2. Colonna intermedia. Carico $S = 40{,}4$ t; altezza della colonna $s_K = 4{,}00$ m. Occorre, sec. Tab. 5.3.3.1, un I 360 con una portata max $S = 42{,}2$ t. $F = 97{,}1$ cm²; $i_y = 2{,}90$ cm; snellezza $\lambda = \frac{400}{2{,}90} = 138$; corrispondente ω (*v. Par. 7.3.2.1, Tab. 1*) = 3,22. Risulta $\sigma_{eff} = \frac{40{,}4 \cdot 3{,}22}{97{,}1} = 1{,}34$ t/cm² o, sec. *Tab. 5.3.6.9*, una colonna con sezione ⊐⊏ composta di 2 ⊏ 100 + 2 ▭ 160 × 8 mm con max $S = 43{,}6$ t.

3. Appoggi su muratura. Reazione max a destra $T_3 = 16{,}1$ t; spessore della muratura a 3 teste = 36,5 cm; lunghezza di appoggio della trave sulla piastra di appoggio = 35 cm; larghezza della piastra per la trave in IPB 300 = 35 cm. Tensione nella muratura $p = \frac{16\,100}{35 \cdot 35} = 13{,}1$ kg/cm², $< p_{am} = 16{,}0$ kg/cm² per mattoni pieni Mz 150 con malta del Gruppo III sec. Tab. 5 DIN 1053, *v. Par. 7.4.3.1.*

10. 2. 4. Capriate a portale con anima piena

10. 2. 4. 1. Generalità

I portali sono strutture unitarie ottenute mediante il collegamento rigido di un elemento superiore orizzontale (traverso) con due elementi laterali (montanti). I montanti vengono sollecitati, oltre che dai carichi verticali, anche dai momenti flettenti indotti dalle reazioni orizzontali H che si verificano agli appoggi.

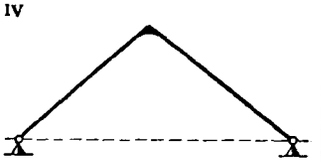
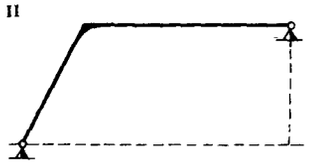
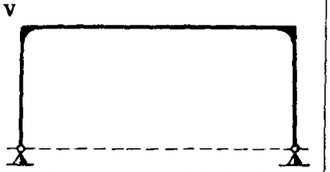
A seconda del tipo di vincolo agli appoggi, i portali si distinguono in portali incernierati ai piedi o a 2 cerniere (una volta iperstatici rispetto all'iperstatica H) e portali incastrati ai 2 piedi o incastrati (3 volte rispetto all'iperstatica H e ai due momenti di incastro), e inoltre, a seconda della forma, in portali semplici, doppi, multipli e chiusi.

La sezione del portale è realizzata in I normali o ad ali larghe, in ⊐⊏ accoppiate o anche in travi composte (prevalentemente saldate).

La Tabella qui accanto illustra le forme fondamentali più in uso nelle costruzioni per i portali a 2 cerniere.

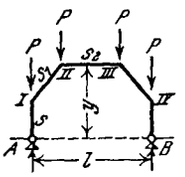
Per i **pesi propri** *v. Tabella: Pesi propri in kg/m² di area coperta, Par. 8.3.*

Schemi di telaio più usuali

Schema n°	Schema n°
I  Calcolo *v. Par. 10.2.4.3.1.* e *Par. 10.2.4.4.3.*	IV  Calcolo *v. Par. 10.2.4.3.4.* e *Par. 10.2.4.4.4.*
II  Calcolo *v. Par. 10.2.4.3.2.*	V  Calcolo *v. Par. 10.2.4.3.6.* e *Par. 10.2.4.1.*
III  Calcolo *v. Par. 10.2.4.3.3.*	VI  Calcolo *v. Par. 10.2.4.3.5.*

10. 2. 4. 2. Calcolo

Trascurando la modesta influenza degli sforzi normali e assumendo il momento d'inerzia J in cm⁴ come costante in tutte le sezioni, col metodo delle deformazioni e supponendo le cerniere come fisse e perfette si determina la spinta laterale agli appoggi



$$H = \frac{\int M^o \, y \, ds}{\int y^2 \, ds}$$

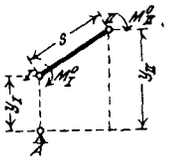
M^o = momento flettente del portale isostatico

y = ordinate riferite alla congiungente le 2 cerniere

È conveniente esprimere tutti i carichi in t e tutte le lunghezze in m.

Particolare cura va posta a che le cerniere non siano soggette a cedimenti, in special modo per i così frequenti appoggi su muratura o su travi maestre. Se ciò non è assicurato, occorre collegare i due appoggi tra di loro mediante adeguati organi.

In un portale sollecitato solo da **carichi concentrati** vale per un elemento qualsiasi del portale quanto segue:



$$\int_0^s M^0\, y\, ds = \frac{s}{6}\left[M^0_I\,(y_{II} + 2y_I) + M^0_{II}\,(y_I + 2y_{II})\right];$$

$$\int_0^s y^2\, ds = s\left(\frac{y_I^2 + y_I y_{II} + y_{II}^2}{3}\right).$$

$M_I = 0,$
$y_I = 0.$

Quanto sopra vale quindi solo per andamento rettilineo del diagramma dei momenti, perciò non per carichi ripartiti; a meno che non si divida la loro lunghezza di ripartizione in tratte molto piccole considerando i relativi carichi parziali come carichi concentrati, riottenendo di nuovo un diagramma dei momenti delimitato da tratti rettilinei.

Procedimento di calcolo. Si determinino anzitutto le reazioni agli appoggi A e B in t in funzione dei carichi, procedendo come per le travi semplicemente appoggiate e i momenti flettenti M^0_I, M^0_{II} ..... in tm indotti nei vari punti; quindi, con l'ausilio delle equazioni contenute nelle Tabelle del *Par. 10. 2. 4. 3. 1* e segg., si calcolino i valori di $\int M^0\, y\, ds$, $\int y^2\, ds$ e, infine, il valore della spinta laterale H.

I momenti effettivamente indotti sono: ..... $M = M^0 - H\,y$.

Il modulo di resistenza del portale in cm³, in funzione della tensione ammissibile $\sigma_{f\,am}$ in t/cm², vale: .... $W_x = \frac{M}{\sigma_{f\,am}}$.
Esprimendo gli sforzi normali in t deve essere:

$$\ldots\ldots\ldots\ldots \sigma_{eff} = \frac{M}{W_x} + \frac{N\,\omega}{F} \ldots \leqq \sigma_{f\,am}$$

W_x = modulo resistente della sezione del portale in cm³,
F = sezione lorda del portale in cm², ω = coefficiente di sbandamento (cfr. *Par. 7. 3. 2. 1, Tab. 1*).

Le singole aste vanno verificate allo sbandamento per i relativi sforzi assiali (calcolo sec. DIN 4114 Criteri di calcolo per i casi di instabilità elastica nelle costruzioni in acciaio, Par. 14 e Istruzioni per detta Norma, Par. Ri. 14).

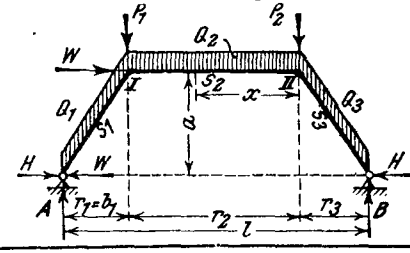
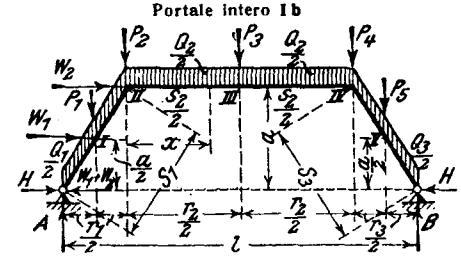
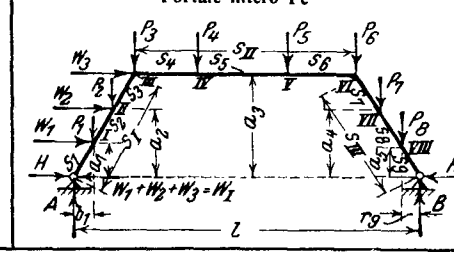
10. 2. 4. 3. Portali soggetti a determinate condizioni di carico [1])

Per i portali secondo gli schemi tipo da I a VI sono riportate, nelle Tabelle seguenti, le formule per il calcolo di H e delle altre grandezze occorrenti a dimensionare le sezioni per le condizioni di carico più usuali nelle costruzioni in acciaio, per effetto di carichi sia concentrati sia uniformemente ripartiti ma trascurando l'influsso di variazioni termiche [2]).

[1]) Kleinlogel: Gebrauchsfertige Formeln für alle statischen Größen in allen praktisch vorkommenden Einfeld-Rahmenformen aus Stahlbeton, Stahl oder Holz (114 Rahmenformen) Anhang Belastungsglieder: Momentenangriffe u. Kragarmlasten; Einflußlinien (*Berlino: Wilh. Ernst u. Sohn*). Mayer: Neue Statik der Tragwerke aus biegesteifen Stäben (Durchlaufende Träger, Stockwerkrahmen usw.) (*Berlino: Bauwelt-Verlag*). Kirchhoff: Statik der Bauwerke, Vol. II. (*Berlino: Wilhelm Ernst u. Sohn*). Bleich: Stahlhochbauten, Vol. 1. (*Berlino: Springer-Verlag*). Gehler: Der Rahmen. Einfaches Verfahren zur Berechnung von Rahmen aus Stahl u. Stahlbeton mit Berechnungsbeispielen. (*Berlino: Gropius'sche Buchhandlung, Wilhelm Ernst u. Sohn*). Su telai di *piano* ecc. cfr. Nota 1 al *Par. 10. 2. 4. 4.* Dernedde: Das Cross'sche Verfahren (*Berlino: Wilhelm Ernst u. Sohn*). Johannson: Das Cross-Verfahren (*Berlino: Springer-Verlag*). Hahn: Durchlaufträger, Rahmen und kreuzweise bewehrte Platten. Eine einfache Berechnungsart mit Leitstart-Beiwerten. (*Düsseldorf-Lohausen: Werner-Verlag*). Von Halasz: Anschauliche Verfahren zur Berechnung von Durchlaufbalken und Rahmen (Ausgleichverfahren). (*Berlino: Wilhelm Ernst u. Sohn*).

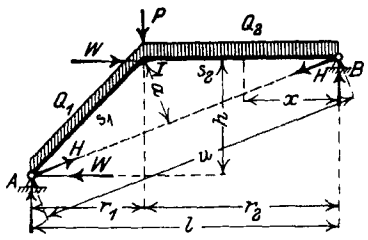
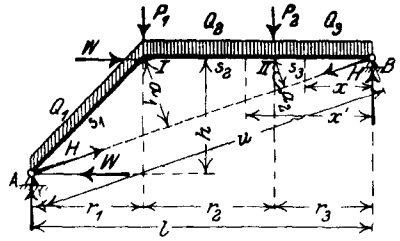
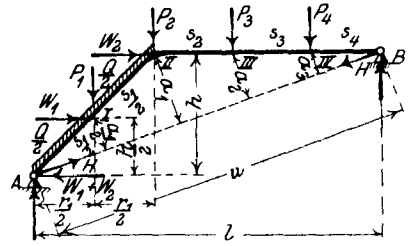
[2]) Formule per la determinazione della spinta orizzontale H_t indotta dalle variazioni termiche, cfr. *Par. 10. 2. 4. 5.*

10. 2. 4. 3. 1. Capriata a portale sec. lo Schema I

	Portale intero I a	Portale intero I b	Portale intero I c
Schema del portale e del carico			
$\int M^0 y\,ds$ = (in tm³)	$\frac{s_1}{3} M^0_I a + \frac{s_2}{2}(M^0_I + M^0_{II})a + \frac{s_2}{3} M^0_{II} a + \frac{s_1}{24} Q_1 r_1 a + \frac{s_2}{12} Q_2 r_2 a + \frac{s_2}{24} Q_2 r_2 a$ . . . . . Per **portale intero I a**		
	$\frac{s_1 a}{24}\left[6 M^0_I + 5 M^0_{II}\right] + \frac{s_2 a}{4}\left[M^0_{II} + 2 M^0_{III} + M^0_{IV}\right] + \frac{s_2 a}{24}\left[5 M^0_{IV} + 6 M^0_V\right] + \frac{s_1}{24}\frac{Q_1}{2}\frac{r_1}{2} a + \frac{s_2}{12}\frac{Q_2}{2}\frac{r_2}{2} a + \frac{s_2}{24}\frac{Q_2}{2}\frac{r_2}{2} a$ . . . . . Per **portale intero I b**		
	$\frac{s_1}{3} a_1 M^0_I + \frac{s_2}{6}\left[M^0_I (a_2 + 2 a_1) + M^0_{II}(a_1 + 2 a_2)\right] + \frac{s_2}{6}\left[M^0_{II}(a_2 + 2 a_3) + M^0_{III}(a_3 + 2 a_2)\right] + \frac{s_4 a_3}{2}\left[M^0_{III} + M^0_{IV}\right] + \ldots$		Per portale intero I c
	$\frac{s_5 a_3}{2}\left[M^0_{IV} + M^0_V\right] + \frac{s_6 a_3}{2}\left[M^0_V + M^0_{VI}\right] + \frac{s_7}{6}\left[M^0_{VI}(a_4 + 2 a_3) + M^0_{VII}(a_3 + 2 a_4)\right] + \frac{s_8}{6}\left[M^0_{VII}(a_3 + 2 a_4) + M^0_{VIII}(a_4 + 2 a_3)\right] + \frac{s_9 a_4}{3} M^0_{VIII}$		
$\int y^2\,ds$ = (in m³)	$\frac{a^3}{3}(s_1 + 3 s_2 + s_3)$	$\frac{a^3}{3}(s_1 + 3 s_2 + s_3)$	$\frac{a_3^3}{3}(s_I + 3 s_{II} + s_{III})$
Ascissa della sezione critica	$x = \frac{B - Q_3 - P_2}{\frac{Q_2}{s_2}}$	$x = \frac{A - (P_1 + P_2 + Q_1)}{\frac{Q_2}{s_2}}$	—
Sforzi normali N (in t)	$N_I = \frac{(A - Q_1)\,a + (H - W)\,b_1}{s_1}$; $N_X = H$; $N_{II} = \frac{(B - Q_3)\,a + H\,r_3}{s_3}$	$N_I = \left[\left(A - \frac{Q_1}{2}\right)a + (H - W_1 - W_2)\,r_1\right]\frac{1}{s_1}$; $N_{II} = N_I - \left[\left(P_1 + \frac{Q_1}{2}\right)a - W_1 r_1\right]\frac{1}{s_1}$; $N_X = N_{III} = H$; $N_{IV} = \left[(B - Q_3 - P_3)\,a + H\,r_3\right]\frac{1}{s_3}$; $N_V = N_{IV} + \left(P_3 + \frac{Q_3}{2}\right)\frac{a}{s}$;	$N_I = \frac{A\,a_1 + (H - W_I)\,b_1}{s_1}$; $N_{II} = \frac{(A - P_1)\,a_1 + [H - (W_2 + W_3)\,b_1]}{s_1}$; $N_{III} = \frac{[A - (P_1 + P_2)]\,a_1 + (H - W_3)\,b_1}{s_1}$; $N_{VI} = \frac{[B - (P_7 + P_8)]\,a_3 + H\,r_9}{s_9}$; $N_{VII} = \frac{(B - P_8)\,a_3 + H\,r_9}{s_9}$; $N_{VIII} = \frac{B\,a_3 + H\,r_9}{s_9}$; $N_{IV} = N_V = H$.

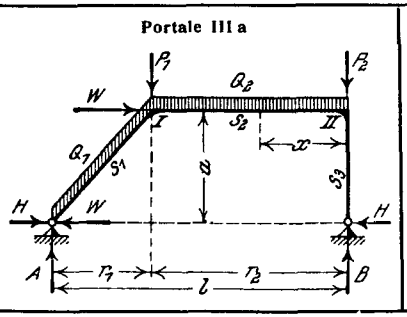
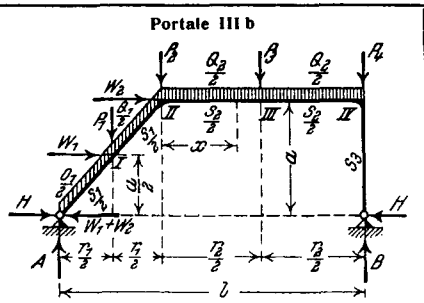
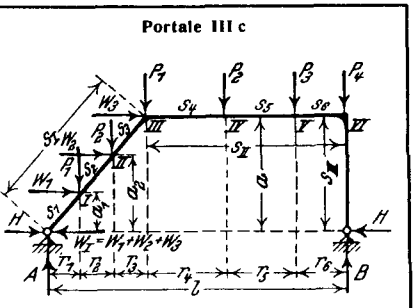
M^0_I, M^0_{II}, M^0_{III} . . . . . = Momenti flettenti della capriata isostatica corrispondente

10. 2. 4. 3. 2. Capriata di portale sec. lo Schema II

	Semiportale II a	Semiportale II b	Semiportale II c
Schema del portale e del carico			
$M^0 y\,ds$ = (in tm²)	$\frac{a}{3} M_I^0 (s_1 + s_2) + \frac{s_1}{24} Q_1 r_1 a + \frac{s_2}{24} Q_2 r_2 a$ . . . . . Per **semiportale II a**		
		$\frac{s_1}{3} M_I^0 a_1 + \frac{s_2}{6}\left[M_I^0 (a_2 + 2a_1) + M_{II}^0 (a_1 + 2a_2)\right] + \frac{s_3}{3} M_{II}^0 a_2 + \frac{s_1}{24} a_1 Q_1 r_1 + \frac{s_2}{24} Q_2 r_2 (a_1 + a_2) + \frac{s_3}{24} Q_3 r_3 a_2$ . . . . . Per **semiportale II b**	
			$\frac{s_1 a_1}{24}\left[6 M_I^0 + 5 M_{II}^0\right] + \frac{s_2}{6}\left[M_{II}^0 (a_2 + 2a_1) + M_{III}^0 (a_1 + 2a_2)\right] + \frac{s_3}{6}\left[M_{III}^0 (a_3 + 2a_2) + M_{IV}^0 (a_2 + 2a_3)\right] + \frac{s_4}{3} a_3 M_{IV}^0 + \frac{s_1}{24} \frac{Q}{2} \frac{r_1}{2} a_1$ } Per **Semiportale II c**
$\int y^2\,ds$ (in m³) =	$\frac{a^2}{3}(s_1 + s_2)$	$\frac{a_1^2}{3}(s_1 + s_2 + s_3)$	$\frac{a_1^2}{3}(s_1 + s_2 + s_3 + s_4)$
Ascissa della sezione critica	$x = \dfrac{B - H\frac{h}{u}}{\frac{Q_2}{s_2}}$	$x = r_1 + r_2$ oppure α) se $B - H\frac{h}{u} < Q_3$, . . . ist $x = \dfrac{B - H\frac{h}{u}}{\frac{Q_3}{s_3}}$; β) se $B - H\frac{h}{u} > Q_3 + P_3$, si ha $x' = \dfrac{\left(B - H\frac{h}{u}\right) - (Q_3 + P_3)}{\frac{Q_3}{s_3}} + s_3$	—
Sforzi normali N (in t)	$N_I = \left(A - Q_1\frac{h}{u}\right)\frac{h}{s_1}$; $N_x = H\frac{l}{u}$	$N_I = \left(A - Q_1 + H\frac{h}{u}\right)\frac{h}{s_1} + \left(H\frac{l}{u} - W\right)\frac{r_1}{s_1}$; $N_{II} = N_x = H\frac{l}{u}$	$N_I = \left(A - \frac{Q}{2} + H\frac{h}{u}\right)\frac{h}{s_1} + \left[H\frac{l}{u} - (W_1 + W_2\right]\frac{r_1}{s_1}$; $N_{II} = N_I - \left(P_1 + \frac{Q}{2}\right)\frac{h}{s_1} + W_1\frac{r_1}{s_1}$; $N_{III\,(IV)} = H\frac{l}{u}$

$M_I^0, M_{II}^0, M_{III}^0 \ldots\ldots$ = Momenti flettenti della capriata isostatica corrispondente.

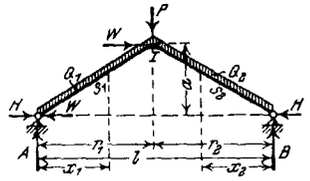
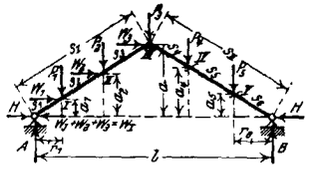
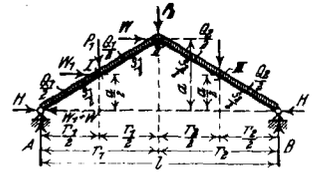
10. 2. 4. 3. 3. Capriata a portale sec. lo Schema III

	Portale III a	Portale III b	Portale III c
Schema del portale e del carico			
$\int M^0 y\, ds =$ (in tm²)	$\frac{a}{6} M^0_I (2 s_1 + 3 s_2) + \frac{s_1}{24} Q_1 r_1 a + \frac{s_2}{12} Q_2 r_2 a$ . . . . . Per **portale III a**		
	$\frac{s_1 a}{24}\left[6 M^0_I + 5 M^0_{II}\right] + \frac{s_2 a}{4}\left[M^0_{II} + 2 M^0_{III}\right] + \frac{s_1}{24}\frac{Q_1}{2}\frac{r_1}{2} a + \frac{s_1}{12}\frac{Q_2}{2}\frac{r_2}{2} a$ . . . . . Per **portale III b**		
	$\frac{s_1}{3} M^0_I a_1 + \frac{s_2}{6}\left[M^0_I (a_2 + 2 a_1) + M^0_{II} (a_1 + 2 a_2)\right] + \frac{s_3}{6}\left[M^0_{II} (a + 2 a_2) + M^0_{III} (a_2 + 2 a)\right] + \frac{s_4}{2}\left[M^0_{III} + M^0_{IV}\right] a +$ $\frac{s_5}{2}\left[M^0_{IV} + M^0_V\right] a + \frac{s_6}{2} M^0_V a$ . . . . . Per **portale III c**		
$\int y^2 ds =$ (in m³)	$\frac{a^2}{3} (s_1 + 3 s_2 + s_3)$	$\frac{a^2}{3} (s_1 + 3 s_2 + s_3)$	$\frac{a^2}{3} (s_I + 3 s_{II} + s_{III})$
Ascissa della sezione critica	$x = \frac{B - P_2}{\frac{Q_2}{s_2}}$	$x = \frac{A - (P_1 + P_2 + Q_1)}{\frac{Q_2}{s_2}}$	—
Sforzi normali N (in t)	$N_I = \frac{(A - Q_1) a + (H - W) r_1}{s_1}$; $N_x = H$; $N_{II} = B$	$N_I = \left[\left(A - \frac{Q_1}{2}\right) a + (H - W_1 - W_2) r_1\right]\frac{1}{s_1}$; $N_{II} = N_I - \left[\left(P_1 + \frac{Q_1}{2}\right) a - W_1 r_1\right]\frac{1}{s_1}$; $N_x = N_{III} = H$; $N_{IV} = B$	$N_I = \left[A a_1 + (H - W_I) r_1\right]\frac{1}{s_1}$; $N_{II} = \left[(A - P_1) a_1 + (H - W_2 - W_3) r_1\right]\frac{1}{s_1}$; $N_{III} = \left[(A - P_1 - P_2) a_1 + (H - W_3) r_1\right]\frac{1}{s_1}$; $N_{IV} = N_V = H$; $N_{VI} = B$

$M^0_I, M^0_{II}, M^0_{III} \ldots\ldots$ = Momenti flettenti della capriata isostatica corrispondente.

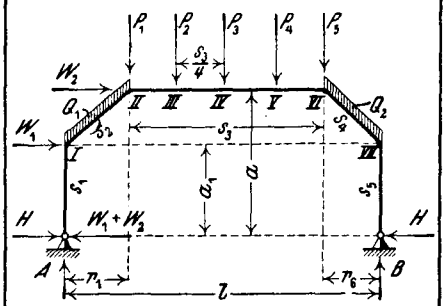
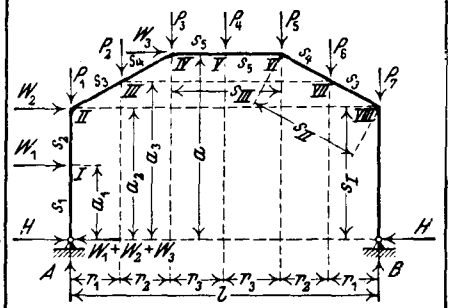
10. 2. 4. 3. 4. Capriata a portale secondo lo Schema IV.

Formule per l'immediato calcolo di A, B, H per portali triangolari isosceli e per condizioni di carico qualsiasi, *v. Par. 10. 2. 4. 4. 4.*

	Portale IV a	Portale IV b	Portale IV c
Schema del portale e del carico			
$\int M^0\, y\, ds =$ (in tm²)	$\frac{a}{3} M_I^0 (s_1 + s_2) + \frac{s_1}{24} Q_1 r_1 a + \frac{s_2}{24} Q_2 r_2 a$ . . . . . Per portale IV a	$\frac{s_1 a}{24}\left[6 M_I^0 + 5 M_{II}^0\right] + \frac{s_2 a}{24}\left[5 M_{II}^0 + 6 M_{III}^0\right] + \frac{s_1}{24}\frac{Q_1}{2}\frac{r_1}{2} a + \frac{s_2}{24}\frac{Q_2}{2}\frac{r_2}{2} a$ . . . . . Per portale IV b	$\frac{s_1}{3} M_I^0 a_1 + \frac{s_2}{6}\left[M_I^0 (a_2 + 2 a_1) + M_{II}^0 (a_1 + 2 a_2)\right] + \frac{s_3}{6}\left[M_{II}^0 (a + 2 a_2) + M_{III}^0 (a_2 + 2 a)\right] + \frac{s_4}{6}\left[M_{III}^0 (a_2 + 2 a) + M_{IV}^0 (a + 2 a_4)\right] + \frac{s_5}{6}\left[M_{IV}^0 (a_4 + 2 a_5) + M_V^0 (a_5 + 2 a_4)\right] + \frac{s_6}{3} M_V^0 a_5$ . . . . . Per portale IV c
$\int y^2\, ds =$ (in m³)	$\frac{a^2}{3} (s_1 + s_2)$	$\frac{a^2}{3} (s_1 + s_2)$	$\frac{a^2}{3} (s_I + s_{II})$
Ascissa della sezione critica	$x_1 = \frac{A - (H - W)\frac{a}{r_1}}{\frac{Q_1}{r_1}}$; $x_2 = \frac{B - H\frac{a}{r_2}}{\frac{Q_2}{r_2}}$	—	—
Sforzi normali N (in t)	$N_{x_1} = \left(A - \frac{Q_1}{r_1} x_1\right)\frac{a}{s_1} + (H - W)\frac{r_1}{s_1}$; $N_I = (B - Q_2)\frac{a}{s_2} + H\frac{r_2}{s_2}$; $N_{x_2} = \left(B - \frac{Q_2}{r_2} x_2\right)\frac{a}{s_2} + H\frac{r_2}{s_2}$	$N_I = \left(A - \frac{Q_1}{2}\right)\frac{a}{s_1} + (H - W_1 - W)\frac{r_1}{s_1}$; $N_{II} = (B - Q_2 - P_2)\frac{a}{s_2} + H\frac{r_2}{s_2}$; $N_{III} = \left(B - \frac{Q_2}{2}\right)\frac{a}{s_2} + H\frac{r_2}{s_2}$	$N_I = \frac{A\, a_1}{s_1} + (H - W_I)\frac{r_1}{s_1}$; $N_{II} = (A - P_1)\frac{a_1}{s_1} + (H - W_2 - W_3)\frac{r_3}{s_1}$; $N_{III} = (B - P_4 - P_5)\frac{a_5}{s_5} + H\frac{r_6}{s_6}$; $N_{IV} = (B - P_5)\frac{a_5}{s_6} + H\frac{r_6}{s_6}$; $N_V = B\frac{a_5}{s_6} + H\frac{r_6}{s_6}$

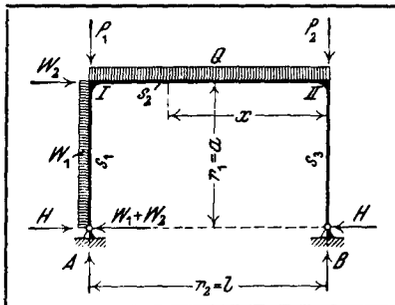
$M_I^0, M_{II}^0, M_{III}^0$ . . . = Momenti flettenti della capriata isostatica corrispondente

10. 2. 4. 3. 5. Capriata a portale secondo lo Schema VI

	Portale VI a	Portale VI b
Schema del portale e del carico		
$\int M^0 y\,ds$ = (in tm²)	$\frac{s_1}{3} M^0_I a_1 +$ $\frac{s_2}{6}\left[M^0_I (a + 2a_1) + M^0_{II} (a_1 + 2a)\right] +$ $\frac{s_3}{8}\left[M^0_{II} + 2M^0_{III} + 2M^0_{IV} + 2M^0_V + M^0_{VI}\right] a +$ $\frac{s_4}{6}\left[M^0_{VI} (a_1 + 2a) + M^0_{VII} (a + 2a_1)\right] +$ $\frac{s_5}{24} Q_1 r_1 (a_1 + a) +$ $\frac{s_6}{24} Q_2 r_6 (a + a_1)$ $M^0_I = (W_1 + W_2) a_1;\ M^0_{VII} = 0$	$\frac{s_1}{3} M^0_I a_1 +$ $\frac{s_2}{6}\left[M^0_I (a_2 + 2a_1) + M^0_{II} (a_1 + 2a_2)\right] +$ $\frac{s_3}{6}\left[M^0_{II} (a_3 + 2a_2) + M^0_{III} (a_2 + 2a_3)\right] +$ $\frac{s_4}{6}\left[M^0_{III} (a + 2a_3) + M^0_{IV} (a_3 + 2a)\right] +$ $\frac{s_5}{6}\left[M^0_{IV} + 2M^0_V + M^0_{VI}\right] a +$ $\frac{s_6}{6}\left[M^0_{VI} (a_3 + 2a) + M^0_{VII} (a + 2a_3)\right] +$ $\frac{s_7}{6}\left[M^0_{VII} (a_3 + 2a_3)\right].$ $M^0_I = (W_1 + W_2 + W_3) a_1;\ M^0_{VIII} = 0;$ $M^0_{II} = (W_1 + W_2 + W_3) a_2 - W_1 (a_2 - a_1)$
$\int y^3\,ds$ = (in m³)	$\frac{a_1^3}{3} (s_1 + s_2 + s_4 + s_5) + \frac{a_1 a}{3} (s_3 + s_4) + \frac{a^3}{3} (s_3 + 3s_5 + s_6)$ . . . . . Per **portale VI a**	
	$\frac{2}{3} a_2^3 (s_I + s_{II}) + \frac{2}{3} a_3 a s_{II} + \frac{a^3}{3} (2s_{II} + 3s_{III})$ . . . . . Per **portale VI b**	
Sforzi normali N (in t)	$Ns_1 = A;$ $Ns_2 = \frac{(A - Q_1)(a - a_1) + (H - W_3) r_1}{s_2};$ $Ns_4 = \frac{(B - Q_2)(a - a_1) + H r_6}{s_4};$ $Ns_3 = H;\ Ns_5 = B$	$N_{s_I} = B;$ $N_{s_{II}} = \frac{(B - P_7)(a_3 - a_2) + H r_1}{s_3};$ $N_{s_{III}} = H$

$M^0_I, M^0_{II}, M^0_{III}$ . . . . . = Momenti flettenti della capriata isostatica corrispondente.

10. 2. 4. 3. 6. Capriate a portale sec. lo schema V



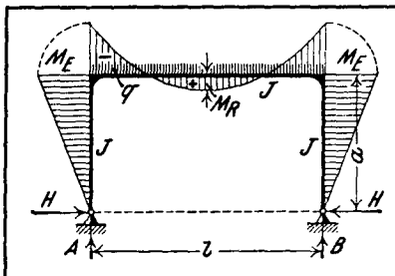
	$\int M^0\, y\, ds =$ (in tm³)		$\frac{a}{6} M_I^0 (2 s_1 + 3 s_2) + \frac{s_1}{24} W_1 r_1 a + \frac{s_2}{12} Q r_2 a.$
	Momenti flettenti		$M_I^0 = (W_1 + W_2) a - W_1 \frac{a}{2}$; $M_{II}^0 = 0$
	$\int y^2\, ds =$ (in m³)		$\frac{a^2}{2} (s_1 + 3 s_2 + s_3)$
Ascissa della sezione critica	$x = \frac{(B - P_2) s_2}{Q}$	Sforzi normali (in t)	$N_I = A$; $N_x = H$; $N_{II} = B$.

10. 2. 4. 4. Formule [1]) per il calcolo delle reazioni agli appoggi e della spinta laterale per qualsiasi carico su qualsiasi tipo di portale

Per calcolare un portale soggetto a uno schema di carico diverso da quelli mostrati, detto schema può venire scomposto negli schemi di carico elementari seguenti. La somma dei valori relativi ai singoli schemi componenti fornisce il valore occorrente per il calcolo dei momenti e per il dimensionamento della sezione.

10. 2. 4. 4. 1. Portale a 2 cerniere con traverso orizzontale e montanti verticali (Schema V)

10. 2. 4. 4. 1. 1. Con lo stesso momento d'inerzia J nei traverso e nei montanti



q = carico sul traverso in kg/m

$\frac{l}{a} = \alpha$ rapporto tra lunghezza del traverso e altezza del portale

Spinta laterale $H = \frac{\alpha^2}{4 (3 \alpha + 2)} q\, l$;

Momento allo spigolo $M_E = \frac{\alpha}{4 (3 \alpha + 2)} q\, l^2 = H a$;

Mom. in mezzeria d. traverso $M_R = \frac{q\, l^2}{8} - M_E$.

Queste formule valgono per portali secondo la Figura; esse valgono però anche per portali con traverso leggermente inclinato ⊓ per una prima determinazione approssimata delle sezioni.

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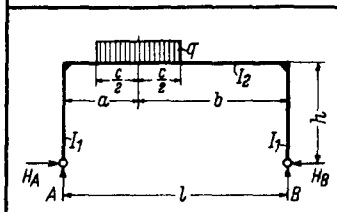
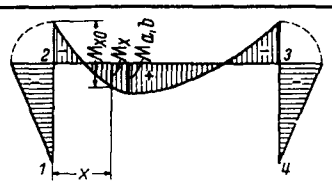
[1]) Per ulteriori tipi di portali e di schemi di carico v. : Kleinlogel u. Haselbach: Mehrfeldrahmen. Fertige Formeln für Durchlaufrahmen, Hallen- u. Stockwerkrahmen, sowie Zahlentafeln u. Sonderformen, (7ª ediz. rielaborata e ampliata dell'opera di Kleinlogel «Mehrstielige Rahmen» in 3 volumi, Vol. 1: Beliebige vielfeldrige, ein- u. zweigeschossige, unverschiebliche u. elastische verschiebliche, elastisch drehbar eingespannte Durchlaufrahmen. (*Berlino: Ernst & Sohn*). Elwitz: Der zweistielige Stockwerkrahmen. (*Düsseldorf: A. Bagel*). Unold: Angenäherte u. genaue Berechnung der Stahlskelettrahmen, *Stahlbau 1931, p. 97* nonché: Die praktische Berechnung der Stahlskelettrahmen. (*Berlino: Wilhelm Ernst u. Sohn*). Sitte: Mehrstufige Rahmenformeln. Flotte Verfahren u. bequeme Formeln für Zehntausende von praktischen Fällen nebst den allgemeinen Grundformeln, einer Anleitung, Zahlenbeispielen u. Quellenangaben Vol. I: Einführung, Anwendungsbeispiele, Belastungsgrößen, Durchlaufbalken. (*Brünn: Rudolf M. Rohrer*). Schnidtmann: Dreigelenkrahmenbinder mit Zugstange in Traufhöhe (Berechnungsformeln), *Stahlbau 1935, p. 71.* Grüning: Spannungsverteilung in stählernen Rahmenecken, *Bauing. 1937 p. 158.* Chwalla: Die Stabilität lotrecht belasteter Rechteckrahmen, *ibidem 1938, p. 69.* Hertwig u. Pohl: Die Stabilität der Brückenendrahmen, *Stahlbau 1936 p. 129.* Schmidt: Die Stabilität des Zweigelenkrahmens mit ungleich langen Stielen, *ibidem 1944, p. 59.* Luetkens: Die Methoden der Rahmenstatik (*Berlino/Göttingen/Heidelberg: Springer-Verlag*). Ulteriori riferimenti bibliografici v. *Par. 10.3.*

10. 2. 4. 4. 2. Con momento d'inerzia del traverso diverso da quello dei montanti

$k = \frac{J_2}{J_1}\frac{h}{l}$ ... $\begin{cases} J_2 = \text{Momento d'inerzia del traverso} \\ J_1 = \text{Momento d'inerzia dei montanti} \end{cases}$ $\quad m = 2h\,(2k+3)$





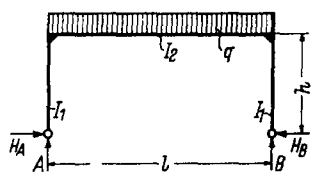
$A = q\frac{bc}{l} \qquad B = q\frac{ac}{l}$

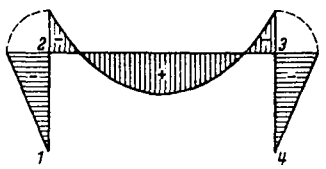
$H_A = H_B = 3q\frac{c}{lm}\left(ab - \frac{1}{12}c^2\right)$

$M_2 = M_3 = -H_A.h$

$M_x = M_{xo} + M_2$

$M_{ab} = \frac{qc}{8l}(8ab - cl) + M_2$



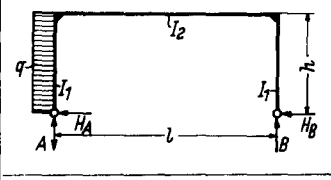


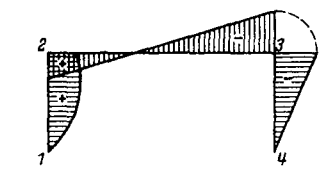
$A = B = \frac{1}{2}ql$

$H_A = H_B = \frac{1}{2}q\frac{l^2}{m}$

$M_2 = M_3 = -H_A h = -\frac{1}{2}qh\frac{l^2}{m}$

$M_{max} = \frac{1}{8}ql^2 + M_2$





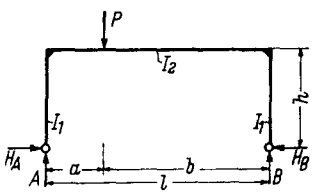
$A = -\frac{1}{2}q\frac{h^2}{l} \qquad B = +\frac{1}{2}q\frac{h^2}{l}$

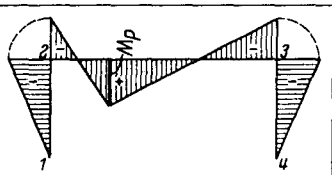
$H_A = qh^2\frac{11k+18}{4m}$

$H_B = \frac{1}{4}qh\left(1 + \frac{hk}{m}\right)$

$M_2 = \frac{1}{2}qh^2 - H_A h = \frac{3}{4}qh^2\frac{k+2}{m}$

$M_3 = -H_B.h = -\frac{1}{4}qh^2\left(1 + \frac{hk}{m}\right)$



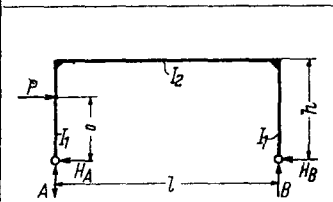


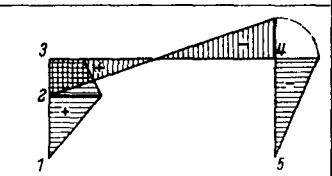
$A = P\frac{b}{l} \qquad B = P\frac{a}{l}$

$H_A = H_B = 3P\frac{ab}{lm}$

$M_2 = M_3 = -H_A h = -3P\frac{abh}{lm}$

$M_x = M_o + M_2,\ M_P = \frac{Pab}{l} + M_2$





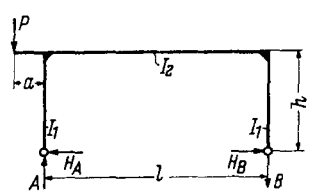
$A = B = P\frac{o}{l}$

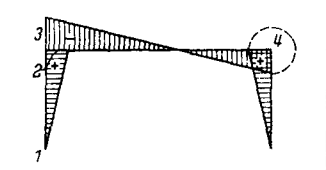
$H_A = P - H_B$

$H_B = \frac{P.a}{m}\left[3 + k\left(3 - \frac{a^2}{h^2}\right)\right]$

Für $o = h$: $A = B = P\frac{h}{l}$

$H_A = H_B = \frac{P}{2},\ M_2 = +\frac{1}{2}Ph,\ M_4 = -\frac{1}{2}Ph$





$A = P\frac{a+l}{l} \qquad B = P\frac{a}{l}$

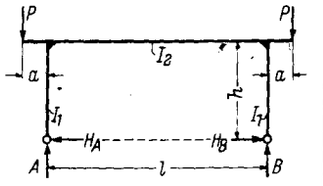
$H_A = H_B = 3P\frac{a}{m}$

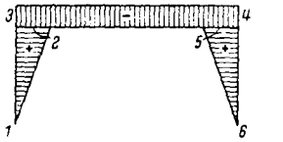
$M_2 = M_4 = H_A.h = 3P\frac{a}{m}h$

$M_3 = -Pa + M_2$

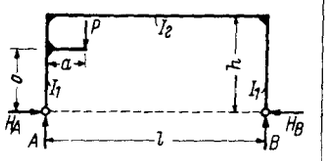
Continuazione: **Con momento d'inerzia del traverso diverso da quello dei montanti**

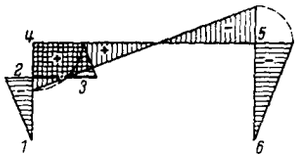
$k = \frac{J_2}{J_1}\frac{h}{l} \cdots \begin{cases} J_2 = \text{Momento d'inerzia del traverso} \\ J_1 = \text{Momento d'inerzia dei montanti} \end{cases}$ $\quad m = 2h\,(2k+3)$



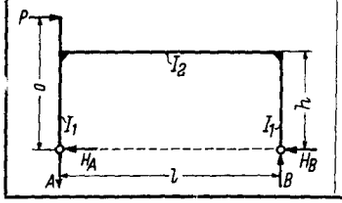


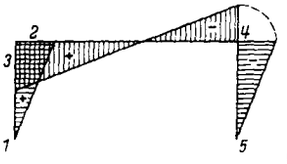
$A = B = P$
$H_A = H_B = 6P\frac{a}{m}$
$M_2 = M_5 = H_A \cdot h$
$M_3 = M_4 = -Pa + H_A h = -2P\frac{2k}{2k+3}$





$A = P\frac{l-a}{l}$
$B = P\frac{a}{l}$
$H_A = H_B = 3Pa\frac{k(h^2-o^2)+h^2}{h^2 m}$
$M_2 = -H_A \cdot o,\ M_3 = Pa - H_A \cdot o$
$M_4 = Pa - H_A h,\ M_5 = -H_B \cdot F$

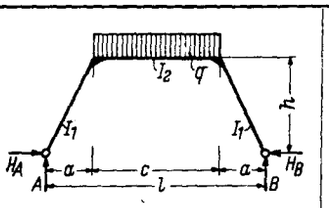


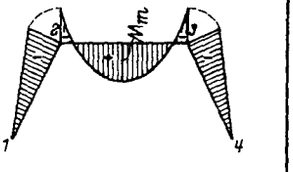


$A = B = P\frac{o}{l}$
$H_A = \frac{1}{2}P - 3P\frac{o-h}{m}$
$H_B = P - H_A = \frac{1}{2}P + 3P\frac{o-h}{m}$
$M_2 = H_A \cdot h$
$M_3 = P\,(o-h)$
$M_4 = -H_B \cdot h$

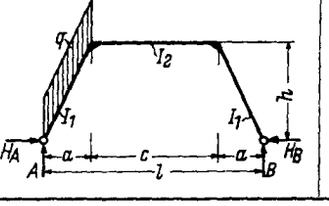
10. 2. 4. 4. 3. Portale simmetrico a due cerniere con traverso orizzontale e montanti inclinati

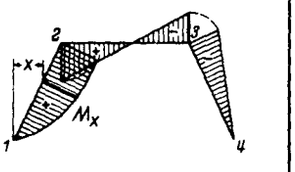
$k = \frac{J_2}{J_1}\frac{s}{c} \begin{cases} J_2 = \text{Momento d'inerzia del traverso} \\ J_1 = \text{Momento d'inerzia dei montanti} \end{cases}$ $\quad m = 2h\,(2k+3)$





$A = B = \frac{1}{2}qc$
$H_A = H_B = \frac{1}{2}qc\left(\frac{a}{h} + \frac{c}{m}\right)$
$M_2 = M_3 = -\frac{1}{2}qc^2\frac{h}{m}$
$M_m = +\frac{1}{8}qc^2\left(1 - 4\frac{h}{m}\right)$

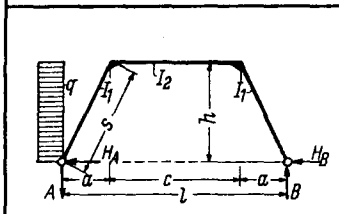


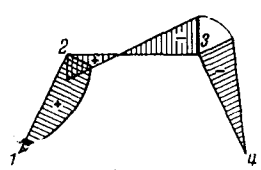


$A = \frac{1}{2}qa\frac{2l-a}{l},\ B = \frac{1}{2}q\frac{z^2}{l}$
$H_A = H_B = \frac{1}{4}q\frac{a^2}{m}(5k+6)$
$M_2 = Aa - H_A h - \frac{1}{2}qa^2 = \frac{1}{4}qa^2\left(\frac{c}{l} - k\frac{h}{m}\right)$
$M_3 = Ba - H_B \cdot h = -\frac{1}{4}qa^2\left(\frac{c}{l} + k\frac{h}{m}\right)$

Continuazione: **Portale simmetrico a due cerniere con traverso orizzontale e montanti inclinati**

$k = \frac{J_2}{J_1}\frac{s}{c}$ $\left\{ \begin{array}{l} J_2 = \text{Momento d'inerzia del traverso} \\ J_1 = \text{Momento d'inerzia dei montanti} \end{array} \right.$ $m = 2h\,(2k+3)$





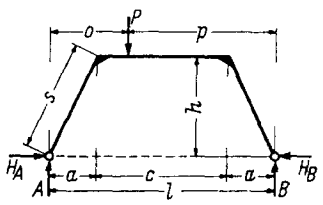
$A = B = \frac{1}{2}\,q\,\frac{h^2}{l}$

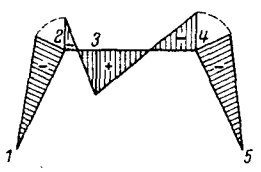
$H_A = qh - H_B$

$H_B = \frac{1}{4}\,qh^2\,\frac{5k+6}{m}$

$M_2 = \frac{1}{4}\,qh^2\left(\frac{c}{l} - \frac{kh}{m}\right)$

$M_3 = -\frac{1}{4}\,qh^2\left(\frac{c}{l} + \frac{kh}{m}\right)$





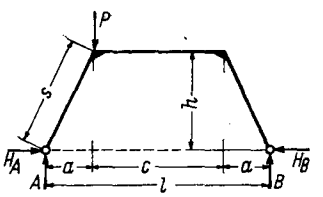
$A = P\,\frac{p}{l}$, $B = P\,\frac{o}{l}$

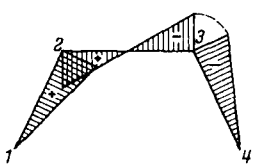
$H_A = H_B = \frac{P}{cm}\,[3(op - a^2) + 2ack]$

$M_2 = -H_A h + Aa = -H_A h + Pa\,\frac{p}{l}$

$M_3 = Ao - H_A h = Po\,\frac{p}{l} - H_A h$

$M_4 = +Ba - H_B h = -Pa\,\frac{o}{l} + H_B h$



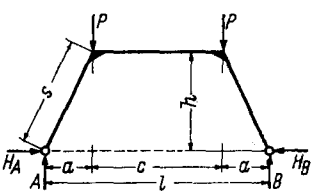


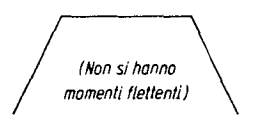
$A = P\,\frac{l-a}{l}$, $B = P\,\frac{a}{l}$

$H_A = H_B = \frac{1}{2}\,P\,\frac{a}{h}$

$M_2 = \frac{1}{2}\,Pa\,\frac{c}{l}$

$M_3 = -\frac{1}{2}\,Pa\,\frac{c}{l}$

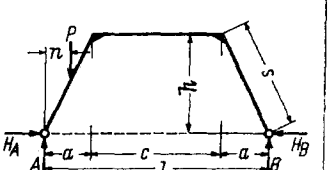


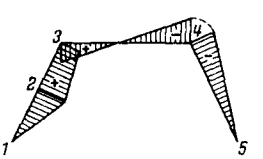


$A = B = P$

$H_A = H_B = P\,\frac{a}{h}$

(Non si hanno momenti flettenti)





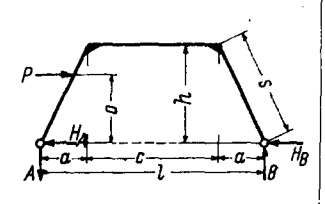
$A = P\,\frac{l-n}{l}$, $B = P\,\frac{n}{l}$

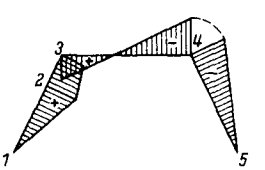
$H_A = H_B = P\,\frac{n}{m}\left[3 + \left(3 - \frac{n^2}{a^2}\right)k\right]$

$M_2 = An - H_A h\,\frac{n}{a}$

$M_3 = Aa - H_A \cdot h - P\,(a - n)$

$M_4 = +Ba - H_B h$





$A = B = P\,\frac{o}{l}$, $H_A = P - H_B$

$H_B = P\,\frac{o}{m}\left[3 + \left(3 - \frac{o^2}{h^2}\right)k\right]$

$M_2 = H_A o - Aa\,\frac{o}{h}$

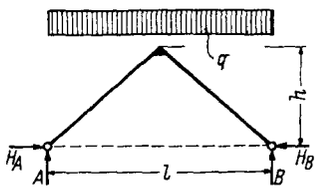
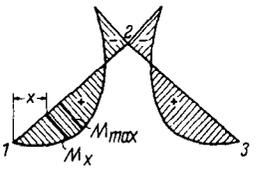
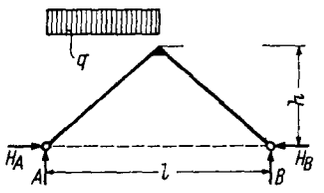
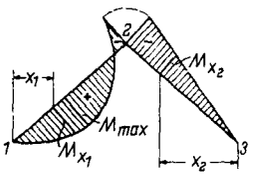
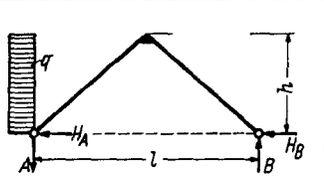
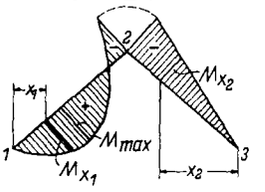
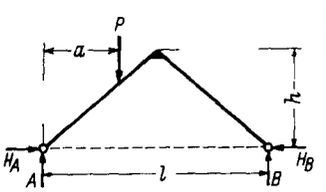
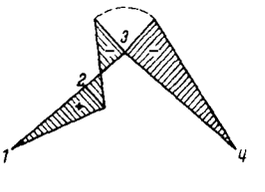
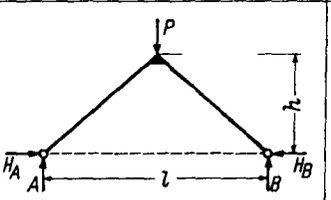
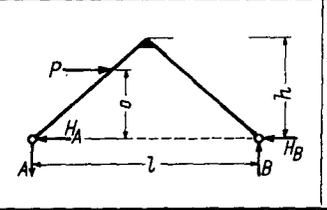
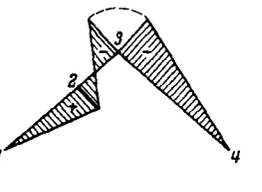
$M_3 = H_A h - Aa - P\,(h - o)$

$M_4 = -H_B h + Ba$

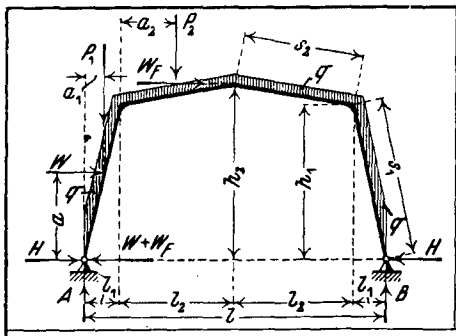
Wird $o = h$, so ist: $A = B = P\,\frac{h}{l}$

$H_A = H_B = \frac{1}{2}\,P$

10. 2. 4. 4. 4. Portale per coperture con falde uguali (Momento d'inerzia J costante)

		$A = B = \frac{1}{2} ql$ $H_A = H_B = \frac{5}{32} q \frac{l^2}{h}$ $M_x = \frac{1}{2} qlx \left(\frac{3}{8} - \frac{x}{l}\right)$ $M_{max} = \frac{9}{512} ql^2$ a $x = \frac{3}{16} l$ $M_3 = -\frac{1}{32} ql^2$
		$A = \frac{3}{8} ql$, $B = \frac{1}{8} ql$ $H_A = H_B = \frac{5}{64} q \frac{l^2}{h}$ $M_{x_1} = \frac{1}{2} qlx_1 \left(\frac{7}{16} - \frac{x_1}{l}\right)$ $M_{max} = \frac{49}{2048} ql^2$ a $x_1 = \frac{7}{32} l$ $M_{x_2} = -\frac{1}{32} qlx_2$ $M_3 = -\frac{1}{34} ql^2$
		$A = B = \frac{1}{2} q \frac{h^2}{l}$ $H_A = \frac{11}{16} qh$, $H_B = \frac{5}{16} qh$ $M_{x_1} = \frac{1}{8} q \frac{h^2}{l} x_1 \left(7 - 16 \frac{x_1}{l}\right)$ $M_{max} = \frac{49}{512} qh^2$ $M_{x_2} = -\frac{1}{8} q \frac{h^2}{l} x_2$ $M_3 = -\frac{1}{16} qh^2$
		$A = P \frac{l-a}{l}$, $B = P \frac{a}{l}$ $H_A = H_B = \frac{1}{4} P \frac{a}{h} \left(3 - 4 \frac{a^2}{l^2}\right)$ $M_3 = \frac{1}{2} P \frac{a^3}{l} \left(4 \frac{a^2}{l^2} + 2 \frac{l}{a} - 5\right)$ $M_3 = -\frac{1}{4} Pa \left(1 - 4 \frac{a^2}{l^2}\right)$
		$A = B = \frac{1}{2} P$ $H_A = H_B = \frac{1}{4} P \frac{l}{h}$ (Non si hanno momenti flettenti)
		$A = B = P \frac{h_1}{l}$ $H_A = P - H_B$ $H_B = \frac{1}{4} P \frac{o}{h} \left(3 - \frac{o^2}{h^2}\right)$ $M_3 = \frac{1}{4} Po \left(4 + \frac{o^3}{h^3} - 5 \frac{o}{h}\right)$ $M_3 = -\frac{1}{4} Po \left(1 - \frac{o^2}{h^2}\right)$

10. 2. 4. 4. 5. Portale a 2 cerniere con traverso a 2 falde e montanti inclinati



J_1 risp. J_2 = momento d'inerzia risp. del montante e del traverso;

$$k = \frac{s_1}{J_1}\frac{h_1^2}{3} + \frac{s_2}{J_2}\left[h_1 h_2 + \frac{(h_2 - h_1)^2}{3}\right]$$

a) Per P_1: $+A = P_1 - B$; $+B = \frac{P_1 a_1}{l}$;

$$-H_A = +H_B = \frac{P_1 l_1}{4k}\left[\frac{s_1}{J_1}\left(\frac{h_1 a_1}{l_1} - \frac{h_1}{3}\frac{a_1^3}{l_1^3}\right) + \frac{s_2}{J_2}\frac{(h_1 + h_2)\, a_1}{l_1}\right]$$

b) Per P_2: $+A = P_2 - B$; $+B = \frac{P_2}{l}(l_1 + a_2)$;

$$-H_A = +H_B = \frac{P_2 l_2}{4k}\left[\frac{2}{3}\frac{s_1 l_1 h_1}{J_1 l_2} + \frac{s_2}{J_2}\left((h_1 + h_2)\left(\frac{l_1}{l_2} + \frac{a_2}{l_2}\right) - h_1\left(\frac{a_2}{l_2}\right)^2 - \frac{h_2 - h_1}{3}\left(\frac{a_2}{l_2}\right)^3\right)\right]$$

c) Per W: $-A = +B = W\frac{a}{l}$; $+H_A = W\left[1 - \frac{1}{4k}\left(\frac{s_1 a}{J_1}\left[h_1 - \frac{a^2}{3h_1}\right] + \frac{s_2 a}{J_2}[h_1 + h_2]\right)\right]$; $+H_B = W - H_A$

Se $W = W_F$ agisce all'altezza del vertice h_2, si ha $-A = +B = W_F\frac{h_2}{l}$; $+H_A = +H_B = \frac{W_F}{2}$

d) Per il carico verticale q uniformemente distribuito sull'intera luce l del portale:

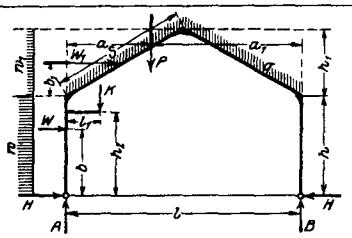
$$+A = +B = q\frac{l}{2}; \quad -H_A = +H_B = \frac{q}{6k}\left[\frac{s_1 l_1}{J_1}\left(\frac{5h_1 l_1}{4} + 2h_1 l_2\right) + \frac{s_2 l_2}{J_2}\left(\frac{3h_1 + 5h_2}{4} l_2 + \frac{3(h_1 + h_2)}{2}\frac{l_1^2 + 2l_1 l_2}{l_2}\right)\right]$$

Se il carico verticale q agisce solo **su metà portale**, ossia sulla luce $l_1 + l_2 = {}^1/_2 l$, si ha:

$$+A = \frac{3ql}{8}; \quad +B = \frac{ql}{8}; \quad -H_A = +H_B = \frac{q}{12k}\left[\frac{s_1 l_1}{J_1}\left(\frac{5h_1 l_1}{4} + 2h_1 l_2\right) + \frac{s_2 l_2}{J_2}\left(\frac{3h_1 + 5h_2}{4} l_2 + \frac{3(h_1 + h_2)}{2}\frac{l_1^2 + 2l_1 l_2}{l_2}\right)\right]$$

10. 2. 4. 4. 6. Portale a 2 cerniere simmetrico a 2 falde con montanti verticali

$m = h^2(3 + k) + h_1(3h + h_1)$; $k = \frac{J_2}{J_1}\frac{h}{s}$ ... { J_2 = Momento d'inerzia del traverso, J_1 = Momento d'inerzia dei montanti.



a) Per P: $+A = P\frac{a_1}{l}$; $+B = P\frac{a}{l}$;

$$-H_A = +H_B = \frac{Pa}{l^2}\frac{6a_1 lh + h_1(3l^2 - 4a^2)}{4m}$$

b) Per W: $-A = +B = W\frac{b}{l}$; $+H_A = W - H_B$;

$$+HB = Wb\frac{k\left(3h - \frac{b^2}{h}\right) + 3(2h + h_1}{4m}$$

c) Per W_1: $-A = +B = W_1\frac{h + b_1}{l}$; $+H_A = W_1 - H_B$;

$$+H_B = W_1\frac{2kh^2 + 3(h + b_1)(2h + h_1) - \frac{b_1^2}{h_1}(3h + b_1)}{4m}$$

Per $b_1 = h_1$ si ha:

$-A = +B = W_1\frac{h + h_1}{l}$;

$+H_A = +H_B = \frac{W_1}{2}$

d) Per carico verticale q uniformemente distribuito:

1. sulla lunghezza l: $+A = +B = \frac{ql}{2}$;

$$-HA = +HB = ql^2\frac{8h + 5h_1}{32m}$$

2. sulla lunghezza $\frac{l}{2}$: $+A = \frac{3}{8}ql$; $+B = \frac{ql}{8}$;

$$-H_A = +H_B = ql^2\frac{8h + 5h_1}{64m}$$

e) Per carico orizzontale w uniformemente distribuito sull'altezza h:

$$-A = +B = w\frac{h^2}{2l}; \quad +H_A = wh - H_B; \quad +H_B = wh^2\frac{5hk + 6(2h + h_1)}{16m}$$

f) Per carico orizzontale w_1 uniformemente distribuito sull'altezza h_1:

$$-A = +B = w_1\frac{h_1(2h + h_1)}{2l}; \quad +H_A = w_1 h_1 - H_B; \quad +H_B = w_1 h_1\frac{8h^2(k + 3) + 5h_1(4h + h_1)}{16m}$$

g) Per il carico concentrato K sulla mensola:

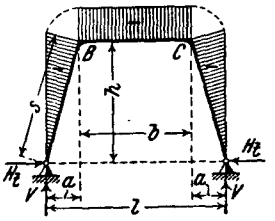
$$+A = K\frac{l - l_1}{l}; \quad +B = K\frac{l_1}{l}; \quad -H_A = +H_B = \frac{3Kl_1}{h}\frac{k(h^2 - h_2^2) + h(2h + h_1)}{4m}$$

10. 2. 4. 5. Formule per il calcolo della spinta orizzontale H_t indotta dalle variazioni di temperatura per diversi tipi di portali con momento d'inerzia J costante

J = momento d'inerzia in cm⁴, | ε = coefficiente di dilatazione termica lineare = 0,000 012
t = variazione di temperatura in °C, | E = modulo di elasticità = 2 100 000 kg/cm² } (per l'acciaio).

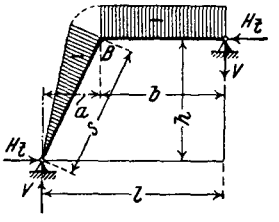
Le formule e i diagrammi dei momenti valgono per aumento di temperatura. Per riduzioni di temperatura le forze agiscono nel senso opposto e i momenti assumono il segno +.

Schema I (*v. Par. 10. 2. 4. 3. 1 e Par. 10. 2. 4. 4. 3*)



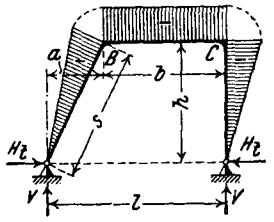
$$H_t = \frac{3\,E\,J\,\varepsilon\,t\,l}{h^3\,(2\,s + 3\,b)};$$
$$V = 0;$$
$$M_B = M_C = -\,H_t\,h.$$

Schema II (*v. Par. 10. 2. 4. 3. 2*)



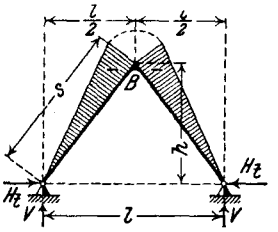
$$H_t = -\frac{M_B\,l}{b\,h};$$
$$V = -\frac{M_B}{b};$$
$$M_B = -\frac{3\,E\,J\,\varepsilon\,t\,(l^2+h^2)}{b\,h\,(s+b)}$$

Schema III (*v. Par. 10. 2. 4. 3. 3*)



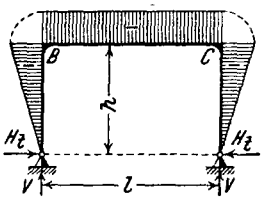
$$H_t = \frac{3\,E\,J\,\varepsilon\,t\,l}{h^3\,(s + 3\,b + h)};$$
$$V = 0;$$
$$M_B = M_C = -\,H_t\,h.$$

Schema IV (*v. Par. 10. 2. 4. 3. 4 e Par. 10. 2. 4. 4. 4*)



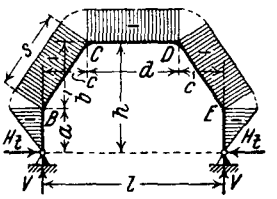
$$H_t = 1{,}5\,\frac{E\,J\,\varepsilon\,t\,l}{s\,h^3};$$
$$V = 0;$$
$$M_B = -\,H_t\,h.$$

Schema V (*v. Par. 10. 2. 4. 3. 6 e Par. 10. 2. 4. 1*)



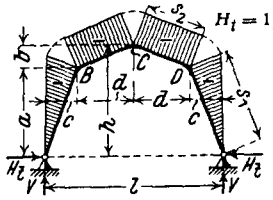
$$H_t = \frac{3\,E\,J\,\varepsilon\,t\,l}{h^3\,(2\,h + 3\,l)};$$
$$V = 0;$$
$$M_B = M_C = -\,H_t\,h.$$

Schema VI (*v. Par. 10. 2. 4. 3. 5*)



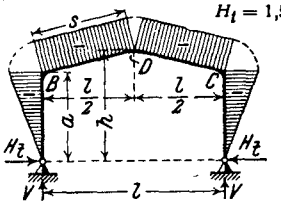
$$H = 1{,}5\,\frac{E\,J\,\varepsilon\,t\,l}{a^3 + s\,a + 1{,}5\,d\,h^2};$$
$$a = a^2 + a\,h + h^2;$$
$$V = 0;$$
$$M_B = M_E = -\,H\,a;$$
$$M_C = M_D = -\,H\,h.$$

Schema sec. Par. 10. 2. 4. 4. 5.



$$H_t = 1{,}5\,\frac{E\,J\,\varepsilon\,t\,l}{a^3\,(s_1+3\,s_2)+s_2\,b\,(3\,a+b)};$$
$$V = 0;$$
$$M_B = M_D = -\,H_t\,a;$$
$$M_C = -\,H_t\,h.$$

Schema sec. Par. 10. 2. 4. 4. 6.



$$H_t = 1{,}5\,\frac{E\,J\,\varepsilon\,t\,l}{a^3 + 3s\left(a^2 + a\,b + \frac{b^2}{3}\right)};$$
$$V = 0;$$
$$M_B = M_C = -\,H_t\,a;$$
$$M_D = -\,H_t\,h.$$