magnitudes are varied. The common factor that multiplies all loads as they vary in fixed proportion is called the *load factor*. The procedure for finding the load factor is as follows [13.5]:

1. Find the locations of the plastic hinges in each component of the frame using the same method as for beams.
2. Form possible failure modes called *mechanisms* by different combinations of plastic hinges. The number of hinges in each mechanism is equal to the number of redundancies plus 1.
3. Calculate the collapse load factor for each mechanism.
4. Calculate the moments in the frame for each collapse load factor to determine the correct load factor. The true load factor should be such that the moment in the frame due to this load should not exceed the plastic moment M_p.

In addition to the collapse load factors that can be determined, a safe-load region can be established. Table 13-6 shows safe-load regions for several frameworks. In Table 13-6, a combination of forces applied on the frame define a point on the xy plane. When this point falls inside the safe region, no collapse occurs. When the point falls on the boundary of the region, collapse occurs and the collapse mode is identified by the location on the boundary, as indicated by the figures in Table 13-6. Loadings leading to points outside the region correspond to a collapsed framework. In fact, an attempt to increase the applied loads beyond that necessary to reach the boundary results in further movements of the plastic hinges without an increase in the collapse loads. See Ref. [13.5] for techniques for calculating the safe-load region.

13.3 GRIDWORKS

A special case of frames is a *gridwork*, or *grillage*, which is a network of beams rigidly connected at the intersections, loaded transversely. That is, a gridwork is a network of closely spaced beams with out-of-plane loading. It may be of any shape and the network of beams may intersect at any angle. These beams need not be uniform.

The gridworks treated here are plane structures (Fig. 13-5), with the beams lying in one direction called *girders* and those lying in the perpendicular direction called *stiffeners*. Either set of gridwork beams can be selected to be the girders. In practice, the wider spaced and heavier set is usually designated as girders, whereas the closer spaced and lighter beams are stiffeners. For a *uniform gridwork*, the girders are identical in size, end conditions, and spacing. However, the set of stiffeners may differ from the set of girders, although the stiffeners are identical to each other. The treatment here is adapted from Ref [13.6].

For the formulas here, the cross section of the beams may be open or closed, although torsional rigidity is not taken into account. For closed cross sections this may lead to an error of up to 5%. Stresses in the girders and stiffeners can be calculated using the formulas for beams in Chapter 11.

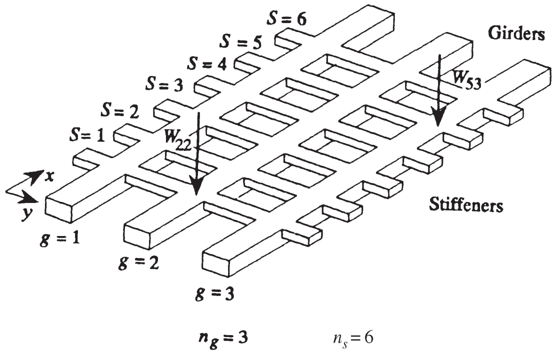

Figure 13-5: Typical gridwork.

For gridworks not covered by the formulas here, use can be made of a framework computer program. The structural matrices, including transfer, stiffness, and mass matrices, for a grillage are provided in Section 13.4. The sign convention of the transfer matrix method for displacements and forces for the beams of Chapter 11 apply to the gridwork beams here.

Static Loading

The deflection, slope, bending moment, and shear force of the gth girder of the gridwork are given in Table 13-7. The ends of both the girders and stiffeners are simply supported. Table 13-8 provides the parameters K_j for particular loadings. Sufficient accuracy is usually achieved if only M terms, where $M \ll \infty$, are included in the formulas for Tables 13-7 and 13-8; that is,

$$\sum_{j=1}^{\infty} = \sum_{j=1}^{M}$$

Example 13.6 Deflection of a Gridwork with Uniform Force The grillage of Fig. 13-6 is loaded with a uniform force of 10 psi. Use the formulas of Tables 13-7 and 13-8 to find the deflections at the intersections of the beams. Assume that the axial forces in both the girders and stiffeners are zero.

As indicated in case 3, Table 13-8, only a single term is needed in the summation of the formulas of Table 13-7. It is reasonable to assume that the loading intensity along either of the stiffeners will be $p_s = (10 \text{ psi})L_g/(n_s + 1) = 10(\frac{100}{3}) = 333.33$ lb/in. Use one term of case 1, Table 13-7:

$$w_g = \sin\frac{\pi g}{n_g + 1} K_1 \sin\frac{\pi x}{L_g} = K_1 \sin\frac{\pi g}{3} \sin\frac{\pi x}{100} \tag{1}$$

where from case 3 of Table 13-8, since $P_g = P_s = 0$,

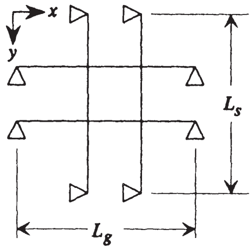

Figure 13-6: Grillage for Examples 13.6–13.8.

$$K_1 = \frac{\frac{4L_s^4}{EI_s\pi^5}\sum_{s=1}^{2} p_s \sin\frac{\pi s}{3}}{\frac{3}{2}+\frac{3}{2}} = \frac{\frac{4L_s^4 p_s}{EI_s\pi^5}(\sqrt{3}/2+\sqrt{3}/2)}{\frac{3}{2}+\frac{3}{2}} = \frac{4L_s^4 p_s}{EI_s\pi^5}\frac{\sqrt{3}}{3} \tag{2}$$

Then

$$w_1|_{x=L_g/3} = w_2|_{x=L_g/3} = w_1|_{x=2L_g/3} = w_2|_{x=2L_g/3}$$

$$= \frac{4L_s^4 p_s}{EI_s\pi^5}\frac{\sqrt{3}}{3}\sin\frac{\pi}{3}\sin\frac{\pi}{3} = 0.062886 \text{ in.} \tag{3}$$

Example 13.7 Moment in a Gridwork with Uniform Force and Axial Loads Find the maximum bending moment in the grillage of Fig. 13-6. The grillage is loaded with a transverse uniform force of 10 psi. In addition, the girders are subject to compressive axial forces of 5000 lb.

The bending moments in the girders are given by case 3, Table 13-7. As noted in case 3 of Table 13-8, only one term in case 3, Table 13-7, is required. Thus

$$M_g = EI_g \sin\frac{\pi g}{n_g+1} K_1 \frac{\pi^2}{L_g^2}\sin\frac{\pi x}{L_g} \tag{1}$$

The coefficient K_1 is taken from case 3, Table 13-8. Use the data $L_s = L_g = 100$ in., $E = 3 \times 10^7$ psi, $I_s = I_g = 100$ in^4, $P_s = 0$, $P_g = 5000$ lb, $n_s = 2$, $n_g = 2$, $p_s = 333.33$ lb/in (Example 13.6).

$$P_e = \frac{\pi^2(3\times 10^7)100}{100^2} = 2{,}960{,}881 = P_c, \qquad \frac{P_g}{P_c} = 1.69\times 10^{-3} \tag{2}$$

$$K_1 = \frac{\frac{4L_s^4 p_s}{EI_s\pi^5}\sum_{s=1}^{2}\sin\frac{\pi s}{3}}{\frac{3}{2}(0.99831)+\frac{3}{2}} = \frac{4L_s^4 p_s}{EI_s\pi^5}(0.57784) \tag{3}$$

It follows from symmetry that the maximum moment occurs at $x = \frac{1}{2}L_g$. Then, for $g = 1$,

$$M_{1,\max} = M_g|_{x=L_g/2} = EI_g \sin\left(\frac{\pi}{3}\right) \frac{4L_s^4 p_s}{EI_s \pi^5}(0.57784)\frac{\pi^2}{L_g^2} = 215{,}190 \text{ in.-lb} \quad (4)$$

Example 13.8 Deflections Due to Concentrated Forces Consider again the grillage of Fig. 13-6. Assume that there are no distributed or in-plane axial forces. Suppose that concentrated forces of 10,000 lb act at each intersection.

With equal concentrated forces, sufficient accuracy is usually achieved with one term of the formulas of Table 13-7:

$$w_g = K_1 \sin\frac{\pi g}{3} \sin\frac{\pi x}{100} \quad (1)$$

with (case 1 of Table 13-8)

$$K_1 = \frac{\frac{2L_s^3}{EI_s\pi^4} \times 10{,}000 \sum_{s=1}^{2} \sum_{g=1}^{2} \sin\frac{\pi g}{3} \sin\frac{\pi s}{3}}{\frac{3}{2} + \frac{3}{2}} = \frac{2L_s^3}{EI_s\pi^4} \times 10{,}000 \quad (2)$$

Substitute (2) into (1):

$$w_1|_{x=L_g/3} = w_2|_{x=L_g/3} = w_1|_{x=2L_g/3} = w_2|_{x=2L_g/3} = 0.0514 \text{ in.} \quad (3)$$

Buckling Loads

The buckling or critical axial loads in the girders of uniform gridworks are given in Tables 13-9 and 13-10. That is, these are formulas for $P_g = P_{\text{cr}}$. The formulas that apply for girders and stiffeners with fixed or simply supported ends are accurate for gridworks with more than five stiffeners. In some cases, the formulas will be sufficiently accurate for as few as three stiffeners.

Example 13.9 Buckling Loads Compute the critical axial forces in the girders of the gridwork of Fig. 13-7 if the girders can be simply supported or fixed. The stiffeners are simply supported. Suppose that $I_g = I_s$ and $L_g = L_s = L$. From Fig. 13-7, $n_g = 3$ and $n_s = 12$.

The girder buckling loads P_{cr} are given by the formulas of Table 13-9 for girders with fixed or simply supported ends. These formulas involve the constant C_1, which is taken from Table 13-10 according to the stiffener end conditions. To use Table 13-9, first calculate D_1. For simply supported stiffeners and $n_g = 3$, the constant C_1 is given as 0.041089 in Table 13-10. Thus,

$$D_3 = \sqrt{C_1 L_g L_s^3 I_g/[I_s(n_s+1)]} = \sqrt{C_1 L^4/13} = L^2\sqrt{C_1/13}$$

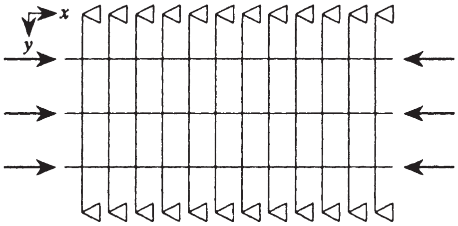

Figure 13-7: Example 13.9.

and

$$D_1 = 0.0866L_g^2/D_3 = 0.0866\sqrt{13/C_1} = 1.54$$

$$D_2 = 0.202L_g^2/D_3 = 3.5930$$

Since $D_1 > 1$, cases 2 and 4 in Table 13-9 are used. These give $P_{\text{cr}} = D_2 P_e = 3.5930P_e$ for simply supported girders and $P_{\text{cr}} = 6.5930P_e$ for fixed girders.

Natural Frequencies

Designate the natural frequencies of a gridwork as ω_{mn}, where the subscript m indicates the number of mode-shape half waves in the y (stiffener) direction and n indicates the number of half waves in the x (girder) direction. Figure 13-8 illustrates typical mode shapes associated with ω_{mn}.

For a uniform grillage with simply supported stiffeners, the lower natural frequencies (radians per time) are given by

$$\omega_{mn}^2 = \frac{EI_s L_g \left(\frac{\pi m}{L_s}\right)^4 + EI_g \frac{n_g+1}{C_n L_g^3} - P_s \left(\frac{m\pi}{L_s}\right)^2 L_g}{\rho_s L_s + \rho_g L_g} \tag{13.1}$$

where n_g is the number of girders; I_g, I_s are the moments of inertia of girders and stiffeners, respectively; L_g, L_s are the length of girders and stiffeners, respectively; ρ_g, ρ_s are the mass per unit length of girders and stiffeners, respectively $(M/L, FT^2/L^2)$; and E is the modulus of elasticity. The stiffener axial force P_s

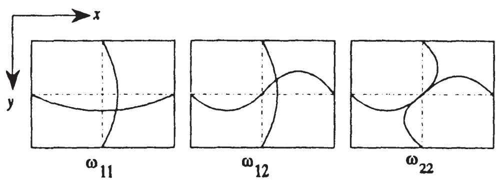

Figure 13-8: Mode shapes corresponding to frequencies ω_{mn}.

is simply set equal to zero if the stiffeners are not subject to axial forces. The parameter C_n is given in Table 13-11 for girders with fixed or simply supported ends. Recall that either set of grillage beams can be selected to be the girders.

If each of the girders is subjected to an axial force P_g, Eq. (13.1) still provides the natural frequencies if C_n is replaced by

$$C_n \frac{P_e}{P_e - P_g} \tag{13.2}$$

where $P_e = \pi^2 E I_g / L_g^2$.

Example 13.10 Natural Frequencies of a Simply Supported Gridwork Find the lower natural frequencies of a 3 × 3 grillage for which all beam ends are simply supported. For this grillage, $n_g = n_s = 3$, $I_g = I_s = 100$ in^4, $\rho_g = \rho_s = 1$ lb-s^2/in^2, $L_g = L_s = 100$ in., and $E = 3 \times 10^7$ psi. There are no axial forces (i.e., $P_s = 0$, $P_g = 0$). From Eq. (13.1),

$$\omega_{mn}^2 = \frac{(3 \times 10^7)100[m^4\pi^4 + (3+1)/C_n]/100^3}{2 \times 100} = 15\left(m^4\pi^4 + \frac{4}{C_n}\right) \tag{1}$$

To calculate ω_{11}, ω_{21}, ω_{12}, and ω_{22}, enter Table 13-11 for $n_s = 3$ and find $C_1 = 0.041089$ and $C_2 = 0.0026042$. Use (1):

$$\begin{aligned}
\omega_{11}^2 &= 15(\pi^4 + 4/C_1) = 2921.37 \quad \text{or} \quad \omega_{11} = 54 \text{ rad/s} \\
\omega_{21}^2 &= 15(16\pi^4 + 4/C_1) = 24{,}838.347 \quad \text{or} \quad \omega_{21} = 157.6 \text{ rad/s} \\
\omega_{12}^2 &= 15(\pi^4 + 4/C_2) = 24{,}500.831 \quad \text{or} \quad \omega_{12} = 156.5 \text{ rad/s} \\
\omega_{22}^2 &= 15(16\pi^4 + 4/C_2) = 46{,}417.89 \quad \text{or} \quad \omega_{22} = 215 \text{ rad/s}
\end{aligned} \tag{2}$$

Other frequencies can be calculated in a similar fashion.

General Grillages

The formulas for uniform gridworks are provided in this section. Since gridworks are a special case of frameworks, use a computer program for the analysis of frames to find the response of complicated grillages. The structural matrices for grillages are listed in Section 13.4 under plane frames with out-of-plane loading.

13.4 MATRIX METHODS

Frames and trusses (both generally referred to as frames) can be considered as assemblages of beams and bars. As a consequence, they can be analyzed using the matrix methods (transfer and displacement) of Appendix III. The displacement method can be employed to obtain the nodal responses, while the displacements and forces between the nodes along the members can be obtained using the transfer matrix method. Such references as [13.7]–[13.10] contain frame analysis formulations.

Frames are often classified as being plane (two-dimensional) and spatial (three-dimensional) in engineering practice.

Transfer Matrix Method

The transfer matrices provided in Chapters 11 and 12 can be combined to obtain the transfer matrices for the analysis of frames or frame members. See Appendixes II and III.

Stiffness and Mass Matrices

In general, the analysis of plane frames requires the inclusion of the axial effects (extension or torsion) as well as bending in the stiffness matrix. As discussed in Appendix III, the analysis also requires a transformation of many variables from local to global coordinates. Then the global system matrix can be assembled. For dynamic problems, the mass matrices can be treated similarly to establish the system mass matrix. The nodal displacements are found by introducing the boundary conditions and solving resulting equations. See the examples in Appendix III.

The stiffness matrices for plane and space trusses and frames are presented in Tables 13-12 to 13-15. Mass matrices for frames are listed in Tables 13-16 and 13-17. All of these matrices use sign convention 2 of Appendix II. Use a frame analysis to analyze a truss for dynamic responses. Stiffness matrices for more complex members can be constructed from the general stiffness matrices of Chapter 11. For example, it is possible to introduce a 4×4 beam stiffness matrix that includes the effect of an axial force on bending. Also, if thin-walled cross sections are of concern, the 4×4 structural matrices of Chapter 14 can replace the 2×2 torsional matrices of this chapter.

Stability Analysis

The stiffness matrices listed in the tables of this chapter do not include the interactions between bending and axial forces. However, in some analyses (e.g., a stability analysis), this interaction must be considered in that the bending moment caused by the axial forces must be included. To do so, introduce the stiffness matrix of Table 11-22 with $P \neq 0$. The buckling loading can be obtained using a determinant search after the global stiffness matrix is assembled and the boundary conditions applied. The details of this instability procedure follow.

1. Perform a static analysis of the frame using the stiffness matrices given in Tables 13-12 to 13-15 to determine the axial forces in each element resulting from a given load.

2. Use element stiffness matrices, such as that given in Table 11-22, that include the effects of bending and the axial force interaction.
3. Assemble the element matrices to form the global stiffness matrix, and impose the boundary conditions on the global matrix using the procedure described in Appendix III.
4. Let all internal axial forces remain in the same fixed proportions to each other throughout the search for the critical applied load. These fixed proportions are determined in step 1. Introduce a single *load factor* λ that holds for global structural matrices that model the entire structure. This λ is a common factor that multiplies all loads as they vary in fixed proportion.
5. Let the determinant of the global stiffness matrix be zero and determine λ, usually employing a numerical search technique. This λ is the critical load factor.

   For examples, see Ref. [13.11].

The stability analysis can also be conducted approximately, but efficiently, by employing the geometric stiffness matrix given in Table 11-23 and using the displacement method of Appendix III.

REFERENCES

13.1. Leontovich, V., *Frames and Arches*, McGraw-Hill, New York, 1959.

13.2. Kleinlogel, A., *Rigid Frame Formulas*, Frederick Ungar, New York, 1952.

13.3. Column Research Committee of Japan, *Handbook of Structural Stability*, Corona Publishing, Tokyo, 1971.

13.4. Timoshenko, S., *Theory of Elastic Stability*, McGraw-Hill, New York, 1936.

13.5. Baker, J., and Heyman, J., *Plastic Design of Frames*, Cambridge University Press, London, 1969.

13.6. Pilkey, W. D., and Chang, P. Y., *Modern Formulas for Statics and Dynamics*, McGraw-Hill, New York, 1978.

13.7. Weaver, W., and Johnston, P. R., *Structural Dynamics by Finite Elements*, Prentice-Hall, Englewood Cliffs, NJ, 1987.

13.8. Paz, M., *Structural Dynamics*, Van Nostrand Reinhold, New York, 1980.

13.9. Ross, C. T. F., *Finite Element Methods in Structural Mechanics*, Ellis Horwood, Chichester, West Sussex, England, 1985.

13.10. Pilkey, W. D., and Okada, Y., *Matrix Methods in Mechanical Vibration*, Corona Publishing, Tokyo, 1989.

13.11. Wunderlich, W., and Pilkey, W. D., *Structural Mechanics: Variational and Computational Methods*, 2nd ed., CRC Press, Boca Raton, FL, 2003.

13.12. Schiff, D., *Dynamic Analysis and Failure Modes of Simple Structures*, Wiley, New York, 1990.

13.13. Blevins, R. D., *Formulas for Natural Frequency and Mode Shape*, Van Nostrand, New York, 1979.

TABLE 13-7 UNIFORM GRIDWORKS[a]

Notation

The ends of both the girders and stiffeners are simply supported.
Girders: beams that lie parallel to the x axis.
Stiffeners: beams that lie parallel to the y axis.

n_g, n_s = total number of girders and stiffeners, respectively

g, s = index for girders and stiffeners, respectively

w_g, θ_g, M_g, V_g = deflection, slope, bending moment, and shear force of gth girder

I_g, I_s = moments of inertia of girders and stiffeners, respectively. All girders have the same I_g and all stiffeners have the same I_s.

L_g, L_s = length of girders and stiffeners, respectively. All girders have the same L_g and all stiffeners have the same L_s.

M = number of terms chosen by user to be included in summation

$$\langle x - x_s\rangle^0 = \begin{cases} 0 & \text{if } x < x_s \\ 1 & \text{if } x \geq x_s \end{cases}$$

K_j = Take from Table 13-8.

	Response
1. Deflection	$w_g = \sin\dfrac{\pi g}{n_g+1}\sum_{j=1}^{\infty} K_j \sin\dfrac{j\pi x}{L_g}$
2. Slope	$\theta_g = -\sin\dfrac{\pi g}{n_g+1}\sum_{j=1}^{\infty} K_j \dfrac{j\pi}{L_g}\cos\dfrac{j\pi x}{L_g}$
3. Bending moment	$M_g = EI_g\sin\dfrac{\pi g}{n_g+1}\sum_{j=1}^{\infty} K_j \left(\dfrac{j\pi}{L_g}\right)^2 \sin\dfrac{j\pi x}{L_g}$
4. Shear force	$V_g = EI_g\sin\dfrac{\pi g}{n_g+1}\sum_{j=1}^{\infty} K_j \times\left[\left(\dfrac{j\pi}{L_g}\right)^3\cos\dfrac{j\pi x}{L_g} + \dfrac{\pi^4 I_s}{(n_g+1)L_s^3 I_g}\sum_{s=1}^{M}\langle x - x_s\rangle^0 \sin\dfrac{j\pi x_s}{L_g}\right]$

[a]From Ref. [13.6].

TABLE 13-8 PARAMETERS K_j OF TABLE 13-7 FOR THE STATIC RESPONSE OF GRIDWORKS

Notation

P_g, P_s = axial forces in girders and stiffeners, respectively (all girders have the same P_g and all stiffeners have the same P_s)

p_s = loading intensity along the sth stiffener (F/L)

W_{sg} = concentrated force at intersection x_s, y_g

$$P_e = \frac{\pi^2 EI_s}{L_s^2} \qquad P_c = \frac{\pi^2 EI_g}{L_g^2}$$

Loading	K_j
1. For concentrated loads W_{sg} at x_s, y_g	$$\frac{\dfrac{2L_s^3}{EI_s\pi^4}\dfrac{P_e}{P_e-P_s}\sum_{s=1}^{n_s}\sum_{g=1}^{n_g} W_{sg}\sin\dfrac{\pi g}{n_g+1}\sin\dfrac{j\pi s}{n_s+1}}{\dfrac{n_g+1}{2}j^4\left(\dfrac{L_s}{L_g}\right)^3\dfrac{I_g}{I_s}\left(1-\dfrac{P_g}{jP_c}\right)+\dfrac{n_s+1}{2}}$$
2. For uniform force p_s along sth stiffener	$$\frac{\dfrac{4L_s^4}{EI_s\pi^5}\dfrac{P_e}{P_e-P_s}\sum_{s=1}^{n_s} p_s\sin\dfrac{j\pi s}{n_s+1}}{\dfrac{n_g+1}{2}j^4\left(\dfrac{L_s}{L_g}\right)^3\dfrac{I_g}{I_s}\left(1-\dfrac{P_g}{jP_c}\right)+\dfrac{n_s+1}{2}}$$
3. If uniform force p_s is same for all stiffeners	Only the first term ($j=1$) in the equations of Table 13-7 is required: $$K_1=\frac{\dfrac{4L_s^4}{EI_s\pi^5}\dfrac{P_e}{P_e-P_s}\sum_{s=1}^{n_s} p_s\sin\dfrac{\pi s}{n_s+1}}{\dfrac{n_g+1}{2}\left(\dfrac{L_s}{L_g}\right)^3\dfrac{I_g}{I_s}\left(1-\dfrac{P_g}{P_c}\right)+\dfrac{n_s+1}{2}}$$

TABLE 13-9 CRITICAL AXIAL LOADS IN GIRDERS[a]

Notation

n_s = number of stiffeners

L_g, L_s = length of girders and stiffeners, respectively

E = modulus of elasticity

I_g, I_s = moments of inertia of girders and stiffeners, respectively

P_{cr} = unstable value of P_g, axial force in girders

The length, moment of inertia, and axial force do not vary from girder to girder.
The lengths and moments of inertia of the stiffeners also do not vary from each other.

$$D_1 = \frac{0.0866L_g^2}{D_3} \qquad D_2 = \frac{0.202L_g^2}{D_3} \qquad D_3 = \sqrt{\frac{C_1 L_g L_s^3 I_g}{I_s(n_s+1)}}$$

$$P_e = \frac{\pi^2 E I_g}{L_g^2}$$

Take C_1 from Table 13-10.

End Conditions of Girders	Case	D_1	P_{cr}
Simply supported	**1**	≤ 1	$(1 + D_1)P_e$
	2	> 1	$D_2 P_e$
Fixed	**3**	≤ 1	$(4 + D_1)P_e$
	4	> 1	$(3 + D_2)P_e$

[a]From Ref. [13.6].

TABLE 13-10 VALUES OF C_1 OF TABLE 13-9 FOR STABILITY[a]

Number of Girders, n_g	End Conditions of Stiffeners, C_1	
	Simply Supported	Fixed
1	0.020833	0.0052083
2	0.030864	0.0061728
3	0.041089	0.0080419
4	0.051342	0.010009
5	0.061603	0.011997
6	0.071866	0.013990
7	0.082131	0.015986
8	0.092396	0.017982
9	0.10266	0.019979
10	0.11293	0.021976

[a] For simply supported stiffeners the formula

$$C_1 = \frac{n_g + 1}{\pi^4}\left(1 + \sum_{j=1}^{\infty}\left\{\left[2j(n_g + 1) + 1\right]^{-4} + \left[2j(n_g + 1) - 1\right]^{-4}\right\}\right)$$

applies for any n_g.

TABLE 13-11 VALUES OF NATURAL FREQUENCY PARAMETERS C_n OF EQS. (13.1) AND (13.2)

Number of Stiffeners, n_s	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
Girders with Simply Supported Ends										
1	0.020833									
2	0.030864	0.0020576								
3	0.041089	0.0026042	0.00057767							
4	0.051342	0.0032240	0.00065790	0.0002462						
5	0.061603	0.0038580	0.00077160	0.00025720	0.00012564					
6	0.071866	0.0044962	0.00089329	0.00028895	0.00012688	0.000073890				
7	0.082131	0.0051361	0.0010177	0.00032552	0.00013769	0.000072209	0.000047321			
8	0.092396	0.0057767	0.0011431	0.00036387	0.00015157	0.000076208	0.000045226	0.000032215		
9	0.10266	0.0064178	0.0012691	0.00040301	0.00016667	0.000082237	0.000046681	0.000030328	0.000022963	
10	0.11293	0.0070590	0.0013954	0.00044252	0.00018233	0.000089133	0.000049521	0.000030753	0.000021400	0.000016967
Any n_s	$C_n = \frac{n_s+1}{\pi^4}\left[\frac{1}{n^4} + \sum_{j=1}^{\infty}\left\{[2j(n_s+1)+n]^{-4} + [2j(n_s+1)-n]^{-4}\right\}\right]$									
Girders with Fixed Ends										
1	0.0052083									
2	0.0061728	0.0011431								
3	0.0080419	0.0011393	0.00042165							
4	0.010009	0.0013459	0.00039075	0.00020078						
5	0.011997	0.0015917	0.00043081	0.00018009	0.00011111					
6	0.013990	0.0018480	0.00048904	0.00018923	0.000098217	0.000067910				
7	0.015986	0.0021078	0.00055303	0.00020779	0.000099794	0.000059682	0.000044545			
8	0.017982	0.0023691	0.00061925	0.00022977	0.00010668	0.000059226	0.000039097	0.000030804		
9	0.019970	0.0026311	0.00068645	0.00025320	0.00011572	0.000061961	0.000038155	0.000027067	0.000022193	
10	0.021976	0.0028934	0.00075415	0.00027732	0.00012573	0.000066109	0.000039232	0.000026101	0.000019547	0.000016522