

The final step uses the orthogonality property of the mode shapes to formulate the equation of motion in terms of the generalized mass and stiffness matrices, which are diagonal matrices. These diagonal matrices do not have the off-diagonal terms that couple the equations of motion. Therefore, in this form the modal equations of motion are uncoupled. In this uncoupled form, the equations of motion are written as a set of uncoupled single degree-of-freedom systems as

$$-\omega^2 m_i \xi_i(\omega) + k_i \xi_i(\omega) = p_i(\omega) \quad \text{Eq. 5-12}$$

where:

$m_i$  = i-th modal mass

$k_i$  = i-th modal stiffness

$p_i$  = i-th modal force

The modal form of the frequency response equation of motion is much faster to solve than the direct method because it is a series of uncoupled single degree-of-freedom systems.

Once the individual modal responses  $\xi_i(\omega)$  are computed, physical responses are recovered as the summation of the modal responses using

$$\{x\} = [\phi] \{\xi(\omega)\} e^{i\omega t} \quad \text{Eq. 5-13}$$

These responses are in complex form (magnitude/phase or real/imaginary) and are used to recover additional output quantities requested in the Case Control Section.

## Damping in Modal Frequency Response

If a damping matrix  $[B]$  exists, the orthogonality property (see “[Overview of Normal Modes Analysis](#)” on page 37) of the modes does not, in general, diagonalize the generalized damping matrix

$$[\phi]^T [B] [\phi] \neq \text{diagonal} \quad \text{Eq. 5-14}$$

If structural damping is used, the orthogonality property does not, in general, diagonalize the generalized stiffness matrix

$$[\phi]^T [K] [\phi] \neq \text{diagonal} \quad \text{Eq. 5-15}$$

where  $K = (1 + iG)[K] + i\sum G_E[K_E]$

In the presence of a  $[B]$  matrix or a complex stiffness matrix, the modal frequency approach solves the coupled problem in terms of modal coordinates using the direct frequency approach described in “[Direct Frequency Response Analysis](#)” on page 106

$$[-\omega^2 [\phi]^T [M] [\phi] + i\omega [\phi]^T [B] [\phi] + [\phi]^T [K] [\phi]] \{\xi(\omega)\} = [\phi]^T \{P(\omega)\} \quad \text{Eq. 5-16}$$

Eq. 5-16 is similar to Eq. 5-5 for the direct frequency response analysis method except that Eq. 5-16 is expressed in terms of modal coordinates  $\xi$ . Since the number of modes used in a solution is typically much less than the number of physical variables, using the coupled solution of the modal equations is less costly than using physical variables.

If damping is applied to each mode separately, the uncoupled equations of motion can be maintained. When modal damping is used, each mode has damping  $b_i$  where  $b_i = 2m_i\omega_i\zeta_i$ . The equations of motion remain uncoupled and have the form

$$-\omega^2 m_i \xi_i(\omega) + i \omega b_i \xi_i(\omega) + k_i \xi_i(\omega) = p_i(\omega)$$

Eq. 5-17

for each mode.

Each of the modal responses is computed using

$$\xi_i(\omega) = \frac{p_i(\omega)}{-m_i \omega^2 + i b_i \omega + k_i}$$

Eq. 5-18

The TABDMP1 Bulk Data entry defines the modal damping ratios. A table is created by the frequency/damping pairs specified on the TABDMP1 entry. The solution refers to this table for the damping value to be applied at a particular frequency. The TABDMP1 Bulk Data entry has a Table ID. A particular TABDMP1 table is activated by selecting the Table ID with the SDAMPING Case Control command.

1	2	3	4	5	6	7	8	9	10
TABDMP1	TID	TYPE							
	f1	g1	f2	g2	f3	g3	-etc.-	ENDT	

Field	Contents
TID	Table identification number.
TYPE	Type of damping units: G (default) CRIT Q
fi	Frequency value (cycles per unit time).
gi	Damping value in the units specified.

At resonance, the three types of damping are related by the following equations:

$$\zeta_i = \frac{b_i}{b_{cr}} = \frac{G_i}{2}$$

$$b_{cr} = 2m_i\omega_i$$

$$Q_i = \frac{1}{(2\zeta_i)} = \frac{1}{G_i}$$

Eq. 5-19

Note that the  $i$  subscript is for the  $i$ -th mode, and not the  $i$ -th excitation frequency.

The values of  $f_i$  and  $g_i$  define pairs of frequencies and dampings. Note that  $g_i$  can be entered as one of the following: structural damping (default), critical damping, or quality factor. The entered damping is converted to structural damping internally using [Eq. 5-19](#). Straight-line interpolation is used for modal frequencies between consecutive  $f_i$  values. Linear extrapolation is used at the ends of the table. ENDT ends the table input.

For example, if modal damping is entered using [Table 5-1](#) and modes exist at 1.0, 2.5, 3.6, and 5.5 Hz, MSC.Nastran interpolates and extrapolates as shown in [Figure 5-3](#) and in the table. Note that there is no table entry at 1.0 Hz; MSC.Nastran uses the first two table entries at  $f = 2.0$  and  $f = 3.0$  to extrapolate the value for  $f = 1.0$ .

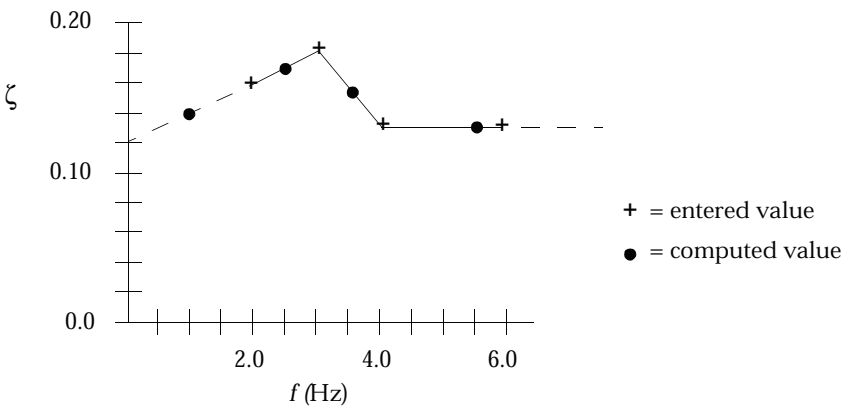


Figure 5-3 Example TABDMP1

Table 5-1 Example TABDMP1 Interpolation/Extrapolation

Entered		Computed	
$f$	$z$	$f$	$z$
2.0	0.16	1.0	0.14
3.0	0.18	2.5	0.17
4.0	0.13	3.6	0.15
6.0	0.13	5.5	0.13