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STUDENT > # Solve HappyGopher's Problem using method suggested by
          zekeman
STUDENT >
STUDENT > # =====ASSUMPTIONS=========
STUDENT > # ASSUME we can neglect mass of beam
STUDENT > # ASSUME small deflections and angles.
STUDENT > # note shear is constant since no loads applied along the
          beam
STUDENT >    # it is everywhere equal to the value on the end of the
          beam
STUDENT > # ASSUMING there is no applied force (only applied torque
          Mapplied)
STUDENT > # assume Euler Bernoulli beam model (neglect shear)
STUDENT >
STUDENT > # =======Symbols================
STUDENT > # V = shear
STUDENT > # M = moment
STUDENT > # T = slope (T for theta)
STUDENT > # Y = transverse displacement
STUDENT > # x = axial coordinate
STUDENT > # t = time
STUDENT > # s = laplace variable transformed from time
STUDENT > # L = length of beam
STUDENT > # C0, C1, C2, C3 - constants in cubic polynomial equation
          for y(x) under Bernoulli model
STUDENT > # m = mass at end of beam
STUDENT > # Id = Diametral mass MOI of item at end of beam
STUDENT > # Mapplied is moment applied at end of beam
STUDENT > # Yend = displacement at end of beam (where disk attaches)
STUDENT > # Tend = Slope at end of beam (where disk attaches)
STUDENT > # Yend1 = d/dt(Yend)  (1 stands for prime)
STUDENT > # Tend1 = d/dt(Tend) (1 stands for prime)
STUDENT > # EOM = Equation Of Motion
STUDENT >
STUDENT >
STUDENT > #============= Approach ==============
STUDENT > #  Euler BernoY(x):=C0+C1*x+ C2*x^2+C3*x^3 has 4 unknown
          coefficients
STUDENT > # 4 equations to "solve" these coeficients
STUDENT > #  1 - Y(0) = 0  (base does not move)
STUDENT > #  2 - T(0) = 0 (base slope is always 0 like fixed bc)
STUDENT > #  3 - Y(L) = Yend (by definition)
STUDENT > #  4 - T(0) = Tend (by definition
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STUDENT >
STUDENT > # Develope expression for KE and PE
STUDENT > # apply lagrangeto determine equations of motion
STUDENT > restart;
STUDENT >
STUDENT > # ===== Find Beam Coefficients=======
STUDENT > Y(x):=C0+C1*x+ C2*x^2+C3*x^3;
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$$\mathrm{Y}(x) := C0 + C1\,x + C2\,x^2 + C3\,x^3$$

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STUDENT > # Differentiate to find slope:
STUDENT > T(x):=diff(Y(x),x);
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$$\mathrm{T}(x) := C1 + 2\,C2\,x + 3\,C3\,x^2$$

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STUDENT > # differentiate to find moment:
STUDENT > M(x):=EI*diff(T(x),x);
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$$\mathrm{M}(x) := EI\,(2\,C2 + 6\,C3\,x)$$

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STUDENT > # differentiate to find shear:
STUDENT > V(x):=diff(M(x),x);
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$$\mathrm{V}(x) := 6\,EI\,C3$$

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STUDENT > # V(x,t) passes sanity check - it does not depend on x...
          constant along the beam since there are no forces applied
          along the beam
STUDENT >
STUDENT >
STUDENT > # We have above 4 unknowns (C1, C2, C3, C4). Will solve
          with 4 equations Eq1, Eq2, Eq3, Eq4
STUDENT >
STUDENT > # C0 and C1 are easily solved by BC at the base
STUDENT > Eq1:=subs(x=0,Y(x))=0;
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$$Eq1 := C0 = 0$$

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STUDENT > Eq2:=subs(x=0,T(x))=0;
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$$Eq2 := C1 = 0$$

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STUDENT >
STUDENT > # Eq 3 and Eq4 are simply defining the end displacement
          Yend and slope Tend as independent variables
STUDENT > Eq3:=subs(x=L,Y(x))=Yend;
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$$Eq3 := C0 + C1\,L + C2\,L^2 + C3\,L^3 = Yend$$

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STUDENT > Eq4:=subs(x=L,T(x))=Tend;
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$$Eq4 := C1 + 2\,C2\,L + 3\,C3\,L^2 = Tend$$

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STUDENT > Csoln:=solve({Eq1,Eq2,Eq3,Eq4},{C0,C1,C2,C3});
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$$Csoln := \{\, C2 = -\frac{L\,Tend - 3\,Yend}{L^2},\ C3 = \frac{L\,Tend - 2\,Yend}{L^3},\ C0 = 0,\ C1 = 0 \,\}$$

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STUDENT > #=== Plug beam coefficients back into general
          expressions:==
STUDENT > Y(x):=subs(Csoln,Y(x));
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$$\mathrm{Y}(x) := -\frac{(L\,Tend - 3\,Yend)\,x^2}{L^2} + \frac{(L\,Tend - 2\,Yend)\,x^3}{L^3}$$

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STUDENT > T(x):=subs(Csoln,T(x));
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$$\mathrm{T}(x) := -2\,\frac{(L\,Tend - 3\,Yend)\,x}{L^2} + 3\,\frac{(L\,Tend - 2\,Yend)\,x^2}{L^3}$$

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STUDENT > M(x):=subs(Csoln,M(x));
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$$\mathrm{M}(x) := EI\left(-2\,\frac{L\,Tend - 3\,Yend}{L^2} + 6\,\frac{(L\,Tend - 2\,Yend)\,x}{L^3}\right)$$

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STUDENT > #===== Use above in Lagrange approach=====
STUDENT > KE:=(1/2)*(m*Yend1^2+Id*Tend1^2); #neglected beam mass!
          # the "1" postscript is like prime.. identifies d/dt
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$$KE := \frac{1}{2}\, m\, Yend1^2 + \frac{1}{2}\, Id\, Tend1^2$$

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STUDENT > PE:=simplify(int(M(x)^2/EI/2,x=0..L));
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$$PE := 2\,\frac{(L^2\, Tend^2 - 3\, L\, Tend\, Yend + 3\, Yend^2)\, EI}{L^3}$$

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STUDENT > # Lagrange formulation:
STUDENT > # d/dt(dKE/dq1) - dKE/dq + dPE/dq = Qk
STUDENT > # where q is coordinate, q' is d/dt(q), Q is associated
          generalized force
STUDENT > # Note in our case all dKE/dq is 0 (KE depends only on q1)
STUDENT > # d/dt(dKE/dq1)  + dPE/dq = Qk
STUDENT >
STUDENT >
STUDENT > # Let q = Yend for EOM1:
STUDENT > # d/dt(dKE/dTend1) + dPE/dTend = 0
STUDENT > # Compute 1st term mentally (chain rule), let Maple do 2nd
          term:
STUDENT > EOM1:=m*diff(Yend(t),t,t)+diff(PE,Yend)=0;
STUDENT >
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$$EOM1 := m\left(\frac{\partial^2}{\partial t^2}\mathrm{Yend}(t)\right) + 2\,\frac{(-3\, L\, Tend + 6\, Yend)\, EI}{L^3} = 0$$

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STUDENT >
STUDENT >
STUDENT > # Let q = Tend for EOM2:
STUDENT > # Compute 1st term mentally (chain rule), let Maple do 2nd
          term:
STUDENT > EOM2:=Id*diff(Tend(t),t,t)+diff(PE,Tend)=Mapplied;
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$$EOM2 := Id\left(\frac{\partial^2}{\partial t^2}\mathrm{Tend}(t)\right) + 2\,\frac{(2\, L^2\, Tend - 3\, L\, Yend)\, EI}{L^3} = Mapplied$$

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STUDENT > # Above are the 2 equations of motion in generalized
          variables Yend and Tend
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