# Fabrication of Welded Steel Wye Branches

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Welded steel wye branches are a key component of most water projects where flows must be divided. They are also a specialty item that requires many engineering considerations. The design of intersecting pipes has been studied by many agencies over the years and good design guidance can be obtained from several sources including the American Water Works Association (AWWA) Manual M11, *Steel Water Pipe: A Guide for Design and Installation,* where the focus is determination of proportions of steel reinforcement necessary to resist internal pressure. However, fabrication of wye branches is often left to the fabricator and little guidance is offered. There are many design related issues requiring a high degree of skill on the part of the fabricator to complete the design and to make it function reliably as a safe, integral part of a conveyance system, as intended by the designer.

This paper will address several areas of wye branch fabrication that are not well covered by other sources including steel selection, geometry, welding, heat treatment, and non-destructive testing. Welding code provisions can be a valuable resource and their contribution to steel wye branch fabrication will be explored.

Figure 1

Source: Ameron International, Rancho Cucamonga, CA.

## Steel Selection

From a design perspective, because pressure vessels are stressed in all directions, it is desirable that steel be homogeneous; in other words, it has consistent mechanical properties in all directions. In general, structural steels (i.e. American Society of Standard Testing Materials [ASTM] A36 *Standard Specification for Carbon Structural Plate*, conforming to ASTM A6/A6M *Standard Specification for General Requirements for Rolled Structural Steel Bars, Plates, Shapes, and Sheet Piling*) have predictable properties in the final direction of rolling only (x direction) while pressure vessel quality (PVQ) steels (i.e. ASTM A516, *Standard Specification for Pressure Vessel Plates, Carbon Steel, for Moderate- and Lower-Temperature Service*, conforming to

ASTM A20/A20M, *Standard Specification for General Requirements for Steel Plates for Pressure Vessels*) receive more hot-working than structural steels resulting in predictable properties in both x and y directions. Also, steels conforming to ASTM A20/A20M receive more testing than steels conforming to ASTM A6/A6M. Pressure vessel steels, because of their homogenous properties and greater degree of predictability, are more desirable than structural steels for wye branch fabrication.

Mechanical properties of carbon and high strength low alloy (HSLA) steels of interest for welded steel wye branches can be reduced to five properties that are suitable for many applications. Welding codes provide good guidance related to steel selection by listing approved steels for welding and by grouping the steels according to their yield and tensile strengths. Other valuable information including matching filler metal and preheat and inter-pass temperature recommendations is also given. Code listed base metals, such as ASTM A516 Grade 70, can typically meet the following criteria:

- Yield: 38 thousands of pounds per square inch (ksi), minimum
- Tensile: 70 ksi, minimum
- Elongation: 21 percent minimum in a 2-inch gauge length and17 percent minimum in an 8-inch gauge length
- Weld-Ability: Carbon Equivalent, CE < 0.45,
  where $CE = C + Mn/6 + (Cr + Mo + V)/5 + (Ni + Cu)/15$
- Toughness: 25 foot-pounds at 30 degrees Fahrenheit (°F)

Crotch plates that externally reinforce the wye branches are loaded in all directions as a result of internal pressure. Achieving predictable properties in x, y, and z directions through heat treatment and control of contaminants such as sulfur improves steel plate performance. Normalizing is a heat treatment where the steel plate is heated to a temperature of approximately 1600-1700°F then air cooled. The heat treatment refines the grain size of the steel and improves its notch toughness. Normalizing is a requirement per ASTM A516 on steels thicker than 1.5 inches.

Controlling contaminants by limiting sulfur content to a very small quantity (maximum 0.005 to 0.010 percent) enhances the mechanical properties of steel plate by reducing the possibility of manganese-sulfide formation where rounded gas pockets are trapped within the molten steel. The gas pockets (called "inclusions") become elongated during the hot rolling process of steel making. Inclusions can lead to plate laminations that, if large enough, can be detrimental to strength and ductility in the through-thickness (z-axis) direction within the steel plates (see Figure 2). It is recommended that steel suppliers be contacted during design to verify availability. For example, low sulfur steels may not be readily available without a special mill run and cannot be economically justified for relatively low quantities of steel needed for typical wye branch fabrication. The author believes that checking the z-axis properties of crotch plates is warranted where plate thickness exceeds 1.5 inches.

Several types of mechanical tests can be performed on crotch plates to verify through thickness strength and check for laminations. These tests are also described in ASTM A20, *Supplementary Requirements*, S4 *Additional Tension Test* and S8 *Ultrasonic Examination in Accordance with A435/435M.*

- Through-thickness tension testing can be performed according to ASTM A770/A770M *Standard Specification for Through-Thickness Tension Testing of Steel Plates for Special Applications* on each plate. The purpose of this test is to determine a measure of resistance of steel plate to lamellar tearing.

- Straight-Beam Ultrasonic Examination can be conducted according to ASTM A435/A435M *Standard Specification for Straight-Beam Ultrasonic Examination of Steel Plates* on each plate. The purpose of this test is to assure delivery of steel plates free of gross internal discontinuities such as pipes, ruptures, or laminations.

**Figure 2**


## Geometry

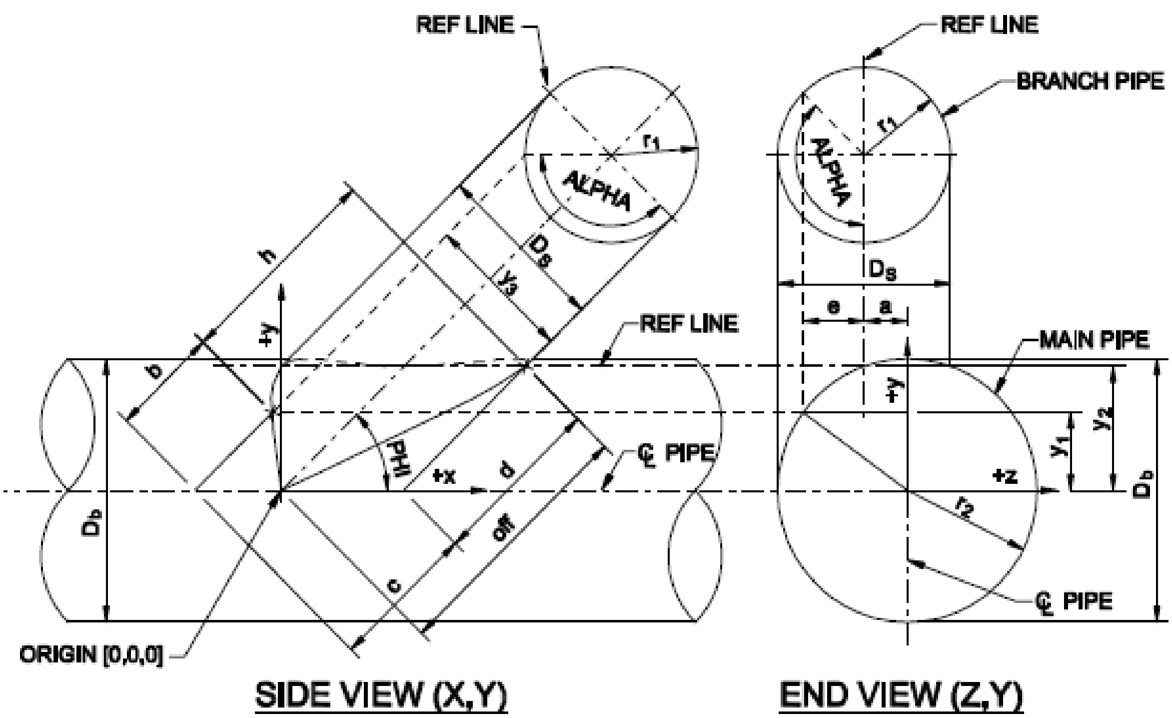
Intersecting cones and cylinders have fascinated mathematicians throughout history. True understanding of the geometry of welded steel wye branches is essential to accurate fabrication and dependable service. However, somewhat rigorous calculations are necessary to accurately design and detail components of a wye branches. Analytic geometry and vector algebra play a significant role in determining templates for cutting and bending the steel plates and for developing groove welding details.

Figure 3 is useful in determining the geometry of intersection of pipes. Figure 4 is used to determine welding details, where angles between planes tangent to the branch and main pipes called "***local dihedral angle***" as indicated in ***AWS D1.1/1.1M:2006, Structural Welding Code-Steel, Annex P, Local Dihedral Angle*** are calculated. Vectors N1 through N5 are also indicated.

Figure 5 describes an analytic procedure recommended by the author for determining the local dihedral angle. An example problem illustrating the procedure is presented at the end of this paper. Note that the branch rotation angle "*ALPHA*" and wye branch angle "PHI" in Figure 3 is the same as "*RHO*" and "THETA" respectively in Figure 4.

Figure 6 illustrates an example of a 96-inch x 66-inch x 45 degree wye branch that is reinforced with a single curved crotch plates of 4-inch thickness. The wye branch is shop hydrostatically tested to 1.5 times working pressure of 220 pounds per square inch (psi) resulting in a test pressure of 330 psi per ASME BPV Code Section VIII, Div. 1, UG-99. Note that crotch plate reinforcement is only practical where the main pipe and branch pipe centerlines lie in the same plane (a = 0 in Figure 3).

FIGURE 3




$\boxed{h + b = c + d}$ Eqs. 1 where $b = \dfrac{y_1}{\sin(\phi)}$ $c = \dfrac{y_3}{\tan(\phi)}$ $d = \dfrac{y_2}{\sin(\phi)}$ $e = r_1 \cdot \sin(\alpha)$

$y_1 = \sqrt{r_2^{\,2} - (a + r_1 \cdot \sin(\alpha))^2}$ $y_2 = \sqrt{r_2^{\,2} - a^2}$ $y_3 = r_1 \cdot (1 - \cos(\alpha))$ $D_s = 2 \cdot r_1$ $D_b = 2 \cdot r_2$

where $D_b$ = diameter_of_main and $D_s$ = diameter_of_branch and $a$ = offset_of_branch_and_main and $\phi$ = deflection_angle and $\alpha$ = polar_rotation_angle_on_branch_axis

Solving Eqs. 1 for "h" results in the following generic formula when branch and main are offset ( $a \neq 0$);

$$h = \frac{\sqrt{D_b^{\,2} - 4 \cdot a^2} - \sqrt{D_b^{\,2} - 4 \cdot a^2 - 4 \cdot a \cdot D_s \cdot \sin(\alpha) - D_s^{\,2} + D_s^{\,2} \cdot \cos(\alpha)^2} + D_s \cdot \cos(\phi) - D_s \cdot \cos(\phi) \cdot \cos(\alpha)}{2 \cdot \sin(\phi)} \qquad \text{Eqs. 2}$$

Solving Eqs. 1 for "h" assuming $a = 0$ (typical case) results in the following simplified formula

$$h = \frac{D_b + D_s \cdot \cos(\phi) - D_s \cdot \cos(\phi) \cdot \cos(\alpha) - \sqrt{D_b^{\,2} - D_s^{\,2} + D_s^{\,2} \cdot \cos(\alpha)^2}}{2 \cdot \sin(\phi)} \qquad \text{Eqs. 3}$$

centerline offset along branch pipe from centerline of main where $off = d + \dfrac{r_1}{\tan(\phi)}$ results in the following simplified formula

$$off = \frac{\sqrt{\frac{1}{4} \cdot D_b^{\,2} - a^2}}{\sin(\phi)} + \frac{1}{2} \cdot \frac{D_s}{\tan(\phi)} \qquad \text{Eqs.4}$$

**Figure 4**

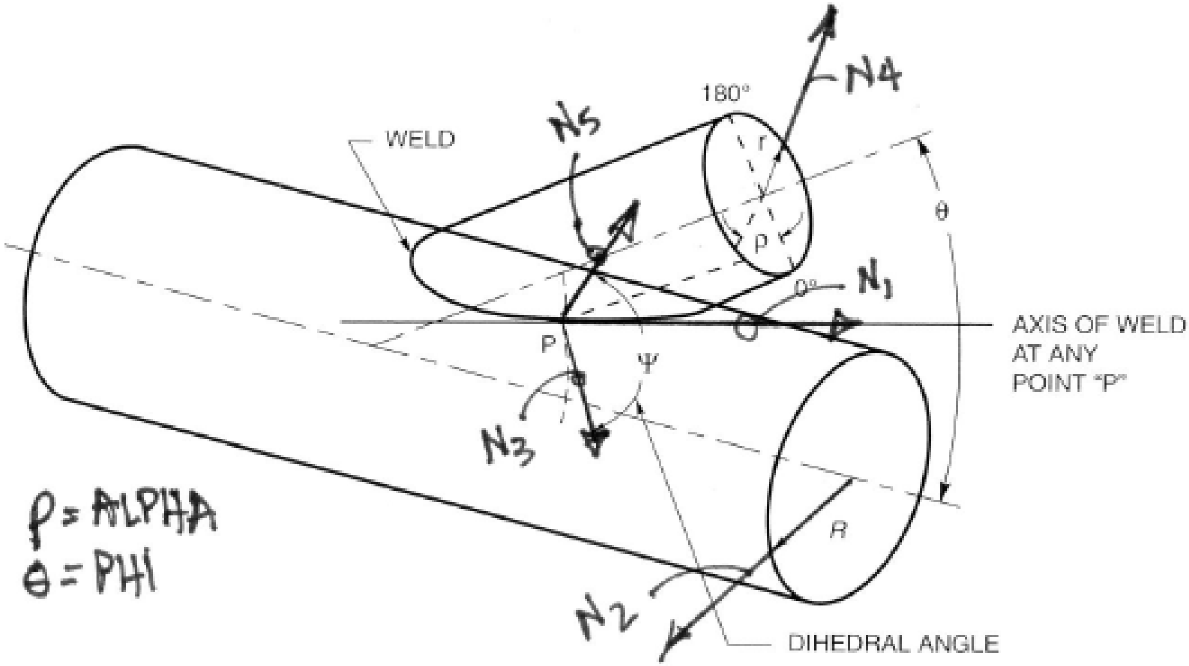



**Figure 5**

## Procedure

1) develop 3D coordinates of pipe intersection using vector algebra and ordinate h along branch calculated using either Eqs. 2 or Eqs. 3 above
2) approximate weld axis vector N1 using differential points at both $(\alpha - 1 \cdot \mathrm{deg})$ and $(\alpha + 1 \cdot \mathrm{deg})$ branch rotation
3) find vector N2, normal to main pipe axis
4) find vector N3, normal to weld axis in plane tangent to main pipe; where N3=N1xN2 (vector cross product)
5) find vector N4, normal to branch pipe axis
6) find vector N5, normal to weld axis in plane tangent to branch pipe; where N5=N1xN4 (vector cross product)
7) calculate local dihedral angle $\psi$, using vector dot product and absolute values; where $\psi = \mathrm{acos}\left(\frac{N5 \cdot N3}{|N5| \cdot |N3|}\right)$

Figure 7 illustrates a developed view of the curved crotch plates where the flat position geometry is described before bending the plates. Figure 8 illustrates the geometry for bending the crotch plates into a curved position to fit the pipe intersection. The radius of curvature of the crotch plates is a minimum value of approximately R = 40 inches which results in a bend radius/plate thickness ratio, R/t = 10; the author believes that, for cold formed crotch plates, the bend ratio should be 10 or greater to avoid structural degradation and possible cracking of the plate. To minimize the crotch plate bend radius the ratio of branch diameter to main diameter, Ds/Db, should not exceed 0.70 or consideration should be given to an alternate design that includes cone/cylinders; branch cones that are tangent to the main cylinder result in straight line intersection that avoid the need for curved crotch plates in favor of planar crotch plates.

**Figure 6**

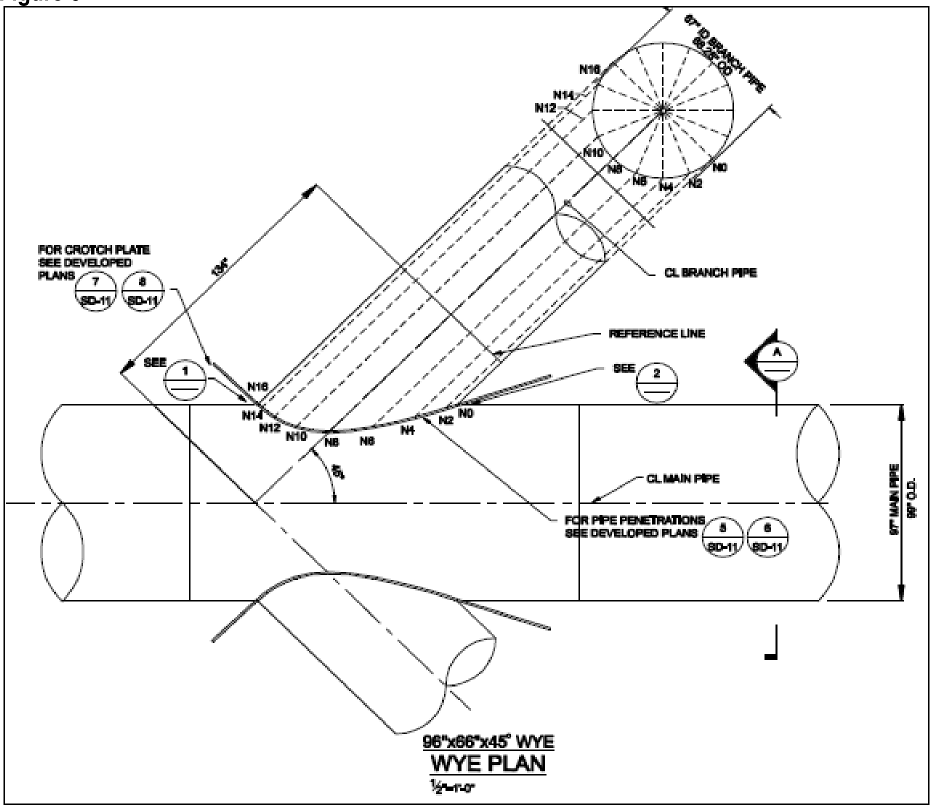



**Figure 7**

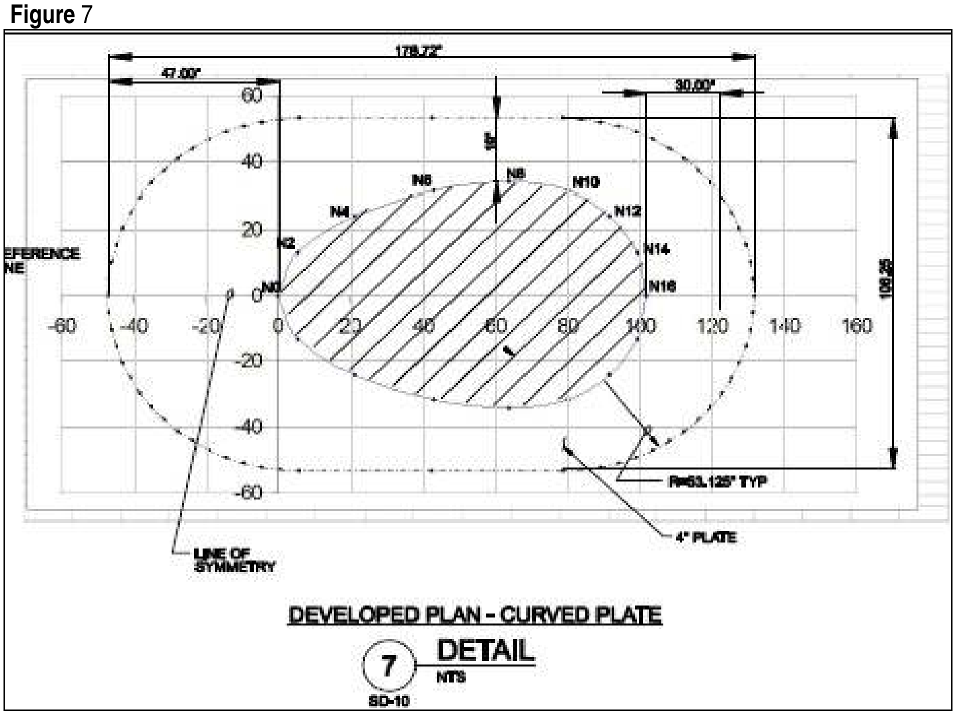
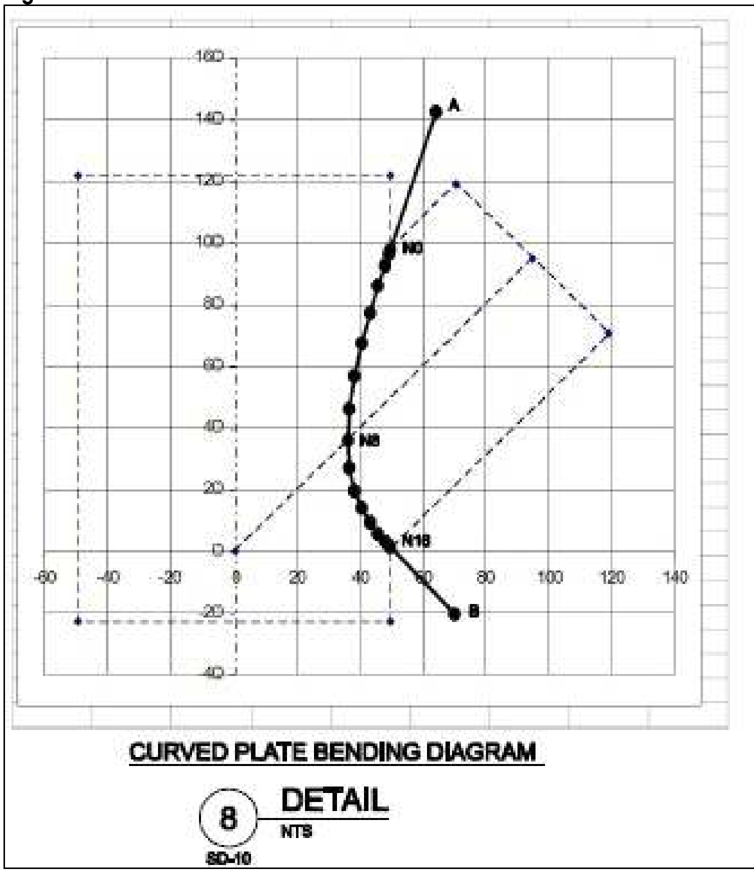


Figure 8




| POINT NO. | X INCHES | Y INCHES |
|---|---|---|
| A | 64.1 | 142.4 |
| N0 | 49.5 | 97.8 |
| N1 | 49.1 | 96.4 |
| N2 | 47.7 | 92.3 |
| N3 | 45.7 | 85.9 |
| N4 | 43.2 | 77.3 |
| N5 | 40.6 | 67.4 |
| N6 | 38.2 | 56.6 |
| N7 | 36.5 | 27.1 |
| N8 | 35.9 | 35.9 |
| N9 | 36.5 | 27.1 |
| N10 | 38.2 | 19.7 |
| N11 | 40.6 | 13.7 |
| N12 | 43.2 | 9.1 |
| N13 | 45.7 | 5.6 |
| N14 | 47.7 | 3.2 |
| N15 | 49.1 | 1.7 |
| N16 | 49.5 | 1.2 |
| B | 70.2 | -20.5 |

## Welding

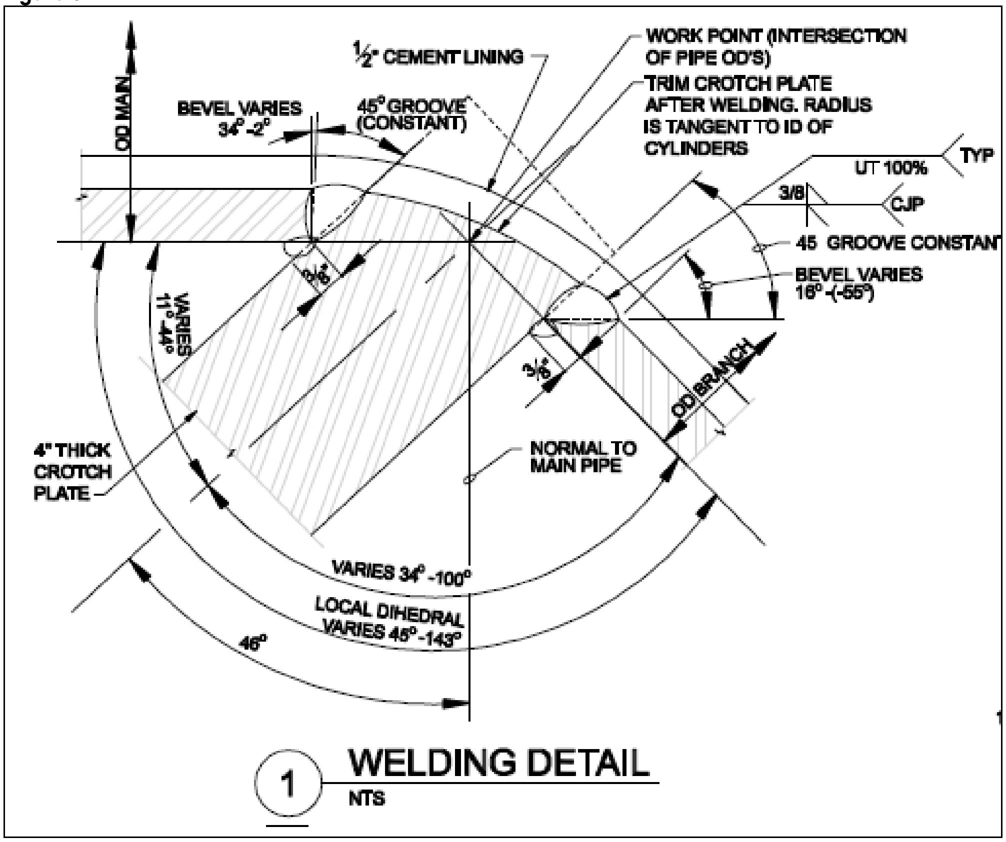
Welding details for wye branches must provide adequate strength to meet design loads, yet permit easy welder access to achieve complete joint penetration and promote good weld quality. Constructability is best if pipe shells are groove welded to sides of crotch plates as indicated in Figures 9 and 10. The pipe shells that are in tension under internal pressure, apply a through-thickness loading to the crotch plates. This loading places added emphasis on the z-axis steel properties as discussed in previous section ***Steel Selection***.

Welding procedure specifications (WPSs) should be qualified per ASME BPV Code Section IX for notch tough welding and Supplementary Essential Variables. Procedure qualification records should document heat input limits and Charpy tests should be conducted in both heat affected zone (HAZ) and weld metal. Charpy acceptance should be the same as base metal; for full-sized 10 millimeter (mm) coupons, acceptance is often 25 ft-lbs energy at a test temperature of 30°F as recommended in AWWA C200, *Steel Water Pipe-6 In. (150 mm) and Larger*.



Figure 9




Welder access is assumed to be primarily from inside the main pipe for application of single bevel groove welds. Exterior fillet welds are applied to balance weld metal shrinkage and control distortion. The local dihedral angle is constantly changing around the welded joint, however the groove angle should remain constant to avoid welding difficulties. For example, a groove angle that becomes too narrow may result in inability of the welder to adequately clean weld passes, thus trapping slag. Too wide a groove angle results in excessive weld metal, greater weld shrinkage and distortion. Geometry requires that weld bevels must vary if groove angles are constant and local dihedral angle varies. Welding symbols alone are not sufficient to describe welding details indicated in Figures 9 and 10. The author believes that a typical wye branch design should include several joint welding details similar to that shown in addition to beveling diagrams (not shown) to convey detailed design information to the shop fabricators and welders.

## Heat Treatment

Post weld heat treatment (PWHT) of the completed wye is beneficial in reducing welding and fabrication stresses. Stress from weld shrinkage that can approach steel yield stress may occur during fabrication. Recommendations for PWHT can be found in the ASME BPV Code Section VIII, Division 1, paragraph USC-56 *Requirements for Post Weld Heat Treatment* and Table UCS-56, *Post Weld Heat Treatment Requirements for Carbon and Low Alloy Steels (*see Figure 11) that requires carbon steel welded joints over 1.5-inch thickness be evenly heated at a uniform rate to a holding temperature of 1,100°F. The holding time can be several hours and is dependant upon the thickness of the steel plate.

**Figure 10**

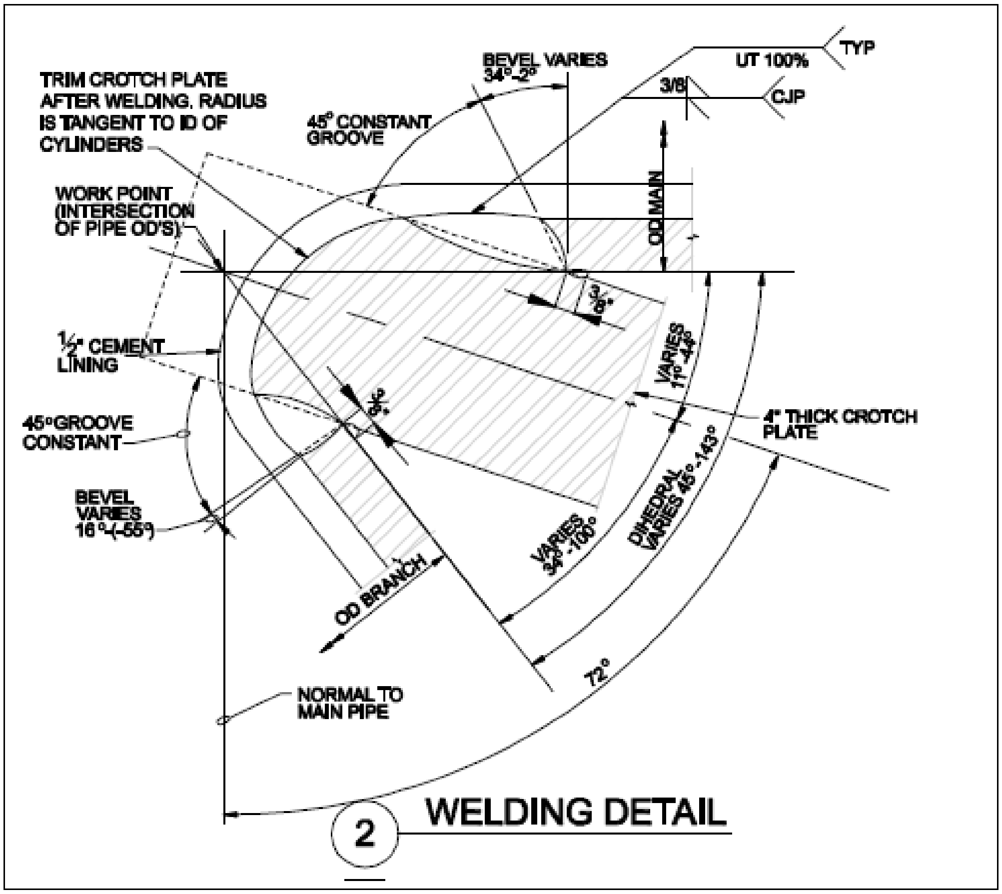



**Figure 11**

TABLE UCS-56
POSTWELD HEAT TREATMENT REQUIREMENTS FOR CARBON AND LOW ALLOY STEELS

| Material | Normal Holding Temperature, °F (°C), Minimum | Minimum Holding Time at Normal Temperature for Nominal Thickness [See UW-40(f)] | | |
|---|---|---|---|---|
| | | Up to 2 in. (50 mm) | Over 2 in. to 5 in. (50 mm to 125 mm) | Over 5 in. (125 mm) |
| P-No. 1 Gr. Nos. 1, 2, 3 | 1,100 (595) | 1 hr/in. (25 mm), 15 min minimum | 2 hr plus 15 min for each additional inch (25 mm) over 2 in. (50 mm) | 2 hr plus 15 min for each additional inch (25 mm) over 2 in. (50 mm) |
| Gr. No. 4 | NA | None | None | None |

NOTES:

(1) When it is impractical to postweld heat treat at the temperature specified in this Table, it is permissible to carry out the postweld heat treatment at lower temperatures for longer periods of time in accordance with Table UCS-56.1.

(2) Postweld heat treatment is mandatory under the following conditions:

(a) for welded joints over 1½ in. (38 mm) nominal thickness;

(b) for welded joints over 1¼ in. (32 mm) nominal thickness through 1½ in. (38 mm) nominal thickness unless preheat is applied at a minimum temperature of 200°F (95°C) during welding;

## Non-Destructive Testing (NDT)

NDT can be an effective tool for verifying weld quality of welded steel wye branches. Nondestructive examination (NDE) of welds is often done by skilled technicians or welding inspectors to obtain information regarding weld quality and to evaluate welds to meet the project requirements. Nondestructive examination does not damage the weld, unlike destructive testing that is often used to qualify welding procedure specifications and welders.

The primary method of nondestructive examination of welds is by visual testing (VT). A certified welding inspector (CWI) is normally required to perform 100 percent VT on all welds. VT is required prior to assembly, during assembly, during welding, and after welding to make sure that the work is done according to project requirements. Acceptance standards for VT are given in the referenced codes: ASME BPVC Section VIII, Div. 1, paragraphs UW-31 through UW-38 provide acceptance of VT, including limits of porosity (pinholes), undercut, cracking (no cracks are permitted) and other welding defects. Other NDE methods in addition to VT include radiographic testing (RT), ultrasonic testing (UT), magnetic particle testing (MT), and liquid penetrant testing (PT).

Discussion of these NDE methods follows. Personnel that perform NDE, other than VT, must be certified to Recommended Practice No. SNT-TC-1A, Level II. Certified welding inspectors (CWI) must meet requirements of AWS QC 1, *Standard for AWS Certification of Welding Inspectors* and must have knowledge of the appropriate welding codes for the work.

Radiographic Testing (RT) is used to verify weld quality by utilizing the penetrating capabilities of X-ray or gamma radiation. RT is well suited for nondestructive examination of complete joint penetration (CJP) butt joints. The weld is exposed to a radioactive source and radiation passes through the part onto photographic film located on the opposite side of weld. The film records an image of the weld created by the radiation. Weld discontinuities (for example cracks, incomplete penetration, slag and porosity) or areas of reduced density will be darker than the surrounding image. The weld discontinuities are compared with code standards to determine weld acceptance. ASME BPVC Section VIII, Div. 1, paragraphs UW-51, *Radiographic and Radioscopic Examination of Welded Joints* provides acceptance of RT.

The limitations of RT include the following:

- Radiation is harmful and special safety precautions are necessary for RT.
- Skilled technicians are required for setup and operation.
- Film interpretation takes much training and experience.
- Access to both front and back sides of the weld are required for RT source and film placement. The shape of the part may make it difficult to produce a meaningful radiograph. Surface discontinuities are difficult to interpret.
- Three RT pictures taken 120 F apart may be required to adequately view pipe welds where double wall exposure techniques are used.
- Discontinuities must be parallel to the beam to be detected; laminations that are planar and located at right angles to the direction of the radiation source cannot be detected with RT; just the opposite is true for UT. Laminations can easily be detected with UT oriented at right angles to the plane of the lamination.

UT is used to verify weld quality by utilizing high frequency sound waves that are pulsed into a weld and the echo that returns can indicate subsurface weld discontinuities. UT is well suited for nondestructive examination of CJP butt joints. Sound waves are directed into the weld via a quartz crystal transducer on a predictable path and the reflected beam can be analyzed to determine location, and sometimes size and shape of weld discontinuities. Weld discontinuities (for example cracks, laminations, incomplete penetration, slag, and porosity) can be detected with UT. The weld discontinuities are compared with Code standards to determine weld quality. ASME BPVC Section VIII, Div. 1, paragraphs UW-53, Technique for Ultrasonic Examination of Welded Joints provides acceptance of UT.

The limitations of UT include the following:

- Transducer must be coupled to surface of part to be inspected.
- Intimate contact between surface of transducer and weld is necessary for signal processing. This requires a couplant, such as glycerin.
- To evaluate weld flaws a calibration block is necessary. The calibration block must be machined with a defect of known size and location.
- UT inspection of groove welds and heat affected zones is normally limited to welded steels of 5/16-inch thickness or greater.
- UT inspection of fillet welds is not practical.
- Results obtained from UT are highly operator dependent; skilled technicians are required for setup and operation.
- Surface discontinuities are difficult to detect.

MT and PT methods are used as enhancements of VT to verify surface or near surface weld quality. MT uses the principal of magnetism to detect discontinuity. Magnetic lines of force (flux) in ferromagnetic material will be distorted by a discontinuity. A magnetic field is applied to weld area and iron powder is sprinkled on the surface. A distinct alignment of the iron powder particles will occur at cracks and other surface defects. However, the orientation of the magnetic lines of flux must be at right angles to the alignment of the flaw for an indication to appear. A complete examination of the weld surface requires moving the magnetic source in several orientations. MT is appropriate for detecting cracks, undercut, slag and other weld flaws. ASME BPVC Section VIII, Div. 1, Appendix 6, *Methods for Magnetic Particle Examination* provides acceptance of MT.

PT is a method that reveals weld discontinuities that are open to the surface. After surface cleaning, liquid penetrant dye, often red in color, is flooded onto weld and allowed to be adsorbed for a period of time. Remaining penetrant is then removed with a clean rag and a developer, often white to contrast with the penetrant, is applied to weld surface. If there are open discontinuities, penetrant will bleed out onto developer to clearly indicate a flaw. ASME BPVC Section VIII, Div. 1, Appendix 8, *Methods for Liquid Penetrant Examination* provides acceptance of PT.

Shop hydrostatic testing should be performed per ASME BPV Code Section VIII, Div. 1, UG-99 at a test pressure of 1.5 times the working pressure so that the fabrication can be installed in the field with confidence that it will perform reliably.

## Summary

Fabrication of wye branches requires a high degree of skill on the part of the fabricator to complete the design and to make it function reliably as a safe, integral part of a conveyance system as intended by the designer. The tools presented in this paper are intended to make both the designer and fabricator's job easier for proper steel selection, geometry, welding, heat treatment, and non-destructive testing. Welding code provisions are a valuable resource and their contribution to steel wye branch fabrication has been demonstrated.

## References

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- ASTM A36, Standard Specification for Carbon Structural Plate
- ASTM A6/A6M, Standard Specification for General Requirements for Rolled Structural Steel Bars, Plates, Shapes, and Sheet Piling
- ASTM A516, Standard Specification for Pressure Vessel Plates, Carbon Steel, for Moderate- and Lower-Temperature Service
- ASTM A20/A20M, Standard Specification for General Requirements for Steel Plates for Pressure Vessels)
- ASTM A770/A770M, Standard Specification for Through-Thickness Tension Testing of Steel Plates for Special Applications
- ASTM A435/A435M, Standard Specification for Straight-Beam Ultrasonic Examination of Steel Plates

American Society of Nondestructive Testing (ASNT): SNT-TC-IA (2006). Personnel Qualification and Certification in Non-destructive Testing.

## Example Problem-Local Dihedral Angle

Given: A 6" OD main pipe is intersected by a 4" OD branch pipe at a 45 degree angle; the branch pipe centerline is offset 1" from main pipe centerline; see Figures 3 & 4

Find: Local dihedral angle at rotation angle $\alpha = 135 \cdot \text{deg}$ on branch pipe

Assume: branch pipe is divided into 16 equal angles; each angle is $\frac{360 \cdot \text{deg}}{16} = 22.50\,\text{deg}$

$i := 7$ work point number $\quad \alpha := (i-1) \cdot \frac{2 \cdot \pi}{16} \quad \alpha = 135.00\,\text{deg}$ rotation angle on branch pipe

$\phi = 45.00\,\text{deg} \quad D_b = 6.00\,\text{in} \quad D_s = 4.00\,\text{in} \quad a = 1.00\,\text{in} \quad r_2 := \frac{D_b}{2} \quad r_2 = 3.00\,\text{in} \quad r_1 := \frac{D_s}{2} \quad r_1 = 2.00\,\text{in}$

$$\text{off} := \frac{\sqrt{\frac{1}{4} \cdot D_b^{\,2} - a^2}}{\sin(\phi)} + \frac{1}{2} \cdot \frac{D_s}{\tan(\phi)} \qquad \text{off} = 6.00\,\text{in}$$

Eqs. 4 distance along branch centerline from intersection of main

$$\text{ctr\_branch} := \begin{pmatrix} \cos(\phi) \cdot \text{off} \\ \sin(\phi) \cdot \text{off} \\ -a \end{pmatrix} \qquad \text{ctr\_branch} = \begin{pmatrix} 4.24 \\ 4.24 \\ -1.00 \end{pmatrix} \text{in}$$

coordinates of branch centerline

$$\text{uub} := \begin{pmatrix} -\cos(\phi) \\ -\sin(\phi) \\ 0 \end{pmatrix} \qquad \text{uub} = \begin{pmatrix} -0.71 \\ -0.71 \\ 0.00 \end{pmatrix}$$

unit vector along branch

$\Delta\alpha := -1 \cdot \text{deg}$

$\alpha + \Delta\alpha = 134.00\,\text{deg}$

$$P_{11} := \begin{bmatrix} \frac{D_s}{2} \cdot \cos(\alpha + \Delta\alpha) \cdot \sin(\phi) \\ -\left(\frac{D_s}{2} \cdot \cos(\alpha + \Delta\alpha)\right) \cdot \cos(\phi) \\ -\left(\frac{D_s}{2} \cdot \sin(\alpha + \Delta\alpha)\right) \end{bmatrix} + \text{ctr\_branch} \qquad P_{11} = \begin{pmatrix} 3.26 \\ 5.23 \\ -2.44 \end{pmatrix} \text{in}$$

point on circle @ end of branch

$$h_1 := \frac{\sqrt{D_b^{\,2} - 4 \cdot a^2} - \sqrt{D_b^{\,2} - 4 \cdot a^2 - 4 \cdot a \cdot D_s \cdot \sin(\alpha + \Delta\alpha) - D_s^{\,2} + D_s^{\,2} \cdot \cos(\alpha + \Delta\alpha)^2} \ldots + D_s \cdot \cos(\phi) - D_s \cdot \cos(\phi) \cdot \cos(\alpha + \Delta\alpha)}{2 \cdot \sin(\phi)} \qquad h_1 = 4.92\,\text{in}$$

Eqs. 2 offset

$$P_{21} := P_{11} + \text{uub} \cdot h_1 \qquad P_{21} = \begin{pmatrix} -0.22 \\ 1.75 \\ -2.44 \end{pmatrix} \text{in}$$

point on intersection (weld) of branch and main

$\Delta\alpha := 0 \cdot \text{deg}$

$\alpha + \Delta\alpha = 135.00\,\text{deg}$

$$P_{12} := \begin{bmatrix} \frac{D_s}{2} \cdot \cos(\alpha + \Delta\alpha) \cdot \sin(\phi) \\ -\left(\frac{D_s}{2} \cdot \cos(\alpha + \Delta\alpha)\right) \cdot \cos(\phi) \\ -\left(\frac{D_s}{2} \cdot \sin(\alpha + \Delta\alpha)\right) \end{bmatrix} + \text{ctr\_branch} \qquad P_{12} = \begin{pmatrix} 3.24 \\ 5.24 \\ -2.41 \end{pmatrix} \text{in}$$

point on circle @ end of branch

$$h_2 := \frac{\sqrt{D_b^{\,2} - 4 \cdot a^2} - \sqrt{D_b^{\,2} - 4 \cdot a^2 - 4 \cdot a \cdot D_s \cdot \sin(\alpha + \Delta\alpha) - D_s^{\,2} + D_s^{\,2} \cdot \cos(\alpha + \Delta\alpha)^2} \ldots + D_s \cdot \cos(\phi) - D_s \cdot \cos(\phi) \cdot \cos(\alpha + \Delta\alpha)}{2 \cdot \sin(\phi)} \qquad h_2 = 4.90\,\text{in}$$

Eqs. 2 offset

$P_{22} := P_{12} + uub \cdot h_2$ $\quad P_{22} = \begin{pmatrix} -0.22 \\ 1.78 \\ -2.41 \end{pmatrix}$ in $\quad$ point on intersection (weld) of branch and main

$\Delta\alpha := 1 \cdot deg$

$\alpha + \Delta\alpha = 136.00\, deg$

$$P_{13} := \begin{bmatrix} \frac{D_s}{2} \cdot \cos(\alpha + \Delta\alpha) \cdot \sin(\phi) \\ -\left(\frac{D_s}{2} \cdot \cos(\alpha + \Delta\alpha)\right) \cdot \cos(\phi) \\ -\left(\frac{D_s}{2} \cdot \sin(\alpha + \Delta\alpha)\right) \end{bmatrix} + ctr\_branch \qquad P_{13} = \begin{pmatrix} 3.23 \\ 5.26 \\ -2.39 \end{pmatrix} \text{in}$$

point on circle @ end of branch

$$h_3 := \frac{\sqrt{D_b^{\,2} - 4 \cdot a^2} - \sqrt{D_b^{\,2} - 4 \cdot a^2 - 4 \cdot a \cdot D_s \cdot \sin(\alpha + \Delta\alpha) - D_s^{\,2} + D_s^{\,2} \cdot \cos(\alpha + \Delta\alpha)^2} \ldots \\ + D_s \cdot \cos(\phi) - D_s \cdot \cos(\phi) \cdot \cos(\alpha + \Delta\alpha)}{2 \cdot \sin(\phi)} \qquad h_3 = 4.87\, \text{in}$$

Eqs. 2 offset

$P_{23} := P_{13} + uub \cdot h_3$ $\quad P_{23} = \begin{pmatrix} -0.22 \\ 1.81 \\ -2.39 \end{pmatrix}$ in $\quad$ point on intersection (weld) of branch and main

$P_3 := P_{22} - \begin{pmatrix} P_{22_1} \\ 0 \\ 0 \end{pmatrix}$ $\quad$ where $P_{22_1} = -0.22\,$in $\quad P_3 = \begin{pmatrix} 0.00 \\ 1.78 \\ -2.41 \end{pmatrix}$ in $\quad$ point on main

$P_4 := P_3$ $\quad P_4 = \begin{pmatrix} 0.00 \\ 1.78 \\ -2.41 \end{pmatrix}$ in $\quad$ vector normal to main

$\Delta\alpha := 0 \cdot deg$

$\alpha + \Delta\alpha = 135.00\, deg$

$$P_5 := \begin{bmatrix} \frac{D_s}{2} \cdot \cos(\alpha + \Delta\alpha) \cdot \sin(\phi) \\ -\left(\frac{D_s}{2} \cdot \cos(\alpha + \Delta\alpha)\right) \cdot \cos(\phi) \\ -\left(\frac{D_s}{2} \cdot \sin(\alpha + \Delta\alpha)\right) \end{bmatrix} \qquad P_5 = \begin{pmatrix} -1.00 \\ 1.00 \\ -1.41 \end{pmatrix} \text{in}$$

vector normal to branch pipe

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$N1 := P_{23} - P_{21}$ $\quad N1 = \begin{pmatrix} -0.00 \\ 0.07 \\ 0.05 \end{pmatrix}$ in $\quad$ approximated weld axis vector

$N2 := P_4$ $\quad N2 = \begin{pmatrix} 0.00 \\ 1.78 \\ -2.41 \end{pmatrix}$ in $\quad$ vector normal to main pipe

$N3 := N1 \times N2$ $\quad N3 = \begin{pmatrix} -0.25 \\ -0.01 \\ -0.01 \end{pmatrix}$ in$^2$ $\quad$ vector normal to weld axis in plane tangent to main pipe

$N4 := P_5$ $\quad N4 = \begin{pmatrix} -1.00 \\ 1.00 \\ -1.41 \end{pmatrix} in$ vector normal to branch pipe

$N5 := N4 \times N1$ $\quad N5 = \begin{pmatrix} 0.14 \\ 0.05 \\ -0.06 \end{pmatrix} in^2$ vector normal to weld axis in plane tangent to branch pipe

## Results

$\psi := acos\left(\frac{N5 \cdot N3}{|N5| \cdot |N3|}\right)$ $\quad \boxed{\psi = 149.98\,deg}$ local dihedral angle between vectors N5 & N3 @ $\alpha = 135\,deg$, work point 7; agrees with graph below that illustrates local dihedral angle for all work points

Local Dihedral Angle Prgm

$$fff = \begin{pmatrix} 1.00 & 48.19 \\ 2.00 & 72.36 \\ 3.00 & 105.80 \\ 4.00 & 142.36 \\ 5.00 & 179.67 \\ 6.00 & 164.18 \\ 7.00 & 149.98 \\ 8.00 & 138.88 \\ 9.00 & 131.81 \\ 10.00 & 128.41 \\ 11.00 & 126.36 \\ 12.00 & 121.38 \\ 13.00 & 109.47 \\ 14.00 & 90.09 \\ 15.00 & 66.57 \\ 16.00 & 47.06 \\ 17.00 & 48.19 \end{pmatrix}$$