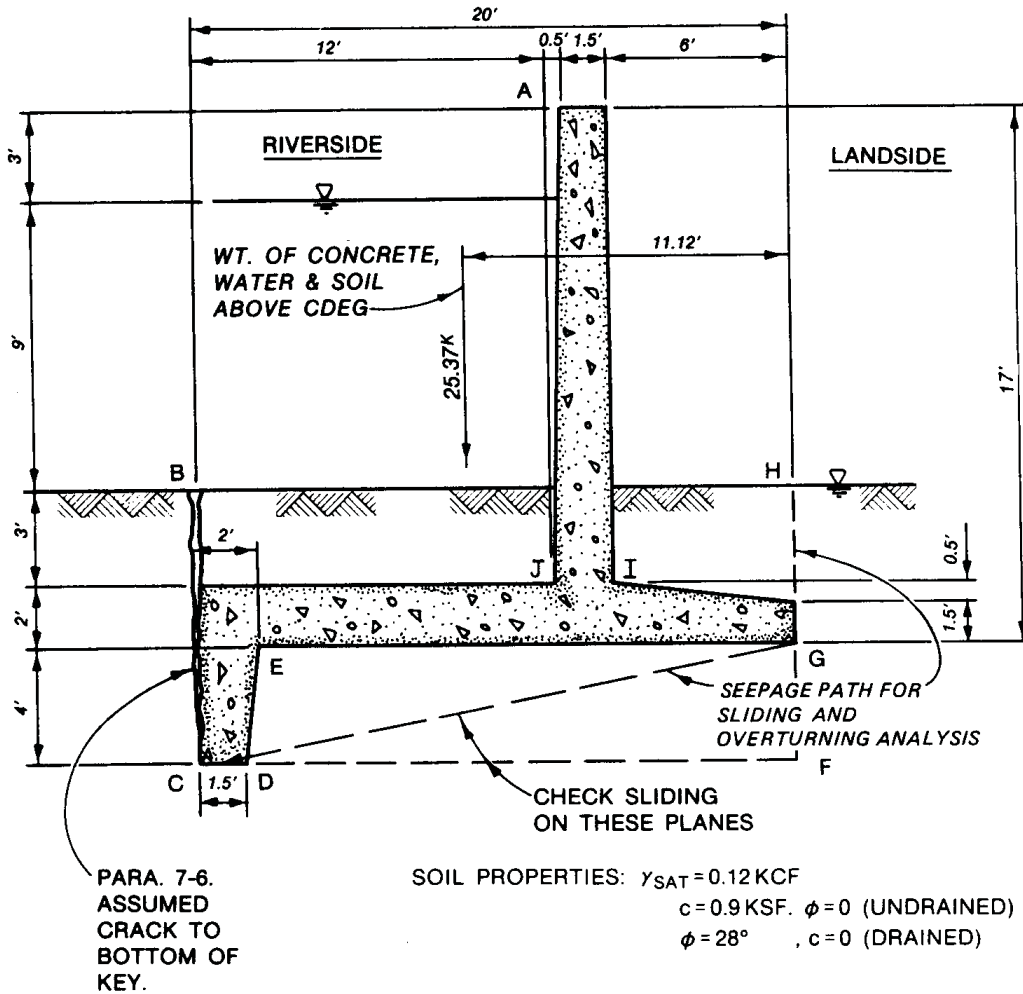


N-3. EXAMPLE 3.

Analyze the wall for overturning stability, sliding and bearing capacity. Determine water pressure using the line of creep method. Calculate reinforcement required at critical sections. Case I1, design flood.



a. Water pressures (line of creep, paragraph 3-19).

$$L_{CG} = \sqrt{4^2 + 20^2} = 20.40 \text{ ft}, \quad L_{BC} = 0, \quad \Delta_h = 9 \text{ ft}$$

$$L_{GH} = 5 \text{ ft}, \quad L_S = 20.4 \text{ ft} + 5 \text{ ft} = 25.4 \text{ ft}$$

$$u_C = \frac{\text{Dist. below Headwater}}{L_S} - \frac{\Delta_h(L_{BC})}{L_S} \quad \gamma_w = \left(18 \text{ ft} - \frac{9 \times 0}{25.4}\right) 0.0625 = 1.125 \text{ ksf}$$

$$u_G = \frac{\text{Dist. below Headwater}}{L_S} - \frac{\Delta_h(L_{CG})}{L_S} \quad \gamma_w = \left(14 \text{ ft} - \frac{9 \times 20.4}{25.4}\right) 0.0625 = 0.4232 \text{ ksf}$$

Compute water pressure at D and E. Prorate head loss along path CDEG. Total head loss along path

$$CG = \frac{9(20.4)}{25.4} = 7.228 \text{ ft}$$

$$L_S \text{ (Concrete surface)} = L_{CD} + L_{DE} + L_{EG}$$

$$L_S = 1.5 + 4.03 + 18 = 23.53 \text{ ft}$$

$$\text{Head loss at D} = \frac{1.5}{23.53} (7.228) = 0.4608 \text{ ft}$$

$$u_D = (18 - 0.4608)0.0625 = 1.0962 \text{ ksf}$$

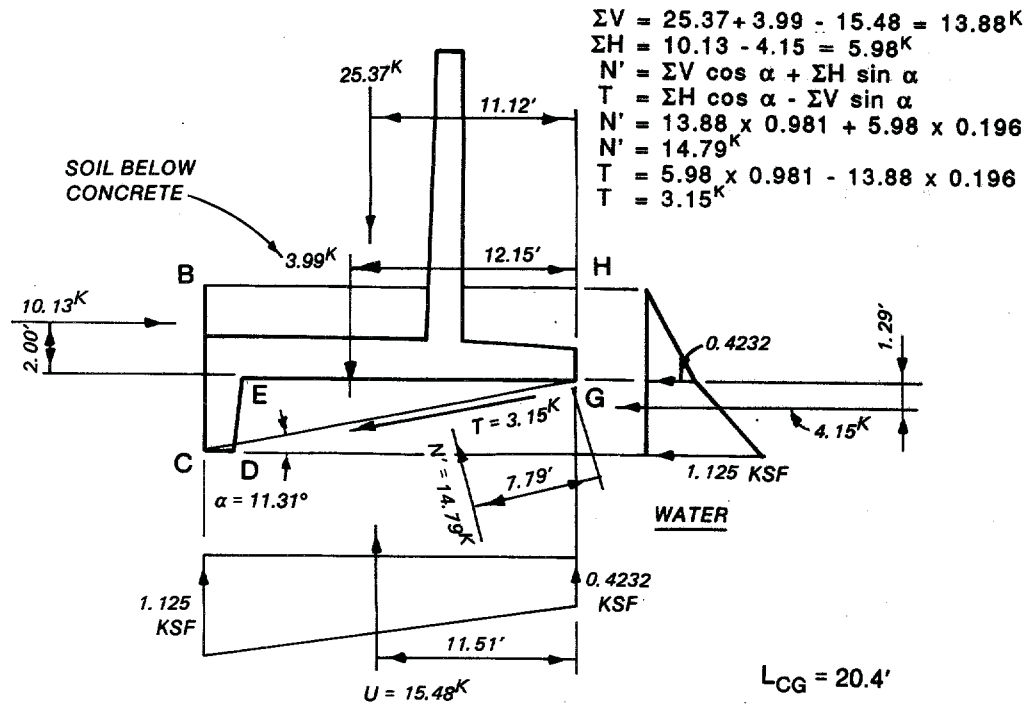
$$\text{Head loss at E} = \frac{(1.5 + 4.03)}{23.53} (7.228) = 1.699 \text{ ft}$$

$$u_E = (14 - 1.699)0.0625 = 0.7688 \text{ ksf}$$



c. Sliding stability analysis (paragraph 4-10).

(1) On Plane CG:



For undrained shear strength:

$$cL_{CG} = 0.9 \times 20.4 = 18.36 \text{ kips}$$

$$T \leq \frac{N' \tan \phi + cL}{FS}$$

[4-12]

From Table 4-2, the minimum FS is 1.5.

$$3.15 \leq \frac{18.36}{1.5}$$

$$3.15 \leq 12.24$$

For drained shear strength,

$$N' \tan \phi = 14.79 \times 0.532 = 7.86 \text{ kips}$$

$$T \leq \frac{N' \tan \phi + cL}{FS} \quad [4-12]$$

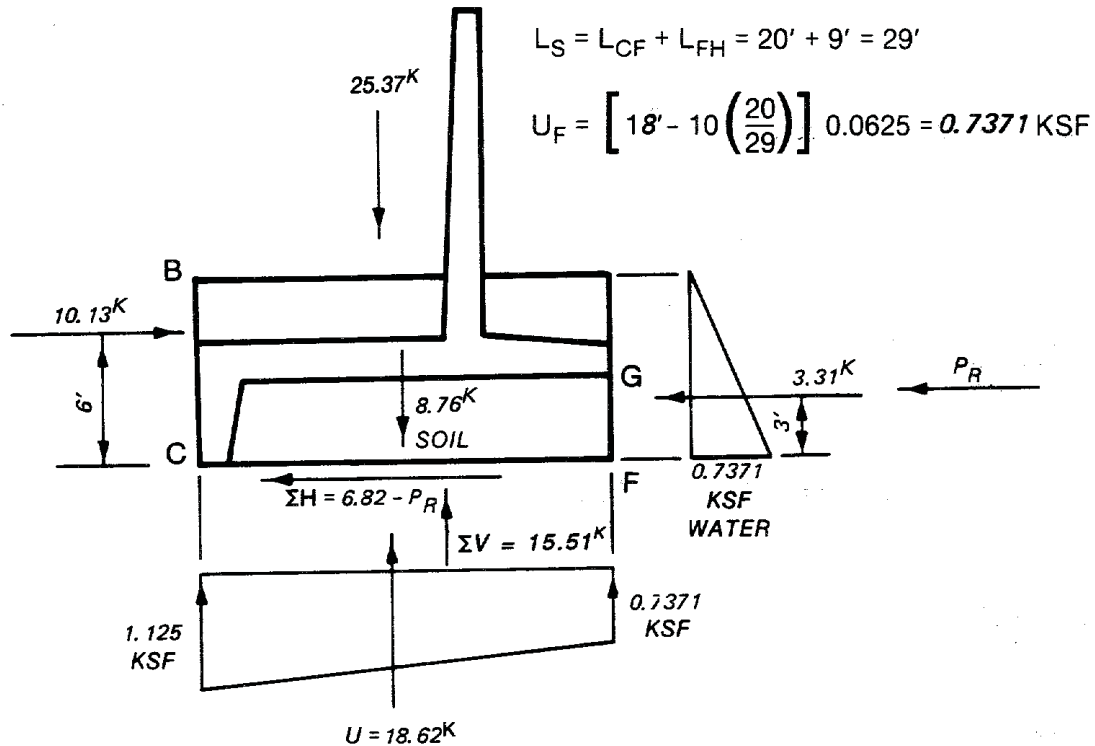
Using the minimum FS of 1.5 from Table 4-2 yields

$$3.15 \leq \frac{7.86}{1.5}$$

$$3.15 \leq 5.24$$

Sliding resistance is adequate without considering soil resistance on landside.

(2) On Plane CF:



$$\begin{aligned}\gamma' &= \text{effective weight on landside} = [9 \text{ ft}(0.12) - 0.7371] \div 9 \\ &= 0.0381 \text{ kcf}\end{aligned}$$

Undrained condition,

$$c = 0.9 \text{ ksf}$$

$$c_d = \text{SMF}(c) \quad [3-10]$$

$$c_d = \frac{2}{3} (0.9) = 0.6 \text{ ksf}$$

$$K_o = 1.0 \quad (\text{from Equation 3-4})$$

The at-rest earth force may be calculated as:

$$P_R = \frac{1}{2} \gamma_b h^2 = \frac{1}{2} (0.0381)(9)^2 = 1.54 \text{ kips}$$

$$c_d^L \text{CF} = 0.6(20) = 12 \text{ kips}$$

$$T \leq \frac{N' \tan \phi + cL}{FS} \quad [4-12]$$

Using the minimum FS of 1.5 from Table 4-2 yields

$$T = \Sigma H = 6.82 - 1.54 = 5.28 \text{ kips}$$

$$5.28 \leq \frac{12}{1.5}$$

$$5.28 \leq 8$$

Sliding resistance is adequate.

Drained condition,

$$\phi = 28^{\circ}$$

$$SMF = \frac{\tan \phi_d}{\tan \phi} = \frac{2}{3} \quad [3-10]$$

$$\phi_d = \tan^{-1} \left( \frac{2}{3} \tan \phi \right) = 19.52^{\circ}$$

$$K_p = \tan^2 \left( 45^{\circ} + \frac{\phi}{2} \right) = 2.77$$

$$\text{Let } K = 1/2 K_p = 1.385 \text{ (Paragraph 3-8b)}$$

$$P_R = \frac{1}{2} K \gamma_b h^2 = \frac{1}{2} (1.385)(0.0381)(9)^2 = 2.14 \text{ kips}$$

$$N' = \Sigma V = 25.37 + 8.76 - 18.62 = 15.51$$

$$T = 10.13 - 3.31 - 2.14 = 4.68 \text{ kips}$$

$$T \leq \frac{N' \tan \phi + cL}{FS} \quad [4-12]$$

Using the minimum FS of 1.5 from Table 4-2 yields

$$4.68 \leq \frac{15.51 \tan 28^{\circ}}{1.5}$$

$$4.68 \leq 5.50$$

Sliding resistance is adequate.

d. Bearing capacity analysis (on Plane CG).

$$N' = 14.79 \text{ kips}$$

$$T = 3.15 \text{ kips}$$

$$\alpha = 11.31^\circ = 0.197 \text{ rad}$$

$$e = \frac{L_{CG}}{2} - a = \frac{20.4}{2} - 7.79 = 2.41$$

$$\bar{B} = L_{CG} - 2e = 15.58 \text{ ft}$$

$$\delta = \tan^{-1} \left( \frac{T}{N'} \right) = \tan^{-1} \left( \frac{3.15}{14.79} \right) = 12.02^\circ \text{ (Figure 5-1)}$$

$$\gamma' = 0.120 - 0.0625 = 0.0575 \text{ kcf}$$

$$D = 5 \text{ ft}$$

$$q_o = \gamma' D = 0.0575(5) = 0.2875 \text{ ksf} \quad [5-8a]$$

$$\xi_{cd}(\phi = 0) = 1 + 0.2 \left( \frac{D}{\bar{B}} \right) = 1 + 0.2 \left( \frac{5}{15.58} \right) = 1.064 \quad [5-4a]$$

$$\xi_{qd} = \xi_{\gamma d} = 1 \quad (\text{for } \phi = 0^\circ) \quad [5-4b]$$

$$\xi_{qd} = \xi_{\gamma d} = 1 + 0.1 \left( \frac{D}{\bar{B}} \right) \tan \left( 45^\circ + \frac{\phi}{2} \right) \quad (\text{for } \phi = 28^\circ) \quad [5-4c]$$

$$\xi_{qd} = \xi_{\gamma d} = 1 + 0.1 \left( \frac{5}{15.58} \right) (1.6643) = 1.053 \quad (\phi = 28^\circ)$$

$$\xi_{qi} = \xi_{ci} = \left( 1 - \frac{\delta}{90} \right)^2 = \left( 1 - \frac{12.02}{90} \right)^2 = 0.7507 \quad [5-5a]$$

$$\xi_{\gamma i} = \left( 1 - \frac{12.02}{28} \right)^2 = 0.3257 \quad [5-5b]$$



$$\xi_{qt} = \xi_{\gamma t} = (1 - \alpha \tan \phi)^2 = (1 - 0.197 \times 0.532)^2 = 0.8015 \quad [5-6a]$$

$$\xi_{ct} = 1 - \left( \frac{2\alpha}{\pi + 2} \right) = 1 - \left( \frac{0.394}{\pi + 2} \right) = 0.9232 \quad (\text{for } \phi = 0) \quad [5-6b]$$

Undrained condition:

$$c = 0.9 \text{ ksf}$$

From Equation 5-2

$$Q = \bar{B} \left( \xi_{cd} \xi_{ci} \xi_{ct} c N_c + \xi_{qd} \xi_{qi} \xi_{qt} q_o N_q \right)$$

$$N_c = 5.14, \quad N_q = 1.00 \quad (\text{Table 5-1})$$

$$Q = 15.58 \left[ 1.064 (0.7507) (0.9232) (5.14) (0.9) + 1 (0.7507) (0.8015) (0.2875) (1) \right]$$

$$Q = 55.84 \text{ kips}, \quad FS = \frac{Q}{N} = \frac{55.84}{14.79} = 3.78 > 3.00 \quad (\text{Table 4-2})$$

Drained condition:

$$\phi = 28^\circ$$

$$Q = \bar{B} \left( \xi_{qd} \xi_{qi} \xi_{qt} q_o N_q + \frac{\xi_{\gamma d} \xi_{\gamma i} \xi_{\gamma t} \bar{B}_\gamma N_\gamma}{2} \right)$$

$$N_q = 14.72, \quad N_\gamma = 11.19 \quad (\text{Table 5-1})$$

$$Q = 15.58 \left[ 1.053 (0.7507) (0.8015) (0.2875) (14.72) + \frac{1.053 (0.3257) (0.8015) (15.58) (0.0575) (11.19)}{2} \right] = 63.24 \text{ kips}$$

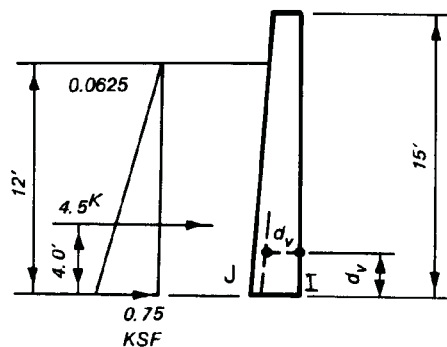
$$FS = \frac{Q}{N'} = \frac{63.24}{14.79} = 4.28 > 3.00 \quad (\text{Table 4-1})$$

Bearing capacity is adequate.

e. Reinforcement (Chapter 9).

$$f'_c = 3 \text{ ksi}, \quad f_y = 40 \text{ ksi}, \quad \text{cover} = 4.5 \text{ in.}$$

(1) At base of stem:



$$M = 4.5 \times 4 = 18.00 \text{ k-ft}$$

$$\frac{M_u}{\phi} = \frac{1.9(18.00)12}{0.9} = 456.0 \text{ k-in.}$$

$$d = 24 \text{ in.} - 4.5 \text{ in.} = 19.5 \text{ in.,}$$

$$b = 12 \text{ in.}$$

$$k_u = 1 - \sqrt{1 - \frac{M_u/\phi}{0.425f'_c b d^2}} \quad (\text{Figure 9-2})$$

$$k_u = 1 - \sqrt{1 - \frac{456.0}{5817.825}} = 0.0400$$

$$C_u = T_u = 0.85f'_c k_u b d = 0.85(3)(0.040)(12)(19.5) = 23.87 \text{ kips}$$

$$A_s = \frac{T_u}{f_y} = \frac{23.87}{40} = \underline{\underline{0.60 \text{ in.}^2/\text{ft}}}$$

Check ductility requirements:

$$\beta_1 = 0.85, \quad \epsilon_c = 0.003$$

$$\rho_b = \frac{0.85f'_c}{f_y} \beta_1 \left( \frac{\epsilon_c}{\epsilon_m + \frac{f_y}{29,000 \text{ ksi}}} \right) \quad (\text{see example 1, paragraph i})$$

$$\rho_b = \frac{0.85(3)}{40} (0.85) \left( \frac{0.003}{0.003 + \frac{40}{29,000}} \right) = 0.03712$$

$$\rho_{\max} = \lambda \rho_b, \quad \lambda = 0.25 \text{ (paragraph 9-8c(2)(a))}$$

$$\rho_{\max} = 0.25(0.03712) = 0.00928$$

$$\rho_{\min} = 200/f_y = 200/40,000 \text{ ksi} = 0.005 \text{ (paragraph 9-8b(4), from ACI 318)}$$

$$\rho = \frac{A_s}{bd} = \frac{0.60}{12(19.5)} = 0.002564$$

$$\rho < \rho_{\min}$$

Instead of using  $\rho_{\min}$ , ACI 318 allows the minimum area of reinforcement to be one-third greater than that required by analysis. Therefore,

$$A_s = 4/3(0.60) = 0.80 \text{ in.}^2/\text{ft}$$

$$\rho = \frac{A_s}{bd} = \frac{0.80}{12(19.5)} = 0.00342$$

$$\rho = \rho_{\max}$$

Ductility is adequate.

(2) Check shear at distance  $d_v$  above base (paragraph 9-8f)

$$d_v = d - \left( \frac{0.5}{15} \right) d_v = 19.5 - 0.0333 d_v$$

$$d_v = \frac{19.5}{1.0333} = 18.87 \text{ in.} = 1.573 \text{ ft}$$

$$V = \frac{1}{2} (0.0625)(12 - 1.573)^2 = 3.40 \text{ kips}$$

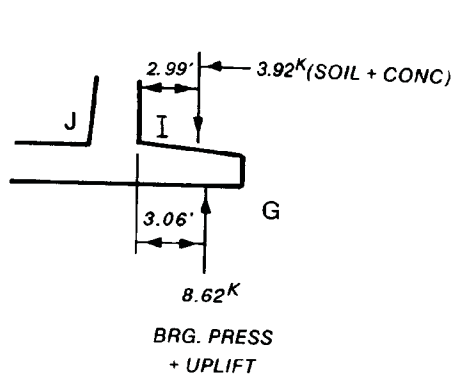
$$V_u = 1.9 \text{ V} = 1.9(3.40) = 6.46 \text{ kips} = 6,460 \text{ lb}$$

$$\phi V_c = \phi 2\sqrt{f'_c} \, b d_v \quad (\text{from ACI-318})$$

$$\phi V_c = 2(0.85)(\sqrt{3000})(12)(18.87) = 21,084 \text{ lb} > 6,460$$

Shear capacity is adequate. Shear is most critical for stem.

(3) Toe at face of stem (Figure 9-2):



$$V_I = \frac{8.62 \times 3.06 = 26.38}{4.70 \text{ kips M}} = \frac{-3.92 \times 2.99 = -11.72}{14.66} \text{ k-ft}$$

$$b = 12 \text{ in.}, \quad d = 19.5 \text{ in.}$$

$$0.425f'_c b d^2 = 5817.825$$

$$\frac{M_u}{\phi} = \frac{1.9(14.66)(12)}{0.9} = 371.39 \text{ k-in.}$$

$$k_u = 1 - \sqrt{1 - \frac{371.39}{5817.825}} = 0.03244$$

$$C_u = T_u = 0.85(3)(0.03244)(12)(19.5) = 19.36 \text{ kips}$$

$$A_s = \frac{T_u}{f_y} = \frac{19.36}{40} = \underline{\underline{0.48 \text{ in.}^2/\text{ft}}}$$

Check ductility:

$$\rho_{\max} = 0.00928, \quad \rho_{\min} = 0.005$$

$$\rho = \frac{A_s}{bd} = \frac{0.48}{12(19.5)} = 0.00205$$

Again, since  $\rho < \rho_{\min}$ , use a one-third increase in the area of reinforcement required by analysis. Therefore,

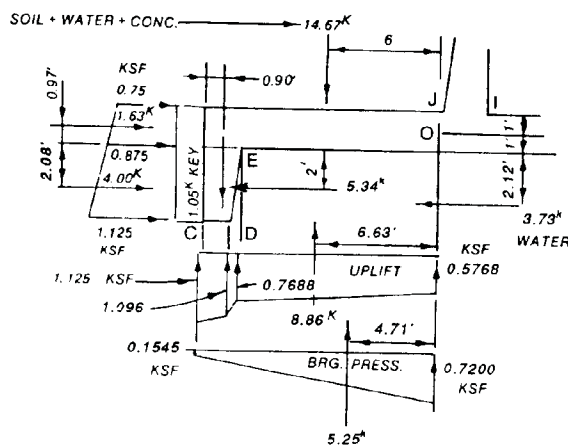
$$A_s = 4/3(0.48) = 0.64 \text{ in.}^2/\text{ft}$$

$$\rho = \frac{A_s}{bd} = \frac{0.64}{12(19.5)} = 0.002735$$

$$\rho < \rho_{\max}$$

Ductility is adequate.

(4) Heel and key reinforcement:



$$\begin{aligned} 14.67 \times 6.00 &= 88.02 \\ 1.05 \times 11.10 &= 11.66 \\ -8.86 \times 6.63 &= -58.74 \\ -5.25 \times 4.71 &= -24.73 \\ V &= \frac{1.61}{1.61} \text{ kips} \\ -3.73 \times 3.12 &= -11.64 \\ -5.34 \times 3.00 &= -16.02 \\ 1.63 \times 0.02 &= 0.03 \\ 4.00 \times 3.08 &= 12.32 \\ P &= \frac{-3.44}{-3.44} \text{ kips} \\ M_o &= -0.90 \text{ k-ft} \end{aligned}$$

$$b = 12 \text{ in.}, h = 24 \text{ in.}, d = 19.5 \text{ in.}$$

$$0.425f'_c b d^2 = 5817.825$$

Reinforcement at "O" with 5.34 kips force on key (Figure 9-4):

$$M_u = 1.9M_o = 1.9(0.90)(12) = 20.52 \text{ in.-k}$$

$$P_u = 1.9P = 1.9(3.44) = 6.54 \text{ kips (tension)}$$

$$\frac{M_u}{P_u} = \frac{20.52}{6.54} = 3.14$$

$$d - \frac{h}{2} = 19.5 - \frac{24}{2} = 7.5$$

$$\frac{M_u}{P_u} < d - \frac{h}{2} \quad \text{Therefore, the conditions of Figure 9-5 apply.}$$

$$\begin{aligned} M_{ue} &= -M_u + P_u \left( d - \frac{h}{2} \right) \\ &= -20.52 + 6.54(7.5) \\ &= 28.53 \text{ in.-k} \end{aligned}$$

$$A'_s = \frac{M_{ue}}{\phi f_y (d - d')} = \frac{28.53}{0.9(40)(19.5 - 4.5)} = 0.053 \text{ in.}^2/\text{ft}$$

$$A_s = \frac{\frac{P_u}{\phi} - f_y A'_s}{f_y} = \frac{\frac{6.54}{0.9} - 40(0.053)}{40} = 0.13 \text{ in.}^2/\text{ft}$$

Reinforcement at "0" neglecting 5.34 kips force on key (Figure 9-3)

$$M_o = 0.90 + 5.34(3.00) = 16.92 \text{ ft-k} = 203.04 \text{ in.-k}$$

$$M_u = 1.9M_o = 385.78 \text{ in.-k}$$

$$P = 1.9 \text{ kips (compression)}$$

$$P_u = 1.9P = 1.9(1.9) = 3.61 \text{ kips}$$

$$\begin{aligned} \phi &= 0.9 - \frac{P_u}{0.1f'_c b h} \quad (0.2) \\ &= 0.9 - \frac{3.61}{0.1(3)(12)(19.5)} \quad (0.2) \\ &= 0.89 \end{aligned}$$

$$\begin{aligned} M_{ue} &= M_u + P_u \left( d - \frac{h}{2} \right) \\ &= 385.78 + 3.61 \left( 19.5 - \frac{24}{2} \right) \\ &= 412.86 \text{ in.-k} \end{aligned}$$

$$\begin{aligned} k_u &= 1 - \sqrt{1 - \frac{M_{ue}}{\phi 0.425 f'_c b d^2}} \\ &= 1 - \sqrt{1 - \frac{412.86}{0.89(5817.825)}} \end{aligned}$$

$$= 0.04070$$

$$\begin{aligned} A_s &= \frac{0.85 f'_c k_u b d - \frac{P_u}{\phi}}{f_y} \\ &= \frac{0.85(3)(0.04070)(12)(19.5) - \frac{3.61}{0.89}}{40} \\ &= 0.51 \text{ in.}^2 \text{ft} \end{aligned}$$

Check ductility:

$$\rho_{\max} = 0.00928, \quad \rho_{\min} = 0.005$$

$$\rho = \frac{A_s}{b d} = \frac{0.51}{12(19.5)} = 0.00218$$

$$\rho < \rho_{\min}$$

$$\text{Use } A_s = 4/3(0.51) = 0.68 \text{ in.}^2/\text{ft}$$

$$\rho = \frac{A_s}{bd} = \frac{0.68}{12(19.5)} = 0.00291$$

$$\rho < \rho_{\max}$$

Ductility is adequate.

Reinforcement in key at "E" (Figure 9-2):

$$M = 5.34(2) + 3.73(2.12) - 4.00(2.08) = 10.27 \text{ ft-k}$$

$$\frac{M_u}{\phi} = \frac{1.9(10.27)(12)}{0.9} = 260.11 \text{ k-in.}$$

$$k_u = 1 - \sqrt{1 - \frac{260.11}{5817.825}} = 0.02261$$

$$C_u = T_u = 0.85(3)(0.02261)(12)(19.5) = 13.49 \text{ kips}$$

$$A_s = \frac{T_u}{f_y} = \frac{13.49}{40} = \underline{\underline{0.34 \text{ in.}^2/\text{ft}}}$$

Check ductility:

$$\rho_{\max} = 0.00928, \quad \rho_{\min} = 0.005$$

$$\rho = \frac{A_s}{bd} = \frac{0.34}{12(19.5)} = 0.00145$$



$$\rho < \rho_{\min}$$

$$\text{Use } A_s = 4/3(0.34) = 0.45 \text{ in.}^2/\text{ft}$$

$$\rho = \frac{A_s}{bd} = \frac{0.45}{12(19.5)} = 0.00194$$

$$\rho < \rho_{\max}$$

Ductility is adequate.