Correlations for Convective Heat Transfer

In many cases it's convenient to have simple equations for estimation of heat transfer coefficients. Below is a collection of recommended correlations for single-phase convective flow in different geometries as well as a few equations for heat transfer processes with change of phase. Note that all equations are for mean Nusselt numbers and mean heat transfer coefficients. The following cases are treated:

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1 Forced Convection Flow Inside a Circular Tube

$$\mathrm{Re} = \frac{\rho u_m D}{\mu}$$

$$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$$

$$\mathrm{Nu} = \frac{hD}{k}$$

All properties at fluid bulk mean temperature (arithmetic mean of inlet and outlet temperature).

Nusselt numbers Nu_0 from sections 1-1 to 1-3 have to be corrected for temperature-dependent fluid properties according to section 1-4. \

1-1 Thermally developing, hydrodynamically developed laminar flow (Re < 2300)

Constant wall temperature:

$$\mathrm{Nu}_0 = 3.657 + \frac{0.19\,(\mathrm{Re}\,\mathrm{Pr}\,\frac{D}{L})^{0.8}}{1+0.117\,(\mathrm{Re}\,\mathrm{Pr}\,\frac{D}{L})^{0.467}}$$ (Hausen)

Constant wall heat flux:

$$\mathrm{Nu}_0 = \begin{cases} 1.953(\mathrm{Re}\,\mathrm{Pr}\frac{D}{L})^{1/3}; & (\mathrm{Re}\,\mathrm{Pr}\,\frac{D}{L}) \geq 33.3 \\ 4.364 + 0.0722\,\mathrm{Re}\,\mathrm{Pr}\,\frac{D}{L}; & (\mathrm{Re}\,\mathrm{Pr}\,\frac{D}{L}) < 33.3 \end{cases}$$ (Shah)

1-2 Simultaneously developing laminar flow (Re < 2300)

Constant wall temperature:

$$\mathrm{Nu}_0 = 3.657 + \frac{0.0677\,(\mathrm{Re}\,\mathrm{Pr}\,\frac{D}{L})^{1.33}}{1+0.1\,\mathrm{Pr}\,(\mathrm{Re}\,\frac{D}{L})^{0.3}}$$ (Stephan)

Constant wall heat flux:

$$\mathrm{Nu}_0 = 4.364 + \frac{0.086\,(\mathrm{Re}\,\mathrm{Pr}\,\frac{D}{L})^{1.33}}{1+0.1\,\mathrm{Pr}\,(\mathrm{Re}\,\frac{D}{L})^{0.83}}$$

which is valid over the range $0.7 < \mathrm{Pr} < 7$ or if $\mathrm{Re}\,\mathrm{Pr}\,D/L < 33$ also for $\mathrm{Pr} > 7$.

1-3 Fully developed turbulent and transition flow (Re > 2300)

Constant wall heat flux:

$$\mathrm{Nu}_0 = \frac{\frac{\xi}{8}(\mathrm{Re}-1000)\,\mathrm{Pr}}{1+12.7\sqrt{\frac{\xi}{8}}\,(\mathrm{Pr}^{2/3}-1)}\left[1+\left(\frac{D}{L}\right)^{2/3}\right]$$ (Petukhov, Gnielinski)

where $\xi = \dfrac{1}{(1.82\log\mathrm{Re}-1.64)^2}$

Constant wall temperature: For fluids with $\mathrm{Pr} > 0.7$ correlation for constant wall heat flux can be used with negligible error.

1-4 Effects of property variation with temperature

Liquids, laminar and turbulent flow:

$$\mathrm{Nu} = \mathrm{Nu}_0 \left(\frac{\mu}{\mu_w} \right)^{0.14}$$

Subscript w: at wall temperature, without subscript: at mean fluid temperature

Gases, laminar flow:

$$\mathrm{Nu} = \mathrm{Nu}_0$$

Gases, turbulent flow:

$$\mathrm{Nu} = \mathrm{Nu}_0 \left(\frac{T_m}{T_w} \right)^{0.36}$$

Temperatures in Kelvin

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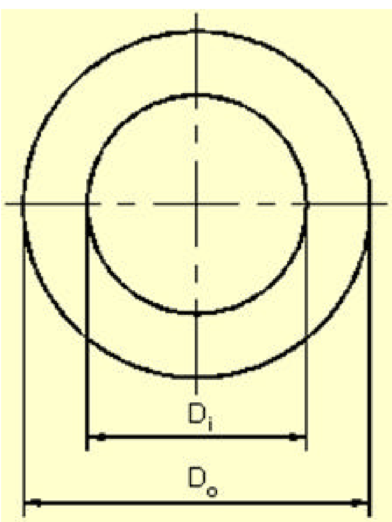
2 Forced Convection Flow Inside Concentric Annular Ducts, Turbulent (Re > 2300)



$$D_h = D_o - D_i$$

$$\mathrm{Re} = \frac{\rho u_m D_h}{\mu}$$

$$\mathrm{Nu} = \frac{h D_h}{k}$$

All properties at fluid bulk mean temperature (arithmetic mean of inlet and outlet temperature).

Heat transfer at the inner wall, outer wall insulated:

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_{tube}} = 0.86 \left(\frac{D_o}{D_i} \right)^{0.16}$$ (Petukhov and Roizen)

Heat transfer at the outer wall, inner wall insulated:

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_{\mathrm{tube}}} = 1 - 0.14\left(\frac{D_i}{D_o}\right)^{0.6}$$ (Petukhov and Roizen)

Heat transfer at both walls, same wall temperatures:

$$\frac{\mathrm{Nu}}{\mathrm{Nu}_{\mathrm{tube}}} = \frac{0.86\left(\frac{D_i}{D_o}\right)^{0.84} + \left[1 - 0.14\left(\frac{D_i}{D_o}\right)^{0.6}\right]}{1 + \frac{D_i}{D_o}}$$ (Stephan)

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3 Forced Convection Flow Inside Non-Circular Ducts, Turbulent (Re > 2300)

Equations for circular tube with hydraulic diameter

$$D_h = \frac{4 \cdot \text{cross} - \text{sectional area}}{\text{wetted perimeter}}$$

$$\mathrm{Re} = \frac{\rho u_m D_h}{\mu}$$

$$\mathrm{Nu} = \frac{h D_h}{k}$$

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4 Forced Convection Flow Across Single Circular Cylinders and Tube Bundles

$$l = \frac{\pi}{2} D$$

$$\mathrm{Re}_l = \frac{\rho u_m l}{\mu}$$

$$\mathrm{Nu}_l = \frac{h l}{k}$$

D = cylinder diameter, u_m = free-stream velocity, all properties at fluid bulk mean temperature. Correction for temperature dependent fluid properties see section 4-4.

4-1 Smooth circular cylinder

$$Nu_{l,o} = 0.3 + \sqrt{Nu_{l,lam}^2 + Nu_{l,turb}^2}$$ (Gnielinski)

where $Nu_{l,lam} = 0.664\ \sqrt{Re_l}\ \sqrt[3]{Pr}$

$$Nu_{l,turb} = \frac{0.037\,Re_l^{0.8}\,Pr}{1 + 2.443\,Re_l^{-0.1}\left(Pr^{2/3} - 1\right)}$$

Valid over the ranges $10 < Re_l < 10^7$ and $0.6 < Pr < 1000$

4-2 Tube bundle

Transverse pitch ratio $a = \frac{S_Q}{D}$

Longitudinal pitch ratio $b = \frac{S_L}{D}$

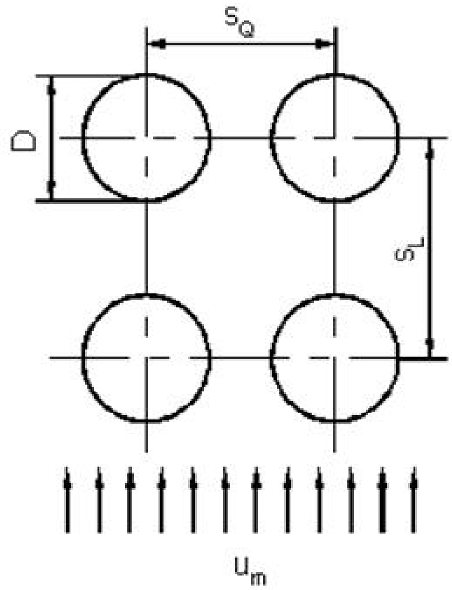
Void ratio $\psi = 1 - \frac{\pi}{4a}$ for $b \geq 1$

$\psi = 1 - \frac{\pi}{4ab}$ for $b < 1$

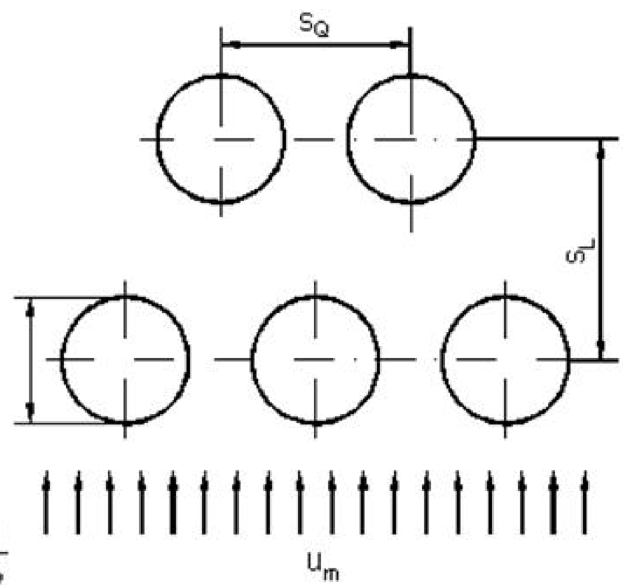
$Nu_{0,bundle} = f_A Nu_{l,0}$ (Gnielinski)

$Nu_{l,0}$ according to section 4-1 with $Re_{\psi,l} = \frac{\rho u_m l}{\psi \mu}$ instead of Re_l.

Arrangement factor f_A depends on tube bundle arrangement.

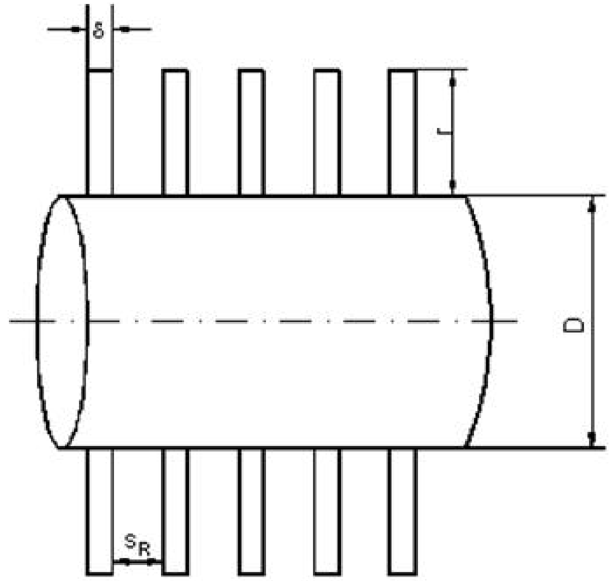


In-line arrangement: $f_A = 1 + \frac{0.7}{\psi^{1.5}} \frac{(b/a - 0.3)}{(b/a + 0.7)^2}$



Staggered arrangement: $f_A = 1 + \frac{2}{3b}$

4-3 Finned tube bundle



$$\frac{A_o}{A_e} = \frac{S_Q(S_R + \delta)}{(S_Q - D)S_R + (S_Q - D - 2r)\delta}$$

$$\frac{A}{A_{Go}} = 1 + \frac{2r(r + D + \delta)}{D(S_R + \delta)}$$

In-line tube bundle arrangement:

$$\mathrm{Nu}_{l,o} = 0.26\,\mathrm{Re}_l^{0.6}\left(\frac{A_o}{A_e}\right)^{0.6}\left(\frac{A}{A_{GO}}\right)^{-0.15}\mathrm{Pr}^{1/3}$$ (Paikert)

Staggered tube bundle arrangement:

$$\mathrm{Nu}_{l,o} = 0.45\,\mathrm{Re}_l^{0.6}\left(\frac{A_o}{A_e}\right)^{0.6}\left(\frac{A}{A_{GO}}\right)^{-0.15}\mathrm{Pr}^{1/3}$$ (Paikert)

4-4 Effects of property variation with temperature

Liquids:

$$\mathrm{Nu}_l = \mathrm{Nu}_{l,o}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

$$\mathrm{Nu}_{\mathrm{bundle}} = \mathrm{Nu}_{0,\mathrm{bundle}}\left(\frac{\mu}{\mu_w}\right)^{0.14}$$

Subscript w: at wall temperature, without subscript: at mean fluid temperature.

Gases:

$$\mathrm{Nu}_l = \mathrm{Nu}_{l,o}\left(\frac{T_m}{T_w}\right)^{0.12}$$

$$\mathrm{Nu}_{\mathrm{bundle}} = \mathrm{Nu}_{0,\mathrm{bundle}}\left(\frac{T_m}{T_w}\right)^{0.12}$$

Temperatures in Kelvin.

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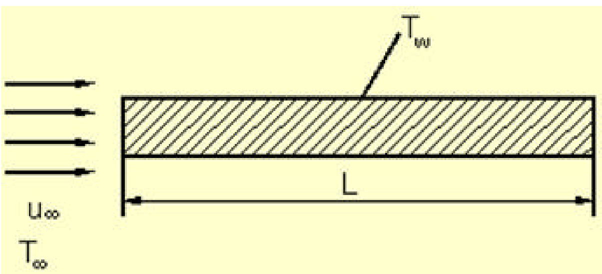
5 Forced Convection Flow over a Flat Plate



$$\mathrm{Re}_L = \frac{\rho u_\infty L}{\mu}$$

$$\mathrm{Nu}_L = \frac{hL}{k}$$

All properties at mean film temperature $T_m = \frac{T_\infty + T_w}{2}$

Laminar boundary layer, constant wall temperature:

$$\mathrm{Nu}_{L,lam} = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$$ (Pohlhausen)

valid for $\mathrm{Re}_L < 2\cdot10^5$, $0.6 < \mathrm{Pr} < 10$

Turbulent boundary layer along the whole plate, constant wall temperature:

$$Nu_{L,turb} = \frac{0.037 Re_L^{0.8} Pr}{1 + 2.443 Re_L^{-0.1} \left(Pr^{2/3} - 1\right)}$$ (Petukhov)

Boundary layer with laminar-turbulent transition:

$$Nu_L = \sqrt{Nu_{L,lam}^2 + Nu_{L,turb}^2}$$ (Gnielinski)

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6 Natural Convection

All properties at $T_M = \frac{1}{2}\left(T_w + T_\infty\right)$

$$Gr = \frac{L^3 \cdot g \cdot \rho^2 \cdot \beta \cdot \left|T_w - T_\infty\right|}{\mu^2}$$

$$Nu = \frac{h \cdot L}{k}$$

L = characteristic length (see below)

	Nu_0	"Length" L
Vertical Wall	0.67	H
Horizontal Cylinder	0.36	D
Sphere	2.00	D

For ideal gases: $\beta = \frac{1}{T_M}$ (temperature in K)

$$\sqrt{Nu} = \sqrt{Nu_o} + \left[\frac{\frac{Gr \cdot Pr}{300}}{\left(1 + \left(\frac{0.5}{Pr}\right)^{9/16}\right)^{16/9}}\right]^{1/6}$$ (Churchill, Thelen)

valid for $10^{-4} \le Gr\,Pr \le 4 \cdot 10^{14}$,

$0.022 \le Pr \le 7640$, and constant wall temperature

7 Film Condensation

All properties without subscript are for condensate at the mean temperature

$$T_m = \frac{3}{4} T_w + \frac{1}{4} T_s$$

Exception: ρ_D= vapor density at saturation temperature T_s

7-1 Laminar film condensation

Vertical wall or tube:

$$\mathrm{Nu} = \frac{hH}{k} = 0.943 \left[\frac{\Delta h (\rho - \rho_D) \cdot g \cdot H^3}{(T_s - T_w) \cdot \nu \cdot k} \right]^{1/4}$$ (Nusselt)

T_w = mean wall temperature

Horizontal cylinder:

$$\mathrm{Nu} = \frac{h \cdot D}{k} = 0.728 \left[\frac{\Delta h (\rho - \rho_D) \cdot g \cdot D^3}{(T_s - T_w) \cdot \nu \cdot k} \right]^{1/4}$$ (Nusselt)

T_w = const.

7-2 Turbulent film condensation

For vertical wall

$\mathrm{Re} = C\,A^m$

$$\mathrm{Re} = \frac{h \cdot H (T_s - T_w)}{\nu \cdot \rho \cdot \Delta h} \;;\; A = \frac{g^{1/3} \cdot k \cdot H \cdot (T_s - T_w)}{\nu^{5/3} \cdot \rho \cdot \Delta h}$$

Re_{crit} = 350

turbulent film: $C = 3 \cdot 10^{-3} \;;\; m = 3/2$ (Grigull)

8 Nucleate Pool Boiling

$$\dot{q} = h(T_w - T_s)$$

T_w = temperature of heating surface

T_s = saturation temperature

Heat transfer at ambient pressure:

$$\mathrm{Nu} = \frac{h d_A}{k'} = 0.1\left(\frac{\dot{q} d_A}{k' T_s}\right)^{0.674}\left(\frac{\rho''}{\rho'}\right)^{0.156}\left(\frac{\Delta h d_A^2}{\alpha'^2}\right)^{0.371}\left(\frac{\alpha'^2 \rho'}{\sigma d_A}\right)^{0.350}(\mathrm{Pr}')^{-0.162}$$

(Stephan and Preußer)

' saturated liquid

" saturated vapor

Bubble departure diameter $d_A = 0.851\beta_o\sqrt{\frac{2\sigma}{g(\rho'-\rho'')}}$

Angle β_o' = $\pi/4$ rad for water

= 0.0175 rad for low-boiling liquids

= 0.611 rad for other liquids

For water in the range of 0.5 bar < p < 20 bar and 10^4 W/m^2 < $\dot{q}$ < 10^6 W/m^2 the following equation may be applied:

$$\frac{h}{\mathrm{W/m^2K}} = 1.95\left(\frac{\dot{q}}{\mathrm{W/m^2}}\right)^{0.72}\left(\frac{p}{\mathrm{bar}}\right)^{0.24}$$

(Fritz)

List of Symbols

c_p	specific heat capacity at constant pressure
D, d	diameter
g	gravitational acceleration
h	mean heat transfer coefficient
Δh	enthalpy of evaporation
H	height
k	thermal conductivity
L	length

$\dot{q}$	heat flux
T	temperature
u	flow velocity
α	thermal diffusivity
β	coefficient of thermal expansion
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
σ	surface tension

Subscripts

h	hydraulic
i	inside
m	mean
o	outside
s	saturation
w	wall

Dimensionless numbers

Gr	Grashof number
Nu	mean Nusselt number
Pr	Prandtl number
Re	Reynolds number

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