Chapter 8 Notes
Cams

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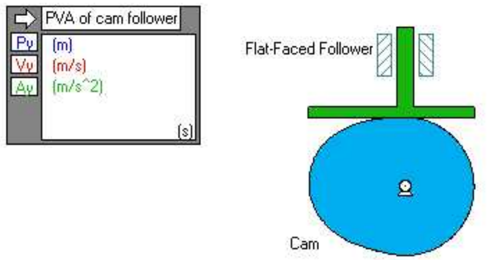
All figures taken from *Design of Machinery*, 3rd ed. Robert Norton 2003

Cams

- Function generator
- Can generate a true dwell

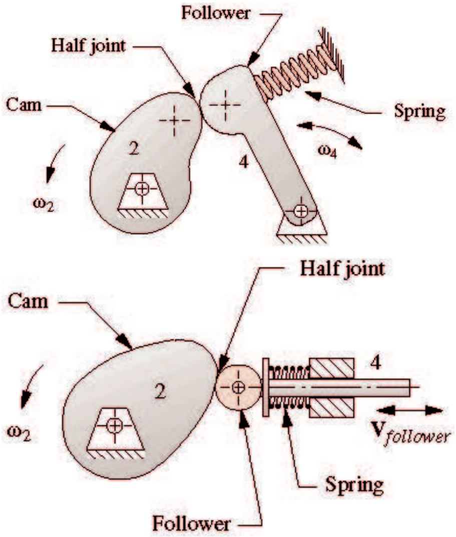


Cam Terminology

- Type of follower motion (rotation, translation)
- Type of joint closure (force, form)
- Type of follower (roller, mushroom, flat)
- Direction of follower motion (radial, axial)
- Type of motion constraints (critical extreme position(CEP) and critical path motion (CPM))
- Type of motion program (rise-fall (RF), rise-fall-dwell (RFD), rise-dwell-fall-dwell (RDFD))

Type of Follower Motion

- Translating and oscillating followers
- Depends on motion required
- Large radius gives approximately straight line

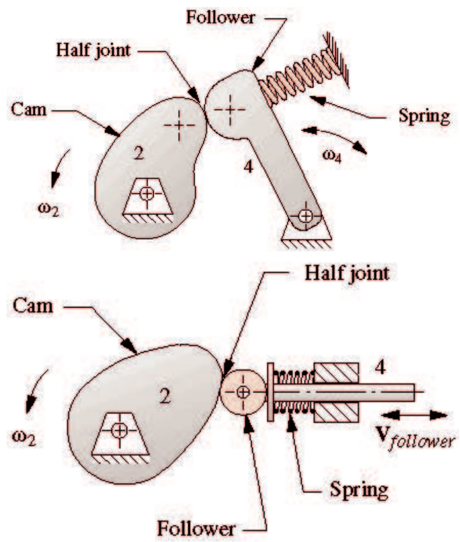


Type of Joint Closure

Force and Form Closed Cams

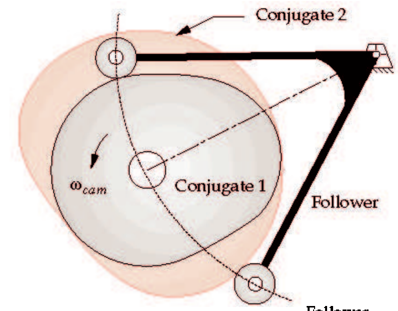
•Force closed cams require an external force to keep the cam in contact with the follower

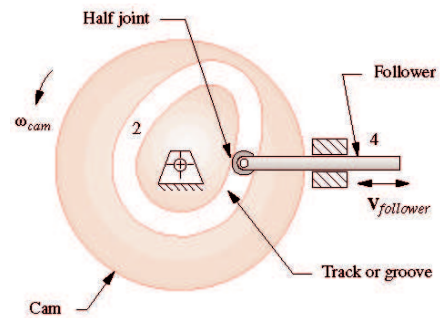
•A spring usually supplies this force

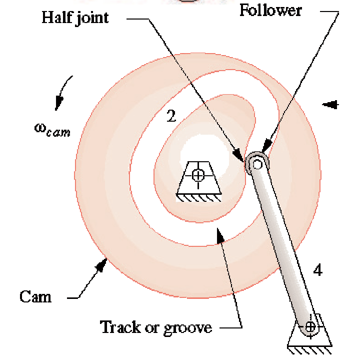


Type of Joint Closure

- Form closed cams are closed by joint geometry
- Slot milled out of the cam

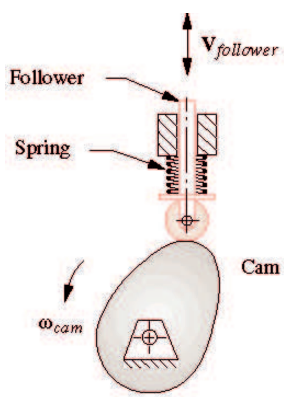




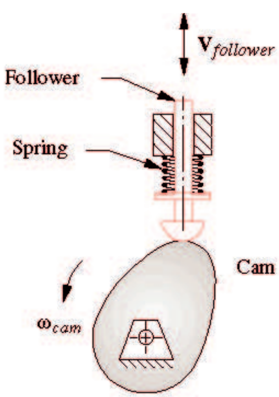


Type of Follower

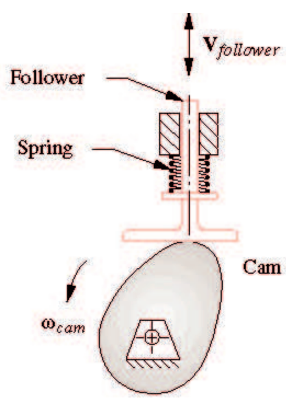
Roller Follower



Mushroom Follower



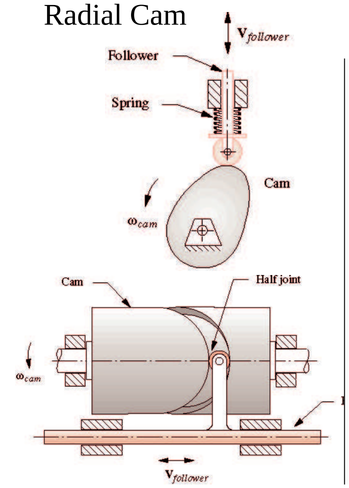
Flat-Faced Follower



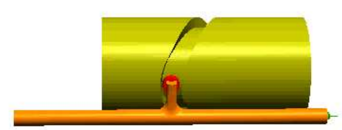
Direction of Follower Motion

- Radial or Axial

Radial Cam

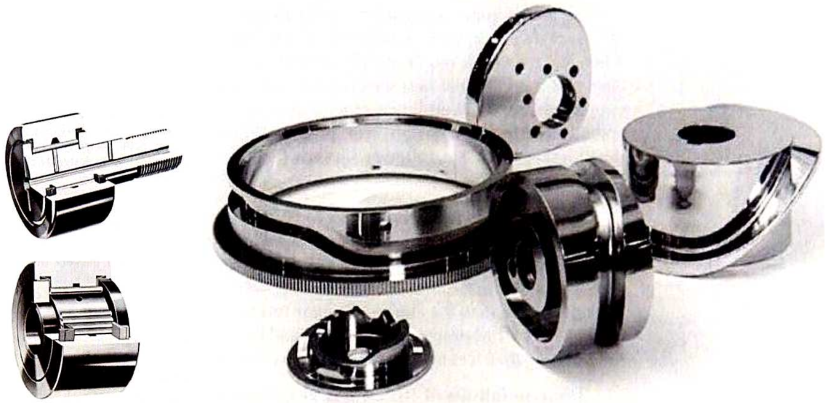


Axial Cam



Cam Terminology (review)

- Type of follower motion (rotation, translation)
- Type of joint closure (force, form)
- Type of follower (roller, mushroom, flat)
- Direction of follower motion (radial, axial)



Type of Motion Constraints

- Critical Extreme Position (CEP) – start and end positions are specified but not the path between
- Critical Path Motion (CPM) – path or derivative is defined over all or part of the cam

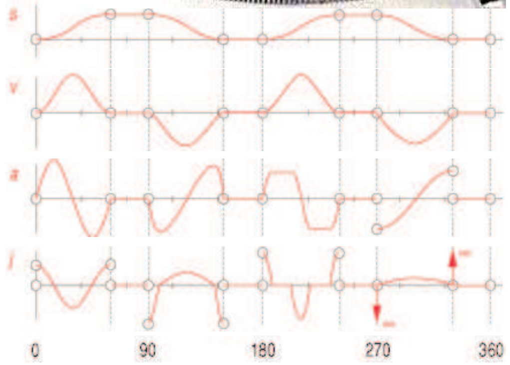
Type of Motion Program

- From the CEP cam profile
- Dwell – period with no output motion with input motion.
- Rise-Fall (RF) – no dwell (think about using a crank-rocker)
- Rise-Fall-Dwell (RFD) – one dwell
- Rise-Dwell-Fall-Dwell (RDFD) – two dwells

- Coke bottling example:

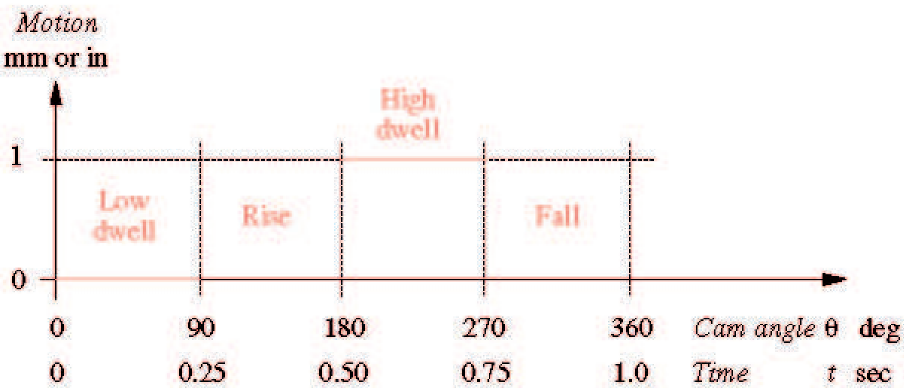
SVAJ Diagrams

- Unwrapping the cam
- Plot of position(s), velocity(v), acceleration(a) and jerk(j) verses cam angle
- Basis for cam design



RDFD Cam Design

- Motion is between two dwells

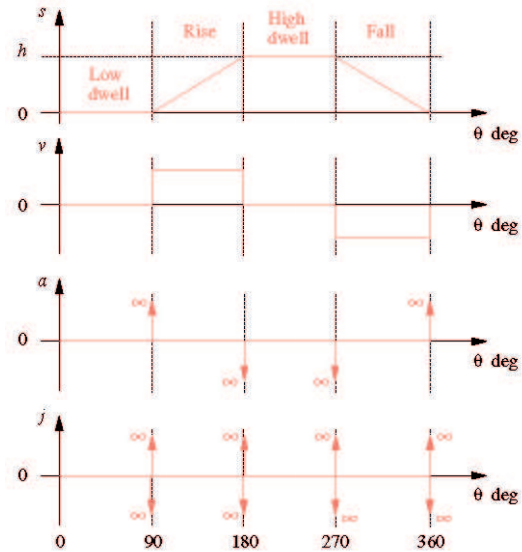


RDFD Cam, Naïve Cam Design

- Connect points using straight line
- Constant velocity

☹Infinite acceleration and jerk

☹Not an acceptable cam program



Fundamental Law of Cam Design

Any cam designed for operation at other than very low speeds must be designed with the following constraints

- The cam function must be continuous through the first and second derivatives of displacement across the entire interval (360 degrees).

Corollary:

- The jerk must be finite across the entire interval (360 degrees).

RDFD Cam Sophomore Design
Simple Harmonic Motion

- Sin function has continuous derivatives

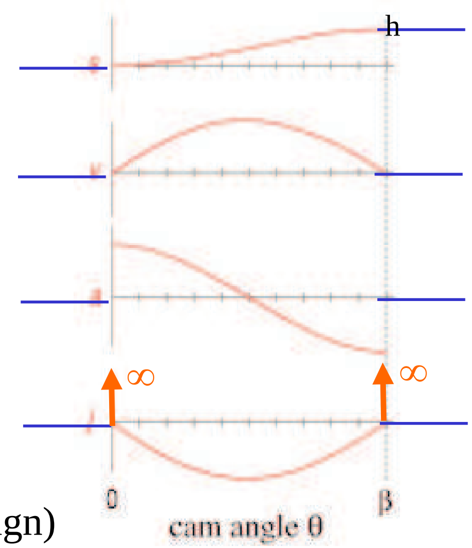
$$s = \frac{h}{2}*\left(1-\cos\left(\frac{\pi\theta}{\beta}\right)\right)$$

$$v = \frac{ds}{d\theta} = \frac{h\pi}{2\beta}\sin\left(\frac{\pi\theta}{\beta}\right)$$

$$a = \frac{dv}{d\theta} = \frac{h\pi^2}{2\beta^2}\cos\left(\frac{\pi\theta}{\beta}\right)$$

$$j = \frac{da}{d\theta} = \frac{-h\pi^3}{2\beta^3}\sin\left(\frac{\pi\theta}{\beta}\right)$$

☹Acceleration is discontinuous, jerk is infinite (Bad Cam Design)



RDFD Cam, Cycloidal

- Need to start with acceleration and integrate

$$a = C\sin\left(\frac{2\pi\theta}{\beta}\right)$$

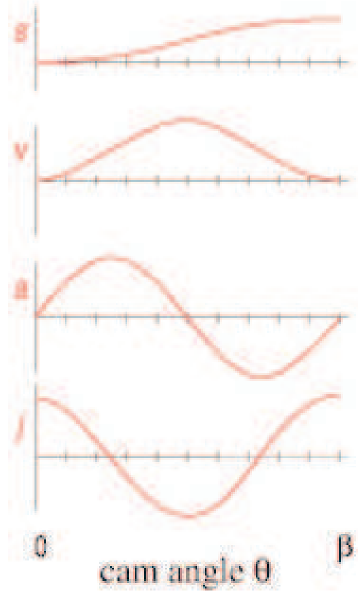
$$v = \frac{-C\beta}{2\pi}\cos\left(\frac{2\pi\theta}{\beta}\right) + k_1$$

since v=0 at $\theta=\beta$,

$$C_1 = \frac{C\beta}{2\pi}$$

$$v = \frac{C\beta}{2\pi}\left(1 - \cos\left(\frac{2\pi\theta}{\beta}\right)\right)$$

$$s = \frac{C\beta}{2\pi}\theta - C\left(\frac{\beta}{2\pi}\right)^2 \sin\left(\frac{2\pi\theta}{\beta}\right) + k_2$$



RDFD Cam, Cycloidal

$$s = \frac{C\beta}{2\pi}\theta - C\left(\frac{\beta}{2\pi}\right)^2 \sin\left(\frac{2\pi\theta}{\beta}\right) + k_2$$

- Since s=0 at θ=0, k_2=0
- Since s=h at θ=β,

$$h = \left(\frac{C\beta}{2\pi}\right)\beta \Rightarrow C = \frac{2\pi h}{\beta^2}$$

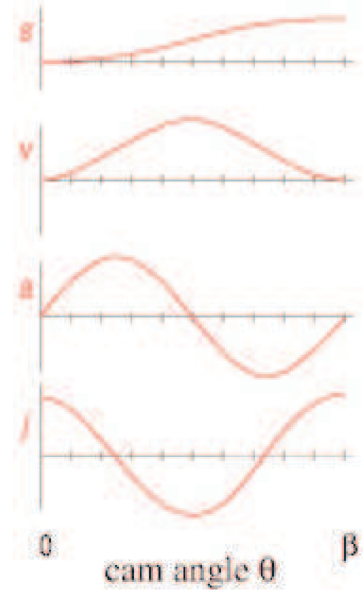
so

$$s = \frac{h}{\beta}\theta - \frac{h}{2\pi}\sin\left(\frac{2\pi\theta}{\beta}\right)$$

$$v = \frac{h}{\beta}\left(1 - \cos\left(\frac{2\pi\theta}{\beta}\right)\right)$$

$$a = \frac{2\pi h}{\beta^2}\sin\left(\frac{2\pi\theta}{\beta}\right)$$

$$j = \frac{h(2\pi)^2}{\beta^3}\cos\left(\frac{2\pi\theta}{\beta}\right)$$

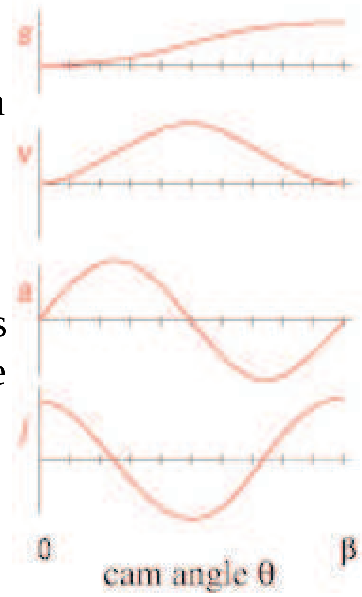


RDFD Cam, Cycloidal

☺Valid cam design (follows fundamental law of cam design

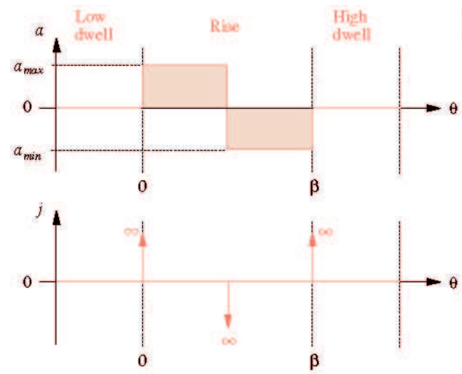
☹Acceleration and velocity are higher than other functions

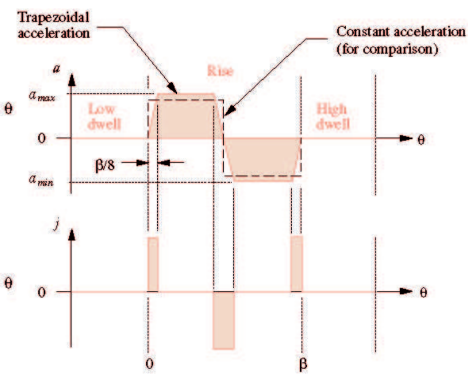
- General procedure for design is to start with a continuous curve for acceleration and integrate.



RDFD Cam, Trapezoidal

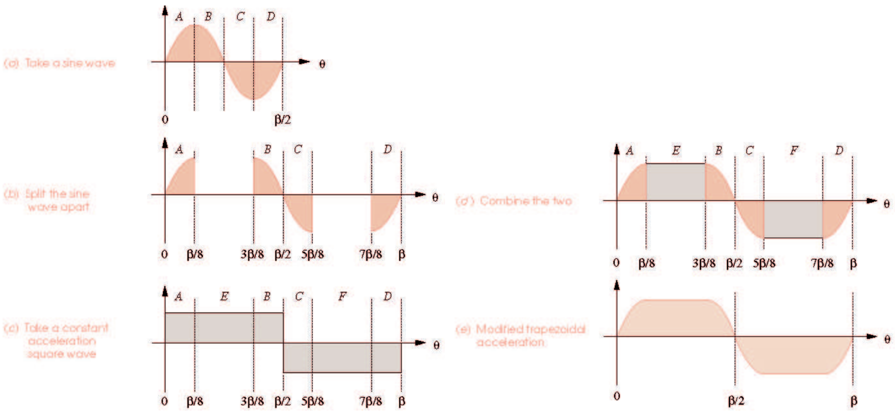
- Constant acceleration gives infinite jerk
- Trapezoidal acceleration gives finite jerk, but the acceleration is higher





RDFD Cam, Modified Trapezoidal

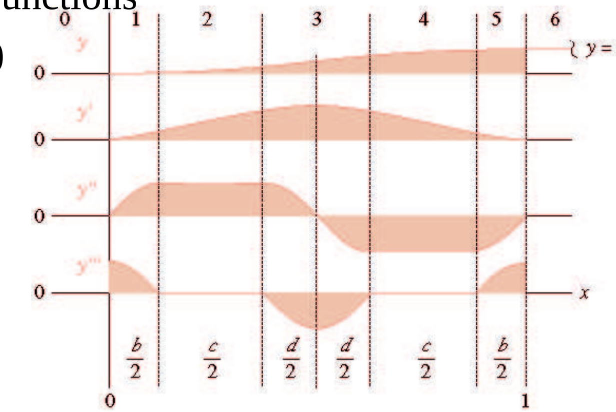
- Combination of sinusoidal and constant acceleration
- Need to integrate to get the magnitude



RDFD Cam, Modified Trapezoidal

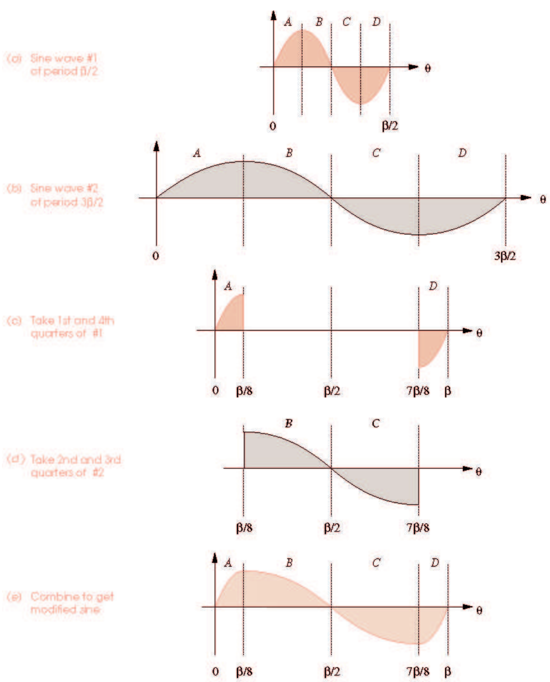
- After integrating, we get the following curves
- Has lowest magnitude of peak acceleration of standard cam functions

  (lowest forces)



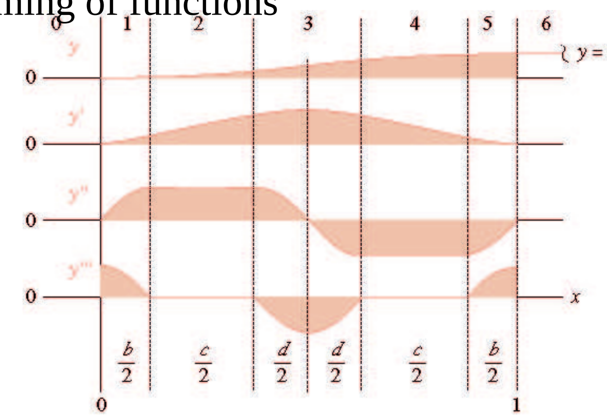
RDFD Cam, Modified Sin

- Combination of a low and high frequency sin function
- Has lowest peak velocity (lowest kinetic energy)



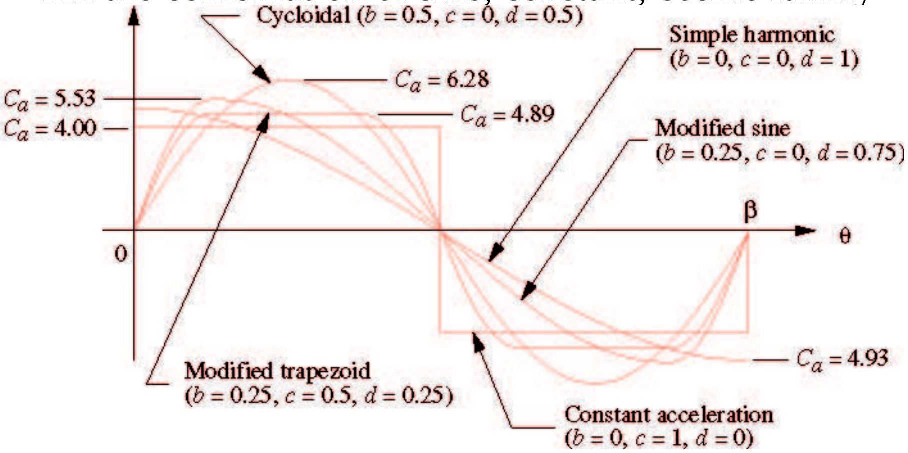
RDFD Cam, SCCA Family

- The cam functions discussed so far belong to the SCCA (family Sine-Constant-Cosine-Acceleration)
- Only change timing of functions



RDFD Cam, SCCA Family

- Comparison of accelerations in SCCA family
- All are combination of sine, constant, cosine family



Polynomial Functions

- Can also choose polynomials as the cam functions
- General form of

$$s = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + \cdots + C_n x^n$$

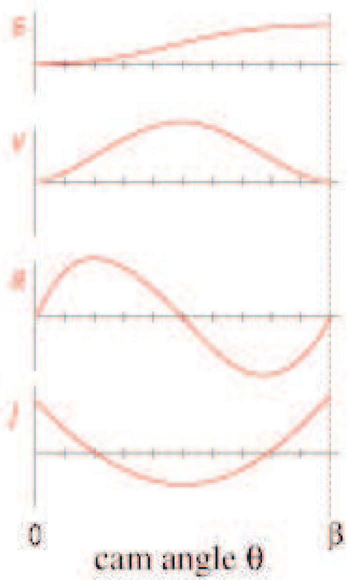
where x=θ/β or t

- Choose the number of boundary conditions (BC's) to satisfy the fundamental law of cam design

3-4-5 Polynomial

- Boundary conditions
  - @θ=0, s=0,v=0,a=0
  - @θ=β, s=h,v=0,a=0
- Six boundry conditions, so order 5 since C_0 term

$$s = C_0 + C_1\left(\frac{\theta}{\beta}\right) + C_2\left(\frac{\theta}{\beta}\right)^2 + C_3\left(\frac{\theta}{\beta}\right)^3 + C_4\left(\frac{\theta}{\beta}\right)^4 + C_5\left(\frac{\theta}{\beta}\right)^5$$

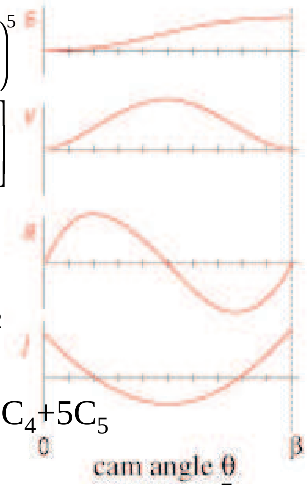


3-4-5 Polynomial

$$s = C_0 + C_1\left(\frac{\theta}{\beta}\right) + C_2\left(\frac{\theta}{\beta}\right)^2 + C_3\left(\frac{\theta}{\beta}\right)^3 + C_4\left(\frac{\theta}{\beta}\right)^4 + C_5\left(\frac{\theta}{\beta}\right)^5$$

$$v = \frac{1}{\beta}\left[C_1 + 2C_2\left(\frac{\theta}{\beta}\right) + 3C_3\left(\frac{\theta}{\beta}\right)^2 + 4C_4\left(\frac{\theta}{\beta}\right)^3 + 5C_5\left(\frac{\theta}{\beta}\right)^4\right]$$

$$a = \frac{1}{\beta^2}\left[2C_2 + 6C_3\left(\frac{\theta}{\beta}\right) + 12C_4\left(\frac{\theta}{\beta}\right)^2 + 20C_5\left(\frac{\theta}{\beta}\right)^3\right]$$



@θ=0, $s=0=C_0$ $v=0=C1/\beta$ $a=0=2C_2/\beta^2$

$C_0=0$ $C_1=0$ $C_2=0$

@θ=β, $s=h= C_3+C_4+C_5$, $v=0=2C_3+3C_4+5C_5$

$a=0= 6C_3+12C_4+20C_5$

Solve the 3 equations to get

$$s = h\left[10\left(\frac{\theta}{\beta}\right)^3 - 15\left(\frac{\theta}{\beta}\right)^4 + 6\left(\frac{\theta}{\beta}\right)^5\right]$$

3-4-5 and 4-5-6-7 Polynomial

- 3-4-5 polynomial
  - Similar in shape to cycloidal
  - Discontinuous jerk

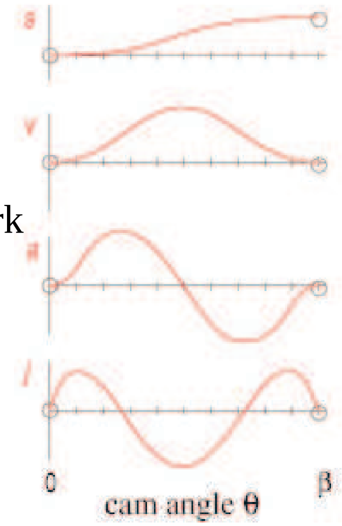
$$s = h\left[10\left(\frac{\theta}{\beta}\right)^3 - 15\left(\frac{\theta}{\beta}\right)^4 + 6\left(\frac{\theta}{\beta}\right)^5\right]$$

- 4-5-6-7 polynomial – set the jerk to be zero at 0 and β

$$s = h\left[35\left(\frac{\theta}{\beta}\right)^4 - 84\left(\frac{\theta}{\beta}\right)^5 + 70\left(\frac{\theta}{\beta}\right)^6 - 20\left(\frac{\theta}{\beta}\right)^7\right]$$

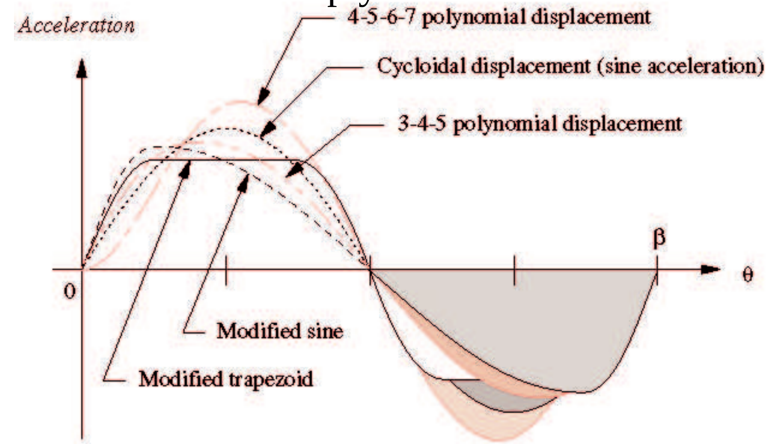
- Has continuous jerk, but everything else is larger

4-5-6-7 Polynomial



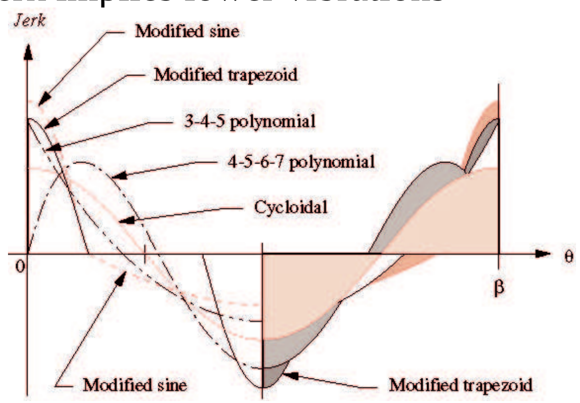
Acceleration Comparisons

- Modified trapezoid is the best, followed by modified sin and 3-4-5
- Low accelerations imply low forces



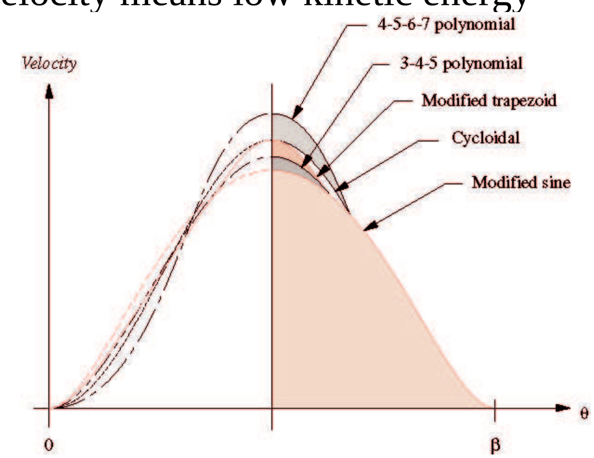
Jerk Comparison

- Cycloidal is lowest, followed by 4-5-6-7 polynomial and 3-4-5 polynomial
- Low jerk implies lower vibrations



Velocity Comparison

- Modified sine is best, followed by 3-4-5 polynomial
- Low velocity means low kinetic energy



Position Comparison

- There is not much difference in the position curves
- Small position changes can lead to large acceleration changes

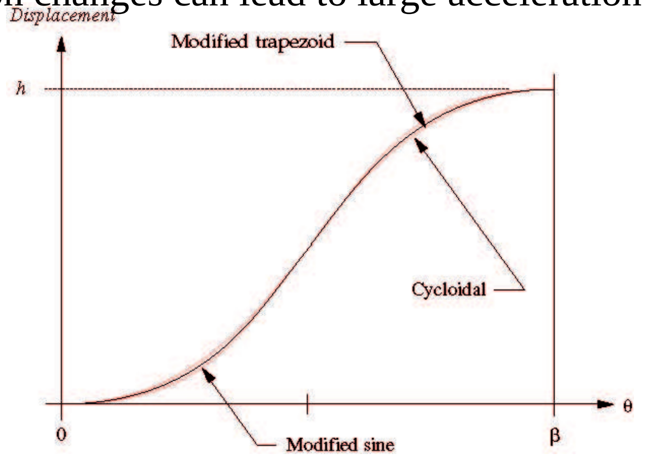


Table for Peak VAJ for Cam Functions

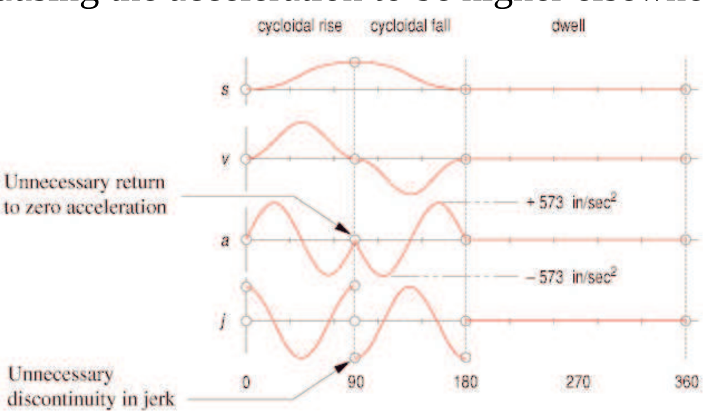
- Velocity is in m/rad, Acceleration is in m/rad^2, Jerk is in m/rad^3.

TABLE 8-3 Factors for Peak Velocity and Acceleration of Some Cam Functions

Function	Max. Veloc.	Max. Accel.	Max. Jerk	Comments
Constant accel.	$2.000\ h/\beta$	$4.000\ h/\beta^2$	Infinite	∞ jerk—not acceptable
Harmonic disp.	$1.571\ h/\beta$	$4.945\ h/\beta^2$	Infinite	∞ jerk—not acceptable
Trapezoid accel.	$2.000\ h/\beta$	$5.300\ h/\beta^2$	$44\ h/\beta^3$	Not as good as mod. trap.
Mod. trap. accel.	$2.000\ h/\beta$	$4.888\ h/\beta^2$	$61\ h/\beta^3$	Low accel. but rough jerk
Mod. sine accel.	$1.760\ h/\beta$	$5.528\ h/\beta^2$	$69\ h/\beta^3$	Low veloc., good accel
3-4-5 poly. disp.	$1.875\ h/\beta$	$5.777\ h/\beta^2$	$60\ h/\beta^3$	Good compromise
Cycloidal disp.	$2.000\ h/\beta$	$6.283\ h/\beta^2$	$40\ h/\beta^3$	Smooth accel. and jerk.
4-5-6-7 poly. disp.	$2.188\ h/\beta$	$7.526\ h/\beta^2$	$52\ h/\beta^3$	Smooth jerk, high accel.

Single Dwell Cam Design, Using Double Dwell Functions

- The double dwell cam functions have an unnecessary return to zero in the acceleration, causing the acceleration to be higher elsewhere.

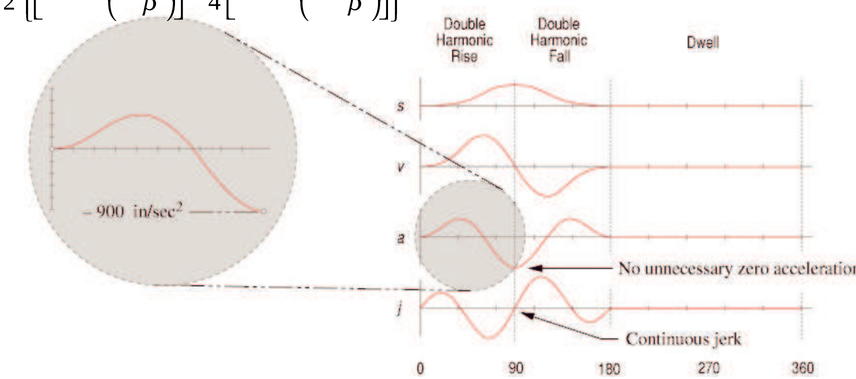


Single Dwell Cam Design, Double Harmonic function

- Large negative acceleration

$$s=\frac{h}{2}\left\{\left[1-\cos\left(\pi\frac{\theta}{\beta}\right)\right]-\frac{1}{4}\left[1-\cos\left(2\pi\frac{\theta}{\beta}\right)\right]\right\}\quad\text{for rise}$$

$$s=\frac{h}{2}\left\{\left[1+\cos\left(\pi\frac{\theta}{\beta}\right)\right]-\frac{1}{4}\left[1-\cos\left(2\pi\frac{\theta}{\beta}\right)\right]\right\}\quad\text{for fall}$$

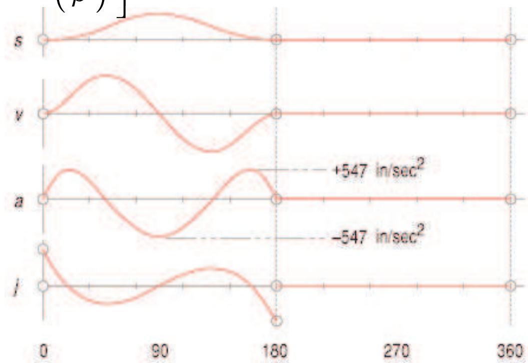


Single Dwell Cam Design, 3-4-5-6 Polynomial

- Boundary conditions @θ=0 s=v=a=0
  @θ=β s=v=a=0 @θ=β/2 s=h

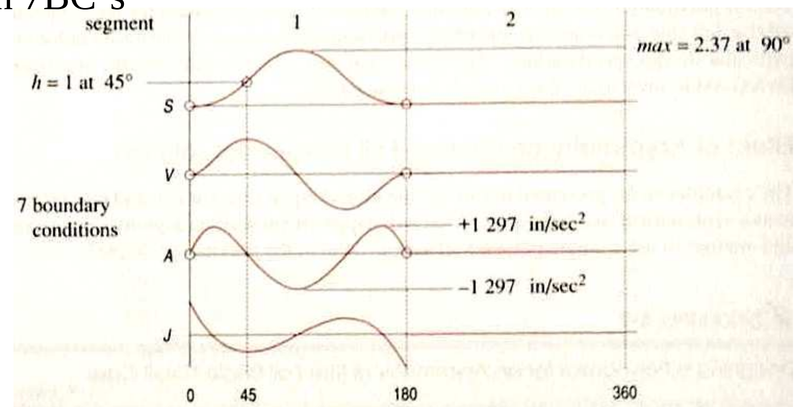
$$s = h\left[64\left(\frac{\theta}{\beta}\right)^3 - 192\left(\frac{\theta}{\beta}\right)^4 + 192\left(\frac{\theta}{\beta}\right)^5 - 64\left(\frac{\theta}{\beta}\right)^6\right]$$

- *Has lower peak acceleration (547) than cycloidal (573) or double harmonic (900)*



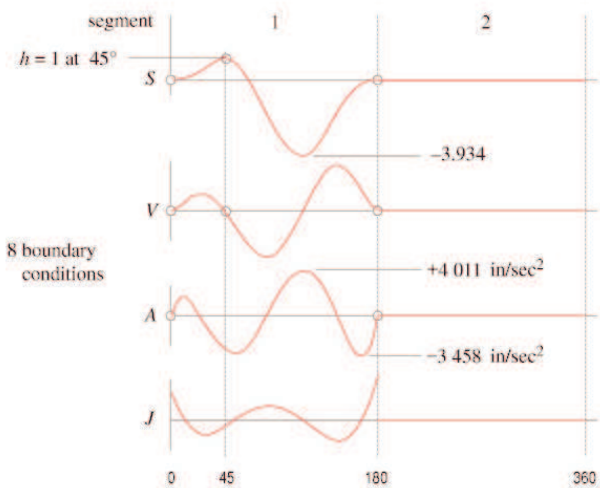
Unsymmetrical RFD Cams

- If the rise has different time than the fall, need more boundary conditions.
- With 7BC's



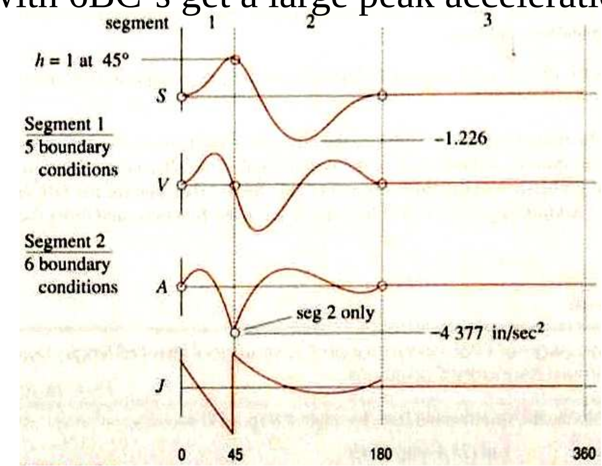
Unsymmetrical RFD Cams

- If you set the velocity to zero at the peak:



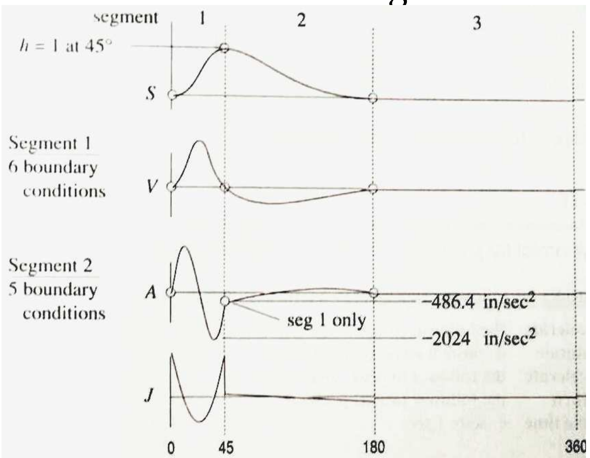
Unsymmetrical RFD Cams

- With 3 segments, segment 1 with 5BC's, segment 2 with 6BC's get a large peak acceleration



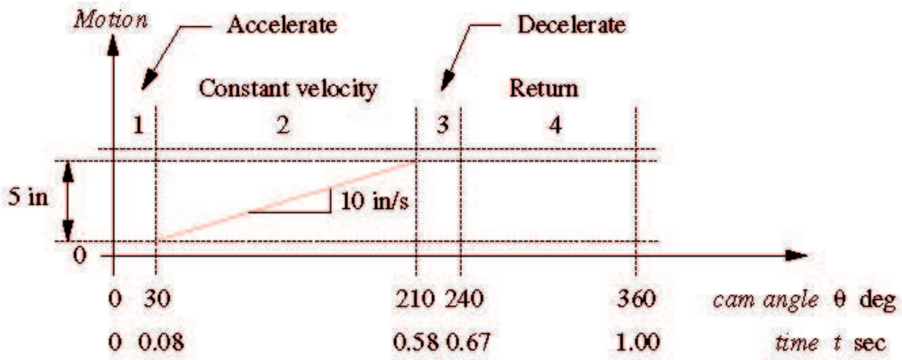
Unsymmetrical RFD Cams

- Best to start with segment with lowest acceleration with 5BC's then do the other segment with 6BC's



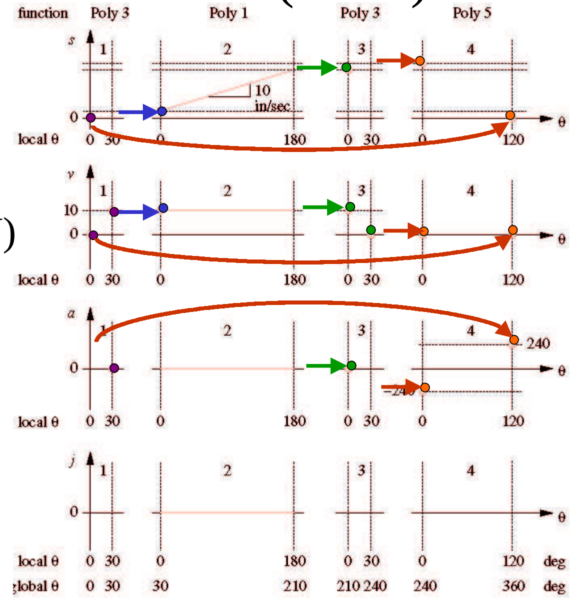
Critical Path Motion (CPM)

- Position or one of its derivatives is specified
- eg. Constant velocity for half the rotation
- Break the motion into the following parts:

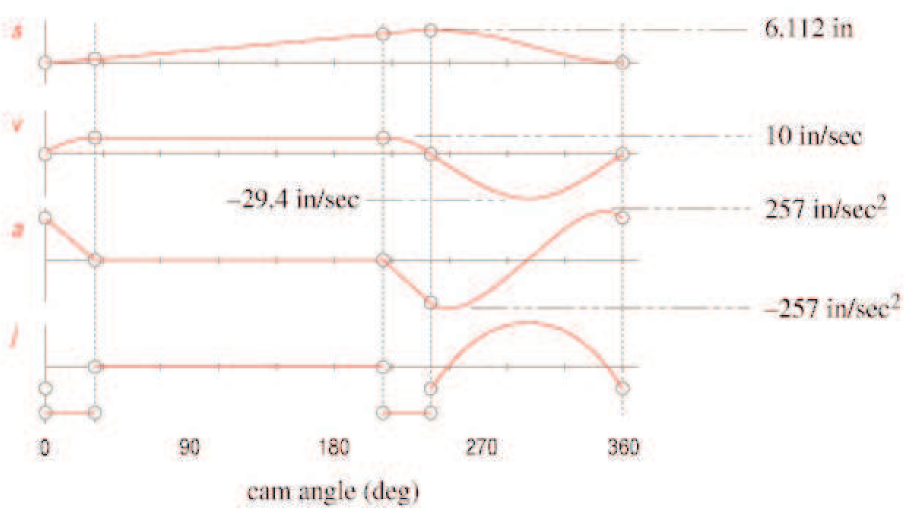


Critical Path Motion (CPM)

- Segment 1 has 4BC's
- Segment 2 has 2BC's (constant V)
- Segment 3 has 4BC's
- Last segment has 6BC's (almost always)

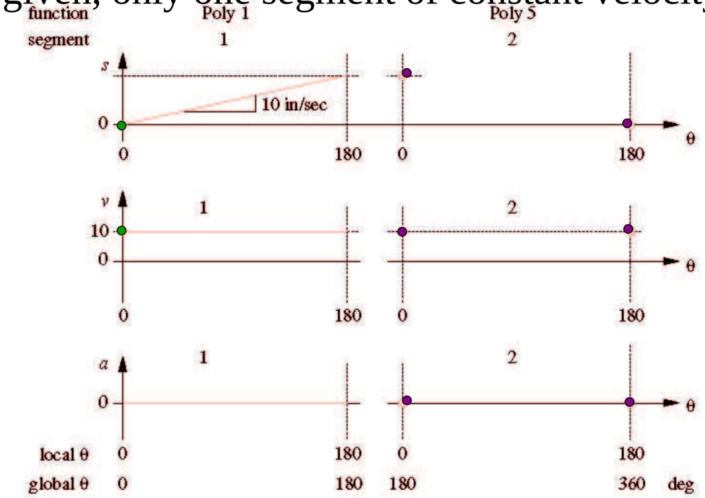


Resulting Curves



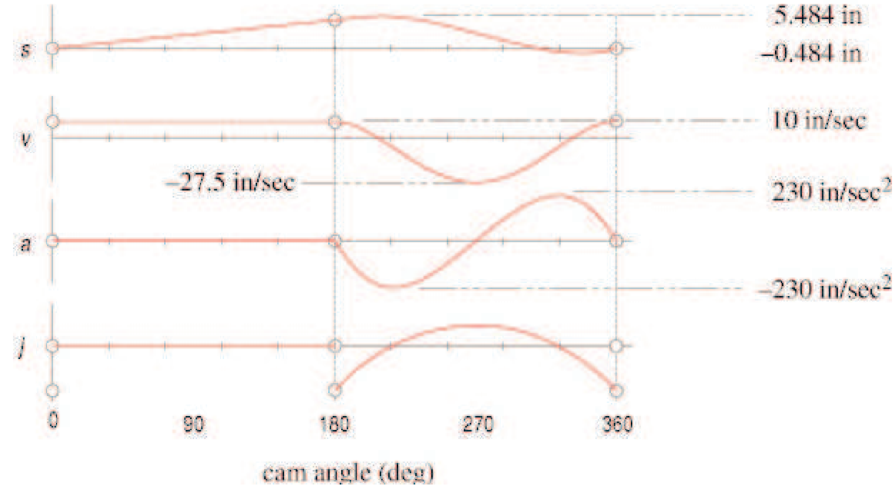
Constant Velocity, 2 Segments

- The divisions on the previous approach are not given, only one segment of constant velocity



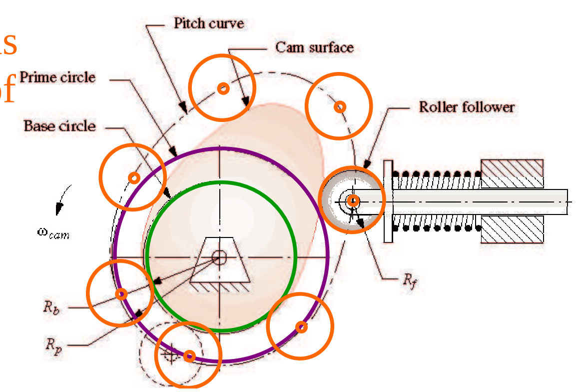
Resulting SVAJ diagram

- 2 segment design has better properties
- 4 segment design had Δs=6.112, v=-29.4, a=257



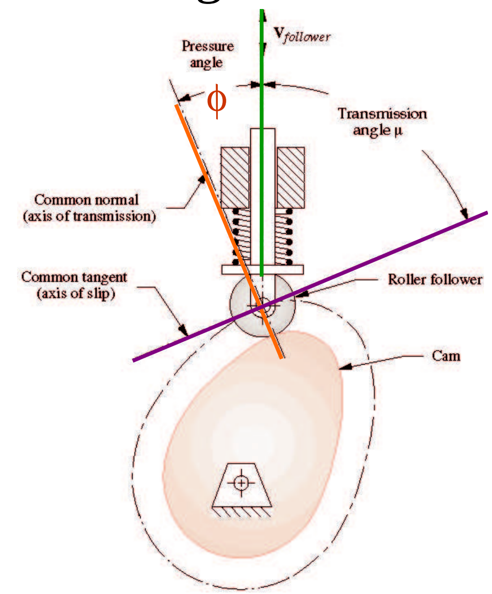
Sizing the Cam, Terminology

- Base circle (R_b) – smallest circle that can be drawn tangent to the physical cam surface
- Prime circle (R_p) – smallest circle that can be drawn tangent to the locus of the centerline of the follower
- Pitch curve – locus of the centerline of the follower



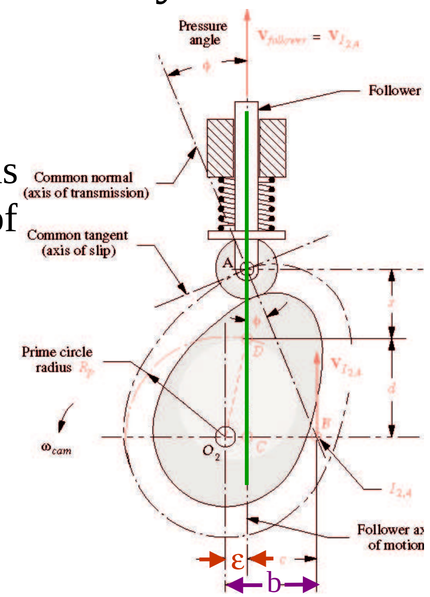
Cam Pressure Angle

- Pressure Angle (ϕ)
  - the angle between the direction of motion (velocity) of the follower and the direction of the axis of transmission
- Want ϕ<30 for translating and ϕ<35 for oscillating followers



Cam Eccentricity

- Eccentricty (ε) – the perpendicular distance between the follower's axis of motion and the center of the cam
- Aligned follower – ε=0
- For velocity, b=v

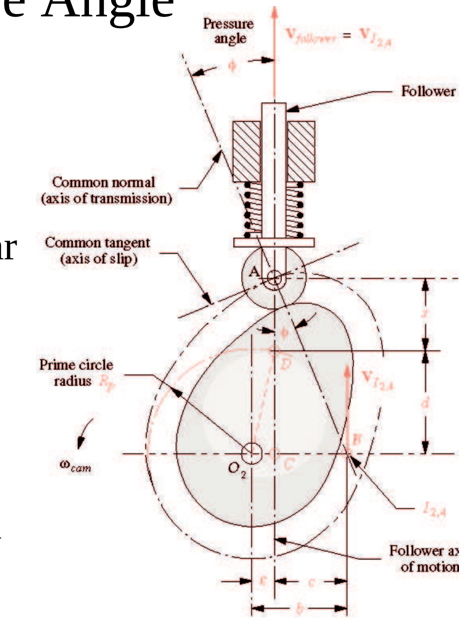


Pressure Angle

- The pressure angle is

$$\phi = \tan^{-1}\frac{v-\varepsilon}{s+\sqrt{R_p^{\,2}-\varepsilon^2}}$$

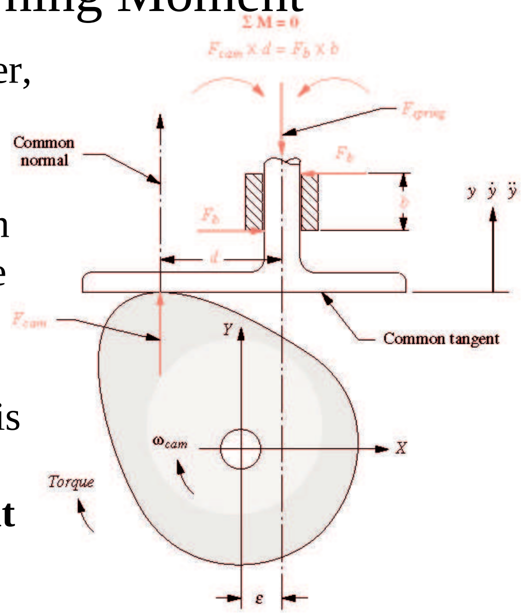
- Pressure angle has similar shape as velocity.
- If you increase R_p, ϕ decreases
- With ε, magnitude of ϕ decreases with positive velocities, increases with negative velocities



Overturning Moment

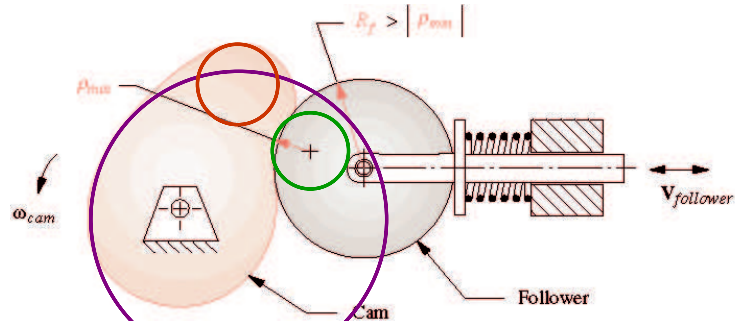
☺For flat faced follower, the pressure angle is zero

☹There is a moment on the follower since the force is not aligned with the direction of follower motion. This is called the **overturning moment**



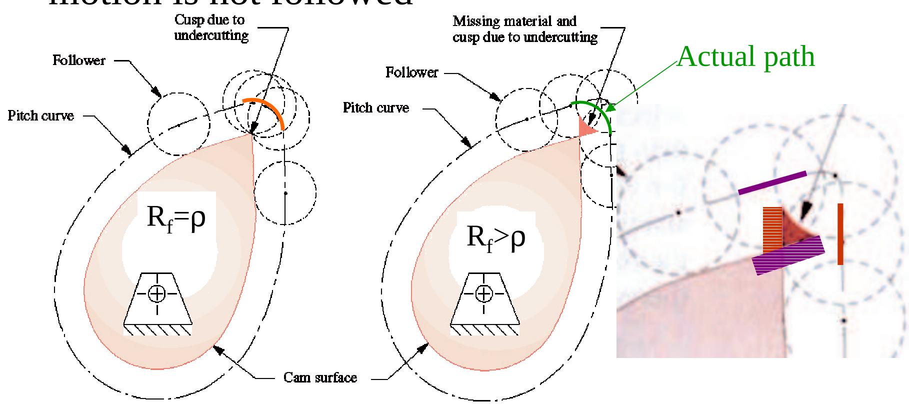
Radius of Curvature

- Every point on the cam has an associated radius of curvature
- If the radius of curvature is smaller than the radius of the follower the follower doesn't move properly



Small Positive Radius of Curvature

- If $R_f=\rho$ then the follower stays on one point for a long time leading to high wear
- If $R_f>\rho$ then there is undercutting and the desired motion is not followed



Sizing Cam for Radius of Curvature

- Want radius of curvature to be 2 to 3 times larger than radius of the follower (R_f)

$$\rho_{pitch}=\frac{\left[(R_p+s)^2+v^2\right]^{3/2}}{(R_p+s)^2+2v^2-a(R_p+s)}$$

$\varepsilon \approx 0$ (book equation 8.33)

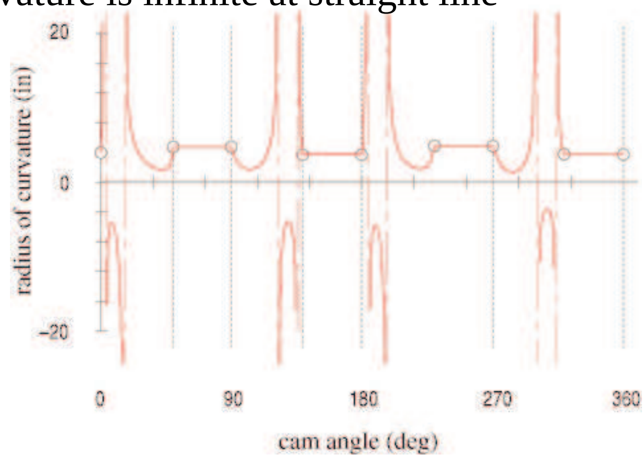
$$\rho_{pitch}=\frac{\left[(d+s)^2+(v-\varepsilon)^2\right]^{3/2}}{(d+s)^2+(2v-\varepsilon)(v-\varepsilon)-a(d+s)}$$

$$d=\sqrt{R_p^{\,2}-\varepsilon^2}$$

- Increasing R_p increases ρ_{pitch}

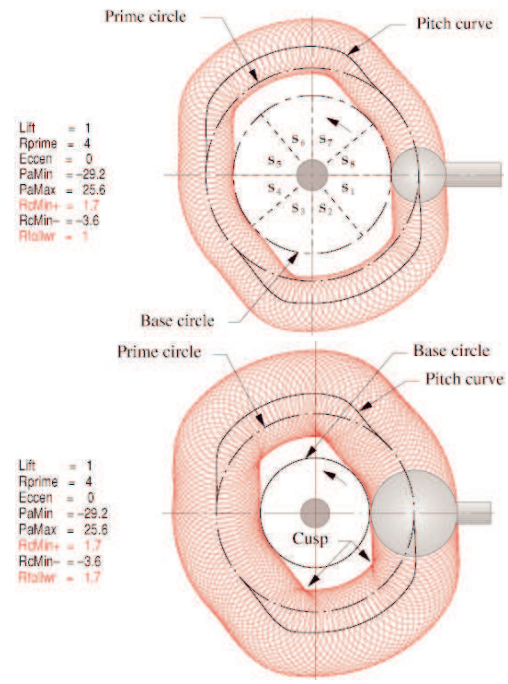
Sample plot of Radius of Curvature

- Radius of curvature can be positive or negative
- Curvature is infinite at straight line



Different Follower Radius

- The top plot with $R_f=1$ is a valid cam design (though the radius of curvature ratio is low.
- The bottom plot has the same SVAJ diagram but is not valid as $R_f=1.7$, the same as ρ_{min}.

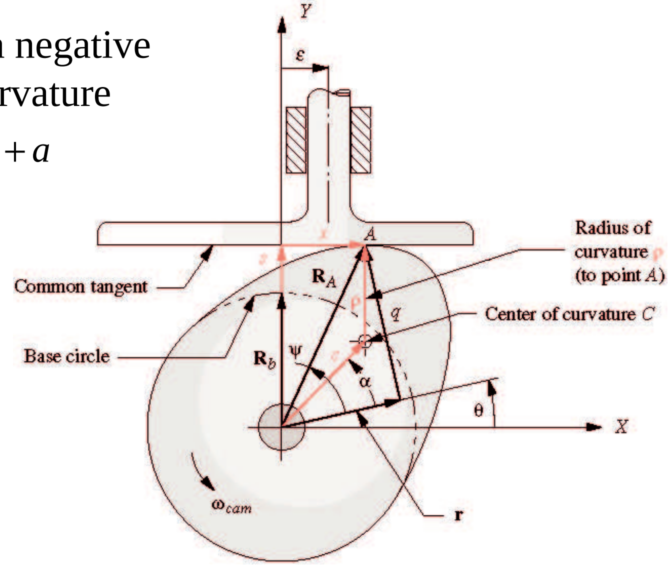


Radius of Curvature – Flat Faced Follower

- Can't have a negative radius of curvature

$\rho_{\min} = R_b + s + a$

x=v



Cam Geometry

- For roller follower, pitch curve

$$x = \cos\lambda\sqrt{(d+s)^2+\varepsilon^2}$$

$$y = \sin\lambda\sqrt{(d+s)^2+\varepsilon^2}$$

where

$$\lambda = (2\pi-\theta) - \tan^{-1}\left(\frac{\varepsilon}{d+s}\right)$$

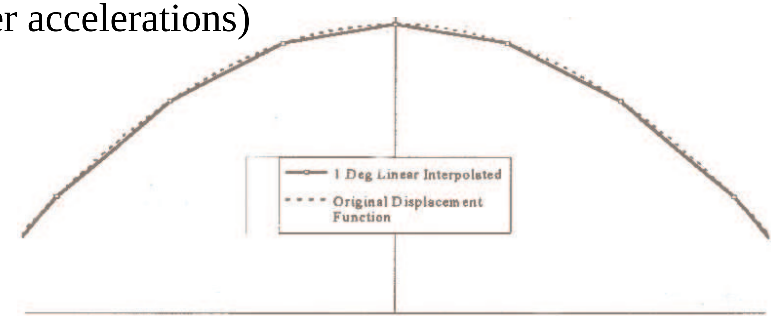
- For flat faced follower

$$x = (R_b+s)\sin\theta + v\cos\theta$$

$$y = (R_b+s)\cos\theta + v\sin\theta$$

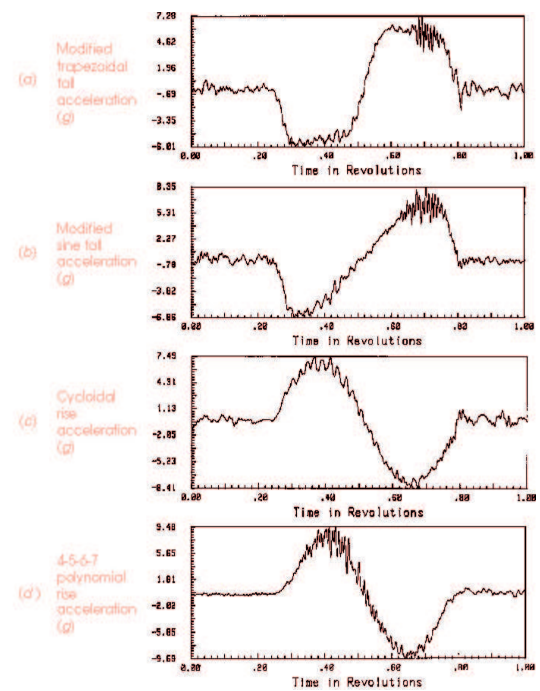
Cam Manufacturing Considerations

- Medium to high carbon steels, or cast ductile iron
- Milled or ground
- Heat treated for hardness (Rockwell HRC 50-55)
- CNC machines frequency use linear interpolation (larger accelerations)



Actual vs. Theoretical Cam Performance

- Larger acceleration due to manufacturing errors, and vibrations from jerk



Practical Design Consideration

- Translating or oscillating follower?
- Force or Form-Closed?
  - Follower Jump vs. Crossover Shock
- Radial or Axial Cam?
- Roller or Flat-Faced Follower?
- To Dwell or Not to Dwell?
- To Grind or not to Grind?
- To Lubricate or Not to Lubricate?