restraint offered by the glazing. This solution gives an increase in the allowable stress over that of Case I of over 100%.

If the elastic restraint afforded by the glazing can be estimated reliably and conservatively, it is certain that higher allowable stresses could be used with great savings of materials. It seems desirable to experimentally evaluate the behavior of curtain wall construction.

EXPERIMENTAL STUDIES

The critical buckling strength of the very slender mullions used in curtain wall construction is very sensitive to the restraint offered by the glazing. As discussed, this restraint is difficult to estimate accurately. However, it is presently the industry standard to carry out full-scale structural mock-up testing on new or major curtain wall designs. This provides an excellent opportunity to collect data that can be used to establish the strength and behavior of the slender mullions.

The Southwell plot was originally proposed as a nondestructive procedure for extrapolating the elastic critical load of concentrically loaded struts from tests in which the load is increased to a fairly high value but not to failure.

The theory is unquestionably correct for the simple s
and application of the Southwell-type procedures f
lems is described by Trahair (1969) and Attard (198
uct wind pressure times measured displacement ve
ment should be a straight line with the slope equal to
The plot may not be a straight line with the corr
pressure reaches a significant percent of the critic.
the stress-strain behavior must be linear. The meas
be the sideways displacement of the compression f
imum displacement. Assuming the mullions have s
if in a test one does not measure any progressivel
placement as one increases the load up to the maxim
would be that the maximum is simply nowhere ne
the several tests witnessed by the senior writer, su
the measured displacements have been negligible. T
curtain walls in these tests depended on the yield s
with no lateral buckling tendencies observed.

SUMMARY AND CONCLUSIONS

Formulas for the elastic lateral buckling strengt
metrical beams that represent curtain wall mullion
and supporting conditions have been derived by a
cedures for four different assumptions of possible el:
(1) No lateral bracing; (2) bracing at midstory only; (
lateral displacement and twisting displacement at tl
maximum bracing that completely prevents lateral d
ing level. The beams span two stories and are subj

A typical design situation is evaluated. The allowa
obtained by applying the two Aluminum Associati
formulas for unbraced beams are shown to be ver
lution based upon no lateral bracing leads to an allov
51% higher than given by AAS. The others are hi
assumption, the third, with very conservative ass
straint leads to an allowable compressive stress abo
value.

Experimental studies performed to verify by test
senior author show no progressive displacement fa
is sufficient bracing restraint by the glazing to prev
ure.

With additional study it should be possible to «
codes that will allow far more economical designs

APPENDIX I. REFERENCES

Anderson, J. M., and Trahair, N. S. (1972). "Stability
and cantilevers." *J. Struct. Div.*, ASCE, 98(1), 269–2
Attard, M. M. (1983). "Extrapolation techniques of bucklin
ASCE, 109(4), 926–935.
Bleich, F. (1952). *Buckling strength of metal structures.*
N.Y.

f safety of 1.65 gives the following allowable stress

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (32)$$

times the value allowed by the Specifications for Alu-
and over twice as much as the suggested allowable stress
beam, Case I.
example a minimum glazing restraint is assumed as well
ent. The true critical load is probably much higher.

um Bracing Restraint

es that the mullion is completely restrained from lateral
lazing plane and elastically restrained from rotation. For
ne the glazing is at the shear center and neglect entirely
raint. Then $a = \bar{a} = 0$ and $K_\theta = 0$. Eq. 18 gives the
$w\lambda^2)_{cr} = -13,420$ in.-kip $(-1,516$ kN $\cdot$ m). Thus, the ab-
^{2} is 35 times larger than the value of Case I. The negative
he wind force would have to be reversed (suction). This
f the given mullion is prevented from lateral displacements
ated at the shear center there is really no lateral buckling
allowable stresses do not have to be reduced.

difference in results between the regular Aluminum Asso-
compressive stress equation, its more precise effective r_t
allowable compressive stress based on the minimum pos-
, Case I, is readily discerned. Case I gives an increased
compared to the effective radius equation and an increase
npared to the regular equation. The contrasts are even greater
as to Cases II, III, and IV are made. In Case III an attempt
-vatively estimate the elastic restraint offered by the glazing.
ves an increase in the allowable stress over that of Case I

aint afforded by the glazing can be estimated reliably
ly, it is certain that higher allowable stresses could be used
s of materials. It seems desirable to experimentally evaluate
curtain wall construction.

STUDIES

uckling strength of the very slender mullions used in curtain
m is very sensitive to the restraint offered by the glazing. As
restraint is difficult to estimate accurately. However, it is
dustry standard to carry out full-scale structural mock-up test-
major curtain wall designs. This provides an excellent op-
lect data that can be used to establish the strength and be-
ender mullions.
ll plot was originally proposed as a nondestructive procedure
g the elastic critical load of concentrically loaded struts from
he load is increased to a fairly high value but not to failure.

The theory is unquestionably correct for the simple strut problem. The theory
and application of the Southwell-type procedures for lateral buckling prob-
lems is described by Trahair (1969) and Attard (1983). The plot of the prod-
uct wind pressure times measured displacement versus measured displace-
ment should be a straight line with the slope equal to the critical wind pressure.
The plot may not be a straight line with the correct slope until the wind
pressure reaches a significant percent of the critical value and, of course,
the stress-strain behavior must be linear. The measured displacement could
be the sideways displacement of the compression flange at a point of max-
imum displacement. Assuming the mullions have some initial crookedness,
if in a test one does not measure any progressively increasing lateral dis-
placement as one increases the load up to the maximum, then the conclusion
would be that the maximum is simply nowhere near the critical value. In
the several tests witnessed by the senior writer, such has been the case, as
the measured displacements have been negligible. The useful strength of the
curtain walls in these tests depended on the yield strength of the mullions,
with no lateral buckling tendencies observed.

SUMMARY AND CONCLUSIONS

Formulas for the elastic lateral buckling strength of slender monosym-
metrical beams that represent curtain wall mullions with practical loading
and supporting conditions have been derived by approximate energy pro-
cedures for four different assumptions of possible elastic restraint. These are:
(1) No lateral bracing; (2) bracing at midstory only; (3) elastic restraint against
lateral displacement and twisting displacement at the glazing level; and (4)
maximum bracing that completely prevents lateral displacements at the glaz-
ing level. The beams span two stories and are subjected to uniform load.

A typical design situation is evaluated. The allowable compressive stresses
obtained by applying the two Aluminum Association Specification (AAS)
formulas for unbraced beams are shown to be very conservative. The so-
lution based upon no lateral bracing leads to an allowable compressive stress
51% higher than given by AAS. The others are higher. The most realistic
assumption, the third, with very conservative assumptions of glazing re-
straint leads to an allowable compressive stress about three times the AAS
value.

Experimental studies performed to verify by test practical designs by the
senior author show no progressive displacement failures and suggest there
is sufficient bracing restraint by the glazing to prevent lateral buckling fail-
ure.

With additional study it should be possible to establish realistic design
codes that will allow far more economical designs than are now possible.

APPENDIX I.   REFERENCES

Anderson, J. M., and Trahair, N. S. (1972). "Stability of monosymmetric beams
  and cantilevers." *J. Struct. Div.*, ASCE, 98(1), 269–286.
Attard, M. M. (1983). "Extrapolation techniques of buckling loads." *J. Struct. Engrg.*,
  ASCE, 109(4), 926–935.
Bleich, F. (1952). *Buckling strength of metal structures.* McGraw-Hill, New York,
  N.Y.

Galambos, T. V. (1968). *Structural members and frames*, Prentice-Hall, Englewood Cliffs, N.J.

Galambos, T. V. (1988). *Guide to stability design criteria for metal structures*. John Wiley and Sons, New York, N.Y.

Masur, E. F., and Milbradt, K. P. (1957). "Collapse strength of redundant beams after lateral buckling." *J. Appl. Mech. Trans. ASME*, 24(2), 283–288.

Roeder, C. W., and Assadi, M. (1982). "Lateral stability of I-beams with partial support." *J. Struct. Div.*, ASCE, 108, 1768–1780.

Specifications for aluminum structures. (1986). 5th Ed., Aluminum Association. Washington, D.C., 42–54.

Timoshenko, S. P., and Gere, J. M. (1961). *Theory of elastic stability*. McGraw-Hill, New York, N.Y.

Trahair, N. S. (1969). "Deformations of geometrically imperfect beams." *J. Struct. Div.*, ASCE, 95(7), 1475–1496.

APPENDIX II. NOTATION

The following symbols are used in this paper:

A = area of cross section of mullion;

a = distance of elastic restraint below shear center;

$\bar{a}$ = distance of point of load application below shear center;

B, C = constants in series expressions;

b, c = dimensions of seal;

C_w = warping-resistance constant;

d = depth of wide-flange or I section;

E = Young's modulus of elasticity;

F_b = allowable design compressive stress;

f_c = maximum compressive stress at elastic buckling;

G = shearing modulus;

I_x, I_y = moments of inertia about x and y axes, respectively;

J = uniform torsion cross-section constant;

K_o = cross-section constant;

K_θ, K_s = stiffness constants for rotational and transverse spring supports, respectively;

k_b = load coefficient;

L_b = unsupported length of compression flange;

M = bending moment about x axis;

n = number of half-waves of buckling displacement;

P = force per unit length of seal;

r_y = radius of gyration about y axis of wide flange beam;

r_{ye} = effective r_y;

S = section modulus for maximum compressive stress for bending about x axis;

u = horizontal displacement of shear center, a function of z;

V = potential energy of slightly displaced system;

w = uniform load;

x, y = principal axes of inertia and Cartesian coordinates;

y_o = y coordinate of shear center;

z = longitudinal coordinate;

β = rotation of cross section about shear center, a function of z;

γ = shearing strain (τ/G);

$\Delta, \Delta_b, \Delta_c$ = displacements of seal;

$\delta_1, \delta_2, \delta_3$ = dimensionless parameters;

λ = story height;

τ = shearing strain in seal;

$(\ )^*$ = quantity computed for I beam with flanges were the same as the comp total depth constant; and

$(\ )_{cr}$ = elastic critical buckling value.

8). *Structural members and frames*, Prentice-Hall, Englewood

9 *Guide to stability design criteria for metal structures*. John
rk, N.Y.
...dt, K. P. (1957). "Collapse strength of redundant beams
g." *J. Appl. Mech. Trans. ASME*, 24(2), 283–288.
Assadi, M. (1982). "Lateral stability of I-beams with partial
Div., ASCE, 108, 1768–1780.
minum structures. (1986). 5th Ed., Aluminum Association,
42–54.
and Gere, J. M. (1961). *Theory of elastic stability*. McGraw-
.Y.
. "Deformations of geometrically imperfect beams." *J. Struct.*
, 1475–1496.

ATION

mbols are used in this paper:

. of cross section of mullion;
ance of elastic restraint below shear center;
ance of point of load application below shear center;
stants in series expressions;
ensions of seal;
ping-resistance constant;
th of wide-flange or I section;
ung's modulus of elasticity;
wable design compressive stress;
ximum compressive stress at elastic buckling;
aring modulus;
ments of inertia about x and y axes, respectively;
form torsion cross-section constant;
ss-section constant;
fness constants for rotational and transverse spring sup-
respectively;
coefficient;
supported length of compression flange;
nding moment about x axis;
mber of half-waves of buckling displacement;
ce per unit length of seal;
lius of gyration about y axis of wide flange beam;
ective r_y;
ction modulus for maximum compressive stress for bending
put x axis;
rizontal displacement of shear center, a function of z;
tential energy of slightly displaced system;
iform load;
incipal axes of inertia and Cartesian coordinates;
coordinate of shear center;
ngitudinal coordinate;
tation of cross section about shear center, a function of z;
earing strain (τ/G);

2494

$\Delta, \Delta_b, \Delta_c$ = displacements of seal;
$\delta_1, \delta_2, \delta_3$ = dimensionless parameters;
λ = story height;
τ = shearing strain in seal;
$(\ )^*$ = quantity computed for I beam with unequal flanges as if both flanges were the same as the compression flange, holding the total depth constant; and
$(\ )_{cr}$ = elastic critical buckling value.

2495