Simple Design Charts for Helicoidal Stair Slabs with Intermediate Landings

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Helicoidal staircases are becoming popular among architects because of their grand appearances. While design charts for simple helicoidal stair slabs are available in design handbooks, no such design aid is available for fixed ended helicoidal slabs with intermediate landings. This paper attempts to propose such design charts. The theoretical background arises from Solanki's (1986) work, which is based on the strain energy method. Because of the symmetric nature of the slab, four of the six stress resultants at mid-span become zero. Thus, if the two redundants at mid-span can be suggested by a design chart, rest of the analysis becomes a statically determinate problem. Based on the analytical work, charts have been proposed to determine these mid-span redundants. Different geometric parameters have been considered and non-dimensional parameters have been used to derive the charts. It is expected that use of the design charts will ease the design engineers' workload.

Keywords: Design charts; Helicoidal stairs

NOTATIONS

b	:	width of the stair slab
EI	:	flexural rigidity about horizontal axis through mid-point
F	:	radial horizontal shear force
GJ	:	torsional rigidity
H	:	radial horizontal shear force at mid-span (redundant)
Ht	:	height of the helicoid
h	:	waist thickness of stair slab
I_h	:	moment of inertia about a vertical axis through mid-point
K	:	ratio of flexural to torsional rigidity
M	:	vertical moment at mid-span (redundant)
M_h	:	lateral moment
M_{sup}	:	vertical moment at support
M_v	:	vertical moment
N	:	thrust
R, R_2	:	centreline radius on horizontal projection
R_i	:	inner radius on horizontal projection
R_o	:	outer radius on horizontal projection
R_1	:	radius of centreline of load
T	:	torsion
V	:	lateral shear force
w	:	dead load and live load per unit length of span, measured along the longitudinal centreline of the plan projection
α	:	slope of the helix contained within the helicoid at radius R
2β	:	total central angle subtended on horizontal projection
θ	:	angular distance from mid-span on a horizontal plane
θ'	:	angle measured from bottom support towards top on a horizontal plane
ϕ	:	angle subtended at the centre by half landing

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INTRODUCTION

One of the most important functional elements of a building, be it residential or commercial, high or low rise, is the stair. Depending on the architectural forms, there may be different types of stairs, such as: simple straight stair, dog-legged stair, saw-tooth/slabless stair, free standing stair, and helicoidal stair. Among these, the helicoidal stair has a grand appearance and is increasingly getting popular among architects.

However, due to the complex geometrical configuration, the analysis and design of helicoidal stair slabs are more difficult than simple type of stairs. The degree of difficulty is further enhanced for helicoidal stairs with an intermediate landing. While design charts for helicoidal stairs are available in literature[1,2] no such aids are found for helicoidal stairs with

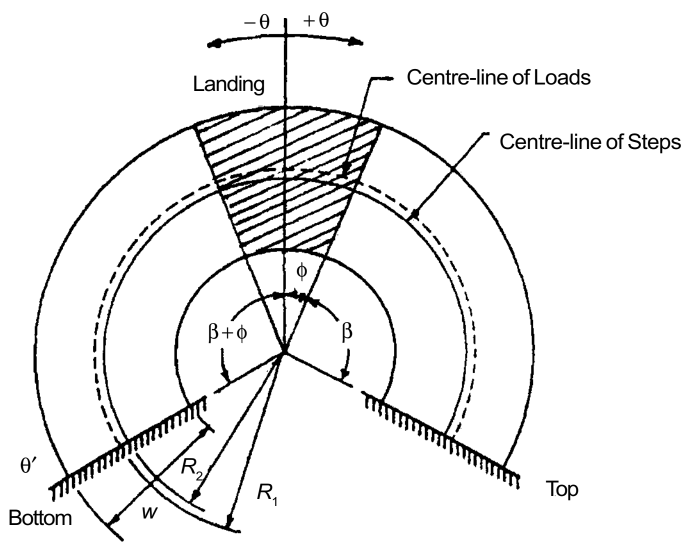

Figure 1 Plan of a helicoidal stair slab with intermediate landinc

intermediate landing. This necessitates the development of a simple design chart and design procedure for this type of elements.

GEOMETRY OF THE HELICOIDAL SLAB WITH INTERMEDIATE LANDING

The geometry of a circular helicoidal stair slab with an intermediate landing at mid-span can be defined as (Figure 1).

$$\alpha' = \tan^{-1} \frac{Ht}{R\,2\beta} \quad (1)$$

The coordinates at the mid-surface can be expressed as:

$$x = R\cos\theta' \quad (2)$$

$$y = R\sin\theta' \quad (3)$$

$$z = \theta' R\tan\alpha' \qquad 0 \le \theta' \le \beta \quad (4)$$

$$z = \beta R\tan\alpha' \qquad \beta \le \theta' \le \beta + 2\phi \quad (5)$$

$$z = (\theta' - 2\phi) R\tan\alpha' \qquad \beta + 2\phi \le \theta' \le 2(\beta + \phi) \quad (6)$$

LOADING AND BOUNDARY CONDITION

The helicoidal stair slab has its self weight. This dead load (self weight) is assumed to be uniformly distributed. In addition, the slab is subjected to live load. The live load could be uniformly distributed over the surface, point loads, line loads or symmetrical loads about the central axis of the slab. However, in this work the live load is considered uniformly distributed over the entire surface on the horizontal projection of the stair.

The ends of the slab may be fixed, partially fixed or hinged. The slab fixed at both ends is six degree indeterminate; there are six equilibrium equations and twelve unknown reactions. Helical slab with one end fixed and one end hinged is indeterminate to third degree. The stair slab here is considered fixed at its ends in all directions.

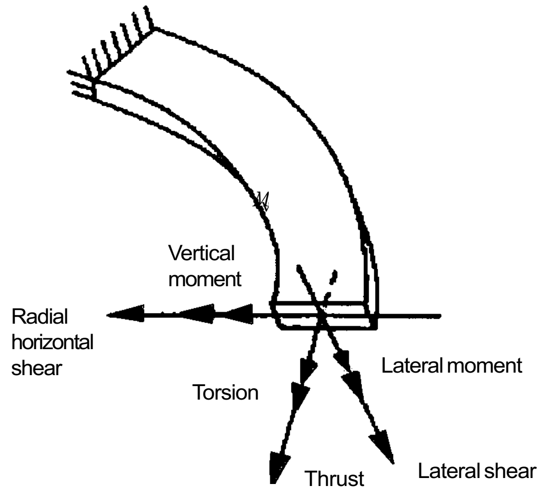

Figure 2 Stress resultants in a helicoidal stair slab

STRESS RESULTANTS

Six stress resultants are available at any section of a space structure. Helicoidal slab, being a space structure, also has six stress resultants at any cross section, which are (a) vertical moment (M_v), (b) lateral moment (M_h), (c) torsion (T), (d) thrust (N), (e) lateral shear force (V), and (f) radial horizontal shear force (F).

The positive directions of these stress resultants have been illustrated in Figure 2.

REVIEW OF PAST WORKS

The complex geometry of a regular helicoidal stair slab has made its analysis quite difficult. The introduction of an intermediate landing further complicates the situation. For the helicoidal stair slabs without landings[3-6], considered the helicoid as a three dimensional (3-d) helical girder. Here, the helicoid is reduced to its elastic line having the same stiffness as that of the original structure. But this simplification neglects the slab action of helicoid and also assumes that the bending stiffness and torsional stiffness of a warped girder are the same as those of a straight beam.

Santathadaporn and Cusens[1] presented 36 design charts for helical stairs with a wide range of geometric parameters. Based on this work, four design charts were compiled in a modified form in the design handbook by Reynolds and Steedman[2]. These design charts now stand as 'helical girder solution' for helicoidal stairs. Evidently, the helical girder solution fail to take into account the 3-d characteristics of helicoid and its inherent structural efficiency.

Arya and Prakash[7] attempted to analyse the case of the helicoidal stairs with intermediate landing. They used flexibility approach to analyse internal forces due to dead and live loads in fixed ended circular stairs having an intermediate landing. Like Scordeilis, they treated the structure as a linearly elastic member in space defined by its longitudinal centroidal axis. Influence lines were drawn at various cross sections for all the six stress resultants found at such sections for unit vertical load and unit moment about the axis of the structure.

Critical positions of loads were determined to obtain the maximum values of the internal forces. From this analysis they suggested some generalized behavior of helicoidal star slabs with intermediate landings.

Solanki[8] analysed the problem of intermediate landing using energy method to find the two unknown redundants at the mid-span section. Other redundants at mid-span have zero values because of symmetry of geometry and loads. He proposed two equations, the simultaneous solution of which gives the values of the redundants. Solanki's findings were similar to those observed by Arya and Prakash. Simply supported or pin jointed helicoids could also be analysed using this method. However, both Arya and Prakash, as well as Solanki did not attempt to develop any simple design methods for helicoidal stair slabs with intermediate landings.

ANALYSIS

Assumptions

To facilitate the analysis procedure, the assumptions made during the analysis were:

(i) Deformation due to shear and direct forces, being small in comparison to the deformations caused by twisting and bending moments, are neglected.

(ii) The cross section is symmetric about the two principal axes of the section.

(iii) The angle subtended at centre by the landing is small compared to the total angle subtended by the stair at the centre.

(iv) The moment of inertia of the helicoidal slab section with respect to a horizontal radial axis is negligible as compared to the moment of inertia with respect to the axis perpendicular to it.

Stress Resultants in the Helicoidal Stair with Intermediate Landings

The analysis method followed here begins from Solanki's Approach. Because of symmetry in geometry and loading, four of the six redundants at the mid-span of a helicoidal stair slab with intermediate landing become zero. Only two, vertical moment *M* and radial horizontal shear *H* remain to be calculated (Figures 1 and 3).

Solanki assumed that at the landing level the bending and torsional moments along the upper half of the stair slab to be:

- Vertical Moment:

$$M_v = M\cos\theta - wR_1^2\,(1-\cos\theta) \tag{7}$$

- Lateral Moment:

$$M_h = -HR_2\sin\theta \tag{8}$$

- Torsion:

$$T = M\sin\theta + wR_1^2\sin\theta - wR_1R_2\,\theta \tag{9}$$

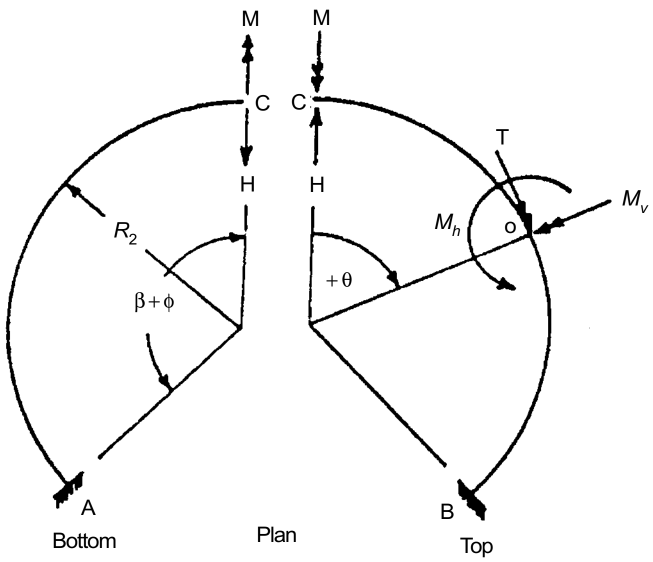

Figure 3 Stress resultants and mid-span redundants

For the flight the moments and other forces were assumed to follow Morgan's[5] derivations for slabs without landing.

- Vertical moment:

$$M_v = M\cos\theta + HR_2\theta\tan\alpha\sin\theta - wR_1^2\,(1-\cos\theta) \tag{10}$$

- Lateral moment:

$$M_h = M\sin\theta\sin\alpha - HR_2\theta\tan\alpha\cos\theta\sin\alpha - HR_2\sin\theta\cos\alpha + \left(wR_1^2\sin\theta - wR_1R\theta\right)\sin\alpha \tag{11}$$

- Torsion:

$$T = (M\sin\theta - HR_2\,\theta\tan\alpha\cos\theta + wR_1^2\sin\theta + HR_2\sin\theta\sin\alpha - wR_1R_2\theta)\cos\alpha + HR_2\sin\theta\sin\alpha \tag{12}$$

- Thrust:

$$N = -H\sin\theta\cos\alpha - wR_1\theta\sin\alpha \tag{13}$$

- Lateral Shear:

$$V = w\theta\cos\alpha - H\sin\theta\sin\alpha \tag{14}$$

- Radial horizontal shear:

$$F = H\cos\theta \tag{15}$$

where the radius of centreline of load is

$$R_1 = \frac{2}{3}\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right) \tag{16}$$

These equations are valid when the angle subtended by the landing at the centre (φ) is small as compared to angle subtended at the centre by the whole stair plus the landing. It should also be noted that the equations for the landing are nothing but the expressions for the flight, with the slope of the helix (α) put to zero.

The Strain Energy Method

The widely used Castigliano's Second theorem states that in any structure, the material of which is elastic and follows Hooke's law and in which the temperature is constant and the supports are unyielding, the first partial derivative of the strain energy with respect to any particular force is equal to the displacement of the point of application of that force in the direction of its line of action. Mathematically expressing,

$$\frac{\partial U}{\partial P} = \delta \tag{17}$$

where U is the strain energy; P the force; and δ the deflection in the direction of force.

The strain energy due to shear stress and axial force is neglected, because they are small. The strain energy stored by the bending moment is given by:

$$U = \Sigma \int \frac{M^2}{2\,EI}\, dL \tag{18}$$

And that by the twisting moment is:

$$U = \Sigma \int \frac{T^2}{2\,GJ}\, dL \tag{19}$$

The Strain Energy Method Applied to the Helicoidal Stair Slabs

The strain energy method has previously been successfully employed by Morgan and Holmes to analyse helicodial stair slabs. Because of symmetry in loading and geometry, in a helicodial stair slab with a landing at the middle, the slope at the mid-span is zero and so is the horizontal deflection. This is why, according to the Castigliano's second theorem, the partial derivatives of the strain energy function with respect to the vertical moment (M) and radial horizontal force (H) is equal to zero. That is,

$$\frac{\partial U}{\partial M} = 0 \tag{20}$$

and

$$\frac{\partial U}{\partial H} = 0 \tag{21}$$

Solution of these equations yield the values of M and H, which can be expressed in the form of:

$$M = k_1\, wR_2^2 \tag{22}$$

and

$$H = k_2\, wR_2^2 \tag{23}$$

Details of the analysis is given in Appendix A.

SUGGESTION FOR A CHART

Once M and H are determined from equations (22) and (23), equations (7) to (15) can be used to determine the six stress resultants at any section of the helicoidal stair slab. However, the derivation of equations (22) and (23) requires tedious mathematical computations. To facilitate the design procedure a series of 21 design charts have been proposed. Figures 4 to 6 present a few of the charts. These charts provide the values of k_1 and k_2 for a wide range of parameters and are available in Wadud[9].

- Total central angle subtended by the stair (range 135° to 360°).
- Slope of the tangent helix centre line with respect to the horizontal plane, α (20° to 40°).
- Ratio of radius of the centre line of load to the mean radius of the stair, R_1 / R_2 (1.01, 1.05 and 1.10).
- Total angle subtended by the landing (10° to 70°).

The vertical moment at the support often becomes the most critical design force. Another factor k_3 has been introduced in order to expedite the design process,

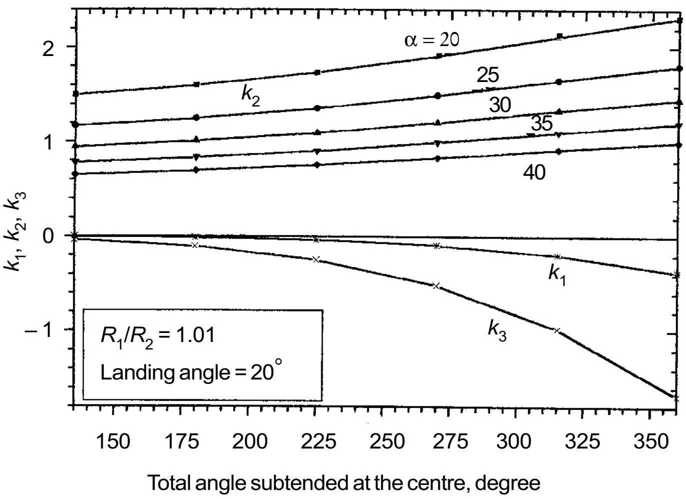

Figure 4 Coefficients k_1, k_2, k_3 for R_1 / R_2 = 1.01, landing angle = 20°

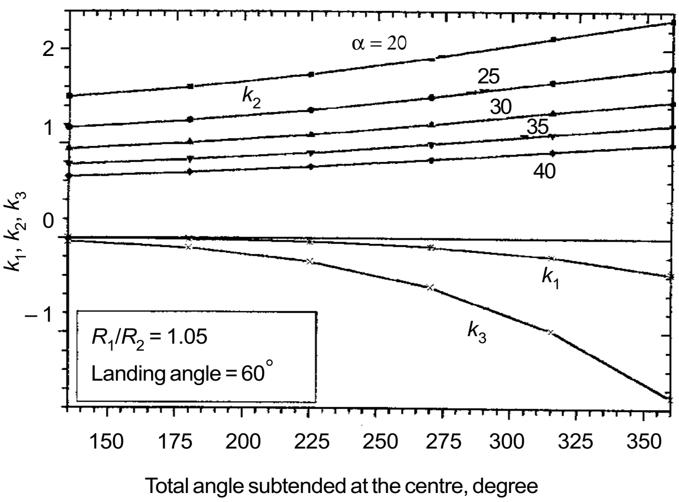

Figure 5 Coefficients k_1, k_2, k_3 for R_1 / R_2 = 1.05, landing angle = 60°

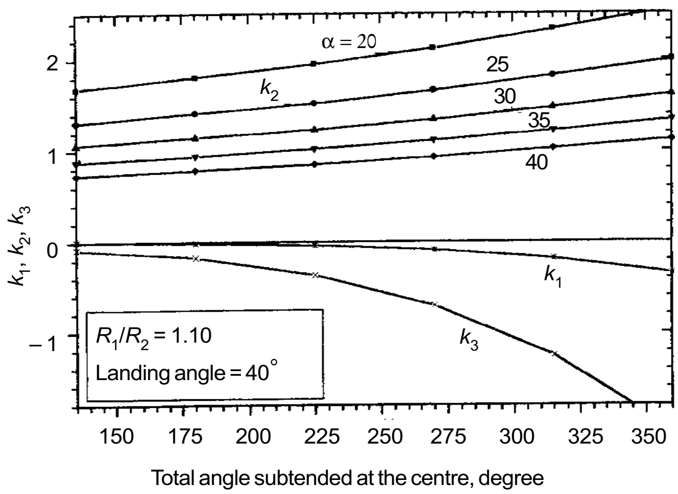

Figure 6 Coefficients k_1, k_2, k_3 for R_1 / R_2 = 1.10, landing angle = 40°

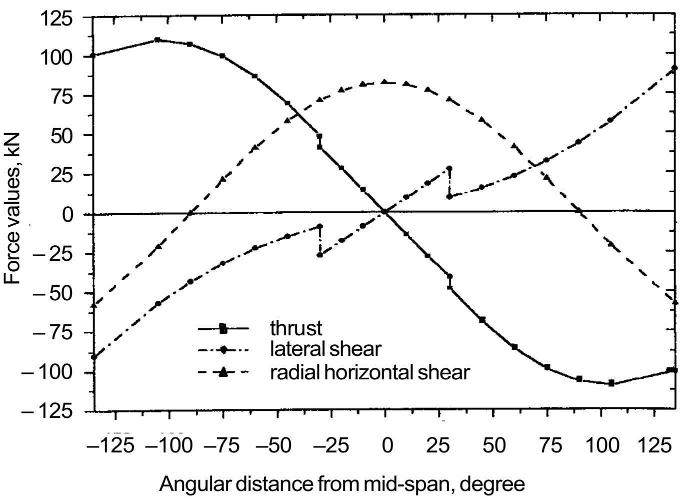

Figure 7 Variation of forces along the span for the design example

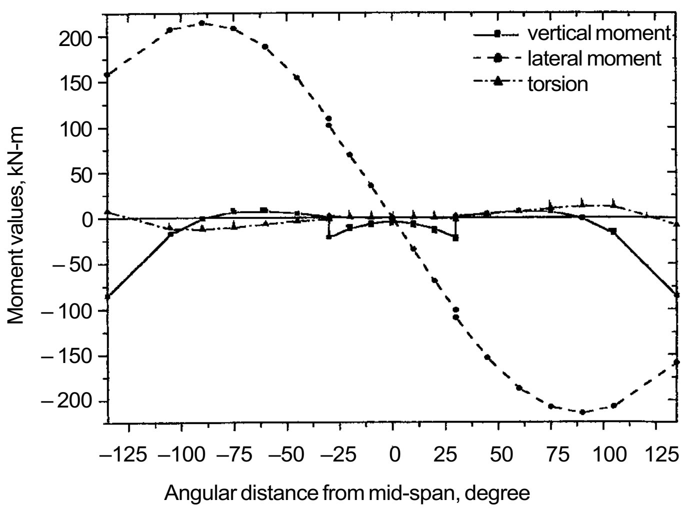

Figure 8 Variation of moments along the span for the design example

where

$$M_{sup} = k_3 . wR_2^2 \quad (35)$$

The values of k_3 are also presented in the proposed design charts. It has been found that k_1 and k_3 do not vary much on the stair inclination, but k_2 does.

CONCLUSION

A simple design chart has been proposed for use in design of helicoidal stair slabs with intermediate landings. The chart also covers the helicoidal slabs without landing as a central landing angle of 0° is equivalent to no landing. Figures 7 and 8 depict the variation of the stress resultants for a prototype stair (Appendix). A step-by-step procedure for analysis using the charts is given in Appendix. It is expected that the use of the charts would expedite the design process.

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APPENDIX A: THE ANALYSIS

As explained before, the basic framework of the analysis is derived from Solanki's work. However, there was some discrepancy in his results and therefore the analysis has been carried out independently beginning from the strain energy principle. For a helicoidal stair slab with intermediate landing, the strain energy function U, is given by,

$$U = \int_0^{\phi} \frac{M_v^2}{2EI} ds + \int_0^{\phi} \frac{M_h^2}{2EI_h} ds + \int_0^{\phi} \frac{T^2}{2GJ} ds + \int_0^{\beta} \frac{M_v^2}{2EI_h} ds + \int_0^{\beta} \frac{M_h^2}{2EI} ds + \int_0^{\beta} \frac{T^2}{2GJ} ds \quad (24)$$

where for the landing

$$ds = R_2 \, d\theta \tag{25}$$

and, for the flight

$$ds = R_2 \sec\alpha \, d\theta \tag{26}$$

The partial derivative of the strain energy function with respect to H is,

$$\frac{\partial U}{\partial H} = \int_0^{\phi} \frac{2M_v}{2EI}\frac{\partial M_v}{\partial H} ds + \int_0^{\phi} \frac{2M_h}{2EI_h}\frac{\partial M_h}{\partial H} ds + \int_0^{\phi} \frac{2T}{2GJ}\frac{\partial T}{\partial H} ds$$

$$+ \int_0^{\beta} \frac{2M_v}{2EI}\frac{\partial M_v}{\partial H} ds + \int_0^{\beta} \frac{2M_h}{2EI_h}\frac{\partial M_h}{\partial H} ds + \int_0^{\beta} \frac{2T}{2GJ}\frac{\partial T}{\partial H} ds \tag{27}$$

Because the stair width is large as compared to its thickness, the moment of inertia with respect to a vertical axis I_h is much greater than the moment of inertia about the horizontal axis I. The ratio I/I_h can therefore be neglected, *ie,*

$$\frac{I}{I_h} \approx 0 \tag{28}$$

As per Solanki, the torsional rigidity can be taken as:

$$GJ = \frac{2\,EII_h}{I + I_h} \tag{29}$$

Then

$$\frac{EI}{GJ} = \frac{1}{2}\left[\frac{I}{I_h} + 1\right] \approx \frac{1}{2} \tag{30}$$

Equation (21) can be rewritten with the help of equations (25), (26) and (28) as:

$$\frac{\partial U}{\partial H} = \int_0^{\phi} M_v \frac{\partial M_v}{\partial H} ds + \frac{1}{2}\int_0^{\phi} T\frac{\partial T}{\partial H} ds + \int_0^{\beta} M_v \frac{\partial M_v}{\partial H} ds + \frac{1}{2}\int_0^{\beta} T\frac{\partial T}{\partial H} ds = 0 \tag{31}$$

Similarly equation (20) stands as:

$$\frac{\partial U}{\partial H} = \int_0^{\phi} M_v \frac{\partial M_v}{\partial M} ds + \frac{1}{2}\int_0^{\phi} T\frac{\partial T}{\partial M} ds + \int_0^{\beta} M_v \frac{\partial M_v}{\partial M} ds + \frac{1}{2}\int_0^{\beta} T\frac{\partial T}{\partial M} ds = 0 \tag{32}$$

Equations (29) and (30) expands to:

$$MR_2^2 \tan\alpha \ \sec\alpha\, A + HR_2^3 \tan^2 \sec\ \alpha\, B - wR_1^2\, R_2^2 \tan\alpha\ \sec\alpha(C - A)$$

$$+ \frac{1}{2}[MR_2^2 \sin\alpha(D - A) + HR_2^3 \sin^2\alpha\sec\alpha(D - 2A + E)$$

$$+ wR_1^2\, R_2^2 \sin\alpha\,(D - A) - wR_1\, R_2^3 \sin\alpha\,(C - F)] = 0$$

$$\Rightarrow M[\tan\alpha\sec\alpha\, A + \frac{1}{2}\sin\alpha(D - A)] + HR_2\,[\tan^2\alpha\sec\alpha\, B$$

$$+ \frac{1}{2}\ \sin^2\alpha\sec\alpha(D - 2A + E)] - wR_1^2\,[\tan\alpha\sec\alpha(C - A)$$

$$- \frac{1}{2}\sin\alpha(D - A) + \frac{1}{2}R_2\sin\alpha(C - F)\,/\,R_1] = 0$$

$$\Rightarrow A_1M + A_2HR_2 = A_3wR_1^2$$

$$\Rightarrow A_1M + A_2HR_2 = A_4wR_2^2 \tag{33}$$

and

$$MR_2\sec\alpha G_1 + HR_2^2\tan\alpha\sec\alpha\, A - wR_1^2\, R_2\sec\alpha(H_1 - G_1)$$

$$+\frac{1}{2}[MR_2\cos\alpha\, D + HR_2^2\sin\alpha(D - A) + wR_1^2\, R_2\cos\alpha\, D - wR_1R_2^2\cos\alpha C]$$

$$+[MR_2\, G' - wR_1^2R_2\,(H' - G')] + \frac{1}{2}[MR_2\, D' + wR_1^2\, R_2\, D' - wR_1\, R_2^2\, C'] = 0$$

$$\Rightarrow M[\sec\alpha G_1 + \frac{1}{2}\cos\alpha\, D + G' + \frac{1}{2}D'] + HR_2\,[\tan\alpha\sec\alpha\, A + \frac{1}{2}\sin\alpha(D - A)]$$

$$- wR_1^2\,[\sec\alpha(H_1 - G_1) - \frac{1}{2}\cos\alpha\, D + \frac{1}{2}R_2\cos\alpha C\,/\,R_1 + H' - G'$$

$$- \frac{1}{2}\ D' + \frac{1}{2}\ R_2\, C'\,/\,R_1] = 0$$

$$\Rightarrow B_1M + B_2HR_2 = B_3wR_1^2$$

$$\Rightarrow B_1M + B_2HR_2 = B_4wR_2^2 \tag{34}$$

where

$$A = (1/8)\sin 2\beta - (1/4)\beta\cos 2\beta$$

$$B = \beta^3/6 - (\beta^2/4 - 1/8)\sin 2\beta - (1/4)\beta\cos 2\beta$$

$$C = \sin\beta - \beta\cos\beta$$

$$C' = \sin\phi - \phi\cos\phi$$

$$D = (1/2)\beta - (1/4)\sin 2\beta$$

$$D' = (1/2)\phi - (1/4)\sin 2\phi$$

$$E = \beta^3/6 + (\beta^2/4 - 1/8)\sin 2\beta + (1/4)\beta\cos 2\beta$$

$$F = 2\beta\cos\beta + (\beta^2 - 2)\sin\beta$$

$$G_1 = (1/2)\beta + (1/4)\sin 2\beta$$

$$G' = (1/2)\phi + (1/4)\sin 2\phi$$

$$H_1 = \sin\beta$$

$$H' = \sin\phi$$

$A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4$ are constants, their values being evident from equations (33) and (34).

The simultaneous solution of equations (33) and (34) yields the values of M and H, mid-span redundant moment and radial horizontal force:

$$M = \frac{A_4\, B_2 - A_2\, B_4}{A_1\, B_2 - A_2\, B_1}\, wR_2^2 = k_1\, wR_2^2 \tag{22}$$

$$H = \frac{A_4\, B_1 - A_1\, B_4}{A_2\, B_1 - A_1\, B_2}\, wR_2 = k_2\, wR_2 \tag{23}$$

APPENDIX B: THE DESIGN PROCESS

B-1 Analysis

The height of the stair, inner radius, outer radius (or alternatively mean radius and width of stair), the total angle (θf) through which the stair is to rotate to reach its height, the length of landing (L) etc. are generally suggested by the architect. Having fixed the geometric parameters, a designer then has to determine the stress resultants. The introduction of the chart will substantially reduce the tedious computations required to find the design forces and moments. The analysis procedure using the charts consists of the following steps:

1. Determine mean radius R_2 from given inner radius R_i and outer radius R_o

   $R_2 = (R_o + R_i)/2$

2. Determine the angle (2ϕ) subtended by the landing of length L at a distance R at the centre

   $L = 2\phi R$

3. Find the total angle subtended at the centre by the flights (2β) from θ_f and 2ϕ

   $2\beta = \theta_f - 2\phi$

4. From the height of stair (Ht), mean radius (R_2), and total angle subtended by flight at the centre (2β) calculate the slope of the tangent to the helix centreline (α) as

   $$\alpha = \tan^{-1} \frac{Ht}{R_2\,2\beta}$$

5. Determine the radius of centre line of loading (R_1) from

   $$R_1 = \frac{2}{3}\,\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$$

6. Find w, total dead and live load per unit length along the centreline
7. With the values of R_1/R_2 and central angle subtended by the landing (2ϕ) go to the appropriate chart, find k_1, k_2, and k_3 for the given value of total angle subtended at the centre (θ_f). Determine mid-span moment, M, mid-span radial horizontal shear, H, and support moment, M_{sup}, from

   $M = k_1 w R_2^2$

   $H = k_2 w R_2$

   $M_{sup} = k_3 w R_2^2$

8. Determine other stress resultants at various distance, θ, from the mid-span toward the top support using equations (10)-(16). Keep in mind that $\alpha = 0$ at the landing.

B-2 Example

It is required to analyse a reinforced concrete helicodial stair slab with a height of 3.81 m (12.5 ft), inner radius of 1.524 m (5 ft), and outer radius of 3.43 m (11.25 ft). The stair is to reach its full height within a 270° turn. The length of landing at the inner edge is 1.6 m (5.25 ft). Live load = 4.788 kN/m^2 (100 psf). Concrete unit weight = 23.563 kN/m^3 (150 pcf). The stair slab is 0.152 m (6 inches) thick and the risers are 0.152 m (6 inches) high.

Step 1: $R_2 = (1.524 + 3.43)/2 = 2.477\text{m}$

Step 2: $\phi = 1.6/(1.524 \times 2) = 0.525 \text{ radian} = 30^\circ$

Step 3: $2\beta = 270 - 2 \times 30 = 210^\circ = 3.665 \text{ radian}$

Step 4: $$\alpha = \tan^{-1} \frac{3.81}{2.477 \times 3.665} = 22.8^\circ$$

Step 5: $$R_1 = \frac{2}{3}\,\frac{3.43^3 - 1.524^3}{3.43^2 - 1.524^2} = 2.6 \text{ m}$$

$R_1 / R_2 = 2.6/2.477 = 1.05$

Step 6: Total thickness in the vertical direction is approximately 0.241 m.

Surface UDL = 0.241 × 23.563 + 4.788 = 10.467 kN/m^2

w = 10.467 × (3.43 – 1.524) = 19.95 kN/m

Step 7: With the calculated values of R_1/R_2, α and β, refering to Figure 5, for a 270° stair,

$k_1 = 0.036 \Rightarrow M = 0.036 \times 19.95 \times 2.477^2 = 4.406$ kN-m (3 250 lb-ft)

$k_2 = 1.657 \Rightarrow H = 1.657 \times 19.95 \times 2.477 = 81.88$ kN (18400 lb)

$k_3 = -0.73 \Rightarrow M_{sup} = -0.73 \times 19.95 \times 2.477^2 = -89.35$ kN-m (– 65 880 lb-ft)

Step 8: The variation of stress resultants along the span, found using the previously stated equations, is depicted through Figures 7 and 8.