1 Background

The aim is to calculate the pressure drop over a tube, delivering air to and recovering air from a pneumatic acutator. Therefor, the flow over the valve shall be calculated and used to evaluate the Darcy equation for turbulent flow.

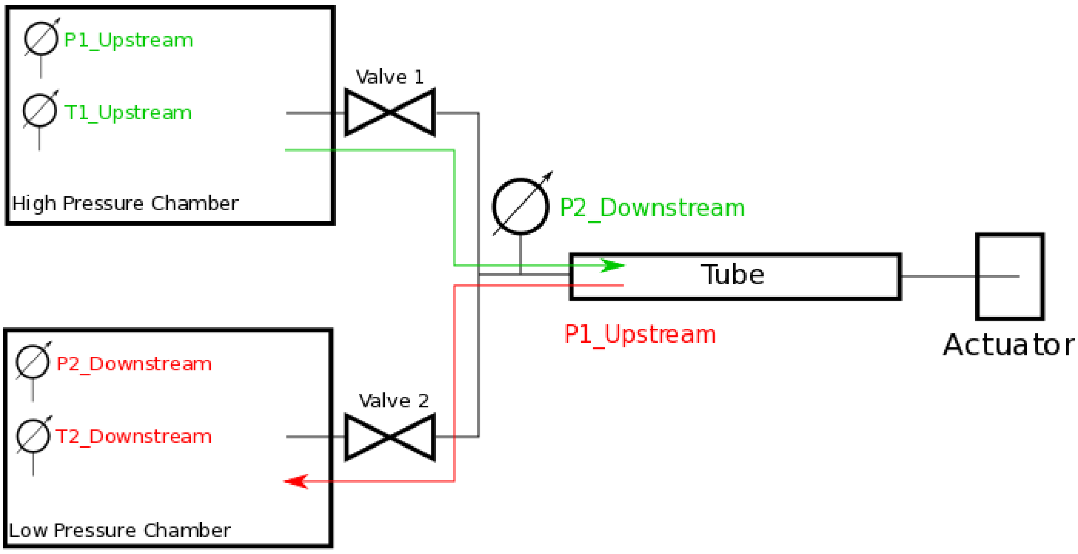

Abbildung 1: Phase 1: Air flows out of the High Pressure Chamber over Vavle 1 into the Tube. Phase 2: Air flows through Valve 2 into Low Pressure Chamber.

The problem is that it works well for the flow out of the High Pressure Chamber over Valve 1 (green arrow) but very bad for the inflow into the Low Pressure Chamber (red arrow).

1.1 Phase 1: Flow from High Pressure Chamber over Valve 1 into Tube

Starting with the energy equation for compressible fluids along a streamline (one dimensional) and assuming adiabatic behaviour and setting the velocity to zero in the chameber

$$\frac{\cancel{c_1^2}}{\cancel{2}} + h_1 = \frac{c_2^2}{2} + h_2 \tag{1}$$

Using the relation $h = c_p T$ gives

$$c_2^2 = 2c_p(T_1 - T_2) \tag{2}$$

Deviding by T_1 and assuming an polytropic change of state $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$ leads to

$$\frac{c_2^2}{T_1} = 2c_p\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right] \tag{3}$$

Using the relation $c_p = R\frac{n}{n-1}$ brings the well known term

$$c_2 = \sqrt{2\frac{n}{n-1}RT_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]} \tag{4}$$

Now, applying $\dot{m}_2 = \rho_2 c_2 A$ and with $\rho_2 = \rho_1\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$

$$\dot{m} = A_{corrected}\sqrt{2\frac{n}{n-1}\rho_1^2 RT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]} \tag{5}$$

1.2 Phase 2: Flow from Tube over Valve 2 into Low Pressure Chamber

Starting again with the energy equation for compressible fluids along a streamline (one dimensional) and assuming adiabatic behaviour and setting the velocity to zero in the chameber

$$\frac{c_1^2}{2} + h_1 = \cancel{\frac{c_2^2}{2}} + h_2 \tag{6}$$

Using the relation $h = c_p T$ gives

$$c_1^2 = 2c_p(T_2 - T_1) \tag{7}$$

Deviding this time by T_2 (because T_2 in Low Pressure Chamber is measured, T_1 is unknown) and assuming an polytropic change of state $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$ or $\frac{T_1}{T_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1-n}{n}}$ leads to

$$\frac{c_1^2}{T_2} = 2c_p\left[\left(1 - \frac{p_2}{p_1}\right)^{\frac{1-n}{n}}\right] \tag{8}$$

Point 2 of Phase 2 is at stagnation condition (inside Low Pressure Chamber), so the temperature of the fluid in the Low Pressure Chamber must be higher as in Point 1. But $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$ allows an increase in temperature only for $n < 1$. Is a change of state with a polytropic exponent <1 possible? Is there an error in reasoning?

Using the relation $c_p = R\frac{n}{n-1}$ gives

$$c_1 = \sqrt{2\frac{n}{n-1}RT_2\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{1-n}{n}}\right]} \tag{9}$$

Now, applying $\dot{m}_1 = \rho_1 c_1 A_1$ and $\rho_1 = \rho_2\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}$

$$\dot{m}_1 = A_{corrected}\sqrt{2\frac{n}{n-1}\rho_2^2 RT_2\left[\left(\frac{p_2}{p_1}\right)^{-\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{-\frac{n+1}{n}}\right]} \tag{10}$$

The problem is, that for typical pressure ratios between 0.65...1 the expression under the root is negative. And also an absolute value of it wouldn't give good results. What is the error?

2 Validation

The mass flow is then used as input for the Darcy equation to calculate the pressure drop over a tube of a certain length.

$$\Delta p = \lambda \frac{L}{D} \frac{1}{2\rho} \left(\frac{\dot{m}}{A} \right)^2 \tag{11}$$

For the validation, pressure was measured also at the end of the tube and was then compared to the calculated pressure drop.