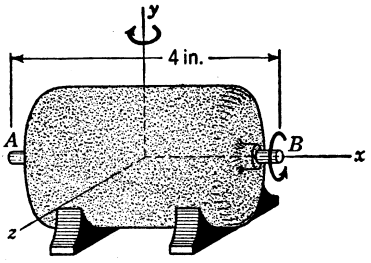
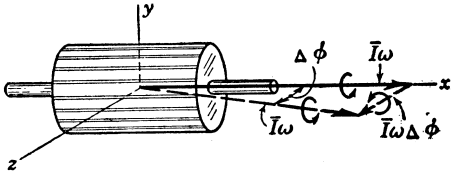
SAMPLE PROBLEM 18.6



The rotor of an electric motor weighs 6 lb and has a radius of gyration of 2 in. The angular velocity of the rotor is 3,600 rpm counterclockwise as viewed from the positive x axis. Determine the reactions exerted by the bearings on the axle AB when the motor is rotated about the y axis clockwise as viewed from above and at a rate of 6 rpm.



Solution. During a time interval Δt the momentum couple $\bar{I}\omega$ of the rotor is rotated in a horizontal plane through an angle $\Delta\phi$. The change in angular momentum about the z axis is thus $\bar{I}\omega\,\Delta\phi$. Denoting by M_z the moment of the couple applied about the z axis, we express that the angular impulse $M_z\,\Delta t$ must equal the change in angular momentum.

$$M_z\,\Delta t = \bar{I}\omega\,\Delta\phi \qquad M_z = \bar{I}\omega\,\frac{\Delta\phi}{\Delta t}$$

Denoting by Ω the angular velocity about the vertical y axis, we write $\Omega = \Delta\phi/\Delta t$ and obtain

$$M_z = \bar{I}\omega\Omega \tag{1}$$

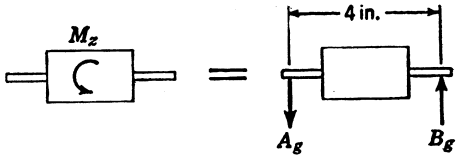
Using the given data, we have

$$\omega = 3{,}600 \text{ rpm} = 377 \text{ radians/sec} \qquad \Omega = 6 \text{ rpm} = 0.628 \text{ radian/sec}$$

$$\bar{I} = m\bar{k}^2 = \frac{6}{32.2}\left(\frac{2}{12}\right)^2 = 0.00518 \text{ lb-ft-sec}^2$$

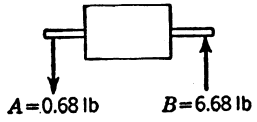
Substituting these values in (1), we obtain the moment of the gyroscopic couple

$$M_z = (0.00518)(377)(0.628) \qquad M_z = 1.226 \text{ lb-ft}$$



Reactions at A and B. The gyroscopic reaction A_g and B_g must be equivalent to the gyroscopic couple M_z.

$$M_z = A_g(\tfrac{4}{12}\text{ ft})$$
$$1.226 = A_g(\tfrac{4}{12})$$
$$A_g = 3.68 \text{ lb} \downarrow$$
$$B_g = 3.68 \text{ lb} \uparrow$$



Since the static reactions are $A_s = 3$ lb $\uparrow$ and $B_s = 3$ lb $\uparrow$, the total reactions are

$$\mathbf{A = 0.68 \text{ lb} \downarrow} \qquad \mathbf{B = 6.68 \text{ lb} \uparrow}$$