

TABLE 8.2 Reaction and deflection formulas for in-plane loading of elastic frames (Continued)

General reaction and deformation expressions for cases 5–12, right end fixed in all eight cases:

Deformation equations:

Horizontal deflection at  $A = \delta_{HA} = C_{HH}H_A + C_{HV}V_A + C_{HM}M_A - LF_H$

Vertical deflection at  $A = \delta_{VA} = C_{VH}H_A + C_{VV}V_A + C_{VM}M_A - LF_V$

Angular rotation at  $A = \psi_A = C_{\psi H}H_A + C_{\psi V}V_A + C_{\psi M}M_A - LF_\psi$

where  $C_{HH} = \frac{l_3^3}{3E_1I_1} + \frac{l_3^3 - (l_1 - l_2)^3}{3E_2I_2} + \frac{l_1^2l_3}{E_3I_3}$

$$C_{HV} = C_{VH} = \frac{l_2l_3}{2E_2I_2}(2l_1 - l_2) + \frac{l_1^2l_3}{2E_3I_3}$$

$$C_{HM} = C_{MH} = \frac{l_2^2}{2E_2I_2} + \frac{l_2}{2E_2I_2}(2l_1 - l_2) + \frac{l_1l_3}{E_3I_3}$$

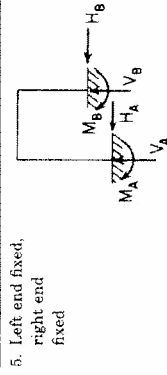
$$C_{VV} = \frac{l_2l_3^2}{E_2I_2} + \frac{l_3^3}{3E_3I_3}$$

$$C_{VM} = C_{MV} = \frac{l_2l_3}{E_2I_2} + \frac{l_3^2}{2E_3I_3}$$

$$C_{MM} = \frac{l_1}{E_1I_1} + \frac{l_2}{E_2I_2} + \frac{l_3}{E_3I_3}$$

and where  $LF_H$ ,  $LF_V$ , and  $LF_\psi$  are loading terms given below for several types of load

(Note: If desired,  $H_B$ ,  $V_B$ , and  $M_B$  are to be evaluated from equilibrium equations after calculating  $H_A$ ,  $V_A$ , and  $M_A$ )



5b. Left end fixed, right end fixed

Since  $\delta_{HA} = 0$ ,  $\delta_{VA} = 0$ , and  $\psi_A = 0$ , these three equations are solved simultaneously for  $H_A$ ,  $V_A$ , and  $M_A$ :

$$C_{HH}H_A + C_{HV}V_A + C_{HM}M_A = LF_H$$

$$C_{VH}H_A + C_{VV}V_A + C_{VM}M_A = LF_V$$

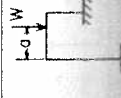
$$C_{\psi H}H_A + C_{\psi V}V_A + C_{\psi M}M_A = LF_\psi$$

The loading terms are given below.

Reference no., loading

Loading terms

5a. Concentrated load on the horizontal member



$$LF_H = W \left[ \frac{l_2}{2E_2I_2}(2l_1 - l_2)(l_3 - a) + \frac{l_1^2}{2E_3I_3}(l_3 - a)^2 \right]$$

$$LF_V = W \left[ C_{VV} - aC_{VM} + \frac{a^3}{6E_3I_3} \right]$$

$$LF_\psi = W \left[ \frac{l_2}{E_2I_2}(l_3 - a) + \frac{1}{2E_3I_3}(l_3 - a)^2 \right]$$

5b. Distributed load on the horizontal member

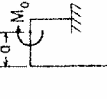


$$LF_H = w_0 \left[ \frac{l_2^2}{4E_2I_2}(2l_1 - l_2) + \frac{l_1^2l_3}{6E_3I_3} \right] + (w_0 - w_a) \left[ \frac{l_1^2l_3}{12E_2I_2}(2l_1 - l_2) + \frac{l_1^2l_3}{24E_3I_3} \right]$$

$$LF_V = w_0 \left( \frac{l_2^3}{2E_2I_2} + \frac{l_1^3}{8E_3I_3} \right) + (w_0 - w_a) \left( \frac{l_2^3}{6E_2I_2} + \frac{l_1^3}{30E_3I_3} \right)$$

$$LF_\psi = w_0 \left( \frac{l_2l_3}{2E_2I_2} + \frac{l_1^2}{6E_3I_3} \right) + (w_0 - w_a) \left( \frac{l_2l_3}{6E_2I_2} + \frac{l_1^2}{24E_3I_3} \right)$$

5c. Concentrated moment on the horizontal member

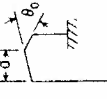


$$LF_H = M_0 \left[ \frac{-l_2}{2E_2I_2}(2l_1 - l_2) - \frac{l_1}{E_3I_3}(l_3 - a) \right]$$

$$LF_V = M_0 \left( -C_{VM} + \frac{a^2}{2E_3I_3} \right)$$

$$LF_\psi = M_0 \left[ \frac{-l_2}{E_2I_2} - \frac{1}{E_3I_3}(l_3 - a) \right]$$

5d. Concentrated angular displacement on the horizontal member

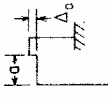


$$LF_H = \theta_0(l_1)$$

$$LF_V = \theta_0(a)$$

$$LF_\psi = \theta_0(1)$$

5e. Concentrated lateral displacement on the horizontal member

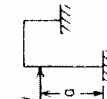


$$LF_H = 0$$

$$LF_V = \Delta_0(1)$$

$$LF_\psi = 0$$

5f. Concentrated load on the left vertical member

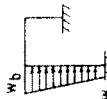


$$LF_H = W \left( C_{HH} - aC_{HM} + \frac{a^3}{6E_1I_1} \right)$$

$$LF_V = W(C_{VH} - aC_{VM})$$

$$LF_\psi = W \left( C_{\psi H} - aC_{\psi M} + \frac{a^2}{2E_1I_1} \right)$$

5g. Distributed load on the left vertical member



$$LF_H = w_0 \left( C_{HH}l_1 - C_{HM} \frac{l_1^2}{2} + \frac{l_1^3}{24E_1I_1} \right) + (w_0 - w_a) \left( C_{HH} \frac{l_1}{2} - C_{HM} \frac{l_1^2}{3} + \frac{l_1^3}{30E_1I_1} \right)$$

$$LF_V = w_0 \left( C_{VH}l_1 - C_{VM} \frac{l_1^2}{2} \right) + (w_0 - w_a) \left( C_{VH} \frac{l_1}{2} - C_{VM} \frac{l_1^2}{3} \right)$$

$$LF_\psi = w_0 \left( C_{\psi H}l_1 - C_{\psi M} \frac{l_1^2}{2} + \frac{l_1^3}{6E_1I_1} \right) + (w_0 - w_a) \left( C_{\psi H} \frac{l_1}{2} - C_{\psi M} \frac{l_1^2}{3} + \frac{l_1^3}{8E_1I_1} \right)$$