HYDRAULIC RESISTANCE

Local (Minor!) Pressure Losses

Local Pressure Losses
(Minor Losses!)

- Although they often account for a major portion of the total pressure loss, the additional losses due to ***entries and exits, fittings and valves*** are traditionally referred to as **minor losses!**
- These losses represent additional energy dissipation in the flow, usually **<u>caused by secondary flows</u>** induced by curvature or recirculation.
- The minor losses are any total pressure loss present *in addition to the total pressure loss for the same length of straight pipe.*

Modified Bernoulli Equation

- Bernoulli equation is valid along any streamline in any frictionless flow
  - However, this is very restrictive.
  - Solid walls introduce friction effects.
  - Fittings in a piping system as well as cross sectional area changes introduce frictional pressure losses.
- Bernoulli's equation can be modified to include these frictional losses present in any fluid system network

$$p_1 + \frac{1}{2} r v_1^{\;2} + rgh_1 = p_2 + \frac{1}{2} r v_2^{\;2} + rgh_2 + \frac{fL}{D_h} \frac{1}{2} r v_2^2 + \sum K \frac{1}{2} r v_2^2$$

Wall Friction

Local Losses

K is the coefficient of flow (hydraulic) resistance

Coefficient of Flow Resistance - K
A Dimensionless Parameter

- Coefficient of flow resistance is defined as the ratio of the total energy lost over the given segment to the kinetic energy in the section
  - Ratio of the total pressure lost over the segment to the dynamic pressure in the segment

$$\text{Flow Resistance} = \frac{\text{Total Pressure Loss}}{\text{Dynamic Pressure (Kinetic Energy)}}$$

- **Flow resistance coefficient K**, for the case of uniform distribution of static pressure and density over the segment but which are variable along the flow

$$K = \frac{\Delta p_{total}}{\frac{1}{2} \rho v_o^2}$$

Base Area for Flow Resistance

- **Flow resistance coefficients** can be expressed in terms of the upstream or downstream velocity of the component
  - Loss term can be added to upstream or downstream velocity with the same net effect

$$p_{up} + \frac{1}{2} r v_{up}^{2} = p_{down} + \frac{1}{2} r v_{down}^{2} + \left(\frac{fL}{D_h} + \sum K_{down}\right) \frac{1}{2} r v_{down}^{2}$$

$$p_{up} + \frac{1}{2} r v_{up}^{2} + \left(\frac{fL}{D_h} + \sum K_{up}\right) \frac{1}{2} r v_{up}^{2} = p_{down} + \frac{1}{2} r v_{down}^{2}$$

$$K_{up} \times \tfrac{1}{2} r v_{up}^{2} = K_{down} \times \tfrac{1}{2} r v_{down}^{2}$$

$$\frac{K_{up}}{K_{down}} = \frac{v_{down}^{2}}{v_{up}^{2}} = \frac{A_{down}^{2}}{A_{up}^{2}}$$

$$K_{up} = K_{down} \times \left(\frac{A_{down}}{A_{up}}\right)^{2}$$

- **Base area** for all flow resistances (K) shown in this section are given based on the **smallest cross sectional area of the component** hence the largest velocity.

$$K = \frac{\Delta p}{\tfrac{1}{2} r v_{base}^{2}}$$

Equivalent Length of Fittings

- Local (minor) losses can be represented in a different way using the the concept of equivalent length
  - Inclusion of local losses in Bernoulli equation can make the iterative type problems labor intensive
  - Local losses can be significantly higher for relatively short piping systems and may not be ignored

$$p_1 + \frac{1}{2} r V_1^2 + r g h_1 = p_3 + \frac{1}{2} r V_3^2 + r g h_3 + \left(\frac{fL}{D_h} \frac{1}{2} r V_3^2 + \sum K \frac{1}{2} r V_3^2\right)$$

$$p_1 + \frac{1}{2} r V_1^2 + r g h_1 = p_3 + \frac{1}{2} r V_3^2 + r g h_3 + \left(\frac{fL}{D_h} + \underbrace{\sum K}\right) \frac{1}{2} r V_3^2$$

$$\sum K = \frac{fL_{eq}}{D_h}$$

Equivalent Length of Fittings

- Concept of equivalent length allows us to replace the local (minor) loss term with equivalent pipe length

$$\Sigma K = \frac{f L_{eq}}{D_h} \Rightarrow L_{eq} = \frac{D_h (\Sigma K)}{f}$$

  - f is the friction factor that applies to the entire pipe
  - D is the pipe hydraulic diameter (characteristic length)
  - **L_{eq} is the equivalent length:** Length of pipe which can replace the fitting (local loss) to obtain the same pressure loss

List of Configurations

- Sudden Expansion
- Sudden Expansion in One Plane
- Sudden Contraction
  - Vena Contracta Effect
- Bend
- Conical Expansion
- Conical Contraction
- Uniformly Distributed Barriers
  - Wire Screen
  - Threaded Screen
  - Two Plane Screen
- Gratings in Line With Flow
- Orifice In a Straight Tube
  - Sharp Edge
  - Rounded
  - Beveled
  - Thick Edge
- Orifice In a Tube Transition
  - Sharp Edge
  - Rounded
  - Beveled
  - Thick Edge

Resistance to Flow Sudden Changes in Flow Area

Sudden Expansion

Sudden Contraction

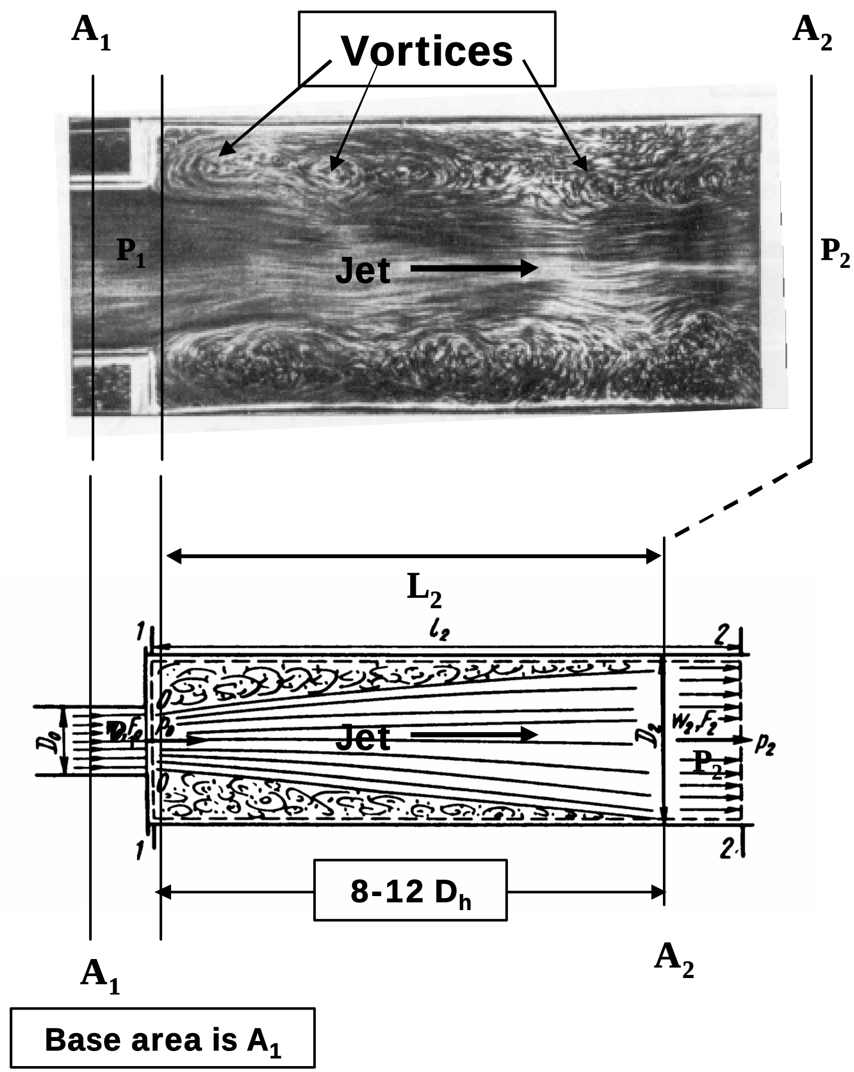
Vena Contracta Effect

Sudden Expansion Pressure Losses

- An abrupt expansion of a tube cross-sectional area gives rise to so-called shock losses.
- The local pressure loss due to this "shock" depends only on the cross sectional area ratio A_1/A_2.

$$K_{\exp} = (1 - \frac{A_1}{A_2})^2$$

  - The area ratio of A1/A2 is less than 1.0
- Assumptions:
  - Uniform velocity distribution at cross section A_1
  - Reynolds number > 10^4

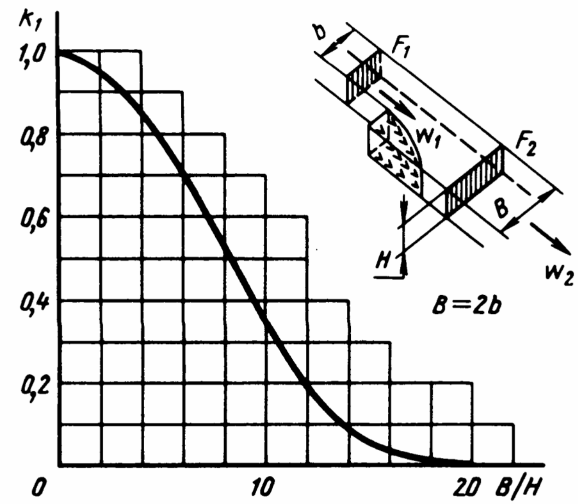


Sudden Expansion Pressure Losses

- The jet formed due to sudden expansion,
  - is separated from the remaining medium
  - Forms a significant turbulence as shown by the vortices near the wall surfaces.
- It requires a pipe length (L_2) of 8-12 hydraulic diameters (D_h) before relatively uniform flow of velocity v_2 is established.

Sudden Expansion in One Plane

- When an abrupt expansion of the tube cross section occurs only in one plane (as shown in the Figure) the shock losses decrease with an increase in the aspect ratio B/H.
  - B is the width of larger cross section
  - H is the constant height of the channel
- Loss coefficient is:



Dependence of k_1 on B/H

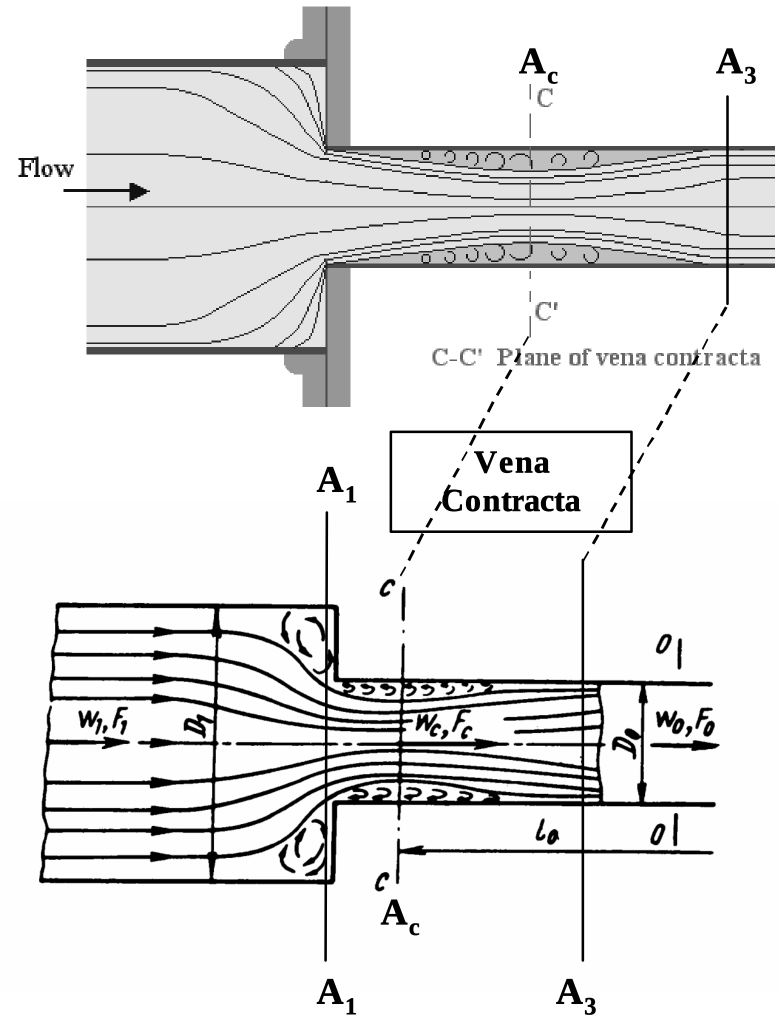
$$K_{\exp} = k_1 (1 - \frac{A_1}{A_2})^2$$

- Where k1 =< 1 is correction factor which depends on the aspect ratio B/H.

Sudden Contraction

- When the cross section abruptly contracts the phenomenon is basically a similar to that observed when shock losses occur during abrupt expansion
- Contraction losses occur when the jet
  - Is further compressed after entering the section A_3 - vena contracta effect
  - Effective flow diameter reduces to A_c
  - Expands until it fills the entire section of the narrow channel A_3

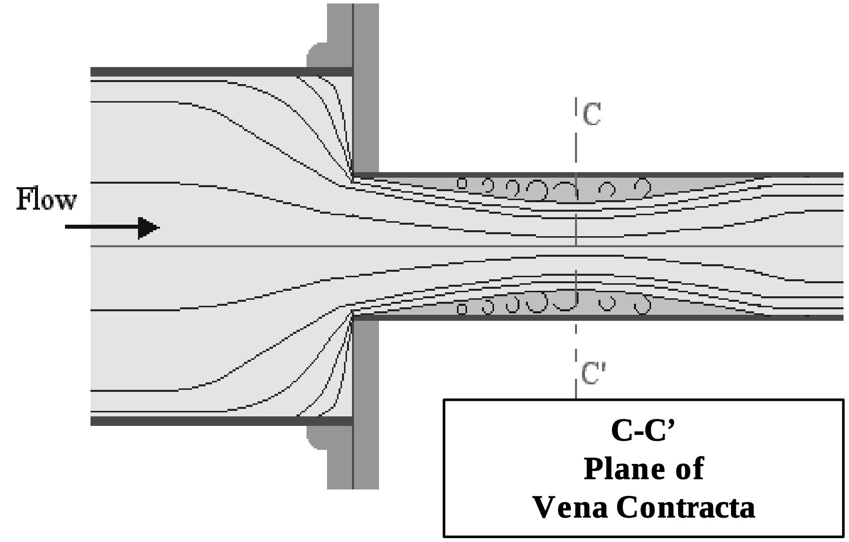
$$K_{Cont} = 0.5\left(1 - \frac{A_3}{A_1}\right)^{0.75}$$



Base area is A_3

Vena Contracta Effect

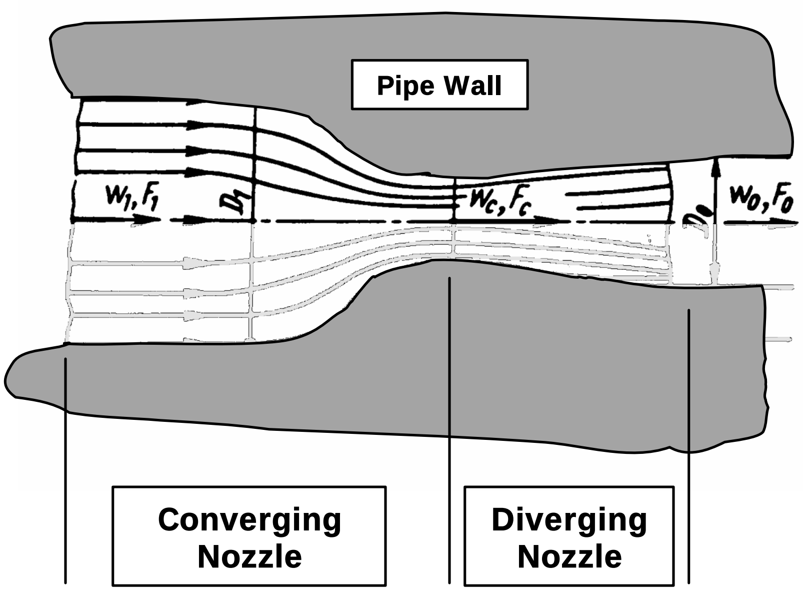
- Flow passage through the contraction is accompanied by distortion of the trajectories of particles with the result that they **continue their motion by inertia** toward the axis of the opening.
- This reduces the initial area of the jet cross section at C-C until the area is smaller than the area of the cross section of the opening. **Plane of vena contracta.**



- Starting mid-section CC, the trajectory of moving particles are straightened.
- Thereafter an abrupt jet expansion takes place.

Tube Walls Following Flow Streamlines

- The **pressure loss** of a contracting cross section **can be significantly reduced** with a tube boundary following the flow streamlines.
  - Allowing a smooth transition from a wide section to the narrow one
  - With the curvilinear boundaries
  - **Converging and diverging (expanding) nozzle**



Resistance to Flow
Changes of the Stream Direction

Flow Losses in an Elbow

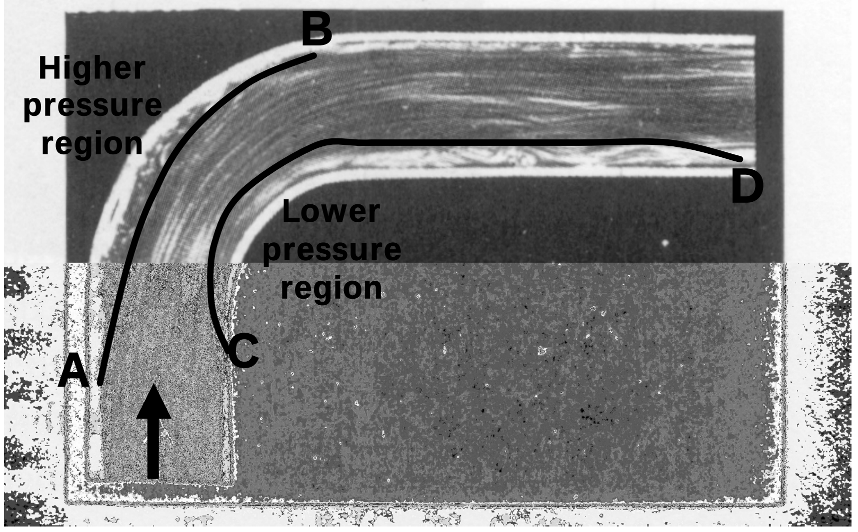
Flanged Elbow

Standard Threaded Elbow

Miter Bend

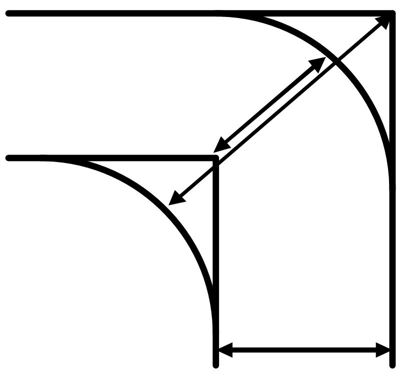
Change of Flow Direction

- In elbows the streamlines are curved and **centrifugal force causes a pressure increase** near the outer wall of the elbow.
- Starting at point **A pressure increases** and rising to a maximum value at **B**.
- In region AB the fluid flow is opposed by an adverse pressure gradient.
- At the inside of the elbow, the **pressure decreases** to point C and then rises again in the exit section **D**.
- For this reason an adverse pressure gradient also exists from C to D at the inside wall.
- These conditions may lead to a separation and turbulence and corresponding losses in flow energy.
- The magnitude of these losses to a large extend depend on the sharpness of the curvature.



Flow Losses In an Elbow

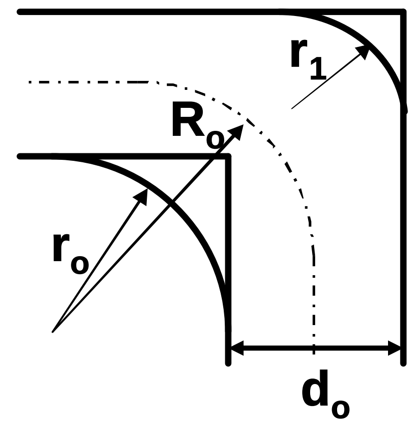
- The main portion of flow pressure losses in curved tubes is due to formation of eddies at the inner wall.
- Other conditions being equal, the curved tube offers the largest resistance in the case when the curvature at the inner wall is a sharp corner.
- Rounding of the elbow corners makes the flow separation much smoother and consequently lowers the resistance.
  - A big majority of flow the flow losses can be recovered by rounding the inner corner and leaving the outer elbow sharp.
    - **Cross sectional area** at the place of bending **increases** sharply (As shown in red x-section)
    - Rounding the outer elbow corner and keeping the inner corner sharp does not reduce the elbow resistance.
    - Large radius on the outer elbow will even increase flow losses since it would **reduce the cross sectional area** significantly (As shown in green x-sec)



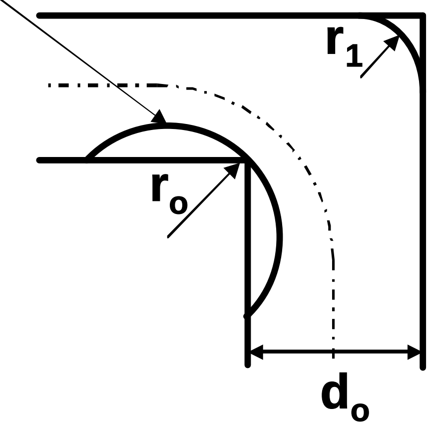
Minimizing Elbow Resistance

- The minimum resistance is achieved by an elbow when outer & inner radii is related as

$$\frac{r_1}{d_o} = \frac{r_o}{d_o} + 0.6$$

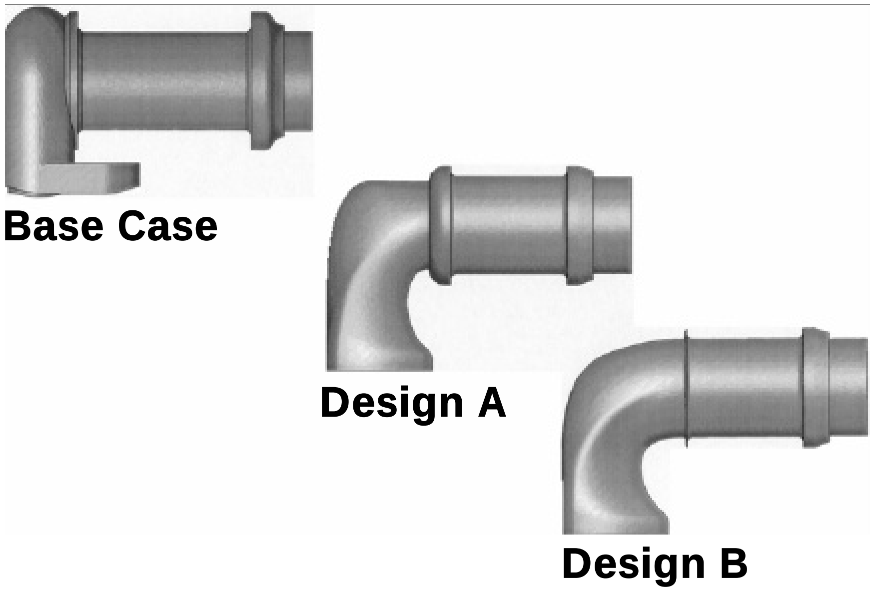


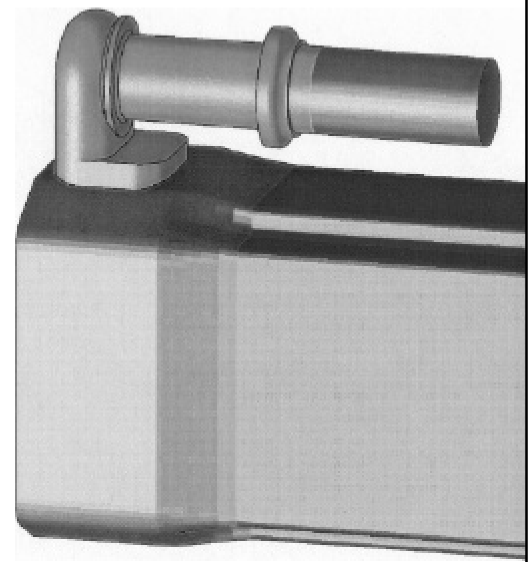
- The resistance of right angle elbows can greatly be reduced by installing a **fairing** on the inner corner
  - An optimum fairing with a ratio of r_o/d_o = 0.45 reduces the pressure losses by approx 50%
  - An additional fairing on the outer corner with a ratio of r_1/d_o = 0.45 reduces the losses by an additional 5%.



- Reduction in the elbow resistance can also be attained by beveling sharp corners of the bend especially the inner corner

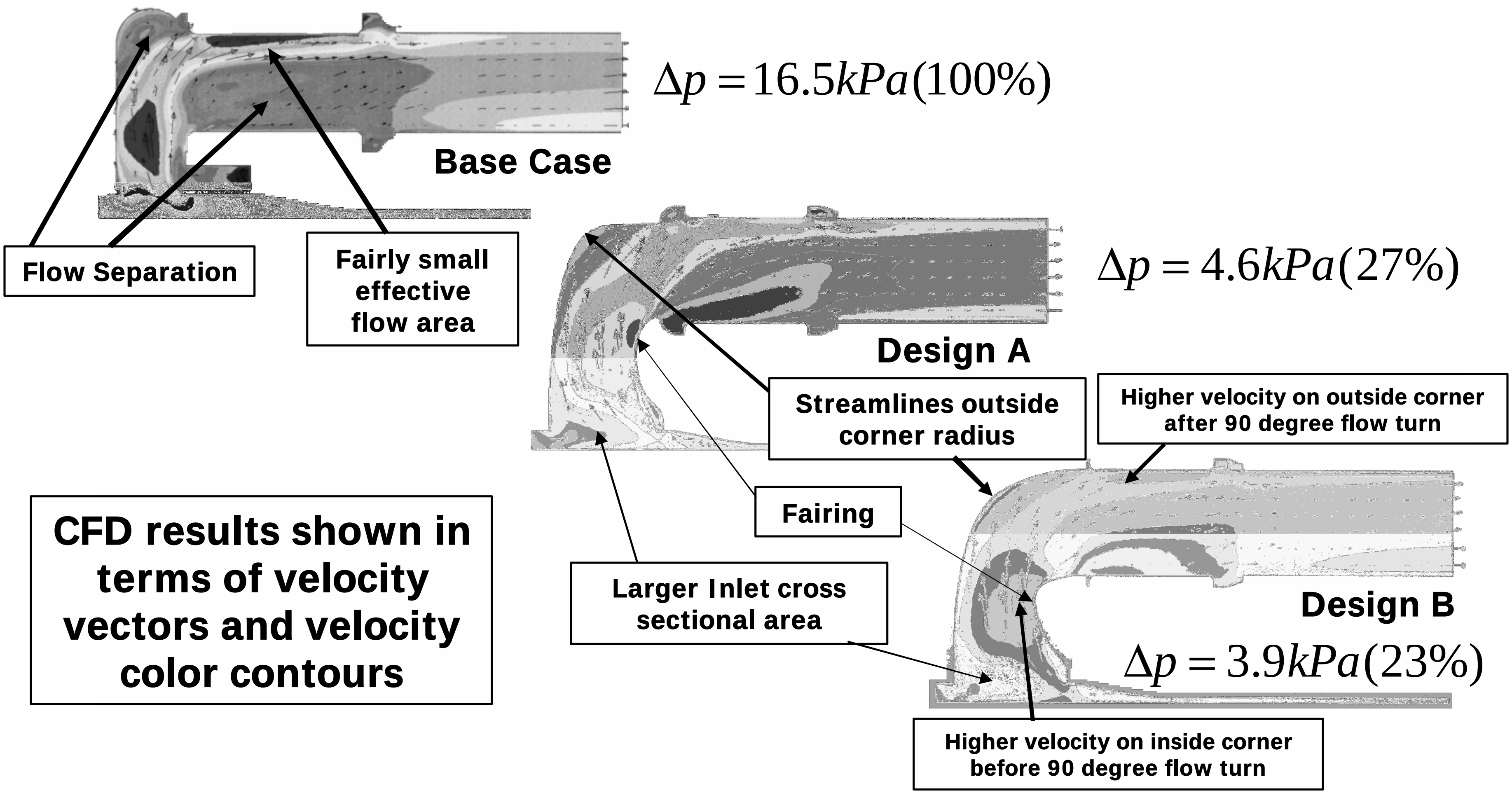
Minimizing Elbow Resistance
Example of a Coolant Return Elbow





- Computational Fluid Dynamics (CFD) was used to minimize the flow losses in a coolant return elbow.
- **Fairing** on the inner corner together with a smooth outer radius reduced the flow losses significantly.

Minimizing Elbow Resistance
Example of a Coolant Return Elbow



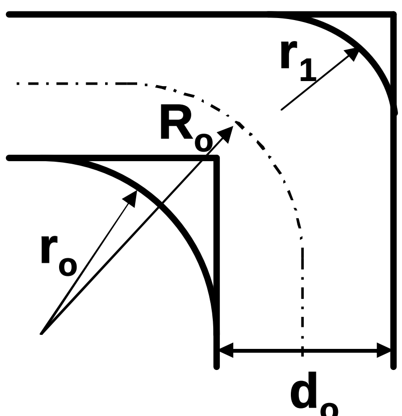
CFD results shown in terms of velocity vectors and velocity color contours

77 % reduction in pressure losses is significant!

Flow Resistance of Elbows

Crane 1988, A-29

- Resistance **(K)** of 90 degree smooth, **flanged** elbow

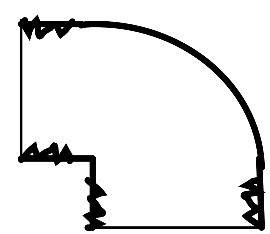


Flanged Elbow

R_o/d_o	K/Ft
1	20
1.5	14
2	12
3	12
4	14
6	17
8	24
10	30
12	34
14	38
16	42
20	50

d_o (mm)	Ft
12.5	0.027
19	0.025
25	0.23
32	0.22
38	0.021
50	0.019
75	0.018
100	0.017
125	0.016
150	0.015
250	0.014
400	0.013
600	0.012

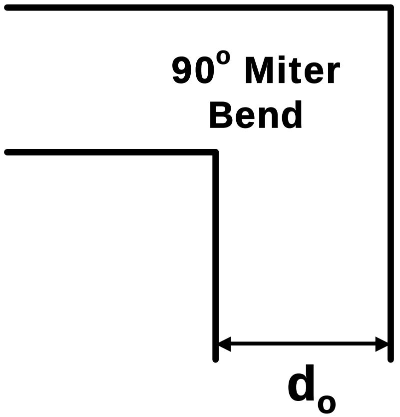
- f_t is the turbulent friction factor given in the Table above
- Resistance **(K)** of standard **threaded elbow➔** $K=30F_t$



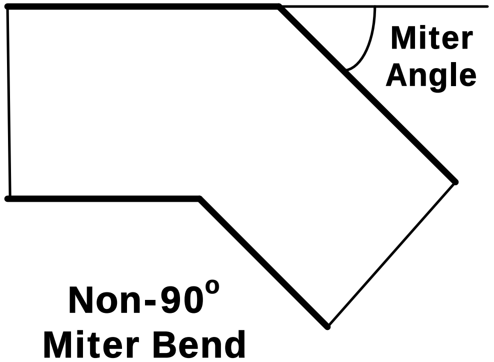
Standard Threaded Elbow

Flow Resistance of Miter Bend

- Resistance of **90 degree Miter Bend ➔** $K=60F_t$



- Resistance of **Non-90 degree** miter bend



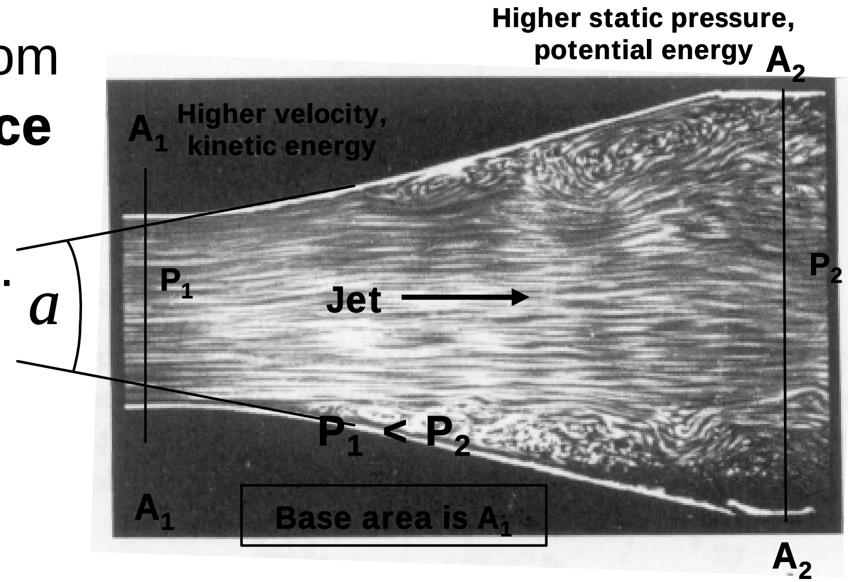
Angle	K/Ft
0	2
15	4
30	8
45	15
60	25
75	40
90	60

Resistance to Flow
Smooth Change in Velocity & Flow Area

Diffuser with Gradual Expansion
Pressure Looses in a Diffuser
Effect of Diffuser Angle on Pressure Looses
Conical Expansion & Contraction
Converging Rectilinear Nozzle

Diffuser - Gradual Expansion

- A **gradual tapered transition** from one diameter to another can **reduce flow losses appreciably** in comparison with abrupt transitions.
- In a conical diffuser the **pressure rises in the direction of flow**
  - Increasing diameter reduces the velocity.



- Kinetic energy at cross section A_1 is converted to potential energy at cross section A_2
- However the slower fluid particles near the diffuser wall tend to stagnate and disturb the theoretical (ideal) pressure rise. This causes an energy loss.
  - Total energy at section A_2 is less than the total energy at section A_1

Pressure Losses in a Diffuser

- The resistance coefficients of the diffusers as well as the flow structure in them and the separation phenomenon depend on many parameters such as
  - Divergence angle, alpha
  - Area ratio
  - The shape of the velocity profile at the entrance
  - Degree of flow turbulence at the entrance
  - Flow regime, at the entrance
  - Flow compressibility
- The effect of the Reynolds number on the resistance coefficients of the diffusers is different for different divergence angles

Effect of Divergence Angle
In Gradual Expansion - Diffuser

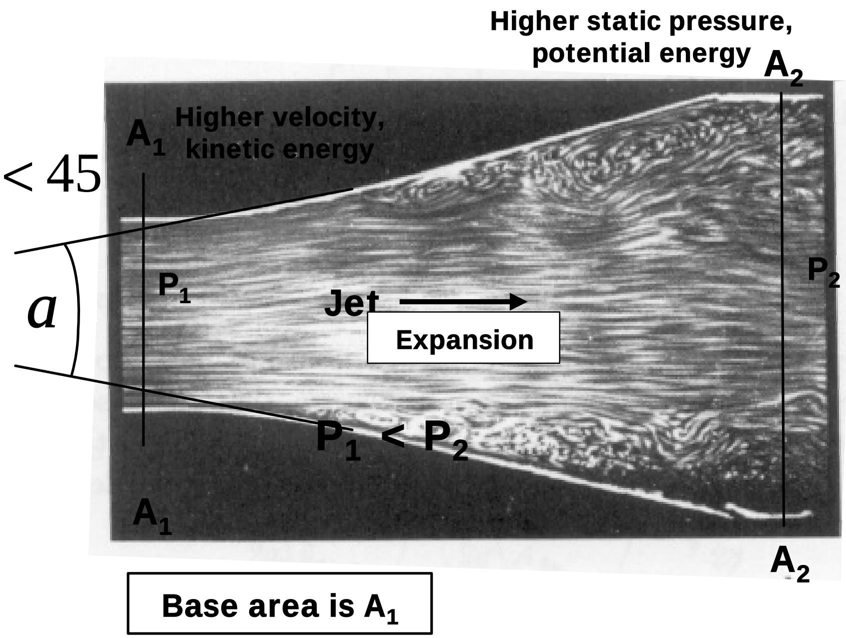
- The total resistance coefficient of a diffuser expressed in terms of the velocity at small section becomes smaller up to certain limits of divergence angle, alpha.
  - Increase in the cross sectional area of the diffuser causes a drop in average velocity.
  - Static pressure gain due to this cross sectional area change is greater than the pressure loss due to flow turbulence up to certain limits of divergence angle, alpha
- If the cone angle is too large, separation of the flow from the wall also occurs and causes an additional energy loss.
- The most advantages cone angle is 8^o degrees
- In flow channels of rectangular cross section, separation occurs if cone angle alpha is greater than 10^o

Conical Expansion & Contraction

- **Expansion**

$$K_{Conical-Exp} = 2.6 \sin\left(\frac{a}{2}\right)\left(1-\frac{A_1}{A_2}\right)^2 \Rightarrow a < 45$$

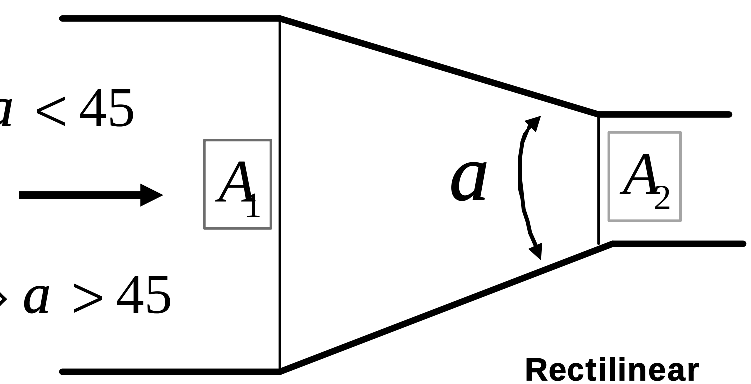
$$K_{Conical-Exp} = \left(1-\frac{A_1}{A_2}\right)^2 \Rightarrow a > 45$$



- **Contraction**

$$K_{Conical-Contraction} = 0.8 \sin\left(\frac{a}{2}\right)\left(1-\frac{A_1}{A_2}\right) \Rightarrow a < 45$$

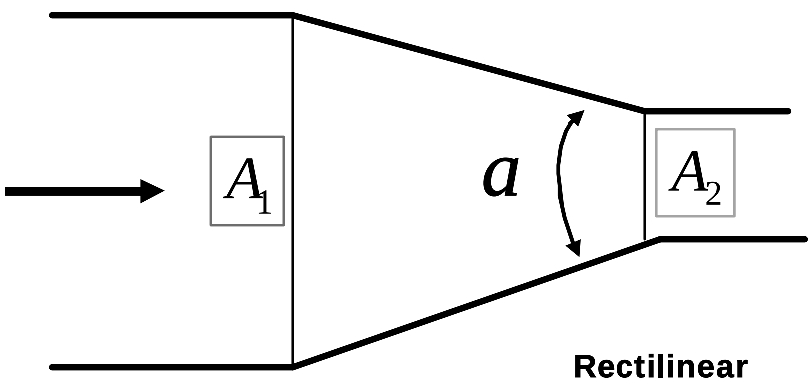
$$K_{Conical-Contraction} = 0.5 \sqrt{\sin\left(\frac{a}{2}\right)\left(1-\frac{A_1}{A_2}\right)} \Rightarrow a > 45$$



Rectilinear Converging Nozzle

Gradual Contraction
Converging Nozzle

- In a convergent section flow losses are small since the pressure gradient is in the same direction as the flow.
- Particles of fluid near the wall are continually accelerated by this pressure gradient so that flow is maintained even in the close vicinity of the wall.
- The **resistance coefficient** of a converging nozzle with rectilinear boundaries at high Re numbers **depends on**:
  - **Convergence angle** alpha
  - **Area ratio** $n_o = \mathbf{A_2/A_1} < 1$



Rectilinear Converging Nozzle

Flow Resistance of
Converging Rectilinear Nozzle

- At sufficiently large angles (alpha > 10º) and area ratios less than 0.3, the flow after passing from the contracting section of the rectilinear converging nozzle to the straight part of the tube separates from the walls which is the main source of the flow losses.
  - The larger the alpha and smaller the area ratio, stronger is the flow separation and greater the pressure losses
  - Pressure losses are at maximum for the limiting case of 180 degree angle sudden contraction.

$$K_{Conv-Nozzle} = (-0.0125n_0^4 + 0.022n_0^3 - 0.00723n_0^2 + 0.044n_0 - 0.00745) \times (a^3 - 2pa^2 + 10a)$$

- where alpha is in radians
  - ➔ 1 degree = (pi/180) = 0.01745 radians

Resistance to Flow at the Exit From Tubes

Uniform Velocity Distribution At Tube Exit

Exponential Velocity Distribution

Sinusoidal Velocity Distribution

Asymmetric Velocity Distribution

Resistance of Exit Sections

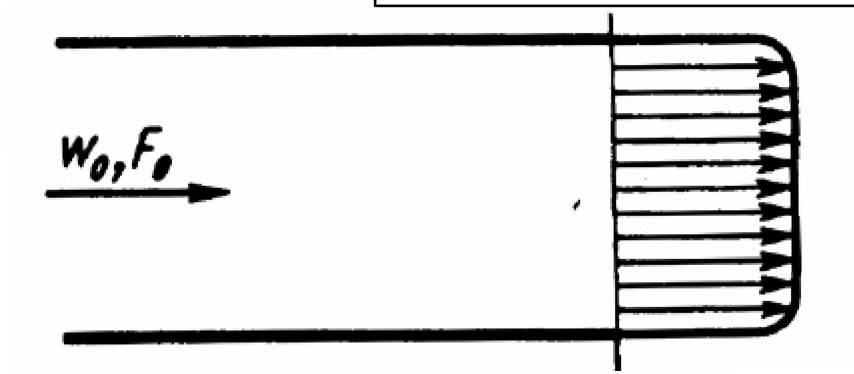
- When fluid flow leaves the piping system, the kinetic energy of the discharged jet is always lost.
- In the case of **free discharge of the flow from a straight section of the tube of constant cross section into a large volume**, the total losses are reduced only to the losses of the velocity pressure at the exit.

$$K = \frac{\Delta p_{velocity}}{\frac{1}{2} r v^2} = N$$

- The coefficient N depends on the nature of the velocity distribution at the exit.
  - In the case of uniform exit velocity distribution, it is equal to unity
  - In the remaining cases where velocity distribution is non-uniform, it always larger than one.
    - Exponential, Parabolic, Sinusoidal, Asymmetric cases are some of the non-uniform velocity distributions.

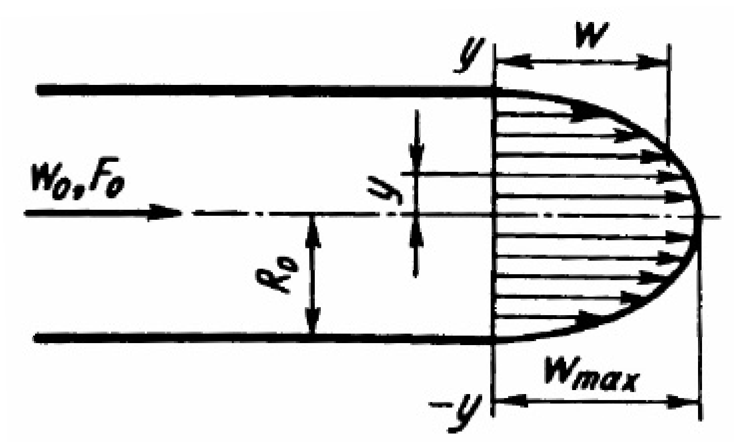
Free Discharge From a Straight Tube
At Different Velocity Distributions

Uniform Velocity Distribution



$$K = \frac{\Delta p}{\frac{1}{2} r w_o^{2}} = 1$$

Exponential Velocity Distribution



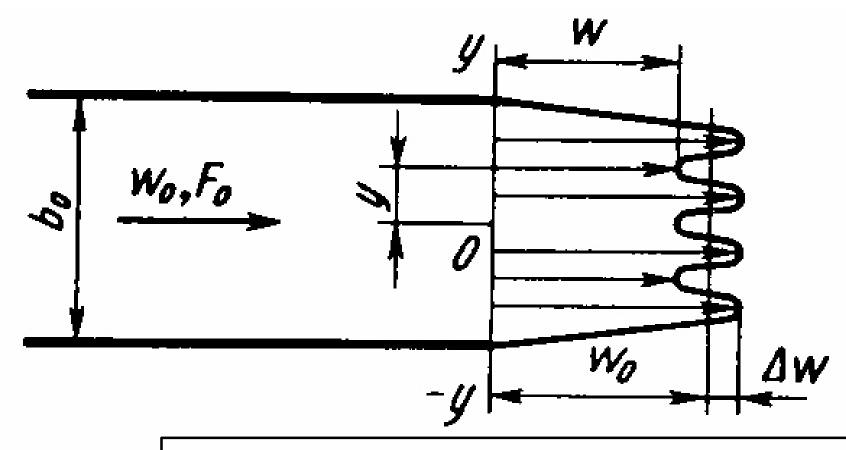
$$\frac{w}{w_{\max}} = \left(1 - \frac{y}{R_0}\right)^{1/m}$$

$$m \geqslant 1.0$$

$$K = \frac{\Delta p}{\frac{1}{2} r w_o^{2}} = \frac{(2m+1)^3 (m+1)^3}{4m^4 (2m+3)(m+3)}$$

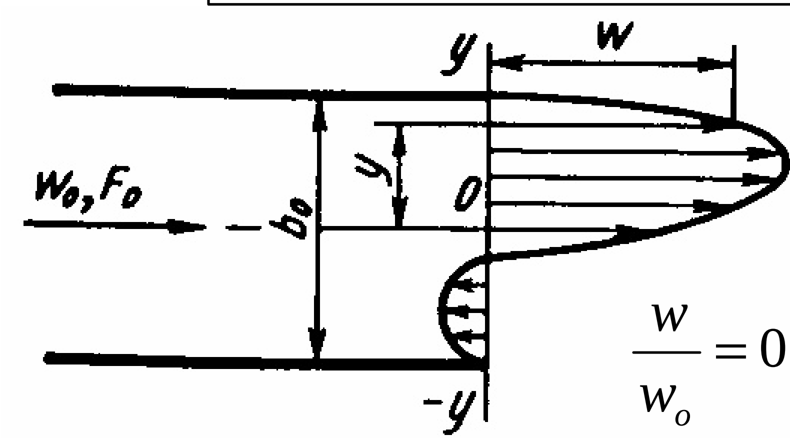
Free Discharge From a Straight Tube
At Different Velocity Distributions

Sinusoidal Velocity Distribution in a plane tube



$$K = 1 + \left(\frac{\Delta w}{w_o} \right)^2$$

Asymmetrical Velocity Distribution in a plane tube



$$K = 3.67$$

$$\frac{w}{w_o} = 0.585 + 164 \sin(0.2 + 3.9 \frac{y}{b_o})$$

Resistance to Flow with Sudden Change in Velocity

Orifice Plate

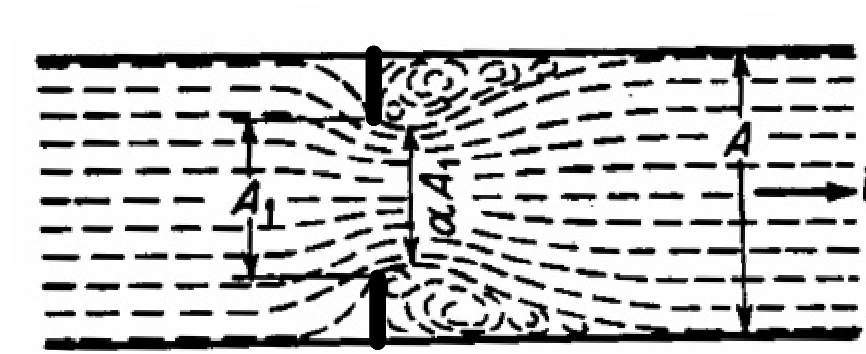
Orifice in A Straight Tube

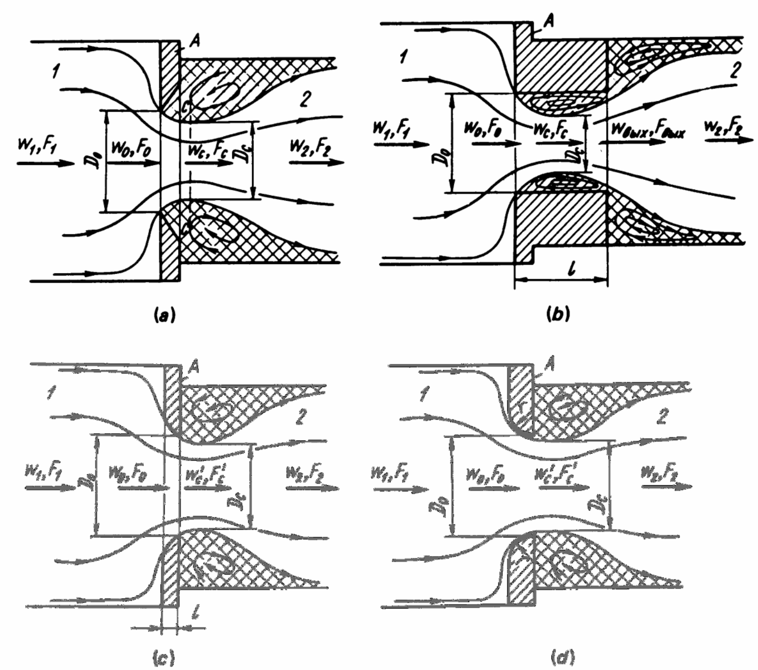
Orifice with Beveled Inlet Edge

Orifice with Rounded Inlet Edge

Orifice Plate

- Flow passage through the opening is accompanied by the **distortion of the trajectories** of particles with the result that they continue their motion by inertia toward the axis of the opening.
- This reduces the initial area jet cross sectional area by a factor alpha. (**Vena contracta effect**)
- **Thickening, beveling and rounding** the orifice edges reduces the effect of jet contraction (reduced jet velocity downstream of orifice plate) and **reduce pressure losses.**



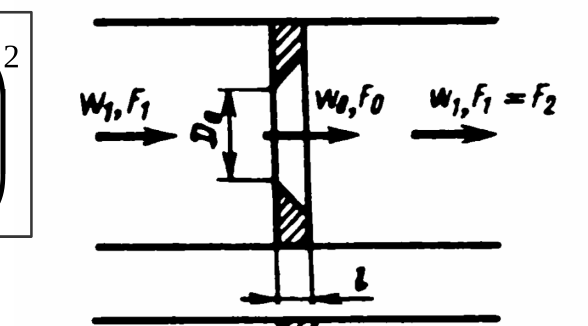


Orifice In A Straight Tube

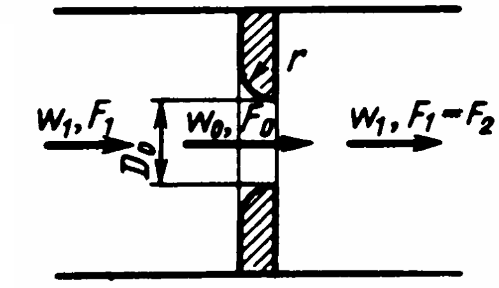
Sharp, Rounded & Beveled Edges

Idelchik Pg 222-224

$$K_{Sharp-Orifice} = \left[\left(1 - \frac{A_o}{A_1} \right) + 0.707 \left(1 - \frac{A_o}{A_1} \right)^{0.375} \right]^2 \left(\frac{A_1}{A_o} \right)^2$$

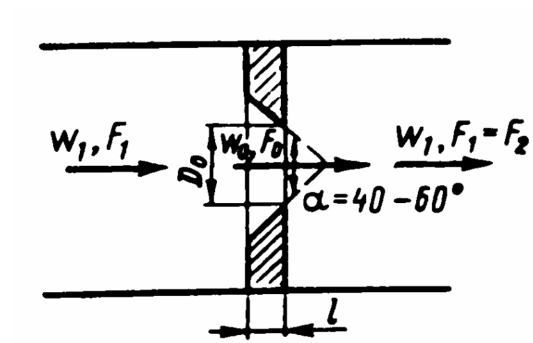


$$K_{Rounded-Orifice} = \left[\left(1 - \frac{A_o}{A_1} \right) + \sqrt{z'} \left(1 - \frac{A_o}{A_1} \right)^{0.75} \right]^2$$



$$z' = 0.03 + 0.47 \times 10^{-7.7 \frac{r}{D_{h_o}}}$$

$$K_{Beveled-Orifice} = \left[\left(1 - \frac{A_o}{A_1} \right) + \sqrt{z'} \left(1 - \frac{A_o}{A_1} \right)^{0.375} \right]^2$$



$$z' = 0.13 + 0.34 \times 10^{-\left[3.4 \frac{l}{D_{h_o}} + 88.4 \left(\frac{l}{D_{h_o}} \right)^{2.3} \right]}$$

Base area is A_o
Flow Thru Orifice

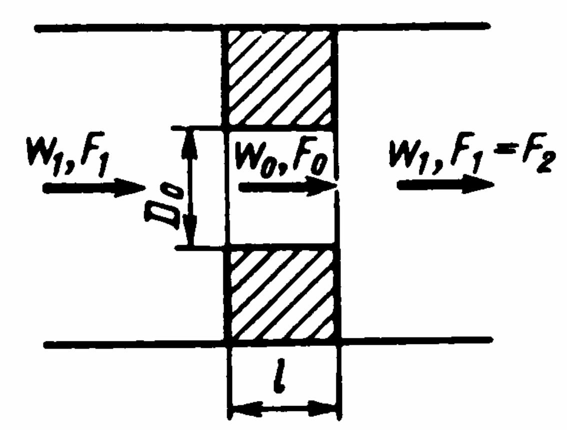
Orifice In A Straight Tube

Thick Edge Orifice

Idelchik Pg 22-224

$$K_{Thick-Edge} = \left[0.5\left(1-\frac{A_o}{A_1}\right)^{0.75} + t\left(1-\frac{A_o}{A_1}\right)^{1.375} + \left(1-\frac{A_o}{A_1}\right)^{2} + \frac{fl}{D_h} \right]$$

$$t = (2.4 - l)\times 10^{\left(0.25+\frac{0.535 l^8}{0.05+l^8}\right)}$$



Base area is A_o
Flow Thru Orifice

Resistance to Flow Through Barriers Uniformly Distributed Over the Tube Cross Section

Circular Metal Wire

Threaded Screens

Two Plane Screens

Grating In Line with Flow

Gratings with an Angle of Attack

Uniformly Distributed Barriers

- Barriers that are distributed uniformly over the tube or duct cross sections create a uniform flow resistance.
- Types of barriers may include:
  - Perforated sheets (grids), screens, beds, fabrics, loose material, crosswise bundles of tubes, etc.
- The flow resistance coefficient of a uniformly distributed barrier depends on:
  - Shape of the barrier
  - Free area coefficient
  - Reynolds number
- Free area coefficient is defined as the percent clear area of the barrier with respect to the upstream pipe area.
- Idelchik pg522

$$A_{free} = \frac{A_{open}}{A_{pipe-upstream}} = \frac{A_o}{A_1}$$

Circular Metal Wire Screens

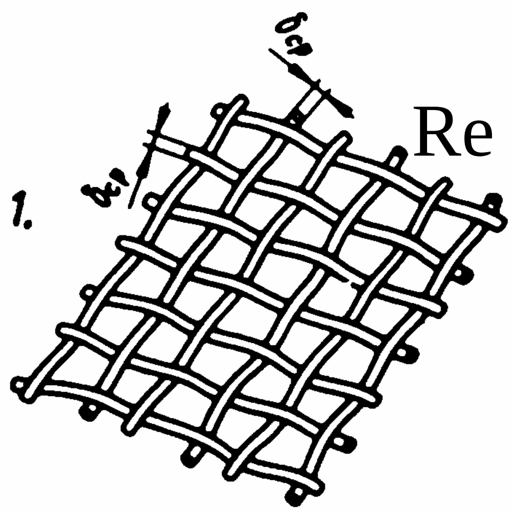
Idelchik Diagram 8.6 Pg 522

$$Re = \frac{v_o d_m}{m} \geq 10^3$$

$$K_{Wire-Screen} = 1.3(1 - \frac{A_0}{A_1}) + (\frac{A_1}{A_0} - 1)^2$$

$$50 < Re < 10^3$$

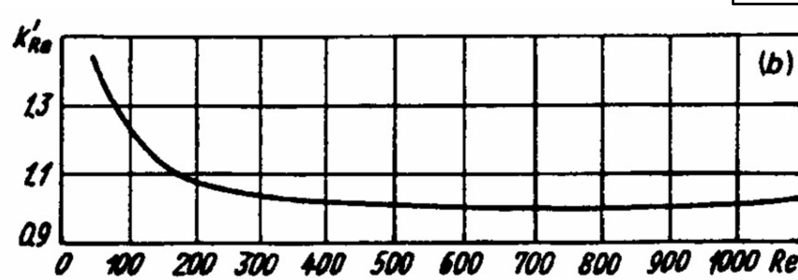
$$K_{Wire-Screen} = k_1 1.3(1 - \frac{A_0}{A_1}) + (\frac{A_1}{A_0} - 1)^2$$



$$Re < 50$$

$$K_{Wire-Screen} = \frac{22}{Re} + 1.3(1 - \frac{A_0}{A_1}) + (\frac{A_1}{A_0} - 1)^2$$

Base area is A_o



A_o is open area on screen

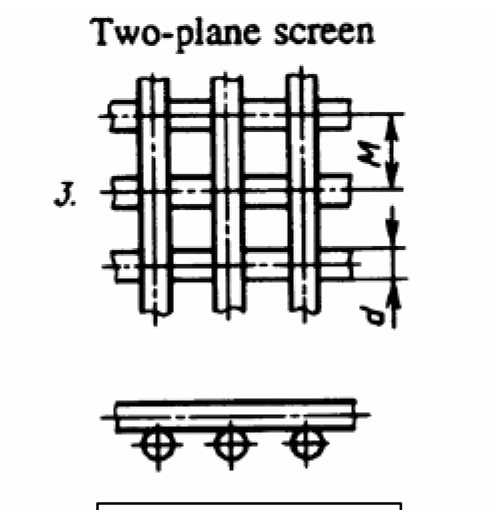
A_1 is pipe upstream flow area

Two Plane & Silk Thread Screen

Idelchik Pg 523

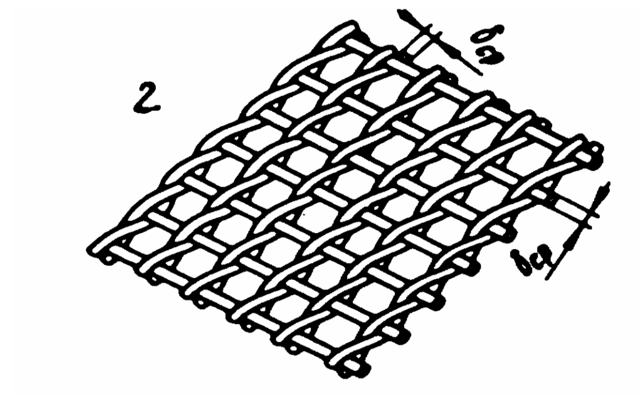
- Two plane screen made from bars of circular cross section

$$K_{2Plane-Screen} = 1.28\frac{1-\left(\frac{A_o}{A_1}\right)}{\left(\frac{A_o}{A_1}\right)^2}$$



Base area is A_o

- Silk Thread Screen



$$K_{Silk-Threads} = 1.62K_{Wire-Screen}$$

$$\mathrm{Re} > 500$$

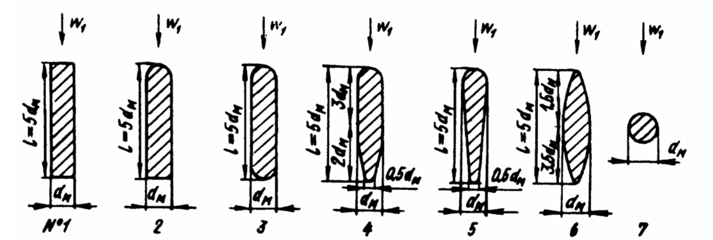
Gratings – In Line with Flow

Idelchik Pg 526

- Special case of $\frac{l}{d_m} = 5 \text{ and } \frac{a_o}{S_1} > 0.5$

$$K_{Grating} = b_1 k_1 \sin q$$

$$k_1 = \left(\frac{S_1}{a_o} - 1 \right)^{4/3}$$

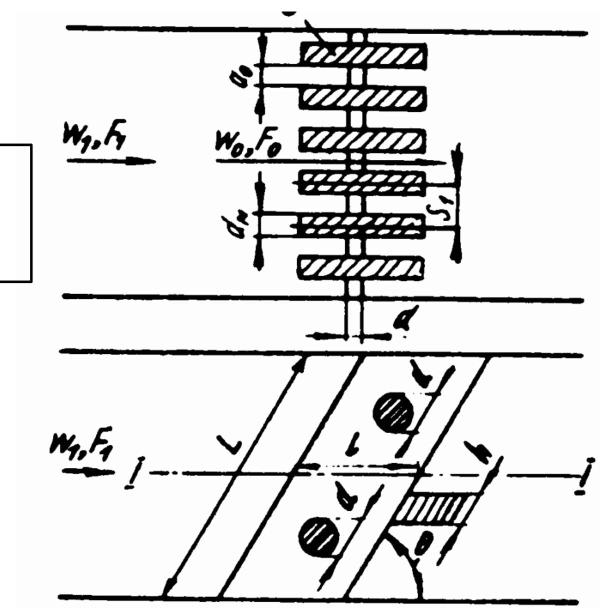


Shape of bars of the grating

Shape	1	2	3	4	5	6	7
$Beta_1$	2.34	1.77	1.77	1	0.87	0.71	1.73
$Beta_2$	1	0.76	0.76	0.43	0.37	0.3	0.74

- a_o is flow space between gratings
- S_1 is pitch of gratings
- l is length of gratings
- d_m is width of gratings
- $Beta_1$ is grating shape constant
- Theta is angle of bar inclination with respect to flow

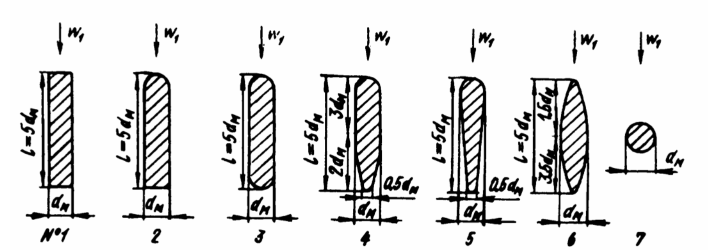
Base area is A_o Flow Between Gratings



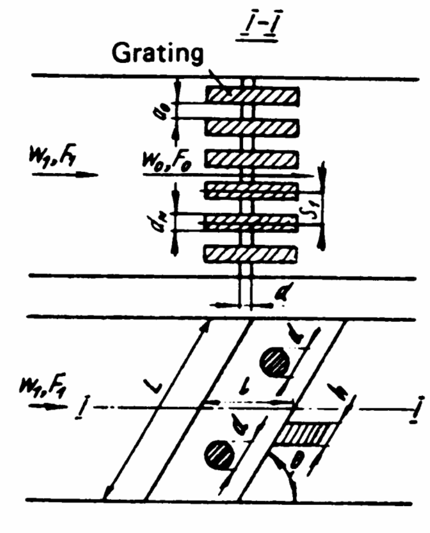
Gratings – In Line with Flow

Idelchik Pg 526

- Flow losses of grids made of bar gratings with different cross sectional shapes consist:
  - Entrance loss
  - Frictional loss over the bars
  - Sudden expansion (shock) losses at the exit



Shape of bars of the grating



- Case of arbitrary $\frac{l}{d_m}$ and $\frac{a_o}{S_1}$

$$K_{Grating} = b_2 K_{Thick-Grid} \sin q$$

$$t = \left[2.4 - \frac{l}{d_h}\right]^{0.25+\frac{0.535 l^8}{0.05+l^7}}$$

$$K_{Thick-Grit} = \left[0.5\left(1-\frac{A_{open}}{A_1}\right)^{0.75} + t\left(1-\frac{A_{open}}{A_1}\right)^{1.375} + \left(1-\frac{A_{open}}{A_1}\right)^2 + f\,\frac{l}{d_h}\right] / \left(\frac{A_{open}}{A_1}\right)^2$$

Gratings with an Angle of Attack

- Reynolds number > 10^4 & $a_0/S_1>0.5$

$$K_{Grating} = s_1 s_2$$

HYDRAULIC RESISTANCE OF NETWORKS

Hydraulic Resistance of Networks

- In flow networks, the portion of the total pressure which is spent in overcoming the forces of hydraulic resistance is irreversibly lost.
  - The molecular and turbulent viscosity of the moving medium irreversibly converts the mechanical work of the resistance forces into heat.
- There are two types of total pressure losses (hydraulic resistance) in flow network (pipeline system)
  - Pressure losses resulting from friction (frictional drag)
  - Local pressure losses

Friction Pressure Losses

- Fluid friction loss is due to the viscosity (both molecular and turbulent) of fluids in motion
- Results from momentum transfer between the molecules and individual particles of adjacent fluid layers moving in different velocities
  - molecules in laminar flow
  - individual particles in turbulent flow

Local Pressure Losses

- The local losses of total pressure are caused by
  - Local disturbance of the flow
  - Separation of flow from walls
  - Formation of vortices and strong turbulent agitation
    - Entrance of a fluid into the pipeline
    - Expansion, contraction, bending and branching of flow
    - Flow through orifices, grids, valves
    - Friction through porous media (filtration)
    - Discharge of fluid into atmosphere or another reservoir

- All of these phenomena contribute to the exchange of momentum between the moving fluid particles
  - Enhancing energy dissipation
  - Increasing total pressure loss

Total Pressure Loss

Friction + Local Pressure Loss

- The total pressure losses in any complex fluids network are inseparable. However, for ease of calculation they are arbitrarily subdivided in each element of the pipeline (network)
  - Into frictional losses
  - Into local losses
- Total pressure loss for the fluid network is the summation of pressure drops in each segment

$$\Delta p_{total} = \sum_{1}^{all-segments} \left(\Delta p_{friction} + \Delta p_{local} \right)$$

- The value of friction loss (relative to local pressure loss) is usually taken into account for
  - Long branch pipes, diffusers with small divergence angles
  - Long fittings

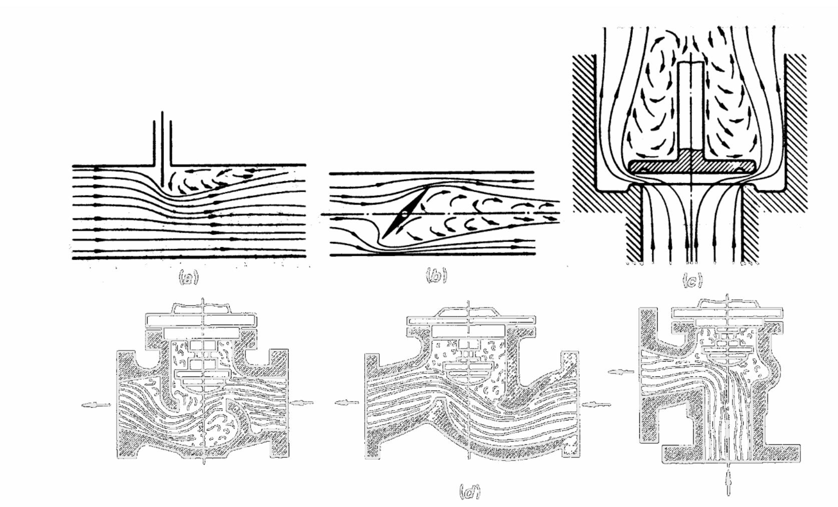
Units of Pressure Loss

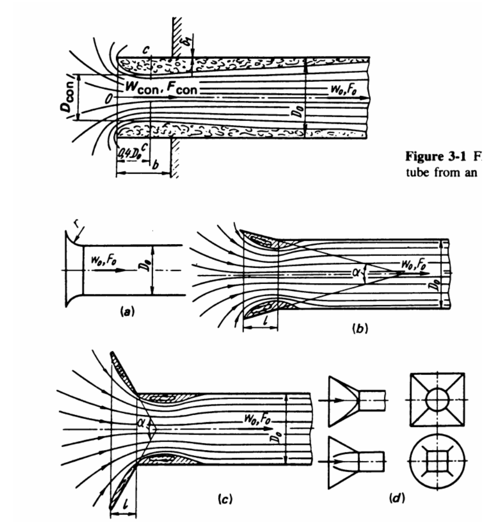
- Head is energy per unit weight of fluid.
  - Force x Length / Weight

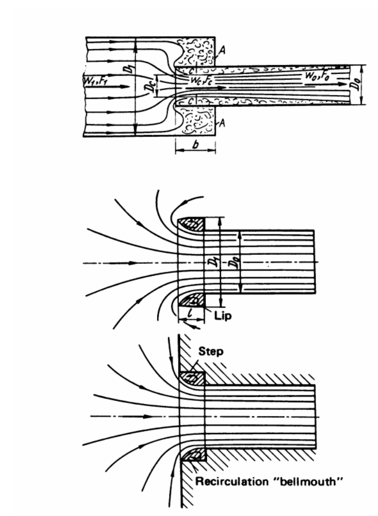
References

- Handbook of Hydraulic Resistance 3rd Edition
  I. E. Idelchik, CRC Begell House - 1994
- Fluid Mechanics
  Dr. Walther Kaufmann, Mc Graw Hill – 1958
- Crane Co., Flow of Fluids Through Valves, Fittings, and Pipe, Technical Paper No. 410, Crane Co., Joliet, IL, 1988.

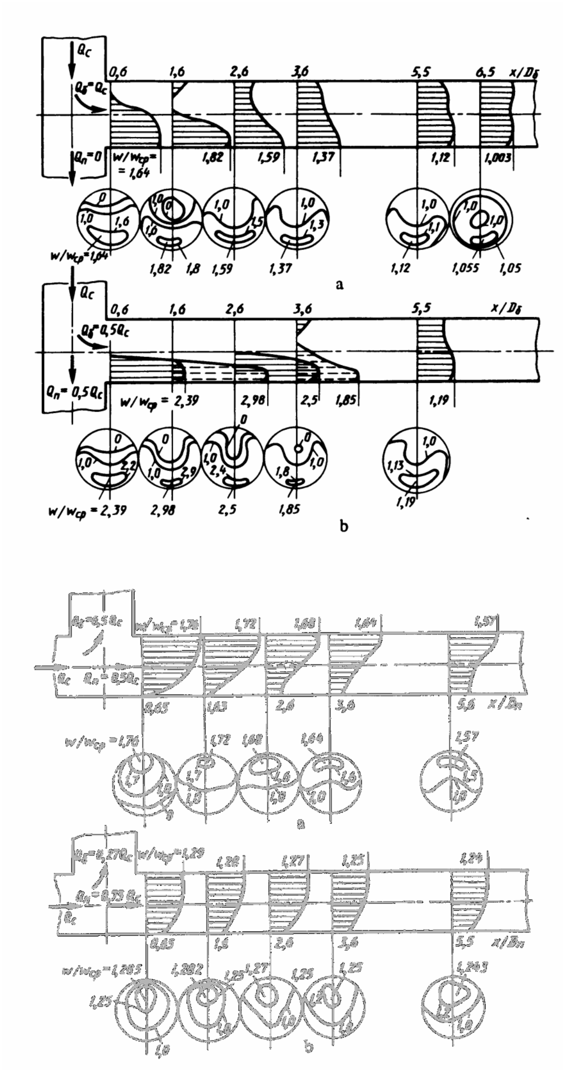
Appendix

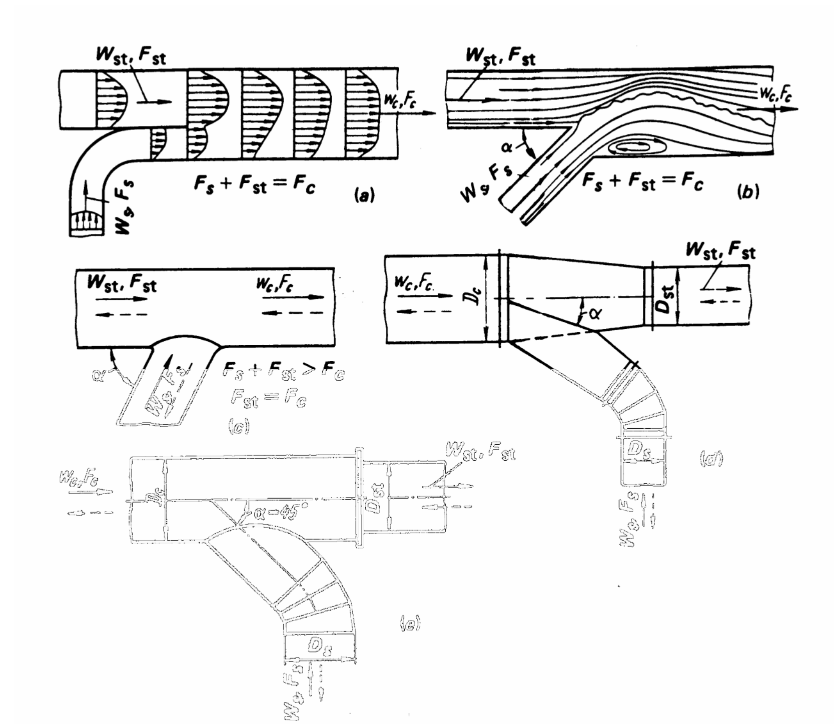






Appendix





- The total resistance coefficient of a diffuser installed in a flow network under any inlet condition can be expressed as

$$\underbrace{K_{Diffuser-L_0>0}}_{\text{Total Resistance Coefficient of a Diffuser with straight section installed upstream}} = k_d \mathbf{z} \underbrace{_{Diffuser-L_0=0}}_{\text{Total Resistance Coefficient of a Diffuser } L_0/D_0=0}$$

$$\boxed{K_{Grad-Exp} = \mathbf{h}\left\{\left(\frac{A_2}{A_1}\right)^2 - 1\right\}}$$

where

$$? = 0.15 - 0.2$$

Sudden Change in Velocity

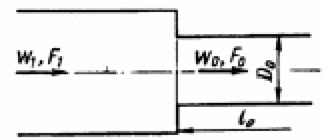
Idelchik Pg 217 Diagram 4-9

Resistance to Flow with Sudden Change in Velocity and Flow Area **217**

Sudden contraction at $b/D_h = 0$;
$Re = w_0 D_h/\nu > 10^4$ [12,13]

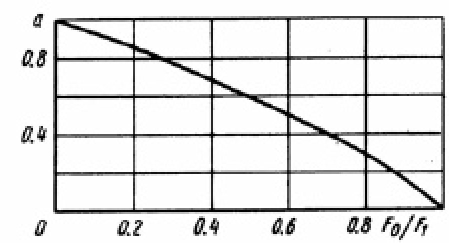
Diagram 4-9

1 Sharp

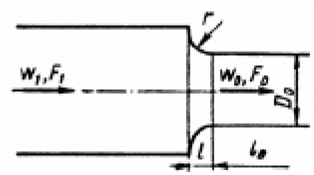


$$D_h = \frac{4F_0}{\Pi_0}$$

$\frac{F_0}{F_1}$	0	0.2	0.4	0.6	0.8	0.9	1.0
a	1.0	0.850	0.680	0.503	0.300	0.178	0



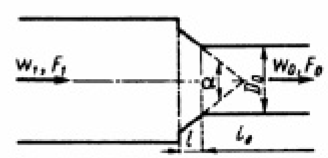
2 Rounded



$$\zeta = \zeta'\left(1 - \frac{F_0}{F_1}\right)^{3/4} + \zeta_{fr} = \zeta' a + \zeta_{fr}$$, where for ζ'

see curve $\zeta = f(b/D_h)$ of Diagram 3-4 (curve c);
for a see curve $a = f(F_0/F_1)$ (para. 1); $\zeta_{fr} = \lambda l_0/D_h$;
for λ see Chapter 2.

3 Beveled

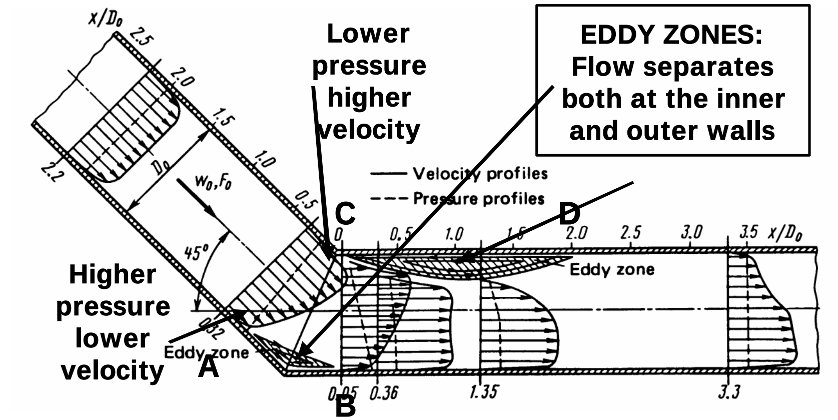


$$\zeta = \zeta''\left(1 - \frac{F_0}{F_1}\right)^{3/4} + \zeta_{fr} = \zeta'' a + \zeta_{fr}$$, where for ζ''

see curve $\zeta = f(\alpha, l/D_h)$ of Diagram 3-7;
for a see curve $a = f(F_0/F_1)$ (para. 1); $\zeta_{fr} = \lambda l_0/D_h$;
for λ see Chapter 2.

Velocity Distribution for a Curved Bend

- Increase of the pressure at the outer wall and decrease at the inner wall will create a flow velocity the lower at the altar wall and larger at the narrow



- **Before the straight section after the turn:** Flow velocity will be lower at the outer wall and larger at the inner wall.
  - Due to higher pressure at the outer wall & lower pressure inside.
- **Downstream of turn - Straight section:** Higher flow at the outer wall and lower velocity at the inner wall.
- **Secondary Flows:**

Derivation of Local Loss Coefficient
Sudden Expansion

- From the momentum law:

$$\Sigma F_{ext} = \dot{m}\Delta v$$

$$(p_1 - p_2)A_2 = rQ(v_2 - v_1)$$

$$(p_1 - p_2)A_2 = rA_2 v_2 (v_2 - v_1)$$

$$(p_1 - p_2) = rv_2(v_2 - v_1)$$

- From Modified Bernoulli Eq.:

$$(p_{2-} p_1) = \tfrac{1}{2}\, r\,(v_1^2 - v_2^2) + K_{\exp}\, \tfrac{1}{2}\, r v_1^2$$

- Eliminating $(p_2$-$p_1)$ & rho

$$-v_2(v_2 - v_1) = \tfrac{1}{2}(v_1^2 - v_2^2) + K_{\exp}\, \tfrac{1}{2}\, v_1^2$$

- Local Loss Coefficient, K_{exp}

$$K_{\exp}\, \tfrac{1}{2} v_1^2 = (-v_2^2 + v_2 v_1 - \tfrac{1}{2} v_1^2 + \tfrac{1}{2} v_2^2)$$

$$K_{\exp} v_1^2 = -2v_2^2 + 2v_2 v_1 - v_1^2 + v_2^2$$

$$= -v_2^2 + 2v_2 v_1 - v_1^2$$

$$= -(v_2 - v_1)^2$$

$$= -v_1^2 \left(\frac{v_2}{v_1} - 1\right)^2$$

$$K_{\exp} = -\left(\frac{A_1}{A_2} - 1\right)^2$$

$$= \left(1 - \frac{A_1}{A_2}\right)^2$$

- Ratio A_1/A_2 is less than 1.0

Flow From A Vessel With Spout

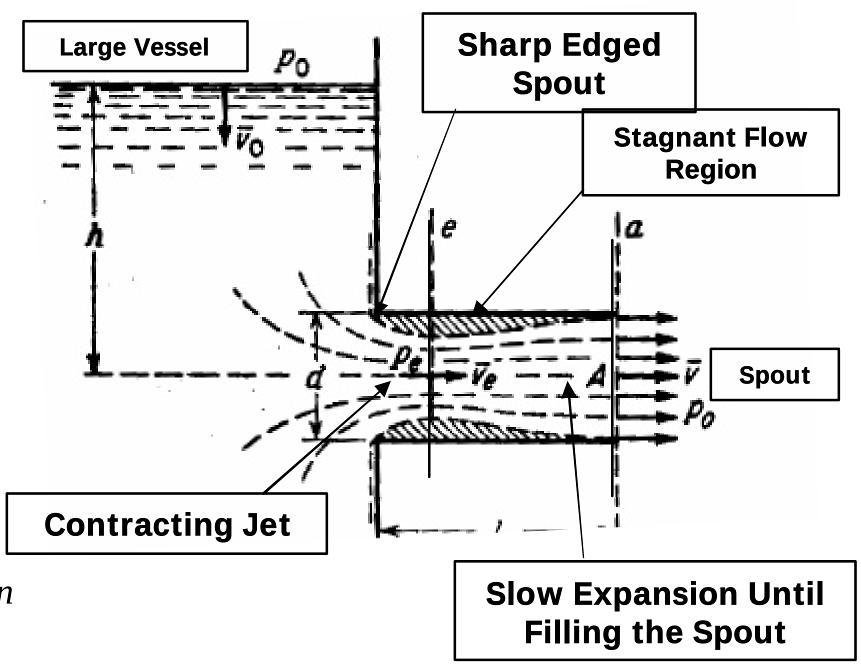
- Momentum Equation Between sections e & a:
  - Forces acting on the fluid must equal to the difference in momenta between fluid leaving and entering the section per unit time

$$(p_e - p_o)A = rAv(v - v_e)$$

- Applying Bernoulli's equation at sections e & a
  - Pressure at e is p_e including the stagnant flow region

$$(p_e - p_o) = \tfrac{1}{2} r\left(v^2 - v_e^2\right) + K_{contraction}$$

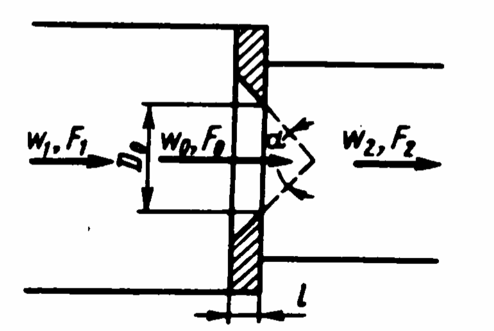
- Combining both equations

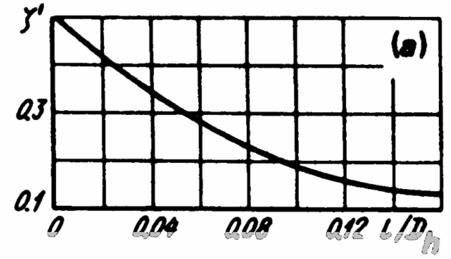


Orifice with Beveled Inlet Edge

In A Tube Transition Section

Idelchik Pg 224



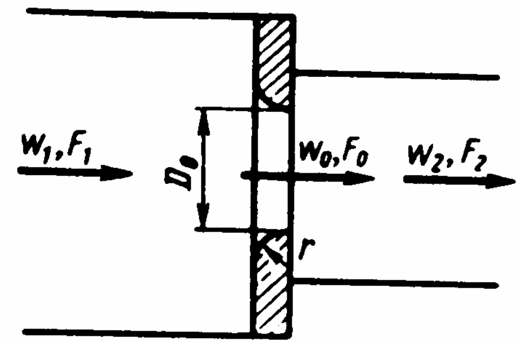


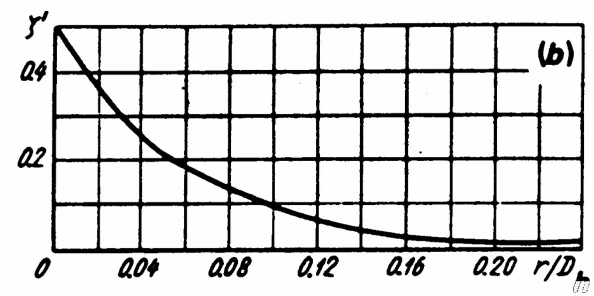
l/D_h	0.01	0.02	0.03	0.04	0.06	0.08	0.12	≥0.16
ζ'	0.46	0.42	0.38	0.35	0.29	0.23	0.16	0.13

Orifice with Rounded Inlet Edge

In A Tube Transition Section

Idelchik Pg 224





r/D_h	0	0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.12	0.16	≥0.2
ζ'	0.50	0.44	0.37	0.31	0.26	0.22	0.19	0.15	0.09	0.06	0.03