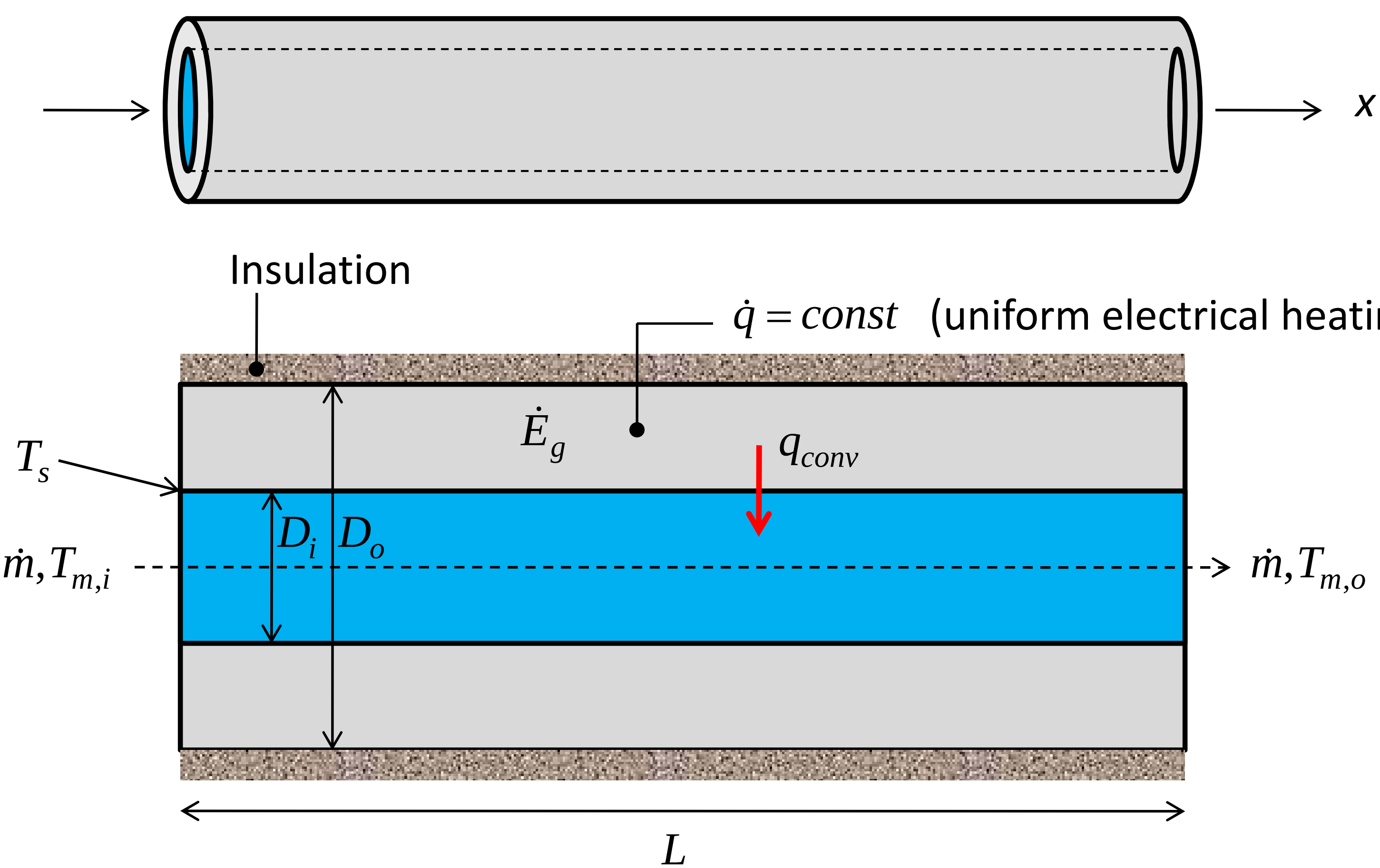
Example 8.2: “Introduction to Heat Transfer”, Incropera, Edition 5E



Known: Internal flow through thick-walled tube having uniform heat generation.

Find:

1. Length of tube needed to achieve the desired outlet temperature
2. Local convection coefficient at the outlet

Assumptions: Steady-state, uniform heat flux, incompressible fluid, adibatic outer tube surface

1. Since the outer surface of the tube is adiabatic, the rate at which energy is generated within the tube wall must equal the rate at which it is convected to the water.

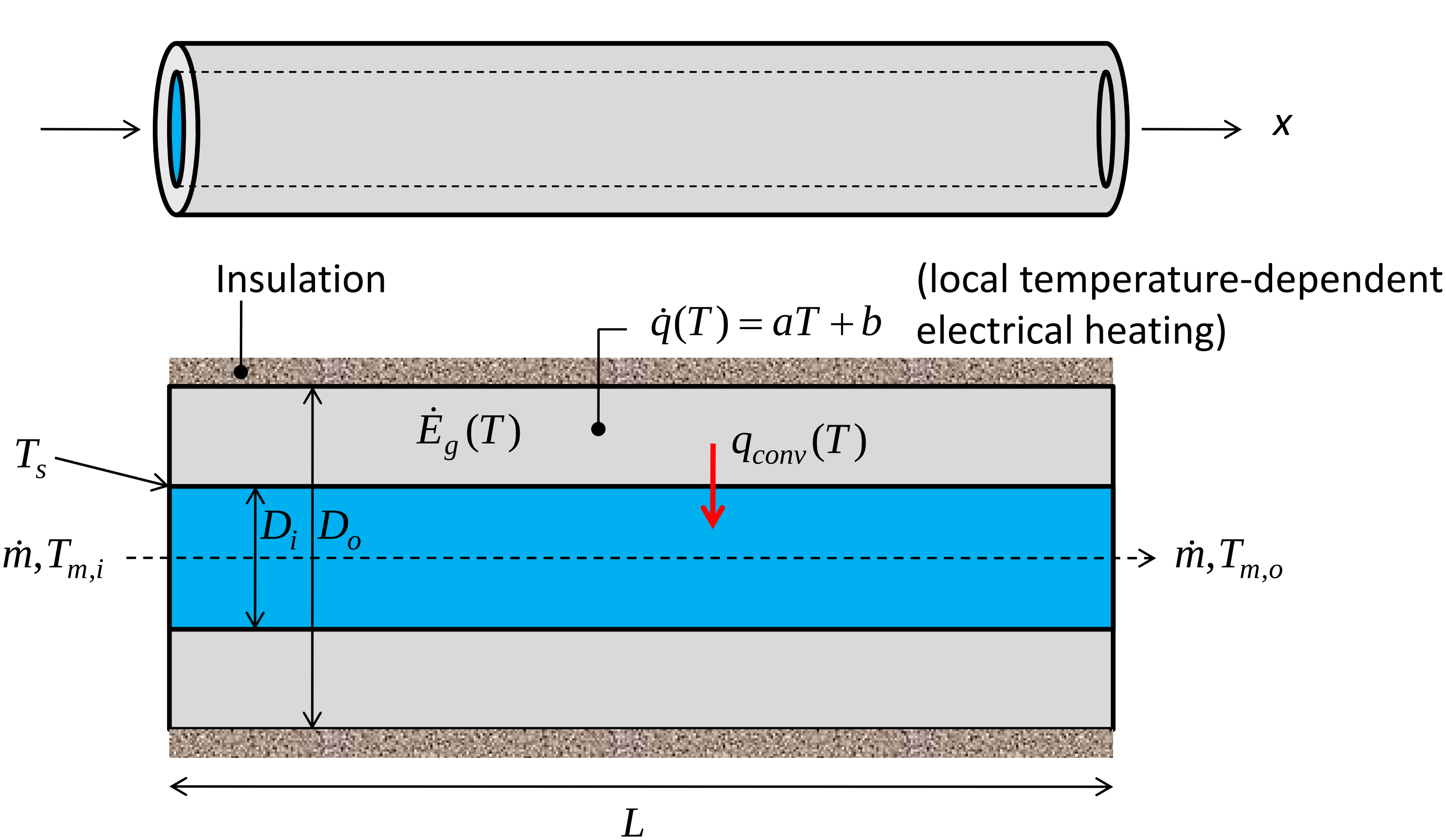
$$\dot{E}_g = q_{conv} \qquad \dot{E}_g = \dot{q}\frac{\pi}{r}(D_o^2 - D_i^2)L = \dot{m}c_p(T_{m,o} - T_{m,i})$$

$$L = \frac{4\dot{m}c_p}{\pi(D_o^2 - D_i^2)\dot{q}}(T_{m,o} - T_{m,i})$$

2. From Newton's law of cooling, the local convection coefficient at the tube exit is

$$h_o = \frac{q_s''}{T_{s,o} - T_{m,o}} \qquad \text{where} \qquad q_s'' = \frac{\dot{E}_g}{\pi D_i L} = \frac{\dot{q}}{4}\frac{D_o^2 - D_i^2}{D_i}$$

Modification of Example 8.2



Known: Internal flow through thick-walled tube having NON-uniform heat generation.

Find:

1. Temperature of pipe wall as a function of axial distance (*x direction*)
2. Find current density

Assumptions: Steady-state, uniform heat flux, incompressible fluid, adiabatic outer tube surface. It can also be assumed that though the temperature changes with x, it does not change with r.

The heat generated by electrical heating can be calculated from Ohm's power law:

$$P = \frac{V^2}{R} = V^2 G$$ where P is power in watts, V is voltage, R is resistance and G is conductance

It is common for R or G to be given in *Ohms/meter* and *Siemens/meter* respectively. Therefore the uniform heat generation rate is

$$\dot{q} = \frac{V^2 (D_o - D_i) g}{\frac{\pi}{4}(D_o^2 - D_i^2) L}$$

where lower-case g is conductance per meter. The top term is in watts, and the bottom term resolves to volume.

The original Example 8.2 treats the heat generation rate as a constant. However, a situation may arise where g is no longer independent, but is dependent on temperature (such as in a material with a high temperature coefficient). If g is linearly modeled, then the above equation becomes

$$\dot{q}(T) = \frac{V^2 (D_o - D_i)(aT + b)}{\frac{\pi}{4}(D_o^2 - D_i^2) dx}$$

q is now also temperature dependent. It is unclear to me where to go from here, since neither the convection heat flux nor the internal surface temperature of the pipe is constant.

$$dx\dot{q}(T) = \frac{V^2 (D_o - D_i)(aT + b)}{\frac{\pi}{4}(D_o^2 - D_i^2)}$$

It also seems to make sense to me that L must become an incremental part of x because temperature also changes with x.