

## (Lat,Long) to Distance

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This is the first in a series of Mathcad documents which describes vessel navigation while carrying out search and rescue operations for the Coast Guard. This article discusses how to find the distance between two points of know latitude and longitude. The author will also try and make clear the concepts of great circles and rhumb lines. Later articles will cover the determination of the search datum (the most likely place a missing boat would be) and the search patterns that would be helpful in finding the missing vessel. Since the Coast Guard Auxiliary vessel's operator must plot this data on nautical charts for each SAR (Search and Rescue) case he's assigned, a P.C. can greatly simplify these calculations.

We will now define a system for specifying your position on the Earth. This is the familiar latitude and longitude used on nautical charts. For this document we will assume that the Earth is a sphere of radius  $r_e$

$$r_e \equiv \frac{21600}{2 \cdot \pi} \quad r_e = 3437.747 \quad \text{Nautical Miles}$$

Definitions for the signs of different directions are:

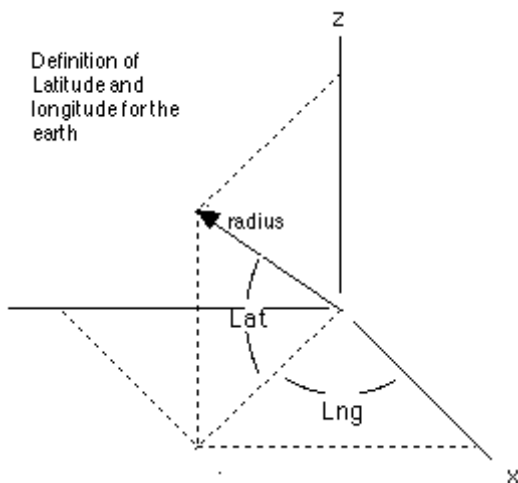
north latitude	$N := 1$	west longitude	$W := 1$
south latitude	$S := -1$	east longitude	$E := -1$

$|\rho| = r_e$  is the simplest vector equation that describes a spherical surface, but the form of this equation using the  $i, j, k$  basis vectors is more useful.

$$\text{Note: } i = (1 \ 0 \ 0)^T \quad j = (0 \ 1 \ 0)^T \quad k = (0 \ 0 \ 1)^T$$

Since the surface of the Earth is two dimensional only two parameters (Lat and Lng) are need to describe any point on its spherical surface. The vector function below, using a modified spherical to rectangular coordinate transform, will generate a position vector for each point on the surface of a sphere of fixed radius. Latitude is the angular position north or south of the equator. Longitude which is the angular position west or east of Greenwich England.

### (radius, Lat, Lng) -> (x, y, z)



Latitude, longitude, and the earth's radius almost from the standard representation of spherical coordinates. The only difference that 0 degrees latitude is at the equator, not the north pole. Also if west longitude is positive, then minus this angle must be used in the transform. LLToR takes care of the conversion. 'R' is the spherical input vector. The first component is the radius of the earth, the second component is latitude, and the third component is longitude. 'units' is the scale factor to convert the angle measurements from degrees to radians. It is one if the angles are already in radians. The function RToLL converts rectangular coordinates back to latitude and longitude.

For radians:  $\text{rad} = 1$  For degrees:  $\text{deg} = 0.017$

For Example, Los Angeles' position is

lat := 34.0·N degrees

lng := 118·W degrees

Convert from latitude and longitude to rectangular coordinates .

$$\text{LLToR}(\mathbf{R}, \text{units}) := \begin{pmatrix} R_0 \cdot \cos(R_1 \cdot \text{units}) \cdot \cos(-R_2 \cdot \text{units}) \\ R_0 \cdot \cos(R_1 \cdot \text{units}) \cdot \sin(-R_2 \cdot \text{units}) \\ R_0 \cdot \sin(R_1 \cdot \text{units}) \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} := \text{LLToR} \left[ \begin{pmatrix} r_e \\ \text{lat} \\ \text{lng} \end{pmatrix}, \text{deg} \right] \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1338.004 \\ -2516.419 \\ 1922.364 \end{pmatrix}$$

$$\mathbf{V}_p(\text{Lat}, \text{Lng}) := \text{LLToR} \left( \begin{pmatrix} r_e & \text{Lat} & \text{Lng} \end{pmatrix}^T, \text{deg} \right) \quad \mathbf{V}_p \text{ is a convenient form of the conversion function.}$$

Convert from rectangular coordinates back to latitude and longitude.  $r_e$  , the radius of the earth is fix

$$\text{RToLL}(\mathbf{V}, \text{units}) := \begin{bmatrix} \frac{\sqrt{(V_0)^2 + (V_1)^2 + (V_2)^2}}{r_e} \\ \frac{\text{asin} \left[ \frac{V_2}{\sqrt{(V_0)^2 + (V_1)^2 + (V_2)^2}} \right]}{\text{units}} \\ \frac{(-\arg(V_0 + V_1 \cdot i))}{\text{units}} \end{bmatrix}$$

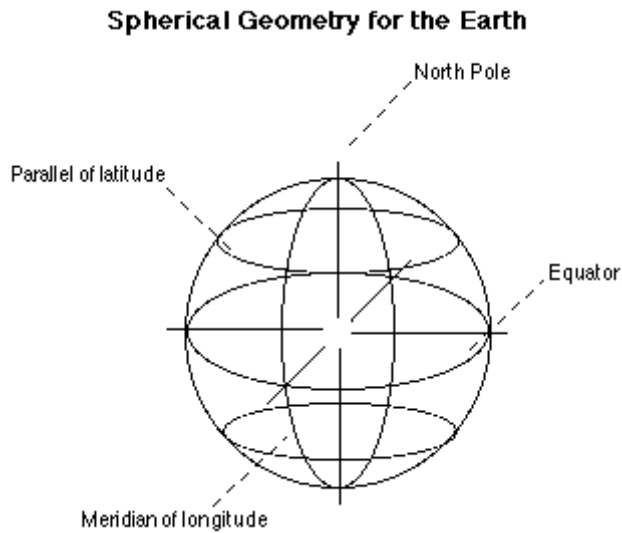
$$\begin{pmatrix} r_{e1} \\ \text{lat} \\ \text{lng} \end{pmatrix} := \text{RToLL} \left[ \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \text{deg} \right] \quad \begin{pmatrix} r_{e1} \\ \text{lat} \\ \text{lng} \end{pmatrix} = \begin{pmatrix} 3437.747 \\ 34 \\ 118 \end{pmatrix}$$

If you examine figure (1) you will see that the semicircles of constant longitude form meridians, which run north and south. If you pair up the two meridians 180 degrees apart they form a great circle. A great circle divides a spherical surface into two equal parts. On a sphere a great circle path is the shortest distance between two point on the surface. On our spherical model of the earth, 1 minute of arc on any great circle is defined as 1 nautical mile. 1/60 of a degree of arc is equal to 1 minute of arc.

The closed curves of constant latitude form the parallels of latitude run east and west around the earth. Except for the equator parallels of latitude are not great circles. The distance around a parallel of latitude is equal to:

$$\Delta x = \frac{60 \cdot \text{nm}}{\text{deg}} \cdot \cos \left( \frac{2 \cdot \pi}{360} \cdot \text{lat} \right) \cdot \Delta \text{lng} \quad \Delta \text{lng} \text{ is the angle in degrees}$$

Figure 1



Below are a couple of functions needed to convert degrees and minutes into degrees and parts of a degree.

$\text{Deg}(d, m) := d + \frac{m}{60}$	Convert degrees & minutes into degrees	definition of a nautical
$\text{DtR}(D) := \frac{2 \cdot \pi}{360} \cdot D$	Convert degrees into radians	$\text{nm} := \frac{6076}{5280} \cdot \text{mi}$
$\text{dtr}(d, m) := \text{DtR}(\text{Deg}(d, m))$	Convert degrees & minutes into radians.	

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Given two points of known latitude and longitude, to calculate the distance between them. First convert the points into position vectors. Then one uses the definition of the dot product to compute the angle between them. Next use Zero and the calculated angle as the limits of integration for the arclength integral to compute the distance between the two points.

### For Example:

Enter (lat,long) positions in degrees and minutes for the two points on the Earth. Convert degrees and minutes into degrees and fractions of a degree for the rest of the calculation. North latitude is positive and west longitude is positive. Note : These two points are just to northeast and southwest of Marin del Rey, Ca.

$$\text{PII0} := (\text{Deg}(33, 58.0) \cdot \text{N} \quad \text{Deg}(118, 28.0) \cdot \text{W})^T \quad \text{PII1} := (\text{Deg}(33, 59.0) \cdot \text{N} \quad \text{Deg}(118, 27.0) \cdot \text{W})^T$$

To calculate the distance between two points on the Earth given as latitude and longitude pairs, you must form the great circle path between those points and measure the distance along that path.

The distance along a great circle is:  $\int_0^{\text{Ang}} r \, d\theta$  where Ang is the angle between the position vectors.

Convert the positions to rectangular coordinate vectors and use the definition of the dot product to calculate the value of the subtended angle.

$$P_{vr0} := V_p(P_{ll0_0}, P_{ll0_1}) \quad P_{vr1} := V_p(P_{ll1_0}, P_{ll1_1})$$

$$\text{ang}(V_0, V_1) := \arccos\left(\frac{V_0 \cdot V_1}{|V_0| \cdot |V_1|}\right) \quad L_h := \text{Re}(\text{ang}(P_{vr0}, P_{vr1})) \quad L_h = 1.2028871063 \times 10^{-4} \pi$$

The distance between  $P_{ll0}^T = (33.967 \ 118.467)$  and  $P_{ll1}^T = (33.983 \ 118.45)$  is:

$$\text{Dist} := \int_0^{L_h} r_e d\Theta \quad \text{Dist} = 1.299 \text{ nautical miles}$$

The measured distance on the 18744 nautical chart between Pll0 and Pll1 is 1.3 nautical miles, Dist is in excellent agreement with reality.

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Dist(p0,p1) nicely packages all the above calculation in one general purpose function.

$$\text{Dist}(p_0, p_1) := \int_0^{\text{Re}(\text{ang}(V_p(p_{0_0}, p_{0_1}), V_p(p_{1_0}, p_{1_1})))} r_e d\Theta \quad \text{Dist}(P_{ll0}, P_{ll1}) = 1.299$$

To save time use the maple engine to symbolically solve the arclength integral and define a faster version of Dist(p0,p1). Since we are assuming the Earth is a spherical, this is an easy integral to evaluate.

$$\int_0^{L_h} r_e d\Theta \quad \text{yields} \quad r_e \cdot L_h$$

Below is the new version of Dist, which doesn't need to evaluate an integral each time the function is called. p0 and p1 are the positions of the two points on the earth's surface. They are two element vectors. The first component is the latitude in degrees and the second is the longitude in degrees.

$$P_{ll0}^T = (33.967 \ 118.467) \quad P_{ll1}^T = (33.983 \ 118.45)$$

$$\text{Dist}(p_0, p_1) := r_e \cdot \text{Re}(\text{ang}(V_p(p_{0_0}, p_{0_1}), V_p(p_{1_0}, p_{1_1})))$$

$$\text{Dist}(P_{ll0}, P_{ll1}) = 1.299$$


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The Dist function allows us to calculate the distance between any two points on the earth given their latitude and longitude. This path is along a great circle so it is the shortest possible distance between two point on the earth's surface. Unfortunately a great circle isn't the most commonly used course for short range navigation. To sail on a great circle one's compass course changes continuously, the Rhumb line course solves this difficulty by sailing a constant compass course. The rhumb line is the most commonly used course for short range navigation, useful on the order of a couple hundred nautical miles. On a Mercator chart a rhumb line is a straight line and a great circle is a curve.

The Rhumb line or Loxodrome or Spherical Helix, is usually defined as the curve cutting the meridians of a sphere at a constant angle. The curve was first conceived by Pedro Nunes in 1550. If  $\beta$  is the constant angle of meridian crossing, and Lat and Lng are the Latitude and Longitude of a point on the loxodrome, then its equations may be written as:

$$\begin{aligned} 1. \quad x &= r_e \cdot \cos(\text{Lat}) \cdot \cos(-\text{Lng}) \\ 2. \quad y &= r_e \cdot \cos(\text{Lat}) \cdot \sin(-\text{Lng}) \\ 3. \quad z &= r_e \cdot \sin(\text{Lat}) \end{aligned} \quad \text{and} \quad 4. \quad \text{Lng} = \tan(\beta) \cdot \ln \left( \tan \left( \frac{\frac{\pi}{2} - \text{lat}}{2} \right) \right)$$

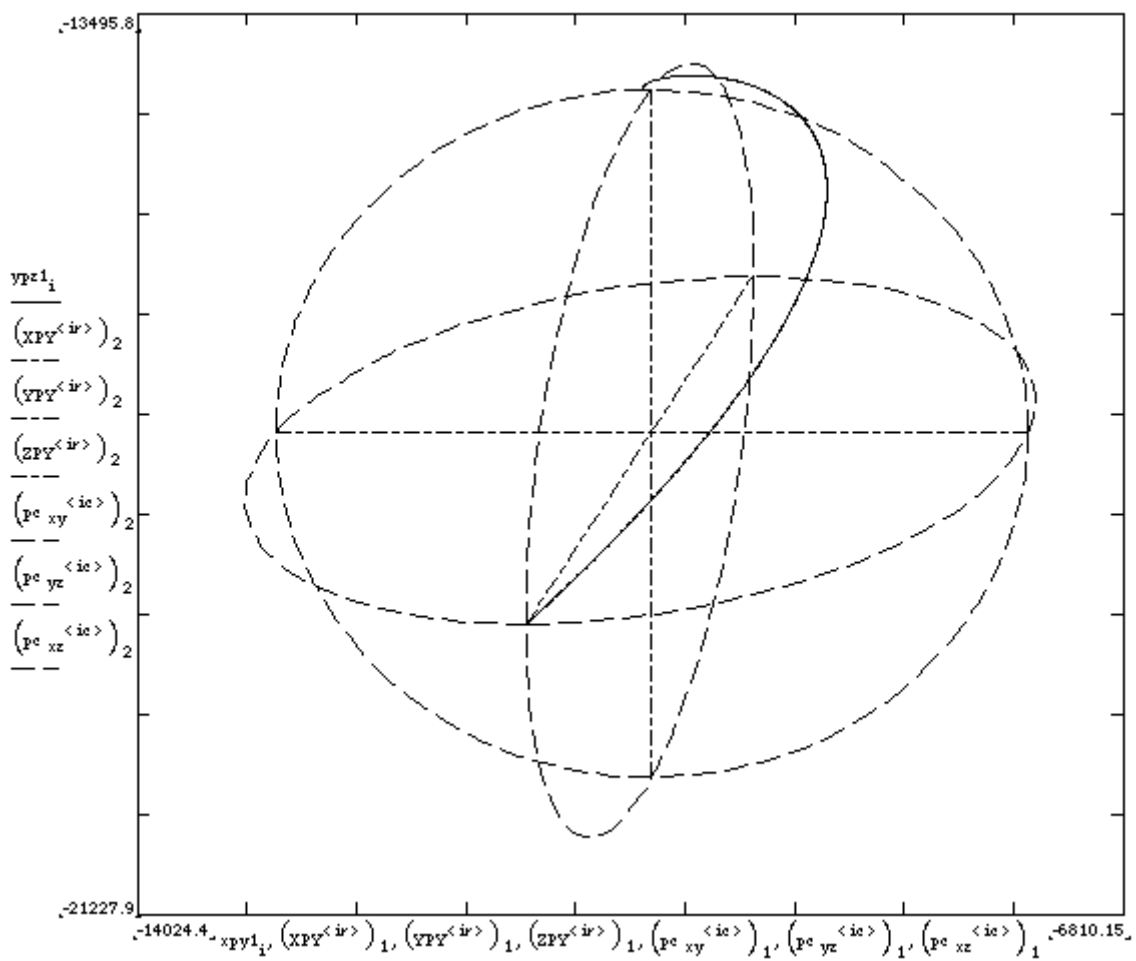
The first three equations map latitude and longitude into (x,y,z) position on the earth. The fourth equation describes longitude as a function of latitude, generating the loxodrome or rhumb line. This curve is the path a vessel follows when it proceeds with a fixed course. The vessel's course is given by  $\beta$ . If navigating in a small area, such as, 10 nautical miles by 10 nautical miles, the difference between a rhumb line and a great circle is insignificant. Below is a plot of a rhumb line or loxodrome sailed at a course of 045 degrees, from a latitude of 0 degrees and a longitude of 0 degrees.

$$\text{Lng}(\text{lat}, \beta) := \tan(\beta) \cdot \ln \left( \tan \left( \frac{\frac{\pi}{2} - \text{lat}}{2} \right) \right) \quad \beta := \frac{\pi}{4}$$

Calculate  $\text{lim} := 600$  points on the loxodrome from latitude  $l_l := 0.0$  to  $l_h := \frac{1}{2} \cdot \pi - .001 \cdot \pi$

This curve is viewed from 10 earth radii out on the X-axis. The view point is also 3 earth radii to the right of the origin and 5 earth radii up from the origin. See (4), Loxodrome generator for the method of drawing the below 3d curve.

**Prospective projection of Loxodrome  
with  $\beta = 45$  degrees on a sphere of radius  $r_e$**



## To Find the distance along a loxodrome we use the following:

From the definition of the loxodrome and the mapping onto a sphere we obtain:

$$\begin{aligned} xsp(Lat, Lng) &:= r_e \cdot \cos(Lat) \cdot \cos(-Lng) \\ ysp(Lat, Lng) &:= r_e \cdot \cos(Lat) \cdot \sin(-Lng) \quad \text{and} \quad Lng(Lat, \beta) := \tan(\beta) \cdot \ln \left( \tan \left( \frac{\frac{\pi}{2} - Lat}{2} \right) \right) \\ zsp(Lat, Lng) &:= r_e \cdot \sin(Lat) \end{aligned}$$

Our basis vectors are:

$$e_x := (1 \ 0 \ 0)^T \quad e_y := (0 \ 1 \ 0)^T \quad e_z := (0 \ 0 \ 1)^T$$

The vector equation of the loxodrome as a function of latitude and course angle.

$$\underline{R}(Lat, \beta) := xsp(Lat, Lng(Lat, \beta)) \cdot e_x + ysp(Lat, Lng(Lat, \beta)) \cdot e_y + zsp(Lat, Lng(Lat, \beta)) \cdot e_z$$

From vector calculus, we use the definition of arc-length of a curve to compute the distance along a loxodrome as a function of latitude and course angle.

$$\begin{aligned} d\mathbf{r} &= e_x \cdot dx + e_y \cdot dy + e_z \cdot dz \\ ds = |d\mathbf{r}| &= \sqrt{dx^2 + dy^2 + dz^2} \quad s = \int_{\phi_1}^{\phi_2} \sqrt{\left(\frac{dx}{d\phi}\right)^2 + \left(\frac{dy}{d\phi}\right)^2 + \left(\frac{dz}{d\phi}\right)^2} d\phi \end{aligned}$$

Use the symbolic processor to substitute the equations for the loxodrome into the definition for arc-length of a vector function. First substitute equation 4 into equation 1 and equation 2, to get x,y,z as a function of lat and  $\beta$

$$\begin{aligned} x &= r_e \cdot \cos(lat) \cdot \cos \left( \tan(\beta) \cdot \ln \left( \cot \left( \frac{1}{4} \cdot \pi + \frac{1}{2} \cdot lat \right) \right) \right) \quad \text{and} \quad z = r_e \cdot \sin(lat) \\ y &= -r_e \cdot \cos(lat) \cdot \sin \left( \tan(\beta) \cdot \ln \left( \cot \left( \frac{1}{4} \cdot \pi + \frac{1}{2} \cdot lat \right) \right) \right) \end{aligned}$$

Differentiate symbolically to derive Dx, Dy, and Dz which are the derivatives of x, y, and z with respect to lat

$$\begin{aligned} Dx_{lat}(lat, \beta) &:= -r_e \cdot \sin(lat) \cdot \cos \left( \tan(\beta) \cdot \ln \left( \cot \left( \frac{1}{4} \cdot \pi + \frac{1}{2} \cdot lat \right) \right) \right) \dots \\ &\quad + 0 - r_e \cdot \cos(lat) \cdot \sin \left( \tan(\beta) \cdot \ln \left( \cot \left( \frac{1}{4} \cdot \pi + \frac{1}{2} \cdot lat \right) \right) \right) \cdot \tan(\beta) \cdot \frac{\left( \frac{-1}{2} - \frac{1}{2} \cdot \cot \left( \frac{1}{4} \cdot \pi + \frac{1}{2} \cdot lat \right) \right)}{\cot \left( \frac{1}{4} \cdot \pi + \frac{1}{2} \cdot lat \right)} \\ Dy_{lat}(lat, \beta) &:= r_e \cdot \sin(lat) \cdot \sin \left( \tan(\beta) \cdot \ln \left( \cot \left( \frac{1}{4} \cdot \pi + \frac{1}{2} \cdot lat \right) \right) \right) \dots \\ &\quad + 0 - r_e \cdot \cos(lat) \cdot \cos \left( \tan(\beta) \cdot \ln \left( \cot \left( \frac{1}{4} \cdot \pi + \frac{1}{2} \cdot lat \right) \right) \right) \cdot \tan(\beta) \cdot \frac{\left( \frac{-1}{2} - \frac{1}{2} \cdot \cot \left( \frac{1}{4} \cdot \pi + \frac{1}{2} \cdot lat \right) \right)}{\cot \left( \frac{1}{4} \cdot \pi + \frac{1}{2} \cdot lat \right)} \\ Dz_{lat}(lat, \beta) &:= r_e \cdot \cos(lat) \end{aligned}$$

From substitution of the derivative functions into the definition of arc-length we obtain:

$$S(lh, ll, \beta) := \int_{ll}^{lh} \sqrt{Dx_{lat}(lat, \beta)^2 + Dy_{lat}(lat, \beta)^2 + Dz_{lat}(lat, \beta)^2} dlat$$

For my two previously defined positions in Marina del Rey, the rhumb line distance is calculated below:

$$PII0^T = (33.967 \quad 118.467) \quad PII1^T = (33.983 \quad 118.45)$$

Rhumb line distance at 040 deg course is:  $S0(PII1_0 \cdot \text{deg}, PII0_0 \cdot \text{deg}, 40 \cdot \text{deg}) = 1.305$  nautical miles

The great circle distance is  $\text{Dist}(PII0, PII1) = 1.299$  nautical miles

As you can see the rhumb line course is about 38 feet farther then the great circle course.

$$S0(PII1_0 \cdot \text{deg}, PII0_0 \cdot \text{deg}, 40 \cdot \text{deg}) - \text{Dist}(PII0, PII1) = 6.289 \times 10^{-3} \text{ nautical miles}$$

As a check, lets see if a purely numeric results agree with the above calculation of arc- length along the loxodrome.

$$S(lh, ll, \beta) := \int_{ll}^{lh} \sqrt{\left(\frac{d}{dLat} xsp(Lat, Lng(Lat, \beta))\right)^2 + \left(\frac{d}{dLat} ysp(Lat, Lng(Lat, \beta))\right)^2 + \left(\frac{d}{dLat} zsp(Lat, Lng(Lat, \beta))\right)^2} dLat$$

$$S(PII1_0 \cdot \text{deg}, PII0_0 \cdot \text{deg}, 40 \cdot \text{deg}) = 1.305 \quad \text{nautical miles} \quad \text{Good!!! both answers agree.}$$

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Now there is one more point to cover in this document. When your vessel is at a known position and you are asked to proceed for a given distance on a given course, what is your the latitude and longitude of your final position? To find your final position, you'll want to solve for the upper limit of integration in the S or S0 function. This limit is the latitude of the final position you are seeking. Next you substitute that latitude in the definition of the loxodrome to calculate the longitude of your final position.

The equation for the loxodrome presented above is only one member of a family of curves with the same  $\beta$  parameter, that covers the entire surface of the earth. The equation presented here is for the loxodrome passing through the point with latitude and longitude equal to zero degrees. To make sure that your longitude is from the correct loxodrome, take the difference initial and final longitudes on the 0,0 loxodrome and add it to the known longitude of your initial position.

For this example we will again use the two points at Marina del Rey.

The initial position is:  $PII0^T = (33.967 \ 118.467)$  degrees  $lat_i := PII0_0$

The course and distance are:  $C := 040 \cdot \text{deg}$  and  $S_d := 1.305$  nautical miles

set the initial guess for the final latitude to:  $lat_f := 0$

Given  $S_d = S0(lat_f \cdot \text{deg}, lat_i \cdot \text{deg}, C)$

$$lat_f := \text{Find}(lat_f)$$

The calculated final latitude is:  $lat_f = 33.983$  degrees

which agrees with the known latitude of the final position :  $PII1_0 = 33.983$

Now lets calculate the final longitude. The initial longitude is :  $lng_i := PII0_1$

The calculated longitudes on the 0,0 loxodrome are:

$$lg_i := \frac{\text{Lng}(lat_i \cdot \text{deg}, C)}{\text{deg}} \quad lg_f := \frac{\text{Lng}(lat_f \cdot \text{deg}, C)}{\text{deg}}$$

The corrected final longitude is:

$$lng_f := lng_i + (lg_f - lg_i) \quad lng_f = 118.45 \text{ degrees}$$

Which is in excellent agreement with  $PII1_1 = 118.45$

The above calculation of your final position, given your initial position and course and distance would be better suited to a parametric form of the Loxodrome and its arc-length. The parametric form of these equations would have no trouble when the change in latitude is zero between the initial and final positions. Also the solution to the above calculation is an easily plotted on a Mercator chart.

## Loxodrome Generator

The equation below describes longitude as a function of latitude, for a loxodrome. Mapping this function onto a sphere of the radius of the earth generates the loxodrome rhumb line. This curve is the path that a vessel follows when it proceeds with a constant speed and a fix course. The vessels' course is give by  $\beta$ . Below are different plots of a rhumb line or loxodrome sailed at a course of 045 degrees, from a position of lat = 0 deg lng = 0 deg.

$$\text{lng}(\text{lat}, \beta) := \tan(\beta) \cdot \ln \left( \tan \left( \frac{\frac{\pi}{2} - \text{lat}}{2} \right) \right) \quad \beta := 45 \cdot \text{deg}$$

Calculate  $\text{lim} := 600$  points on the loxodrome from latitude  $l_1 := 0.0$  to  $l_h := \frac{1}{2} \cdot \pi - .001 \cdot \pi$

$$\text{delta} := \frac{l_h - l_1}{\text{lim} - 1} \quad i := 0.. \text{lim} - 1$$

Calculate vector of latitudes

$$l_i := l_1 + i \cdot \text{delta}$$

Calculate vector of longitudes for the loxodrome

$$\text{lng}_i := \text{lng}(l_i, \beta)$$

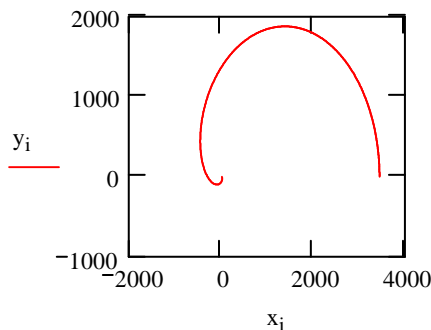
The radius of the earth is:  $r_e \equiv \frac{21600}{2 \cdot \pi}$   $r_e = 3437.747$  Nautical Miles

Map the curve onto a sphere ( the earth ) using rectangular coordinates.

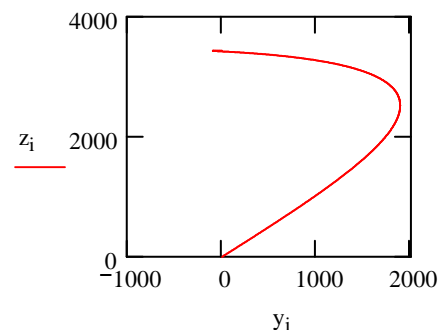
$$x_i := r_e \cdot \cos(l_i) \cdot \cos(-\text{lng}_i) \quad y_i := r_e \cdot \cos(l_i) \cdot \sin(-\text{lng}_i) \quad z_i := r_e \cdot \sin(l_i)$$

## Axonometric Projections of the loxodrome

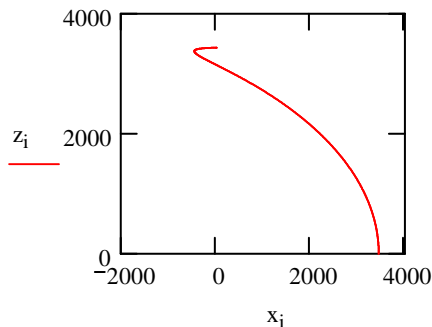
Projection to the X,Y plane



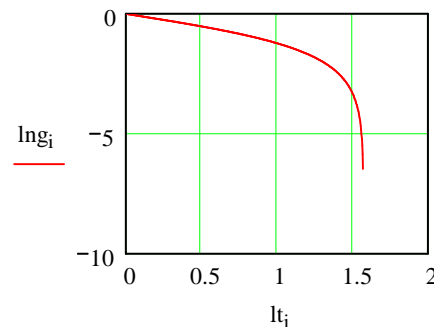
Projection to the Z,Y plane



Projection to the Z,X plane



Longitude vs Latitude



Now we will define the prospective projection along the x axis with the eye at the point :  $(e_x, e_y, e_z)$   
Using Homogeneous coordinates and matrix multiplication obtain :

$$(x \ y \ z \ 1) \cdot \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -e_y & -e_z & 1 \end{pmatrix} \text{ yields } \begin{bmatrix} 0 & y - e_y & z - e_z & \frac{(-x + e_x)}{e_x} \end{bmatrix} \quad e_x \text{ is the eye distance or the projection axis}$$

Normalize the homogeneous vector representing of the point projected into the  $x = 0$  plane

$$\frac{\begin{bmatrix} 0 & y - e_y & z - e_z & \frac{(-x + e_x)}{e_x} \end{bmatrix}}{\frac{(-x + e_x)}{e_x}} \text{ yields } \begin{bmatrix} 0 & \frac{1}{(-x + e_x)} \cdot e_x \cdot (y - e_y) & \frac{1}{(-x + e_x)} \cdot e_x \cdot (z - e_z) & 1 \end{bmatrix}$$

After normalization yields

$$\text{projl}_x(x, y, z, e_x, e_y, e_z) := \begin{bmatrix} 0 & \frac{1}{(-x + e_x)} \cdot e_x \cdot (y - e_y) & \frac{1}{(-x + e_x)} \cdot e_x \cdot (z - e_z) & 1 \end{bmatrix}$$

Define A per coordinate from of the transform is:  $\text{Projl}_x(\xi, x, e_x, e_y) := \frac{1}{(-x + e_x)} \cdot e_x \cdot (\xi - e_y)$

Set the eye position to  $e_x \equiv 10 \cdot r_e$   $e_y \equiv 3 \cdot r_e$   $e_z \equiv 5 \cdot r_e$

project the loxodrome from 3-space to the 2d plot  $\text{xpyl}_i := \text{Projl}_x(y_i, x_i, e_x, e_y)$

$$\text{ypzl}_i := \text{Projl}_x(z_i, x_i, e_x, e_z)$$

Now generate axes for the plot

$$\text{xaxis} := \begin{pmatrix} r_e & 0 & 0 & 1 \\ -r_e & 0 & 0 & 1 \end{pmatrix} \quad \text{yaxis} := \begin{pmatrix} 0 & r_e & 0 & 1 \\ 0 & -r_e & 0 & 1 \end{pmatrix} \quad \text{zaxis} := \begin{pmatrix} 0 & 0 & r_e & 1 \\ 0 & 0 & -r_e & 1 \end{pmatrix}$$

$$\text{Ax} := \text{rows}(\text{xaxis}) \quad \text{ir} := 0.. \text{Ax} - 1$$

$$\text{XPY}^{\langle \text{ir} \rangle} := \text{projl}_x(\text{xaxis}_{\text{ir},0}, \text{xaxis}_{\text{ir},1}, \text{xaxis}_{\text{ir},2}, e_x, e_y, e_z)^T$$

$$\text{YPY}^{\langle \text{ir} \rangle} := \text{projl}_x(\text{yaxis}_{\text{ir},0}, \text{yaxis}_{\text{ir},1}, \text{yaxis}_{\text{ir},2}, e_x, e_y, e_z)^T$$

$$\text{ZPY}^{\langle \text{ir} \rangle} := \text{projl}_x(\text{zaxis}_{\text{ir},0}, \text{zaxis}_{\text{ir},1}, \text{zaxis}_{\text{ir},2}, e_x, e_y, e_z)^T$$

$$\text{iclim} := 50 \qquad \Phi_1 := 0 \qquad \Phi_h := 2 \cdot \pi \qquad \Delta \Phi := \frac{\Phi_h - \Phi_1}{\text{iclim} - 1}$$

$$\text{ic} := 0.. \text{iclim} - 1 \qquad \Phi_{\text{ic}} := \Phi_1 + \Delta \Phi \cdot \text{ic}$$

$$c_{xy}^{\langle \text{ic} \rangle} := \left( r_e \cdot \cos(\Phi_{\text{ic}}) \quad r_e \cdot \sin(\Phi_{\text{ic}}) \quad 0 \quad 1 \right)^T$$

$$c_{yz}^{\langle \text{ic} \rangle} := \left( 0 \quad r_e \cdot \cos(\Phi_{\text{ic}}) \quad r_e \cdot \sin(\Phi_{\text{ic}}) \quad 1 \right)^T$$

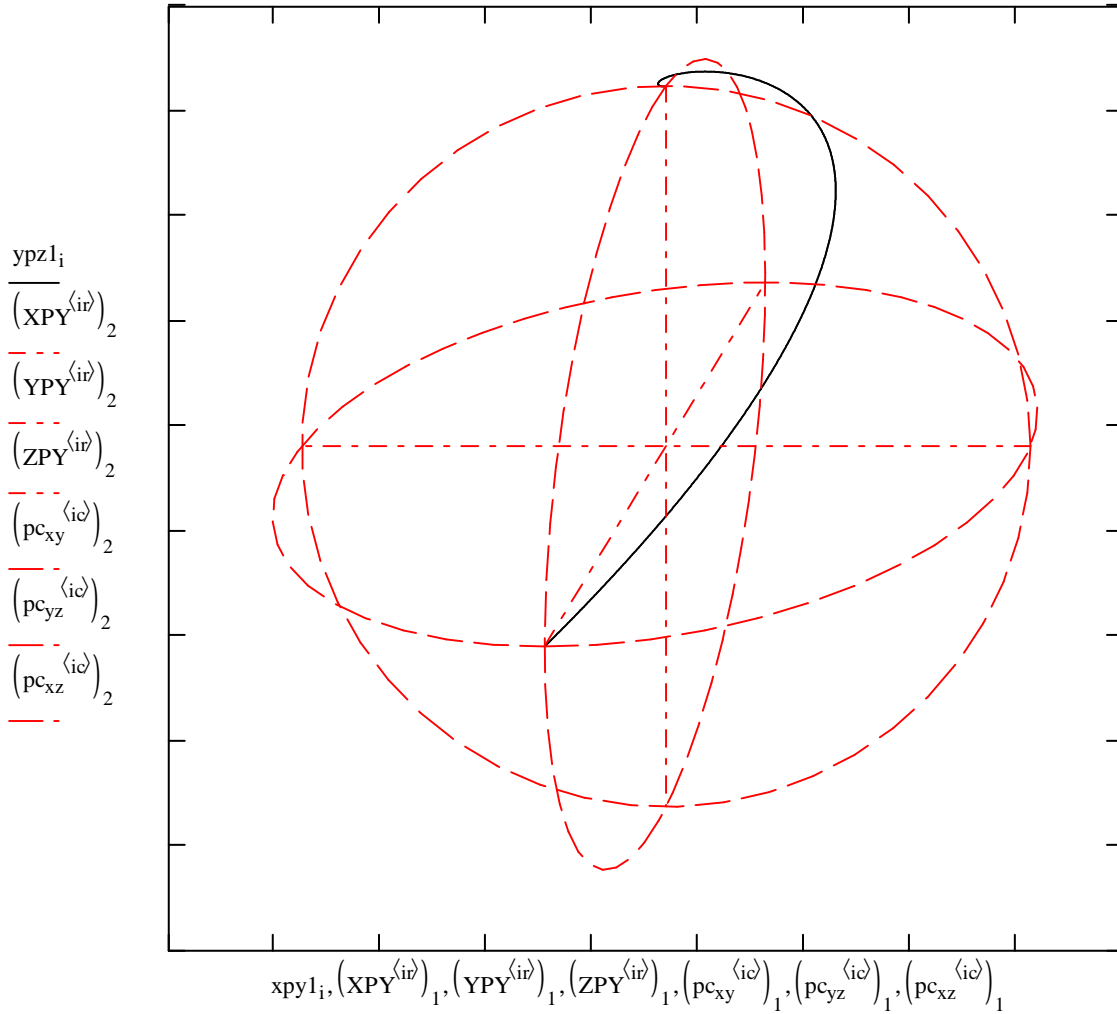
$$c_{xz}^{\langle \text{ic} \rangle} := \left( r_e \cdot \sin(\Phi_{\text{ic}}) \quad 0 \quad r_e \cdot \cos(\Phi_{\text{ic}}) \quad 1 \right)^T$$

$$pc_{xy}^{\langle \text{ic} \rangle} := \text{projl}_x \left( c_{xy_{0,\text{ic}}}, c_{xy_{1,\text{ic}}}, c_{xy_{2,\text{ic}}}, e_x, e_y, e_z \right)^T$$

$$pc_{yz}^{\langle \text{ic} \rangle} := \text{projl}_x \left( c_{yz_{0,\text{ic}}}, c_{yz_{1,\text{ic}}}, c_{yz_{2,\text{ic}}}, e_x, e_y, e_z \right)^T$$

$$pc_{xz}^{\langle \text{ic} \rangle} := \text{projl}_x \left( c_{xz_{0,\text{ic}}}, c_{xz_{1,\text{ic}}}, c_{xz_{2,\text{ic}}}, e_x, e_y, e_z \right)^T$$

**Prospective projection of Loxodrome  
with  $\beta = 45$  degrees on a sphere of radius  $r_e$**



**References**

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