

Study on dimensionless criterion of fracture of closed pipe due to freezing of water

S. Oiwake, H. Saito, H. Inaba, and I. Tokura, Hokkaido, Japan

Abstract. The pressure rise due to freezing of water enclosed in pipes has been studied, by means of theoretical computation and experiments, under the various heat transfer conditions and ambient temperatures, taking deformation of pipe walls into account. It has been found that the presented models well represent the actual phenomena and dimensionless parameters proposed are effective for correlating the pressure rises, the conditions of heat transfer, dimensions of the pipes. The tangential stress has been analyzed in dimensionless form to clarify its dependabilities on the modified Biot number and the ratio of thickness of a pipe wall and its radius. It has also shown that F_{crit} , the critical Fourier number, can be considerably increased by introducing a void area at the center of pipes and the criterion of freezing fracture of pipes can be expressed as a combination of the dimensionless parameters presented in the report.

Studie über ein dimensionsloses Kriterium zur Vorhersage des Berstens eines geschlossenen Rohres durch frierendes Wasser

Zusammenfassung. Es wurde der Druckanstieg bei Einfrieren eines in Rohren eingeschlossenen Wassers experimentell und mittels theoretischer Berechnungen bei unterschiedlichen Werten von Wärmübergang und Umgebungstemperatur und unter Berücksichtigung der Verformung der Rohrwand berechnet. Dabei konnte festgestellt werden, daß das vorgestellte Modell die tatsächlichen Phänomene gut wiedergibt und die vorgeschlagenen dimensionslosen Parameter für die Berechnung des Druckanstieges und die Berücksichtigung der Wärmeübergangsbedingungen geeignet sind. Die Tangentialspannung wurde in dimensionsloser Form analysiert, um die Abhängigkeit von der modifizierten Biot-Zahl und von dem Verhältnis von Wandstärke zu Radius des Rohres zu klären. Es wird auch gezeigt, daß die kritische modifizierte Fourier-Zahl beträchtlich erhöht werden kann, wenn man eine Gaszone im Zentrum des Rohres einführt. Weiterhin wird gezeigt, daß das Kriterium für den Einfrierbruch des Rohres als Kombination dimensionsloser Parameter ausgedrückt werden kann.

1 Introduction

Freezing in city water systems causes the failure of water supply and, sometimes, fracture of the pipe line system, including joints or valves, and floods would happen by flowing out of water from the damaged parts of the systems, when the frozen pipe melts due to temperature rise in daytime. Especially when the system is for emer-

gency water, its stoppage would cause fatal danger to city life. Therefore, freezing of water in pipes is a very important problem to solve for every day life in northern areas.

Freezing of water in tubes is known as a Stefan problem and has been investigated by many researchers [1–3]. Those works treated, however, problems such that freezing occurs at the triple point of water and the deformation of containers caused by volumetric change of water can be negligible in freezing. This is for simplification of the problem in describing and the results thus obtained is effective to prediction of such processes as in ice production systems where freezing occurs at constant pressure. However, the volumetric change of water in freezing has to be taken into account in the case that the frozen fracture of pipes is to be considered as mentioned above since the fracture or deformation of a vessel is caused by the increase in volume with the phase change of water. Recent works by Sugawara et al. [4, 5] treated a freezing problem in tubes exposed to the natural convection conditions and found that the water pressure begins to rise very rapidly immediately after the starting of the ice formation inside and the ice formation and the pressure rise stop as time progresses in small pipes because of freezing temperature drop due to the high pressure inside them.

This paper studies on stresses in pipe walls induced by the freezing pressure under the relation of heat transfer conditions and simulates the pressure rises in pipes made of different materials to find the dimensionless correlation between the pressure rise and the heat transfer conditions. A countermeasure is also proposed in this paper to prevent the frozen fracture of pipes by introducing small amount of damping space in pipes.

2 Theoretical model for calculation

2.1 Outline of the model

Assuming a pipe with inner radius R_b , wall thickness ΔR filled with water and having an ice layer with radius R_i on

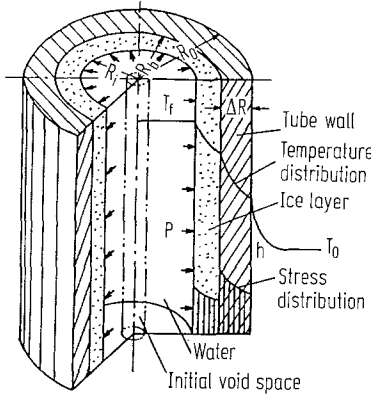


Fig. 1. Theoretical model

the inner surface exposed to the ambient temperature T_0 , and letting ΔR_i be the increase in thickness during time interval $\Delta \tau$, T_f be drop in freezing temperature caused by pressure rise ΔP , the energy flowing through the ice layer and the wall of the pipe to the ambient is equal to the energy corresponding the sensible heat drop of water and the latent heat required for ice growth of ΔR_i during $\Delta \tau$ (Fig. 1).

If the temperature of water is assumed to be uniform, the energy balance can be written as,

$$L \varrho_i \frac{dR_i}{d\tau} = \lambda_i \frac{dT}{dr} - \frac{1}{2} \varrho_w C_w R_i \frac{dT_f}{d\tau} \quad (1)$$

where ϱ and L are density and latent heat of freezing and subscriptions i and w denote ice and water, respectively.

Let ΔR_w be the thickness occupied by the ice layer of ΔR_i before freezing, the volume in the pipe increases by $\Delta W = \Delta R_i - \Delta R_w$. If the water is enclosed in the pipe, the pressure rises. If the deformation caused by the pressure change be ΔQ , and the water in the pipe compressed by volume ΔV ,

$$\Delta V = \Delta W - \Delta Q. \quad (2)$$

The compression of water can be expressed as follows in terms of P [4].

$$\frac{\Delta V}{\Delta P} = \frac{1}{2} \alpha R_i. \quad (3)$$

where α is compressibility of water.

Since ΔQ is the deformation of the inner surface of the ice layer during $\Delta \tau$, it can easily be obtained, by putting $\bar{R}_0 = R_0/R_b$ and $\bar{R}_b = R_b/R_i$, from the composite tubes theory of elasticity as,

$$\Delta Q = \left(\frac{\bar{R}_b^2 + 1}{\bar{R}_b^2 - 1} + \frac{1}{m_i} \right) \frac{\Delta P}{E_i} R_i - \frac{2 \bar{R}_b^2}{\bar{R}_b^2 - 1} \frac{\Delta P_b}{E_i} R_i \quad (4)$$

where E : Young's modulus, m : Poisson number, suffix s denotes pipe wall. ΔP_b in Eq. (4) is the pressure change applying on the inner surface of the pipe and, therefore, it

can be estimated from the following equation.

$$\Delta P_b = \frac{2 \Delta P / \{E_i (\bar{R}_b^2 - 1)\}}{\frac{1}{E_s} \left(\frac{\bar{R}_0^2 + 1}{\bar{R}_0^2 - 1} + \frac{1}{m_s} \right) + \frac{1}{E_i} \left(\frac{\bar{R}_b^2 + 1}{\bar{R}_b^2 - 1} - \frac{1}{m_i} \right)} \quad (5)$$

The tangential stress σ is, on the inner surface of the pipe,

$$\sigma = \frac{2 P (\bar{R}_0^2 + 1) / \{E_i (\bar{R}_b^2 - 1) (\bar{R}_0^2 - 1)\}}{\frac{1}{E_s} \left(\frac{\bar{R}_0^2 + 1}{\bar{R}_0^2 - 1} + \frac{1}{m_s} \right) + \frac{1}{E_i} \left(\frac{\bar{R}_b^2 + 1}{\bar{R}_b^2 - 1} - \frac{1}{m_i} \right)}. \quad (6)$$

From Eqs. (2)–(5), the relation $\varrho_i \Delta R_i = \varrho_w \Delta R_w$, and the expression $\alpha = (\Delta Q / \Delta P) / \varrho$, obtained by substituting $\Delta Q = \varrho_2 - \varrho_1$ to the definition of compressibility α , ΔP can be solved as,

$$\Delta P = \frac{(1 - \varrho_i / \varrho_w) \Delta R_i}{\frac{1}{2} \alpha R_i + \frac{R_i}{E_i} (A - B)} \quad (7)$$

where,

$$A = \frac{\bar{R}_b^2 + 1}{\bar{R}_b^2 - 1} + \frac{1}{m_i}$$

$$B = \frac{4 \bar{R}_b^2 / (\bar{R}_b^2 - 1)^2}{\frac{E_i}{E_s} \left(\frac{\bar{R}_0^2 + 1}{\bar{R}_0^2 - 1} + \frac{1}{m_s} \right) + \left(\frac{\bar{R}_b^2 + 1}{\bar{R}_b^2 - 1} - \frac{1}{m_i} \right)}$$

and suffix 1 and 2 indicate values at the present time and the preceding time, respectively.

The drop in temperature of freezing can be estimated from the Clausius-Clapeyron equation,

$$\frac{dT_f}{dP} = \frac{T_f (v_w - v_i)}{L} \quad (8)$$

where v_i and v_w are specific volumes of ice and water, respectively, which vary with the pressure applied.

2.2 Outline of calculation procedure

The term $dT_f/d\tau$ in Eq. (1) is assumed to be zero and then the first approximation of ΔP can be obtained from Eq. (7) by regarding the temperature profile in the ice layer to be steady in each time step. Based on this value of ΔP , the freezing temperature drop ΔT_f can be estimated as the first approximation.

Using the results thus obtained, the first approximation of $dT_f/d\tau$ is corrected, and this procedure is repeated until ΔP converges within an accuracy desired.

In this calculation, all the independent variables, T_f , R_i and P have to be converged; however, it is only enough that one should monitor the values of P at each step of time interval since P is least convergible of the variables.

In the present case, the authors stopped the calculation when the relative error in P 's between the values of the

last repetition and the values immediately before the last became less than 10^{-5} .

The Runge-Kutta-Gill method was used for integrating Eq. (1).

In the case of large values of modified Biot numbers (to be mentioned later), the integrated results changed depending upon the values of the time step $\Delta\tau$. Therefore, the authors integrated those basic equations by using various values of time step $\Delta\tau$ and checked the results to confirm the appropriate $\Delta\tau$ for each modified Biot number. The initial temperature of water in pipes was assumed 0°C in the present study.

3 Results and discussion

3.1 Examination of the calculated results

Figure 2 shows the pressure changes in pipes having different radii and compares them with the results obtained by Sugawara [4]. It can be seen that the pressures increase almost linearly with increasing time and seem to reach to their maximum values as time passes. This tendency becomes more evident in pipes having relatively small radii. This is, as mentioned before, because of the drop in freezing temperature due to the pressure increase of water, and if the computation is continued for long time under the same conditions as in Fig. 2, the temperature of water in the pipe becomes equal to that of the ambient and the growth of the ice layer stops, for example in the case of $R_b = 5\text{ mm}$ and $R = 3\text{ mm}$, at 4.5 hours after

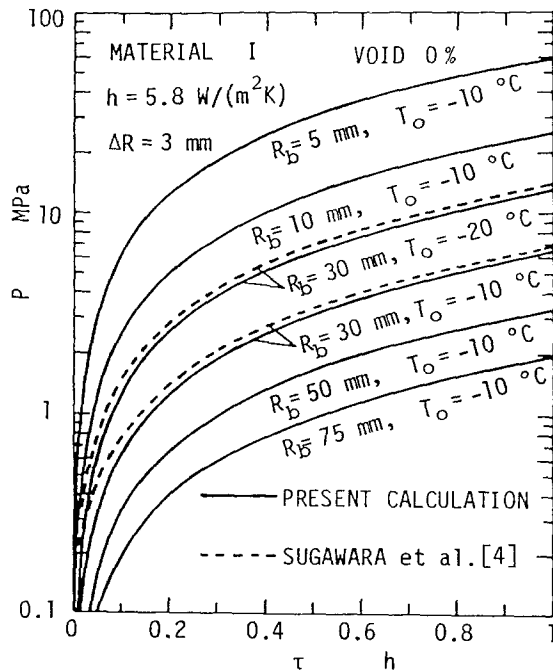


Fig. 2. Pressure change in pipes with time

the beginning of freezing. In comparing the present calculation with the Sugawara's results, both are in good agreement as whole tendency. However, the present results are different from the latter especially in the case of pipes with thinner wall. The reason of this is not always clear but the authors consider that the discrepancies came from the difference of repetitions in numerical integration since, in the case of the thinner tube wall, the displacement of the tube wall is more than in the thicker-wall tube so that the thinner the wall is, the more the number of repetition in calculation is. As will be seen later, the present results also represents the experimental data as whole. However, freezing occurs when the temperature of water reaches a limit temperature of supercooling and then the temperature of water rises due to the delivery of the latent heat, and it is not until the water which rose in temperature in the previous freezing reaches again at the limit temperature of supercooling that the next freezing occurs [7]. So, the actual process does not occur continuously at the triple point temperature but discontinuously.

3.2 Dimensionless parameters

The growth of the ice layer causes the pressure increase in pipes. Therefore, it is necessary to derive a parameter which describes the formation of an ice layer inside a tube. From the dimensional analysis of heat transfer rates and growing rates of ice layers, the following number can be obtained as a parameter governing the process [6].

$$F^* = Fo \cdot Ste$$

where, Fo : Fourier number $= a_i \tau / R_b^2$, a_i : thermal diffusivity of ice, Ste : Stefan number $= C_i (T_f - T_0) / L$, C_i : specific heat of ice, $T_f - T_0$: difference in temperatures of the ambient and the initial state of water. This should be called the modified Fourier number.

In order to discuss the fracture of pipes due to freezing, it is important to know the maximum stresses in pipes induced by pressure inside. When a critical stress σ_{crit} is induced on the inner surface of a pipe, having wall thickness of ΔR and radius R_b , and the ice layer of $R_b - R_i$ in thickness, by a critical pressure P_{crit} , σ_{crit} can be obtained from Eq. (6) by putting $P = P_{crit}$. Defining a new dimensionless parameter P^* as $P^* = P / P_{crit}$, it is equal to σ / σ_{crit} . Therefore, P^* can be considered as a parameter governing the magnitude of tangential stresses in pipes.

On the other hand, taking the ratio of the resistance of convective heat transfer to that of heat conduction through the wall and using a logarithmic temperature distribution in the wall, a new parameter can be derived as follows.

$$B^* = Bi / \log_e (1 + \Delta R / R_b)$$

where Bi : Biot number $= h R_0 / \lambda_s$, h : heat transfer coefficient on the outer surface of a pipe, λ_s : thermal conductivity of a pipe, which is considered as a dimensionless

Table 1. Properties of metals and ice

Material	Young's modulus GPa	Poisson number	σ_{crit} GPa	λ_s W/mK
I	206	3.45	0.245	46.5
II	206	2.95	0.425	40.0
III	150	3.7	0.290	34.5
ice	9.8	2.94	—	2.2

Table 2. Condition of calculation

T_0	B^*	F^*	$\Delta\tau$
$-30 \sim -5^\circ\text{C}$	$0.01 \sim 100$	$10^{-3} \sim 100$	$0.0001 \sim 0.01 \text{ h}$

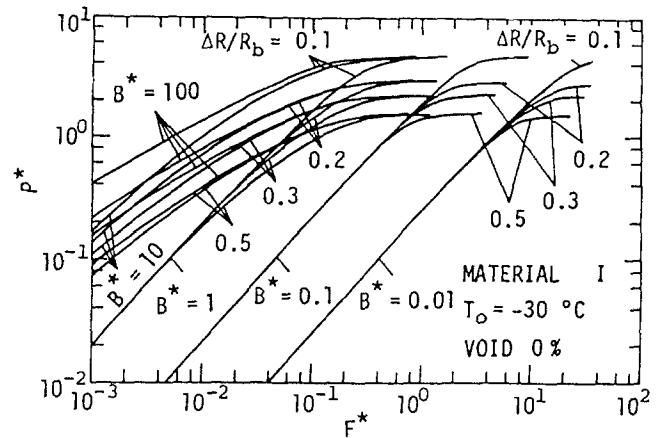
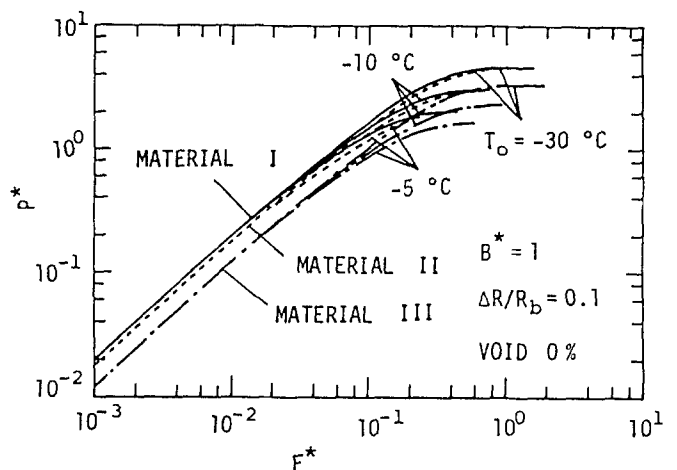
parameter correlating heat transfer conditions on the outer surface of the pipe and should be called a modified Biot number. The results correlated by the parameters presented above will be discussed below. The materials and their properties used in the calculation are tabulated in Table 1 and conditions for calculation are shown in Table 2.

3.3 Pressure rise in dimensionless form

The calculated pressure rises are correlated by the dimensionless parameters mentioned above and shown in Fig. 3 as a function of the modified Fourier number F^* . It shows that P^* increases rapidly with increasing F^* in early period of time, slows down the increasing rates as time advances and approaches their peak values. A tendency can be seen the peak values are reduced by increasing of the ratio R/R_b . The change in pressure rises of pipes with different materials are shown in Fig. 4. In this figure, it can be seen that P^* is higher when the Young's modulus E is large. This is natural because the deformation of the pipe with large E is small comparing when the same pressure is applied to a pipe with small E . The pressure rises can be correlated well by the presented dimensionless parameters except the ranges where the effect of the drop in freezing temperature appears.

3.4 Effect of heat transfer rate on the surface

In order to clarify the effect of heat transfer conditions on the surface of pipes, dimensionless pressure rise P^* are also presented in Fig. 3 with different modified Biot number as a parameter. It is clear in the figure that P^* increases relatively slowly with time when B^* is small while the larger the value B^* is, the more steeply it increases. This can be understood as the result that in the case of large B^* the ice formation rate is high since large B^* means low resistance to heat flow through the pipe wall. In the case of B^* larger than 10, the ice layer grows

**Fig. 3.** Relation between dimensionless pressure change and modified Fourier number**Fig. 4.** Influence of properties of materials upon dimensionless pressure change

very rapidly so that the drop in freezing point appears early and the behavior in the pressure rise is considerably different from others. In any case, since the behavior of pressure like this is due to the ice formation rate and the rigidity of the pipe, in order to prevent the freezing fracture, it is necessary to increase the heat transfer resistance and use materials with low E and high critical stresses. In that case, those figures are available to predict the change in time of fracture by changing the surface resistance and properties of the pipe material. When the actual pressure in pipes is required to estimate, it is also necessary to know P_{crit} for each corresponding condition. P_{crit} for various conditions are plotted in Fig. 5 as a function of F^* . The magnitude of P_{crit} depends upon the values of σ_{crit} and the values employed in this calculation are shown in Table 1.

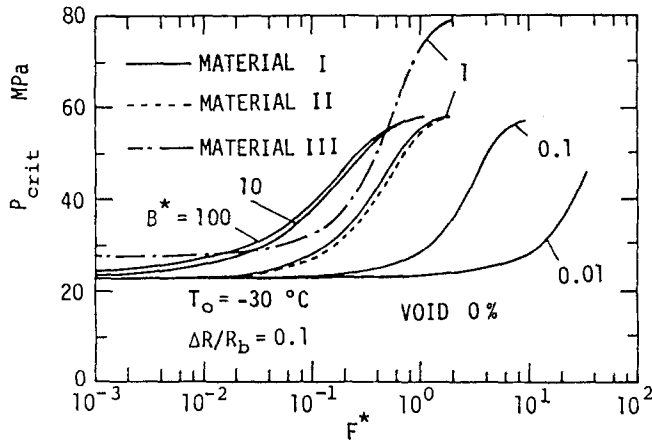


Fig. 5. Critical pressure P_{crit} and modified Fourier number F^*

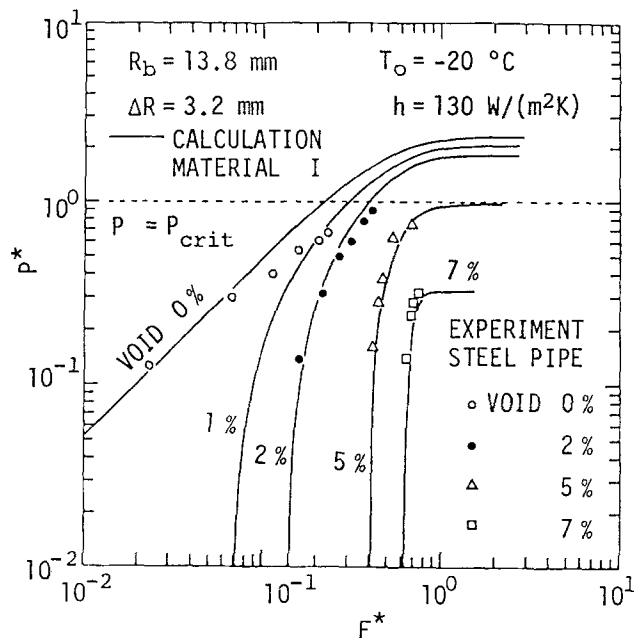


Fig. 6. Effect of void in pipes on dimensionless pressure rise

3.5 Effect of void in pipes on critical modified Fourier number F_{crit}^*

As mentioned above, the fracture of pipes is resulted from the increase in volume in freezing. Therefore, it is possible to delay the critical time for fracture by introducing a space which absorbs the expansion of water. In this study, the authors tried to examine the effect of the damper volume introduced in pipes on the pressure rises due to freezing. The relation between the modified Fourier numbers and the dimensionless pressure rises are shown in Fig. 6. The plotted data in the figure were obtained by immersing a pipe which has a small amount of an air space at its center into a brine with a prescribed temperature, and measuring strains on the outer surface of the pipe by strain gauge.

It can be seen that the calculations almost represent the tendency of the experimental data as whole, but little difference can still be recognized between them. This is understood by considering that the actual freezing does not occur at a temperature which is expected in a thermodynamic equilibrium state and the material of pipes employed in the experiments was not always uniform microscopically. From this figure, one can see that the onset of pressure rises are delayed very much by introducing the voids inside. Also the dimensionless pressure P^* is considerably reduced by the voids, comparing with the case of no-void pipes. Especially in the case of pipes having void more than 5%, P^* does not increase beyond the line $P = P_{crit}$. This means that the maximum stresses in the walls induced by freezing remain still lower than the critical value even after F_{crit}^* and, therefore, the pipes do not break at any time under those conditions. This reduction in the dimensionless pressure rise is due to the ice layers which have already existed on the inner surface of pipes at the onset of pressure rise and increases P_{crit} in comparing with a critical pressure which would be in a pipe without void. This can be said in other words that the ice layer existing at the onset of pressure increase acts as a reinforcing part of the pipe and apparently increases its strength. While it is needless to say that if the volume of a void is more than 8% of the volume of water contained in a pipe the fracture does not occur in any condition, the probabilities of fracture of a pipe can be lowered even in the case that void ratio is less than 8% because of the delay of the critical time and the reduction of the maximum P^* .

3.6 Prediction of critical time for fracture

The critical time for fracture, which is defined as an interval from the beginning of freezing to the instant at which the tangential stress reaches a prescribed critical value, can be obtained by reading values of F^* at intersections of P^* lines with $P^* = 1$ line in Figs. 3 and 4. Defining F_{crit}^* as values of F^* at critical time, F_{crit}^* can be expressed as a function of B^* as in Fig. 7. For comparison, F_{crit}^* in the case of $P^* = 0.5$ is also shown in the figure (dashed lines). The figure shows that the modified Fourier number F_{crit}^* very rapidly decreases with increasing modified Biot number B^* and approaches some lower limiting values at B^* ranging from 5 to 10. This means a pipe fractures very early when its modified Biot number is large and it takes longer time for a pipe to fracture with decreasing B^* . Also the effect of a void in pipes can be seen in the figure. Comparing the lines of void 0% and 2%, one can see that this amount of void in a pipe is enough to double the F_{crit}^* even in the region of small B^* and more effective in the region of B^* larger than 1. In the case of pipes having voids more than 6%, P^* did not exceed 1 under any condition in the present calculation. This results can be understood pipes do not break because of freezing

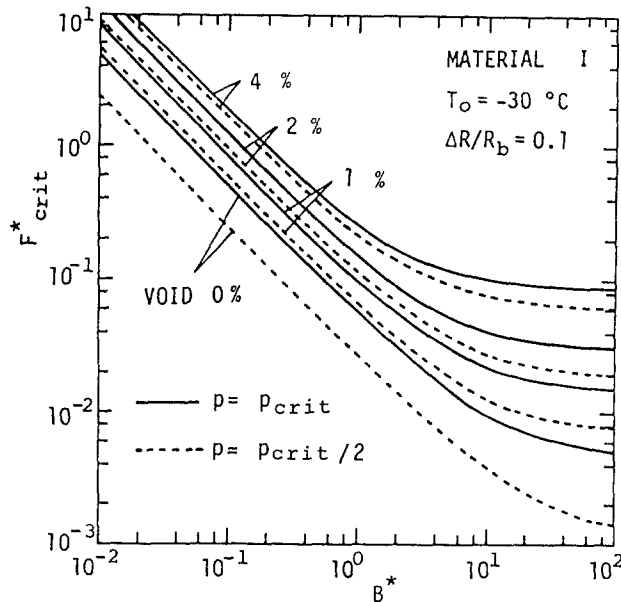


Fig. 7. Relation between F_{crit}^* and B^*

under any heat transfer conditions and the ambient temperatures ($T_0 > -30\text{ °C}$), regardless of their radii and thickness.

4 Conclusion

In order to investigate the freezing fracture problems of water pipe lines, the characteristics of pressure behavior in frozen pipes were studied and clarified that the pressure rise can be correlated with geometry of pipes, heat transfer conditions and temperatures of the ambient by using dimensionless parameters presented. It was also shown that introducing a small void in a pipe can be a countermeasure to avoid the freezing fracture.

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S. Oiwake, Assist. Prof. and
H. Inaba, Assist. Prof.
Department of Mechanical Engineering
Kitami Institute of Technology
Kitami, Hokkaido 090
Japan

H. Saito, Prof. and
I. Tokura, Assist.
Department of Mechanical Engineering
Muroran Institute of Technology
Muroran, Hokkaido 050
Japan