

The material is assumed to have three orthogonal planes of symmetry.

Assuming the material coordinate system is perpendicular to these planes of symmetry, the plastic compliance matrix [M] can be written

$$[M] = \begin{bmatrix} G+H & -H & -G & 0 & 0 & 0 \\ -H & F+H & -F & 0 & 0 & 0 \\ -G & -F & F+G & 0 & 0 & 0 \\ 0 & 0 & 0 & 2N & 0 & 0 \\ 0 & 0 & 0 & 0 & 2L & 0 \\ 0 & 0 & 0 & 0 & 0 & 2M \end{bmatrix}$$

F, G, H, L, M and N are material constants that can be determined experimentally. They are defined as:

$$F = \frac{1}{2} \left(\frac{1}{R_{yy}^2} + \frac{1}{R_{zz}^2} - \frac{1}{R_{xx}^2} \right)$$

$$G = \frac{1}{2} \left(\frac{1}{R_{zz}^2} + \frac{1}{R_{xx}^2} - \frac{1}{R_{yy}^2} \right)$$

$$H = \frac{1}{2} \left(\frac{1}{R_{xx}^2} + \frac{1}{R_{yy}^2} - \frac{1}{R_{zz}^2} \right)$$

$$L = \frac{3}{2} \left(\frac{1}{R_{yz}^2} \right)$$

$$M = \frac{3}{2} \left(\frac{1}{R_{xz}^2} \right)$$

$$N = \frac{3}{2} \left(\frac{1}{R_{xy}^2} \right)$$

The yield stress ratios R_{xx} , R_{yy} , R_{zz} , R_{xy} , R_{yz} and R_{xz} are specified by the user and can be calculated as:

$$R_{xx} = \frac{\sigma_{xx}^y}{\sigma_0}$$

$$R_{yy} = \frac{\sigma_{yy}^y}{\sigma_0}$$

$$R_{zz} = \frac{\sigma_{zz}^y}{\sigma_0}$$

$$R_{xy} = \sqrt{3} \frac{\sigma_{xy}^y}{\sigma_0}$$

$$R_{yz} = \sqrt{3} \frac{\sigma_{yz}^y}{\sigma_0}$$

$$R_{xz} = \sqrt{3} \frac{\sigma_{xz}^y}{\sigma_0}$$

Lamina Compliance Matrix for Arbitrary Orientation

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [T]^T [S] [T] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ & \bar{S}_{22} & \bar{S}_{26} \\ sym. & & \bar{S}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}$$

where

$$\bar{S}_{11} = S_{11} \cos^4 \theta + (2S_{12} + S_{66}) \sin^2 \theta \cos^2 \theta + S_{22} \sin^4 \theta$$

$$\bar{S}_{12} = (S_{11} + S_{22} - S_{66}) \sin^2 \theta \cos^2 \theta + S_{12} (\sin^4 \theta + \cos^4 \theta) = S_{21}$$

$$\bar{S}_{16} = (2S_{11} - 2S_{12} - S_{66}) \sin \theta \cos^3 \theta + (2S_{12} - 2S_{22} + S_{66}) \sin^3 \theta \cos \theta = S_{61}$$

$$\bar{S}_{22} = S_{11} \sin^4 \theta + (2S_{12} + S_{66}) \sin^2 \theta \cos^2 \theta + S_{22} \cos^4 \theta$$

$$\bar{S}_{26} = (2S_{11} - 2S_{12} - S_{66}) \sin^3 \theta \cos \theta + (2S_{12} - 2S_{22} + S_{66}) \sin \theta \cos^3 \theta = S_{62}$$

$$\bar{S}_{66} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66}) \sin^2 \theta \cos^2 \theta + S_{66} (\sin^4 \theta + \cos^4 \theta)$$

and

$$S_{11} = \frac{1}{E_1}$$

$$S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2} = S_{21}$$

$$S_{22} = \frac{1}{E_2}$$

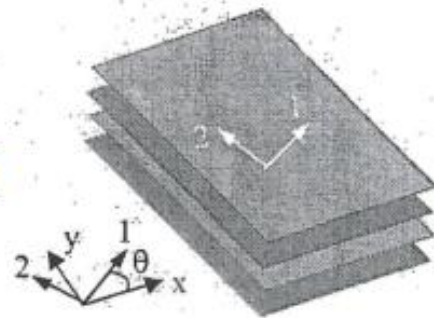
$$S_{66} = \frac{1}{G_{12}}$$

$$S_{16} = S_{26} = S_{61} = S_{62} = 0$$

Generally Orthotropic Ply

4. Strain-Stress Relationship in x-y System

Details of the transformed reduced **compliance** terms are shown in the teaching notes and can be determined from S_{ij} as follows



$$\begin{Bmatrix} \bar{S}_{11} \\ \bar{S}_{22} \\ \bar{S}_{33} \\ \bar{S}_{12} \\ \bar{S}_{13} \\ \bar{S}_{23} \end{Bmatrix} = \begin{bmatrix} m^4 & n^4 & 2m^2n^2 & m^2n^2 \\ n^4 & m^4 & 2m^2n^2 & m^2n^2 \\ 4m^2n^2 & 4m^2n^2 & -8m^2n^2 & (m^2 - n^2)^2 \\ m^2n^2 & m^2n^2 & m^4 + n^4 & -m^2n^2 \\ 2m^3n & -2mn^3 & 2(mn^3 - m^3n) & mn^3 - m^3n \\ 2mn^3 & -2m^3n & 2(m^3n - mn^3) & m^3n - mn^3 \end{bmatrix} \begin{Bmatrix} S_{11} \\ S_{22} \\ S_{33} \\ S_{12} \end{Bmatrix} \quad (17.8)$$

From Eq(16.3)

$$S_{11} = \frac{1}{E_1} \quad S_{12} = \frac{-\nu_{21}}{E_2} = \frac{-\nu_{12}}{E_1}$$

$$S_{22} = \frac{1}{E_2} \quad S_{33} = \frac{1}{G_{12}}$$

$$m = \cos \theta$$

$$n = \sin \theta$$

reduced compliance matrix:

$$\{\epsilon\} = [S] \{\sigma\}$$