May 22, 2006
Splash Version 3.0 Quality Control Computer Results
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General Notes

1) The current Splash version number appears in the window handle, and for this test set should be Version 3.0.

2) Generally increasing the pressure iterations will improve the solution, although the user is encouraged to experiment with both small and large values of the allowed number of pressure cycles to observe the effect on a particular solution. The default number of allowed pressure cycles is 40. This number can practically be reduced to as few as (5), and to greater than (1000) or more. It is the program user's responsibility to assure that critical problem variables are not affected by solution convergence parameters or assumptions.

3) Depending on the size of the solution, once the Sola Vof algorithm has completed, several seconds of output processing time will be required before data files will be available for review.

4) The printed output report is only interactively available from the last run of the model. Changing the filename or rereading the input file for a particular model will not update the available interactive output results.

5) A more benign way to determine the sloshing frequency of a vessel is to provide a single fluid cell against the left or right side of the wall instead of a step change to the middle. Each method can excide the fundamental sloshing modes of the vessel but the user should adjust the critical tolerances to be sure that they do not have an effect on the solution. Critical tolerances may be: 1-Cell Flow Imbalance (0.001 recommended), 2-Full Cell Tolerance (1e-8 recommended) 3-Maximum Allowed Pressure Iterations (10 or 2000).

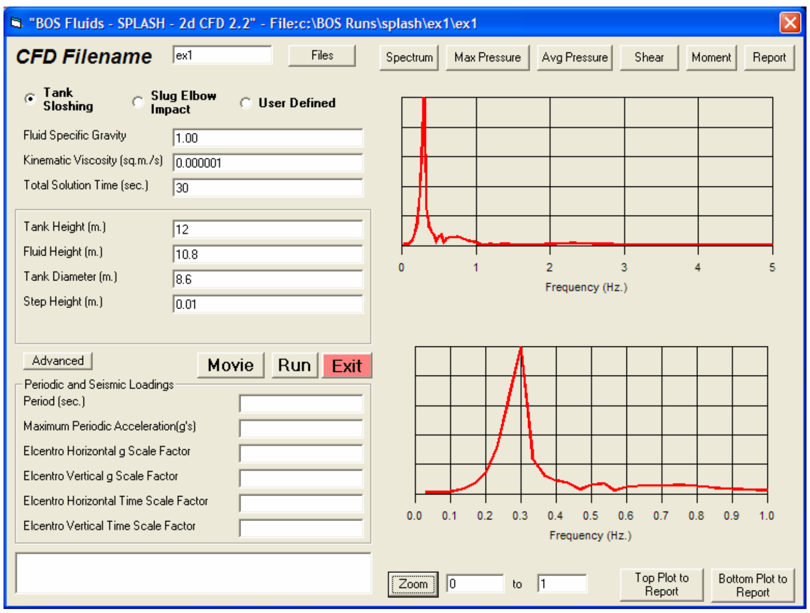
6) Virus scanners can interfere with the file handling. It is not unusual to close and reopen Splash in between runs to make sure that virus scanners have not locked intermediate files for access.

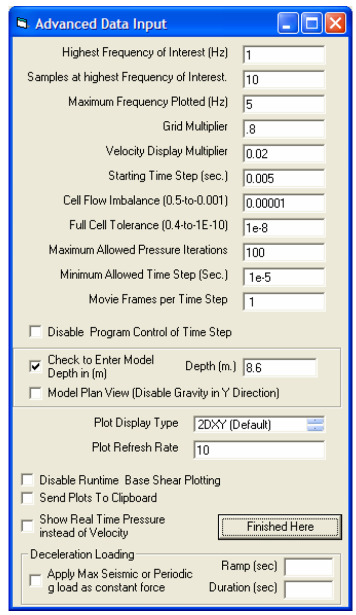
Problem #1:

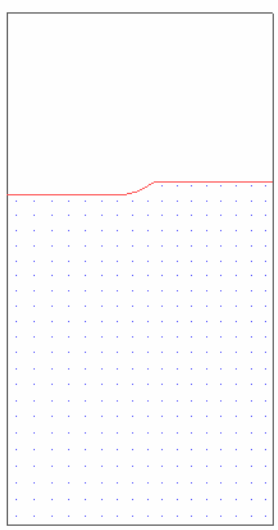
Using the geometry of the emergency water tank described in reference 2, "A Seismic Analysis of the Emergency Water Tank", Hu Yong Tao, Gu Tang Yu, PVP-Vol 314, ASME 1995, enter the following data into "Splash".

Diameter	= 8.6 m.
Liquid Height	= 10.8 m.
Fundamental Period	= 3.066 sec.

From Reference 2, the first sloshing mode should be approximately (1/3.066) = 0.326 Hz. The model for this tank and its input is shown below. The default value for the kinematic viscosity of water is 0.000001 m^2/sec.



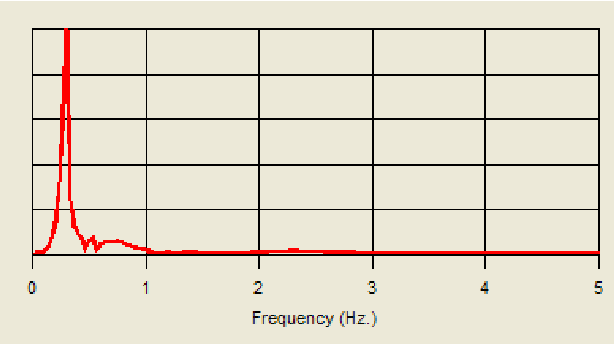


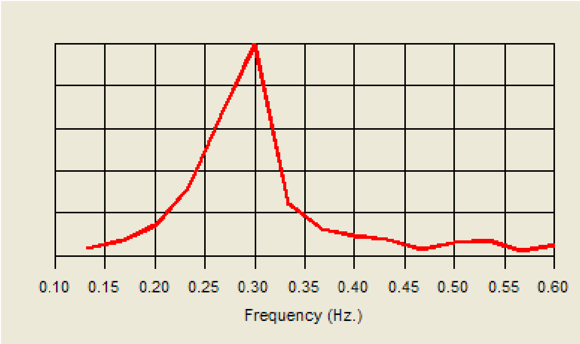


The total solution time is 30 seconds.

To trap the lowest frequency at 0.326 Hz. the Fourier analysis must be focused in this range.

The maximum plotted frequency will be set to 5 Hz. This will limit the plot size and make it easier to see the low frequency range. Other parameters on the Advanced Data Input screen are set to produce an accurate solution. Plotted results are shown below:





API 650 (Reference. 3) also gives an equation for the fundamental period of large storage tanks in it's Appendix E

$$T = k(D^{0.5})$$

Where D is the tank diameter in feet, and "k" is a factor from Figure E-4 which can be approximated by:

$k = 0.6$ for $D/H = 2.2$ or less
$k = (0.6) + (0.045)[(D/H) - 2.0]$ for D/H greater than 2.2 and less than 8.0

Reference (4) also gives the following equations for the sloshing modes in tanks:

$$w = 1/[(2)(3.14)] \; [\; L(g/R) \tanh(L\, H/R)\;]^{0.5}$$

Where L is the 1.841 for mode 1, 5.331 for mode 2 and 8.536 for mode 3.
g is the acceleration due to gravity.

Using these equations for the tank system analyzed above:

<u>From API 650:</u>
D = 8.6m (28.215 ft.)
D/H = 8.6/10.8 = 0.7963

k = 0.6 for D/H less than 2.2
$T = (0.6)(28.215^{0.5}) = 3.18$ sec. (which corresponds to a frequency of 1/3.18 = 0.314 Hz.

<u>From Ref. 4</u> "Guidelines for the Seismic Design of Oil and Gas Pipeline Systems", ASCE Technical Council on Lifeline Earthquake Engineering, ASCE, 1984, the second sloshing frequency can also be obtained.

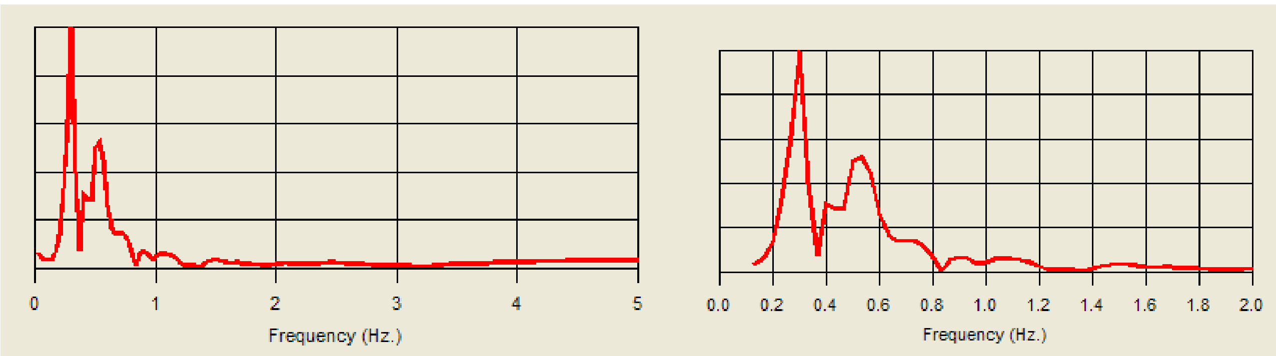
L1 = 1.841
g/R = 32.2 / 14.11 = 2.2825

H/R = 10.8 / 4.3 = 2.512

w_1 $= 1 / [(2)(3.14)] [(1.841)(2.2825) \tanh[(1.841)(2.512)]]^{0.5}$
$= 0.3262$ Hz.

w_2 $= 1 / [(2)(3.14)][(5.331)(2.2825) \tanh[(5.331)(2.512)]]^{0.5}$
$= 0.555$ Hz.

Increasing the size of the step height introduces more energy in the potentially higher modes, and the resulting plots are obtained that show the presence of the predicted second mode. Results from the "ex2" run are shown below:



Values obtained from the API Code and simplified theory (0.326 and 0.318 Hz.) are very close to the frequency calculated by SPLASH (0.3 Hz.)

Problem #2 Seismic Response of Storage Tanks

Taken from *"Seismic Design of Liquid Storage Tanks"*, George W. Housner and Medhat Haroun, Preprint 80-085 American Society of Civil Engineers, Portland ASCE Convention April 14-18, 1980.

Example (1) from the paper:

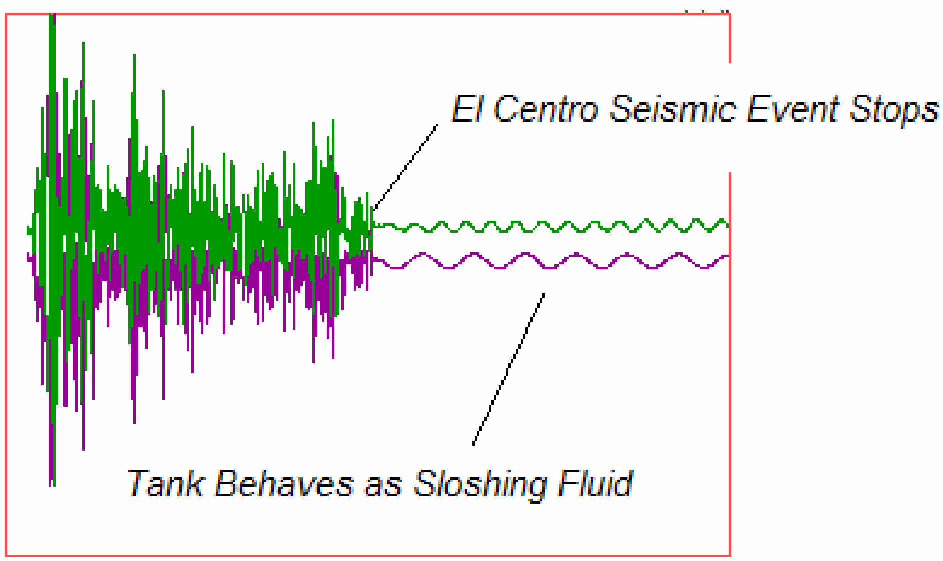
Consider an open top tall tank whose dimensions are:

R=24 ft. (7.32 m), L = 72 ft (21.96 m), and h = 1 inch (2.54 cm). The tank is assumed to be full of water and to be subjected to the N-S component of the 1940 El Centro Earthquake. The fundamental natural frequency of sloshing is calculated from the paper Equation (2), and for this tank is 4 seconds, (0.25 Hz.).

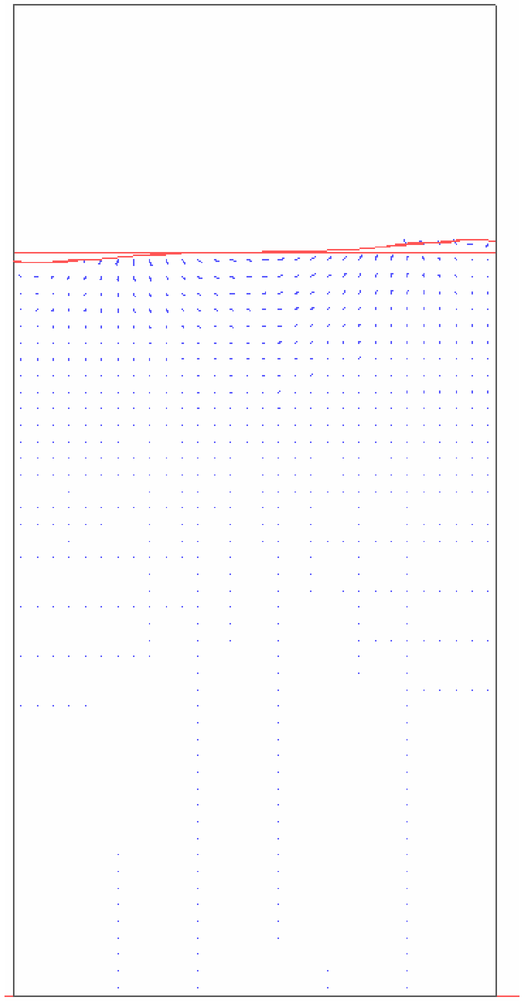
The maximum free surface wave height is given by the paper Equation (3) and is found to be: 15.91 in. (0.386 m.). The maximum base shear is found from the paper Equation (10) to be 22.98E6 N. These values are found from the SPLASH solution and compared.

This model is stored in folder: \ex3, and the job named: *"example"* contains the results for the seismic analysis. The job named *"exampleH"* contains the results for the sloshing frequency analysis.

The plot below shows the SPLASH interactive pressure and shear load plot. The effect of the seismic excitation is seen in the left portion of the plot, and the harmonic motion in the tank that occurs after the seismic shaking forces have stopped are seen on the right side of the plot.

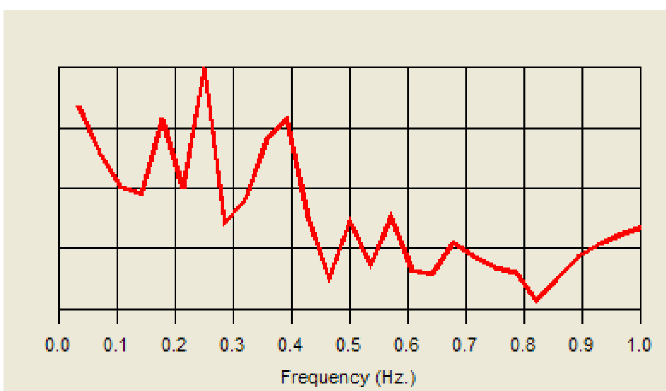


Seismic Shear Loads at Base of Liquid Filled Tank



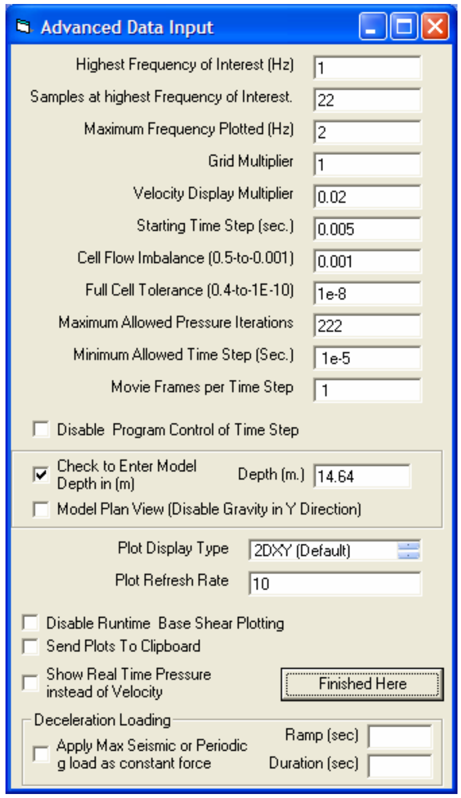
Tank Liquid Height Profile at Approximately Maximum Waveheight.

The maximum liquid height during the seismic time history is shown in the above plot. As can be seen, there is approximately one cell of increase in height, where one cell is 0.5 m. For the accuracy of the example problem this is considered a suitable result. The estimated height from the Reference 5 paper is 0.4 m.

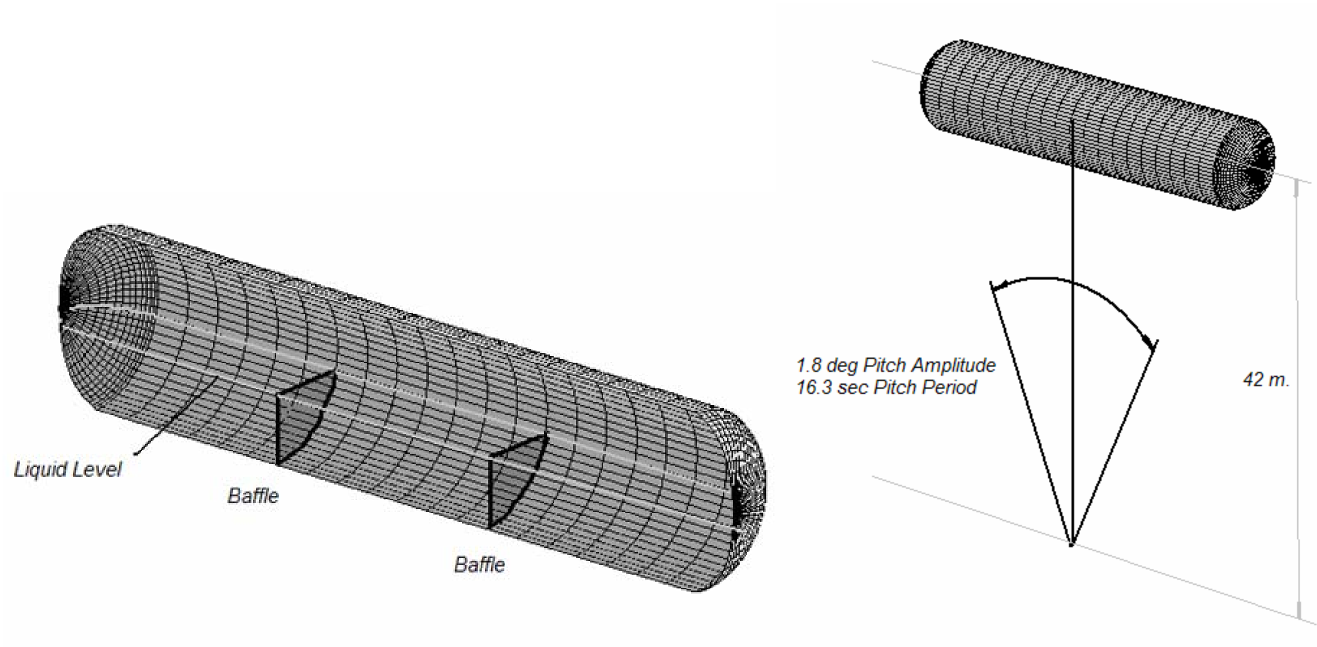


Transformed frequency plots from the "sloshing" analysis in problem "ExampleH" are shown above. The highest modal peak occurs at 0.25 Hz. In a tank of this size and depth there are many low sloshing modes as can be seen by applying the equation from Reference 5: 0.25 Hz, 0.433 Hz, and 0.538 Hz. (Note that for this depth ratio, the frequency expression in Reference 5 simplifies to: $1/[(2)(\pi)]\ [(\lambda_j)g/R]^{1/2}$, where λ_j is a mode multiplier, equal to: (1.841 for the first mode), (5.331 for the second mode), and (8.536 for the third mode.)

The "Advanced" Form input used for this problem is shown below:



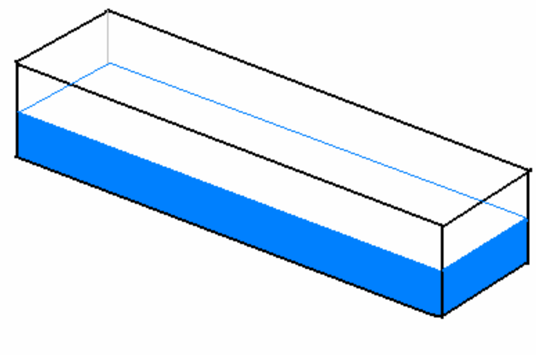
Problem #3 Baffle Loading



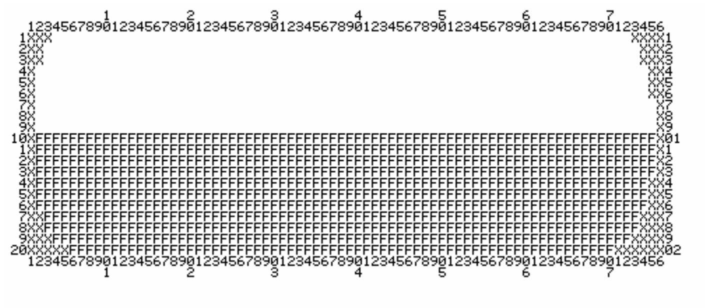
Maximum horizontal sloshing load = 17 kN
Length to centerline of pitch axis = 42 m.
1.8 deg total pitch amplitude
16.3 sec. pitch period
4.9 m. diameter vessel
19 m long vessel
Half filled with SG = 0.9
20% baffles (open area/total area = 0.2)
3 baffles equally spaced
Baffle Height = 3.6 m.

Vessel axial movement (x) can be described by: x = $R\theta$ sine ($2\pi\omega t$).
The acceleration in the (x) direction is the second derivative of the displacement with respect to time:
Acceleration in (x) = $d^2x/dt^2 = \omega^2 R\theta$ sine ($2\pi\omega t$).
Amplitude of the acceleration = $\omega^2 R\theta$.
$R\theta$ = (42 m)(1.8 deg)(π/180) = 1.319 m.
Frequency (Hertz) = 1 / period = 1/16.3 sec = 0.0613 Hz.
Frequency (Radians/second) = (0.0613)(2π) = 0.38547 rad/sec.
$\omega^2 R\theta = (0.38547^2)$(1.319 m.) = 0.196 m/sq.sec.
The acceleration amplitude in g's = 0.196 mpss / 9.81 = 0.01998 g's.

The general Splash model for this problem can be visualized in the sketch below:

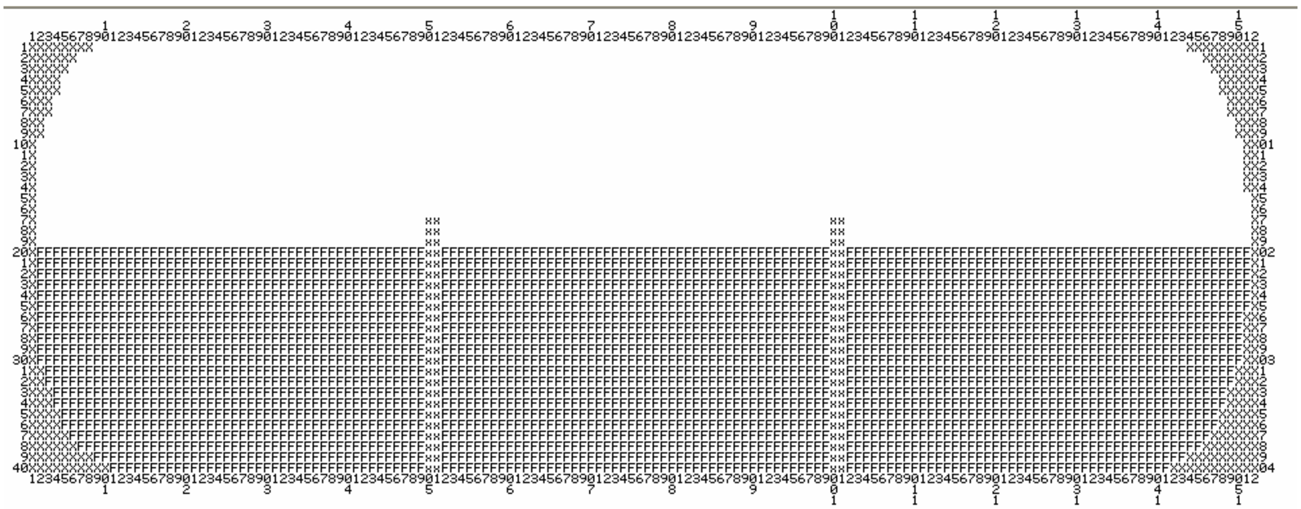


3-D Schematic

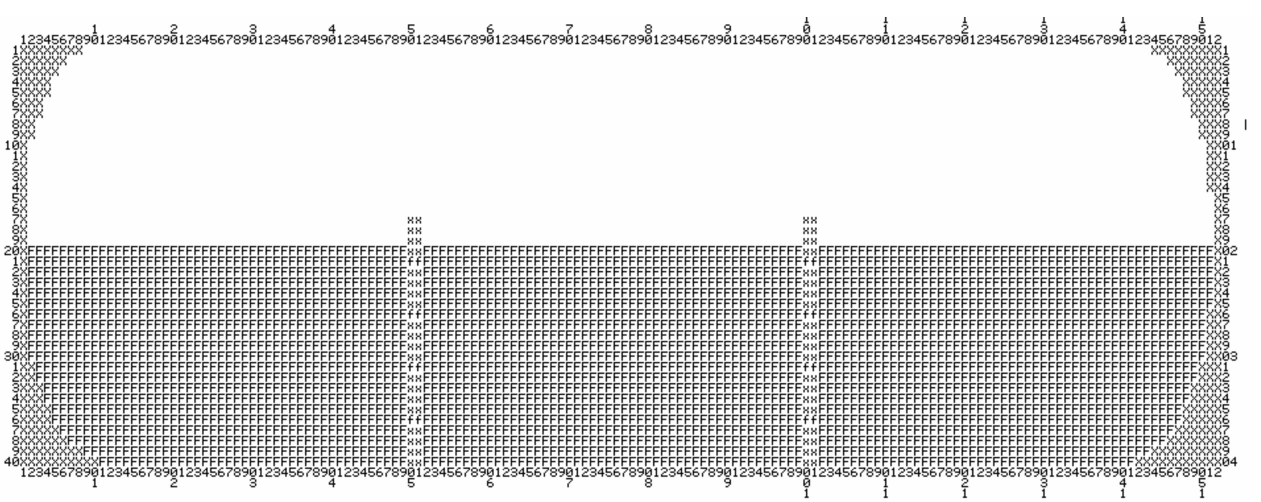


SPLASH 2-D Eulerian Grid

The splash grids for the solid and perforated models is shown below.



Solid Baffle Model ("example4g20p")



Perforated Baffle Model ("example4h")

The total volume of liquid in the contained space is:

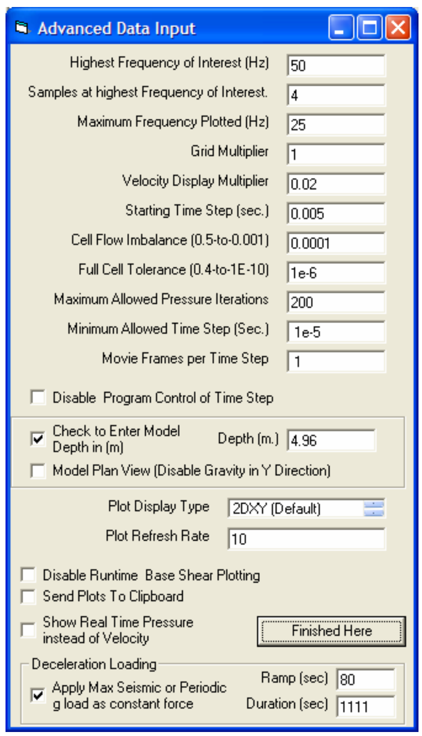
(19)(4.9)(4.9)(0.5)(1000 kg/cu.m)(0.9 SG) = 205,285 kg.

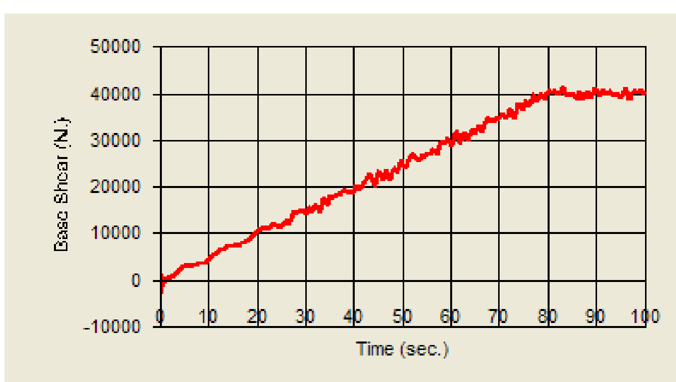
The load applied in the horizontal direction due to a static 0.02 g acceleration is:

(205,285) kg (0.02 g)(9.81 m/sq.s) = 40,277 N.

The Splash program allows for any specified periodic or spectral load to be applied in a ramped manner with a fixed duration. This can be used to validate the method by slowly ramping any maximum horizontal "g" load and then leaving this load applied for a constant period. The ramp time should be significantly longer than the sloshing period of the container.

The basic sloshing period for this vessel estimated using the Equations in Reference 4 is 5.6 seconds. Use 80 seconds as the ramp time (80/5.6 = 14) to eliminate dynamic effects and run a simulation for 100 seconds. The advanced data input and user defined geometry for this problem is shown below:



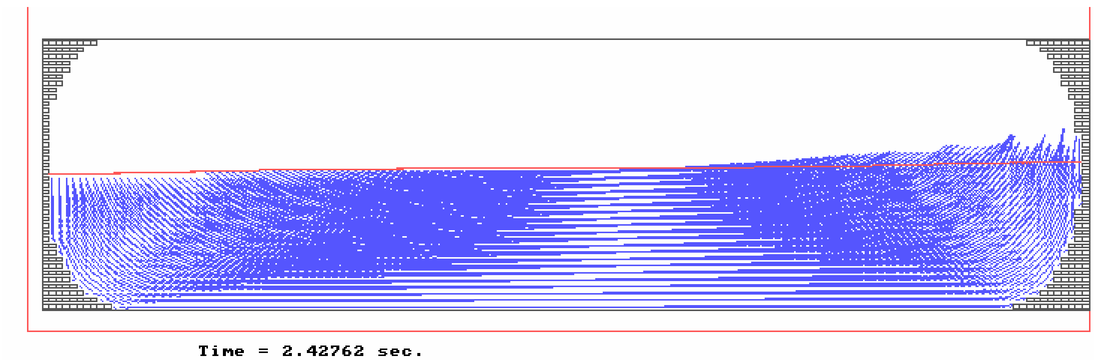


"example4c Base Shear Result"

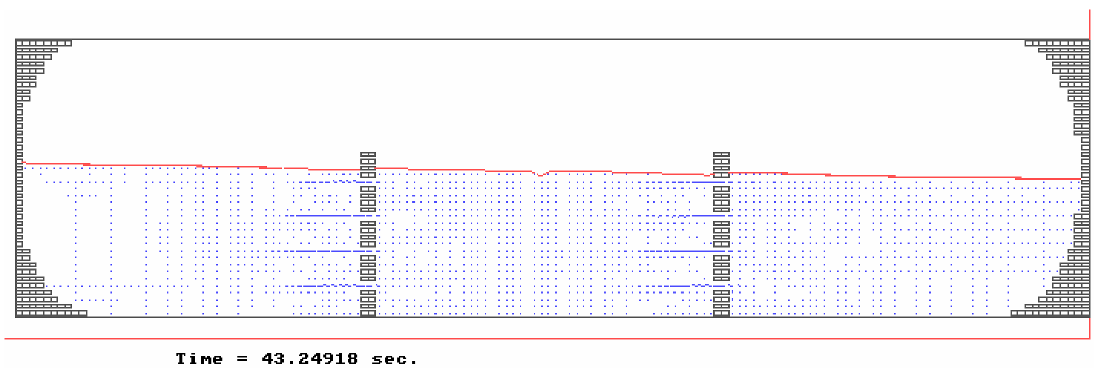
The resulting shear load on the vessel due to the static horizontal load is shown in the figure above. The 40,000 N result matches the hand calculation to within the precision of the plotted result.

Remove the slow ramp and expose the vessel to the horizontal shaking force. Three run results are reported:

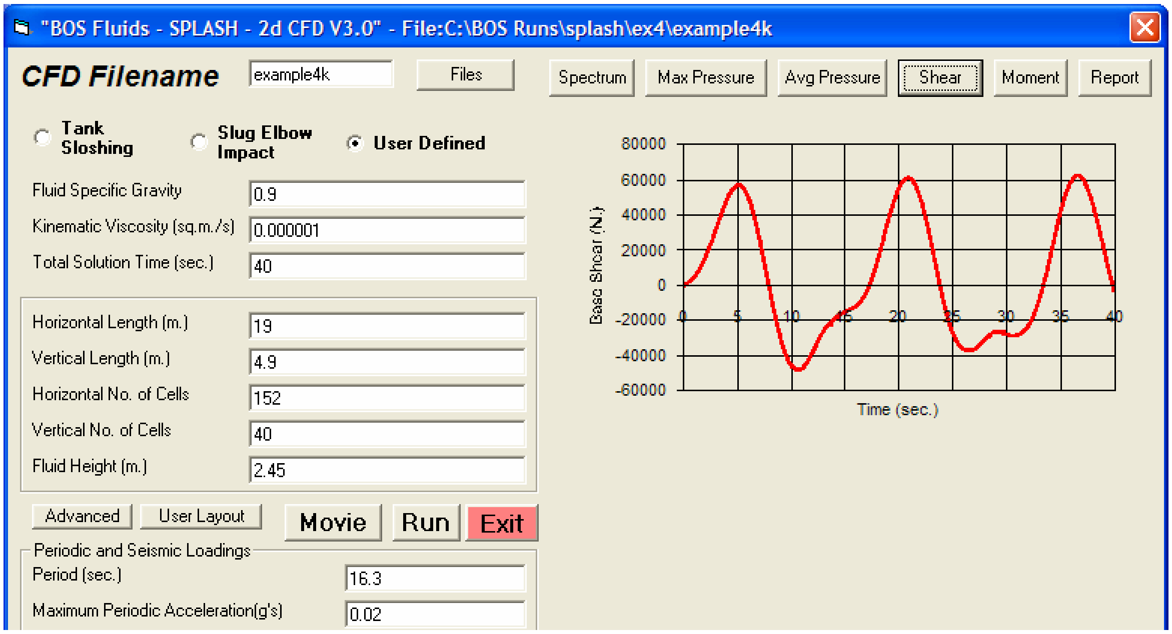
1) Result for no baffles
2) Result for solid baffles (2)
3) Result for 20% perforated baffles (2)



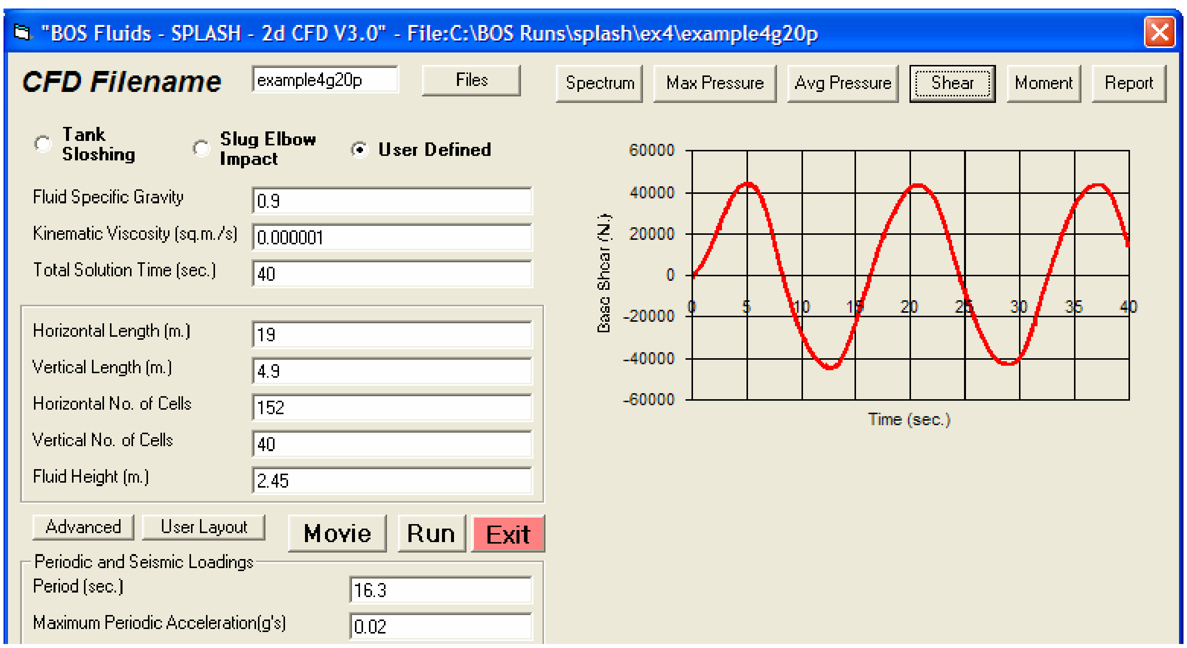
Interactive Velocity Plot – No Baffles



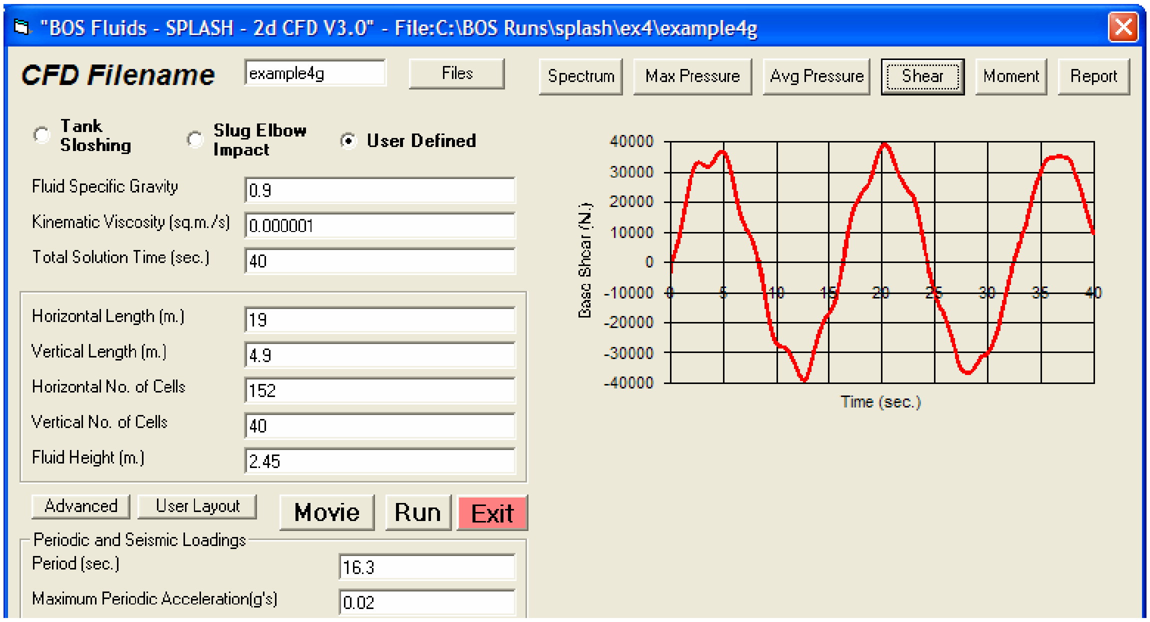
Interactive Velocity Plot – Perforated Baffles (20%)



Example4k maximum shear plot. F = 62376 N. (From Report)



"Example4f" with Perforated Baffles. Max shear (From Report) = 45,271 N.



"Example4g20p" – Solution with Solid Baffles. Shear = 39,215 N. (From Report)

The effect of two baffles can be seen to reduce the maximum shear load on the vessel from 60,000 N to approximately 40,000 N.

Baffles with 20% openings can also be simulated as shown in the following simulation:

The Reference 4 equations for sloshing frequency are:

$L_1 = 1.841 \quad L_2 = 5.331 \quad L_3 = 8.536$

$f_l(Hz) = (1/[2\pi])[(L_j)(g/R) \tanh(L_j H/R)]^{1/2}$

where:

f_l = Frequency in Hz.
L_j = Mode Factor (1.841 for the first sloshing mode)
g = Acceleration due to gravity times the specific gravity of the liquid (9.81 m/sq.s)
R = Radius of the tank (19/2 = 9.5 m.)
H = Height of liquid (4.9/2 = 2.45 m.)

For the axial sloshing mode in an horizontal vessel the "radius" (R) is equal to half the length of the horizontal vessel. For the horizontal vessel above:

$g/R = (9.81)(0.9) / (9.5) = 0.929$
$H/R = 2.45 / 9.5 = 0.258$

$f_l = (1/[2\pi]) [(1.841)(0.929) \tanh[(1.841)(0.258)]]^{1/2} = 0.176$ Hz.

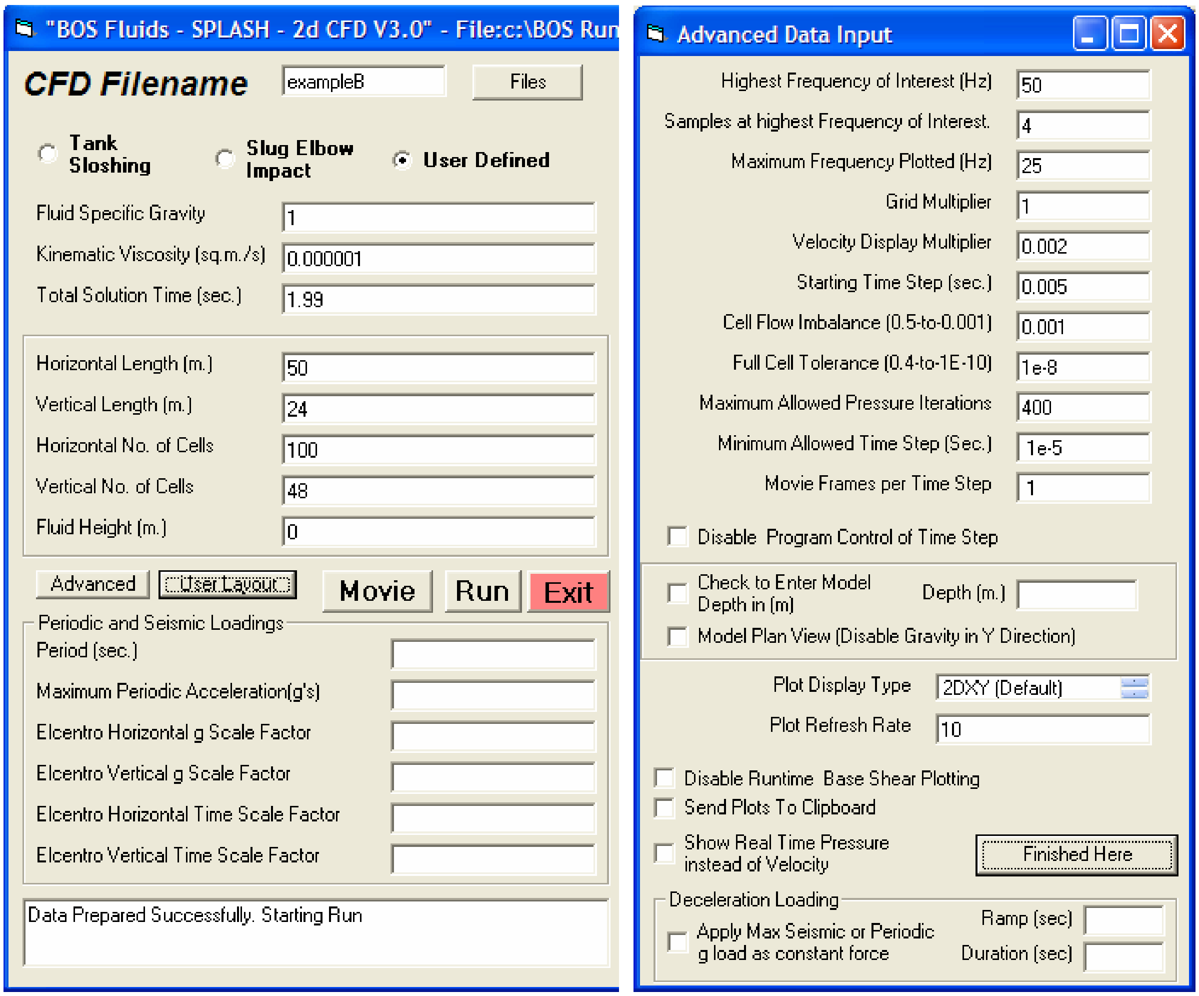
The period associated with this frequency is: 1/f1 = 5.6 s.

Problem #4 Free Fall

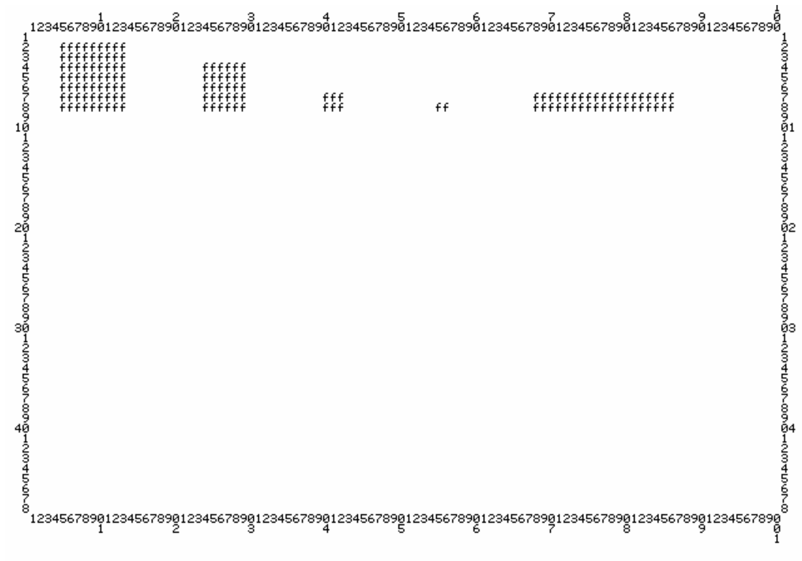
This problem is taken from the calculation provided in the manual and validates the solvers ability to properly evaluate the effect of gravity on a fluid filled region.

The simulation shows that several differently sized droplets falling through 20 meters contact the base of the tank at the same time and in 2.019 seconds.

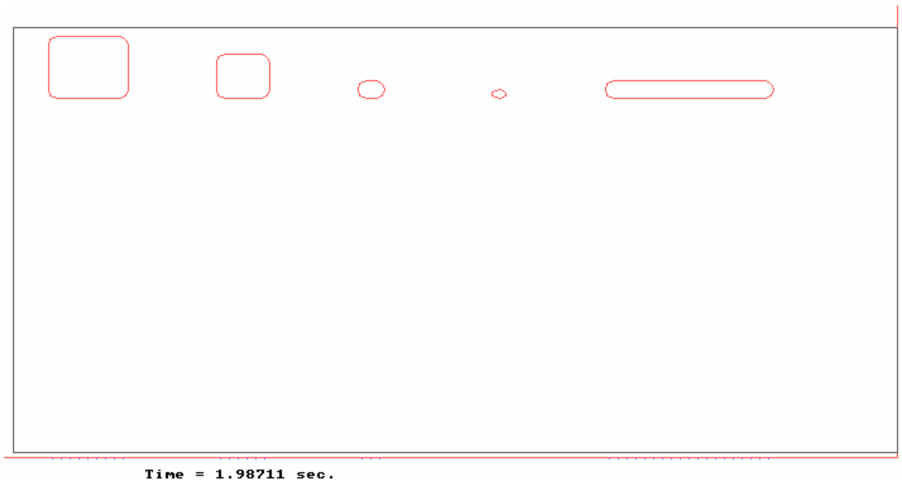
The example model is shown below:



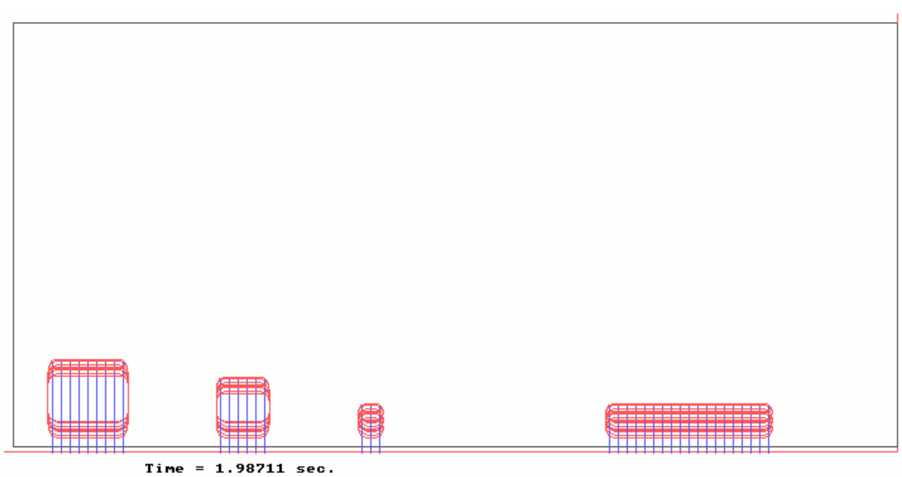
Model Input for Free-Fall Control



Free-Fall User-Defined Model Screen



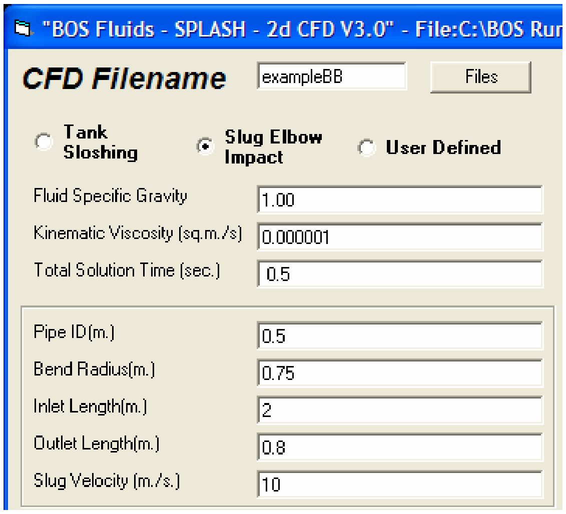
Interactive SPLASH Screen at Beginning of Simulation (Fluid Droplets Free Surface in Red)

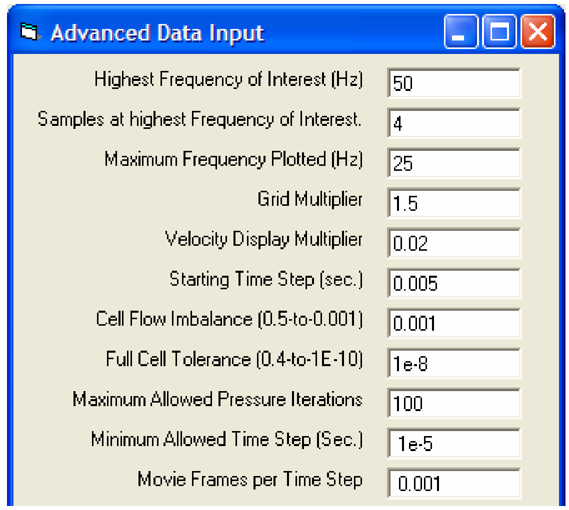


Fluid Droplets just Prior to 2.0 seconds.

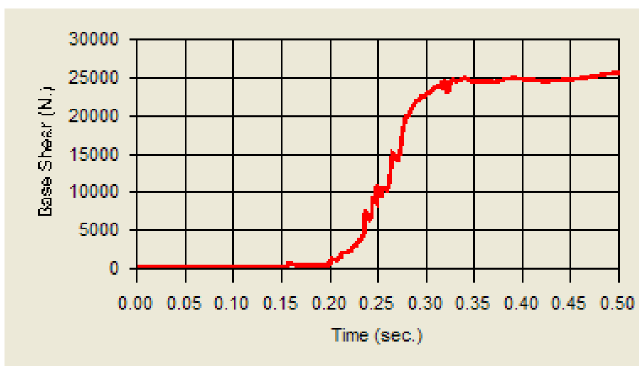
Problem #5 – Slug Impact on Elbow

This problem is take from the Splash User's Guide (3.1.22) and the results are compared against a hand calculation for the typical slug loading on an elbow.

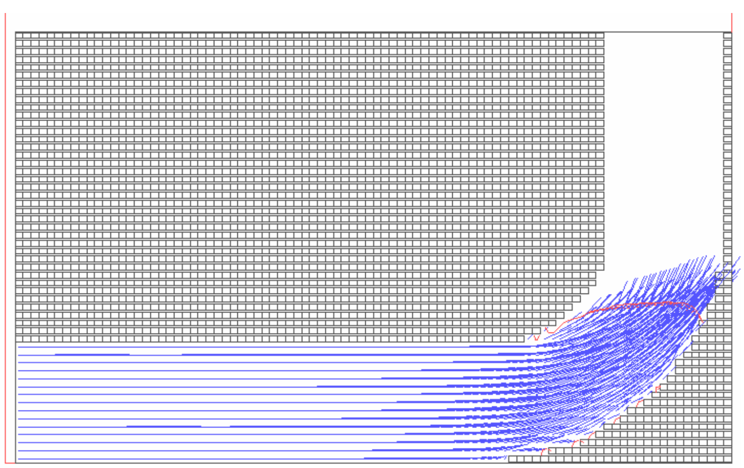




Slug Model Input and Advanced Control



Base Shear as Slug Enters and Leaves Elbow 25592. N. (From Report)



The calculated slug load is = ρAV^2 = (1000) kg/m^3(0.25) m^2 (100) m^2/s^2 = 25,000 N

References:

1) *"In-Tank Fluid Sloshing Impact Effects During Earthquakes: A Preliminary Computational Simulation,"* James E. Park, Mohamad A. Rezvani, PVP-Vol. 314, ASME 1995.
2) *"Aseismic Analysis of the Emergency Water Tank"*, Hu Yong Tao, Gu Tang Yu, PVP-Vol 314, ASME 1995.
3) *"Welded Steel Tanks for Oil Storage"*, API Standard 650 9th Edition, 1993, American Petroleum Institute
4) *"Guidelines for the Seismic Design of Oil and Gas Pipeline Systems"*, ASCE Technical Council on Lifeline Earthquake Engineering, ASCE, 1984.
5) *"Seismic Design of Liquid Storage Tanks"*, George W. Housner and Medhat Haroun, Prepint 80-085 American Society of Civil Engineers, Portland ASCE Convention April 14-18, 1980.
6) *"Dynamic Pressures on Accelerated Fluid Containers",* G.W. Housner, Bulletin of the Seismological Society of America, October 1977, pp. 15-35, Vol. 47.

Splash – Load Factor Development

A cylindrical tank with free water surface Fig.1 subjected to horizontal ground acceleration has two kinds forces by the water. First, when the walls of the tank accelerate a certain amount of water is forced to participate in the motion, which exerts a reactive force on the tank. The force is called impulsive force. Second, the motion of the tank excites the water into oscillations which exert an oscillating force on the tank. The oscillating force is called convective force. In engineering, the convective force corresponding to the fundamental mode of oscillation of the water is most often considered (DynaVessel). Computational fluid mechanics (CFD) programs (Splash) include the effect of multiple modes and the results can be compared with single mode models to guarantee conservatism of the results.

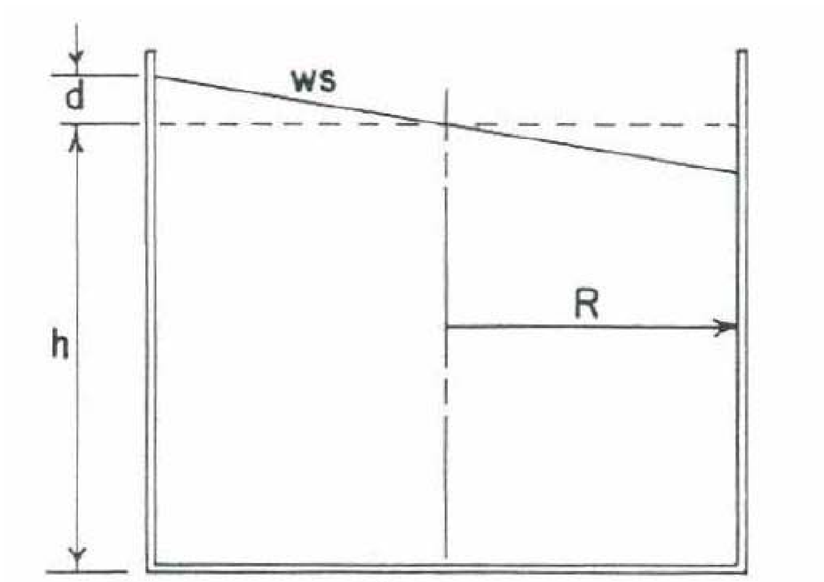


Figure 1. A tank with free water surface.

For an axisymmetric tank, it is possible to avoid three-dimensional computation by multiplying the two-dimensional result by a geometric factor. The two-dimensional result is obtained by cutting one slice of the tank along its cylindrical axis with unit thickness. There are four types of tanks under consideration as illustrated in the following graphs (Fig.2, Fig.3):

1. Vertically cylindrical tank with flat bottom;
2. Vertically tank with hemispherical bottom;
3. Horizontally cylindrical tank with flat head;
4. Horizontally cylindrical tank with hemispherical bottom.

The graphs are the geometry used in the two-dimensional computation with the unit thickness along the direction perpendicular to the paper. The vertical tanks are under horizontal excitation while the horizontally tanks under longitudinal excitation.

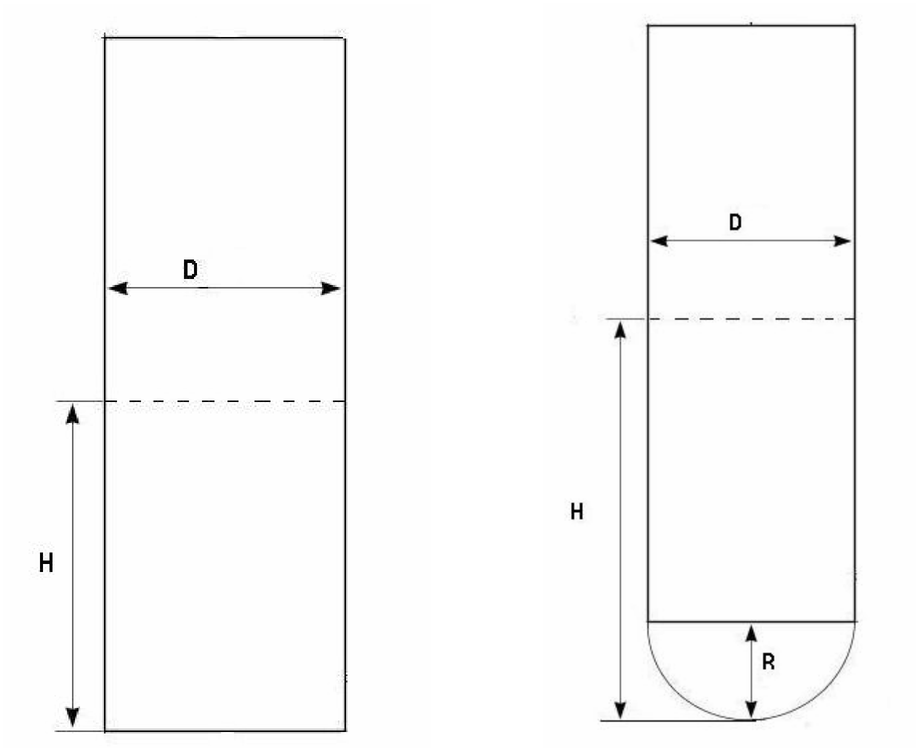


Figure 2. Vertical tanks with flat bottom and spherical bottom

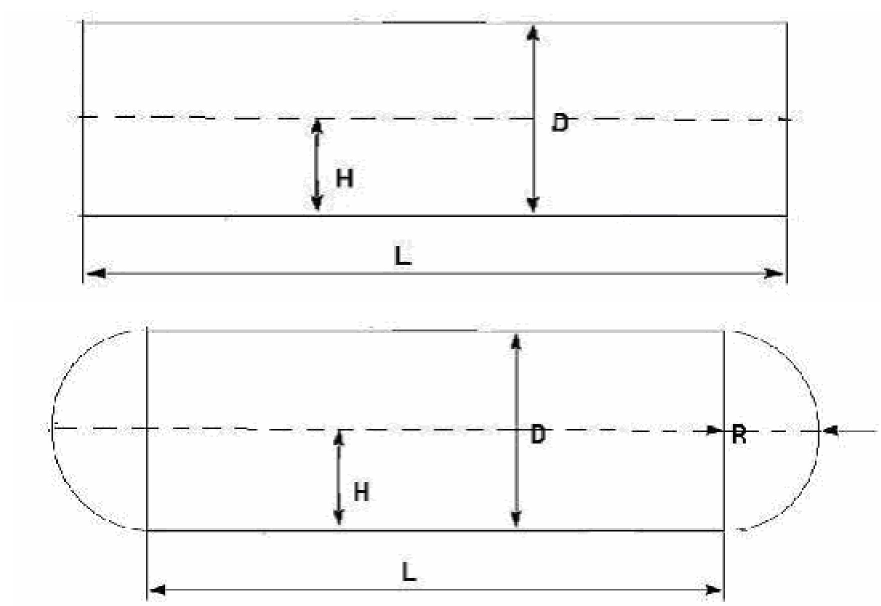


Figure 3. Horizontal tanks with flat head and spherical head

In two dimensional calculation, we can take one slice of liquid with unit thickness along the direction perpendicular to the paper and the slice of the liquid has the geometry as shown in Fig. 2 and Fig. 3. The two dimensional calculation is carried out on this slice of liquid and the three dimensional results are obtained by multiplying the two dimensional result by αD where D is the diameter of the cylinder. See Fig. 4 and Fig. 5. Even though the factors in Fig.5 was generated for the case that the height of the liquid is equal to half of the diameter, they are applicable to the liquid of any height.

The procedures of obtaining Fig.4 and Fig.5 are elaborated in the following:

For example, for a vertically cylindrical tank with flat bottom subjected to ground acceleration of amplitude a, the maxmum base shear of the tank due to the acceleration is:

$$P_{cy} = M_0 a + \frac{12}{11} M_1 g \theta_h$$

where M_0, M_1 are impulsive mass and sloshing mass respectively which be addressed in the following section. g is gravity acceleration. θ_h is the maximum rotation angle for the free surface of the liquid in the cylindrical tank.

Along the axis of the tank, a rectangular slice is taken with unit thickness, which is also the model used in **Splash** as shown later. The base shear of the rectangular slice due to the same ground acceleration can be put in the following form

$$P_{rec} = M_0^{'} a + M_1^{'} g \theta_h^{'}$$

where $M_0^{'}, M_1^{'}$ are the impulsive mass and sloshing mass for the liquid in the rectangular slice respectively. $\theta_h^{'}$ is the maximum rotation angle for the free surface of the liquid in the rectangular tank.

The ratio of $\dfrac{P_{cy}}{P_{rec}}$ is the factor used in **Splash** to convert the result from the rectangular slice into the result for the three-dimensional vertically cylindrical tank.

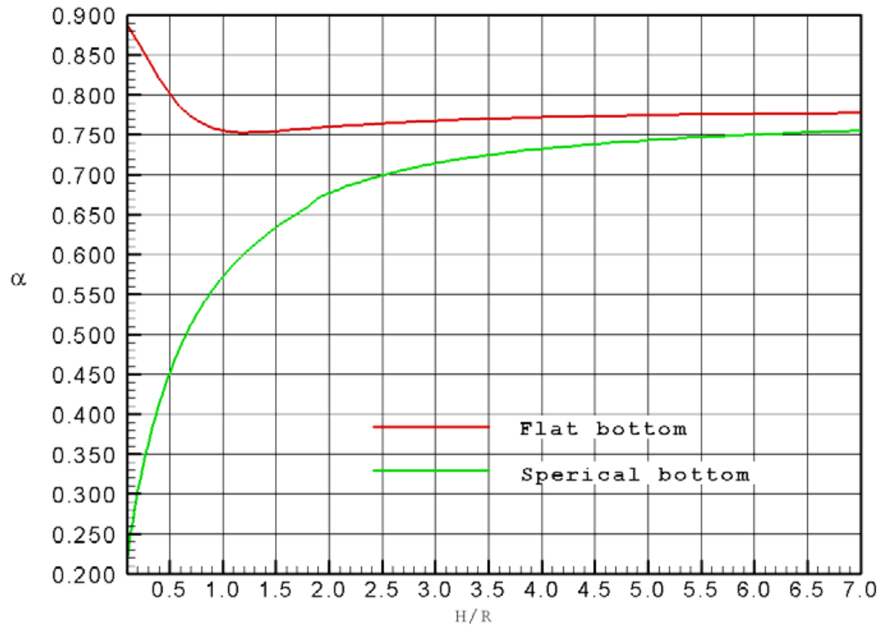


Figure 4. Factor α vs. $\frac{H}{R}$ used to convert the 2D results into 3D ones for vertically cylindrical tank where H is illustrated in Fig.2 and $R = \frac{D}{2}$

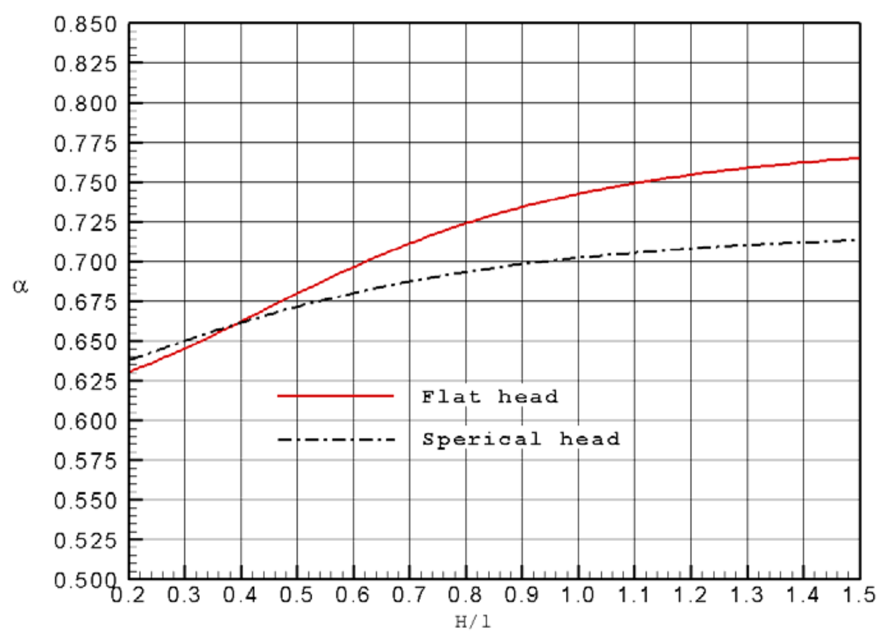


Figure 5. Factor α vs. $\frac{H}{l}$ used to convert the 2D results into 3D ones for horizontally cylindrical tank where H is illustrated in Fig.3 and $l = \frac{L}{2}$

For the volume of sloshing liquid it should be noted that the impulsive force can be equivalent to the one exerted by a mass M_0 that is attached rigidly to the tank at the proper height h_0. The mass M_0 is attached at a height h_0 so that the horizontal force exerted by it is the same as the resultant force exerted by the equivalent water. The convective force is the same the one exerted by a mass M_1 that can oscillate horizontally against a restraining spring as show in Figure 8. Table 1 and 2 are the parameters used to define the equivalent system with respect to different tank geometry.

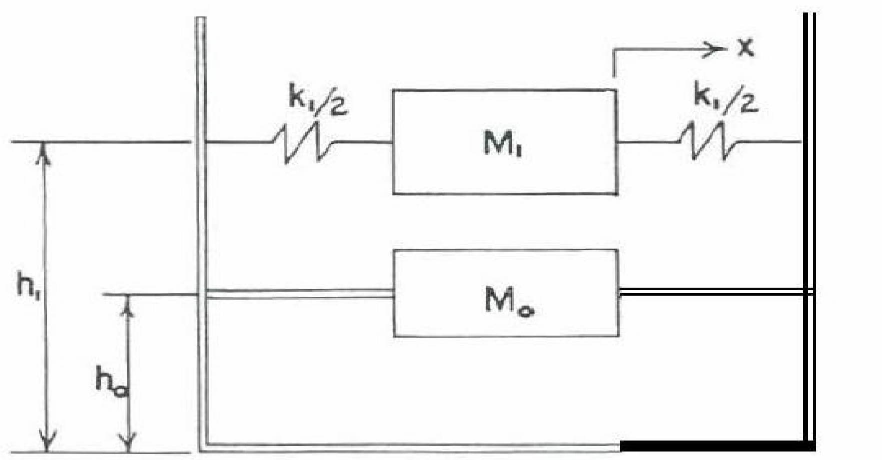


Figure 8. The equivalent system for water tank where M_0 and M_1 produce dynamic forces equivalent to those produced by the water.

Table 1. Parameters to define the equivalent system for rectangular tank and vertically cylindrical tank.

	Rectangular tank M : Mass of the sloshing liquid h : Height of the sloshing liquid l : Half length of the rectangle	Vertical tank with flat bottom M : Mass of the sloshing liquid h : Height of the sloshing liquid R : Radius of the tank
M_0	$M\dfrac{\tanh\left(\sqrt{3}\dfrac{l}{h}\right)}{\sqrt{3}\dfrac{l}{h}}$	$M\dfrac{\tanh\left(\sqrt{3}\dfrac{R}{h}\right)}{\sqrt{3}\dfrac{R}{h}}$
h_0	$\dfrac{3h}{8}\left(1+\dfrac{4}{3}\left(\dfrac{\sqrt{3}\dfrac{l}{h}}{\tanh\left(\sqrt{3}\dfrac{l}{h}\right)}-1\right)\right)$	$\dfrac{3h}{8}\left(1+\dfrac{4}{3}\left(\dfrac{\sqrt{3}\dfrac{R}{h}}{\tanh\left(\sqrt{3}\dfrac{R}{h}\right)}-1\right)\right)$
ω^2	$\dfrac{g}{l}\sqrt{\dfrac{5}{2}}\tanh\left(\sqrt{\dfrac{5}{2}}\dfrac{h}{l}\right)$	$\dfrac{g}{R}\sqrt{\dfrac{27}{8}}\tanh\left(\sqrt{\dfrac{27}{8}}\dfrac{h}{R}\right)$
M_1	$\dfrac{M}{3}\sqrt{\dfrac{5}{2}}\dfrac{l}{h}\tanh\left(\sqrt{\dfrac{5}{2}}\dfrac{h}{l}\right)$	$\dfrac{M}{4}\left(\dfrac{11}{12}\right)^2\sqrt{\dfrac{27}{8}}\dfrac{R}{h}\tanh\left(\sqrt{\dfrac{27}{8}}\dfrac{h}{R}\right)$
h_1	$h\left(1-\dfrac{\cosh\left(\sqrt{\dfrac{5}{2}}\dfrac{h}{l}\right)-2}{\sqrt{\dfrac{5}{2}}\dfrac{h}{l}\sinh\left(\sqrt{\dfrac{5}{2}}\dfrac{h}{l}\right)}\right)$	$h\left(1-\dfrac{\cosh\left(\sqrt{\dfrac{27}{8}}\dfrac{h}{R}\right)-\dfrac{135}{88}}{\sqrt{\dfrac{27}{8}}\dfrac{h}{R}\sinh\left(\sqrt{\dfrac{27}{8}}\dfrac{h}{R}\right)}\right)$
K_1	$\omega_1^2 M_1$	$\omega_1^2 M_1$

Comments:

1. The above formulae are only applicable to the amount of liquid with $\frac{h}{l}$ or $\frac{h}{R} < 1.5$.
2. It should be noted here the rectangular tank has unit width and in calculating the force the calculated force from the above formulate needs to be multiplied by the width of the rectangle.
3. For vertically cylindrical tank with flat bottom, the height of the sloshing liquid is defined as $h = H$ while for vertically cylindrical tank with hemispherical bottom, all the above formulae for the flat bottom can be applied with the height of the liquid h replaced by $H - \frac{R}{3}$ which is the height of the liquid for an equivalent cylindrical tank.
4. In the case $\frac{h}{l}$ or $\frac{h}{R} < 1.5$, the calculations of ω^2, M_1, h_1, K_1 still use the full depth of the liquid and whole volume of the mass. The calculations of M_0, h_0 have to be modified. It can be considered that at $h, l = 1.5R$ there is a fictitious bottom. The mass between the fictitious bottom and the actual bottom just has rigid body motion. For the mass between the fictitious bottom and the free surface of the liquid the M_0, h_0 has the following expression.

M_0	$M'\dfrac{\tanh\left(\frac{\sqrt{3}}{1.5}\right)}{\frac{\sqrt{3}}{1.5}}$	$M'\dfrac{\tanh\left(\frac{\sqrt{3}}{1.5}\right)}{\frac{\sqrt{3}}{1.5}}$
h_0	$\dfrac{4.5R}{8}\left(1+\dfrac{4}{3}\left(\dfrac{\frac{\sqrt{3}}{1.5}}{\tanh\left(\frac{\sqrt{3}}{1.5}\right)}-1\right)\right)$	$\dfrac{4.5R}{8}\left(1+\dfrac{4}{3}\left(\dfrac{\frac{\sqrt{3}}{1.5}}{\tanh\left(\frac{\sqrt{3}}{1.5}\right)}-1\right)\right)$

$M' = 1.5\rho\pi R^2$ for cylindrical tank and $M' = 1.5\rho L^2$ for the rectangular tank.

Table 2. The parameters used to define the equivalent system for horizontally cylindrical tank.

	Tank with flat head	Tank with hemispherical head
	Tank with flat head M : Mass of the sloshing liquid R : Radius of the tank h : Height of the sloshing liquid defined as $\frac{\pi R}{4}$. l : Half length of the rectangle defined as $\frac{L}{2}$	Tank with hemispherical head M : Mass of the sloshing liquid R : Radius of the tank h : Height of the sloshing liquid defined as $\frac{\pi R}{4}$. l : Half length of the rectangle defined as $\frac{L}{2}+\frac{2}{3}R$
M_0	$M\frac{\tanh\left(\sqrt{3}\frac{l}{h}\right)}{\sqrt{3}\frac{l}{h}}$	$M\frac{\tanh\left(\sqrt{3}\frac{l}{h}\right)}{\sqrt{3}\frac{l}{h}}$
h_0	$\frac{3h}{8}\left(1+\frac{4}{3}\left(\frac{\sqrt{3}\frac{l}{h}}{\tanh\left(\sqrt{3}\frac{l}{h}\right)}-1\right)\right)$	$\frac{3h}{8}\left(1+\frac{4}{3}\left(\frac{\sqrt{3}\frac{l}{h}}{\tanh\left(\sqrt{3}\frac{l}{h}\right)}-1\right)\right)$
ω^2	$\frac{g}{l}\sqrt{\frac{5}{2}}\tanh\left(\sqrt{\frac{5}{2}}\frac{h}{l}\right)$	$\frac{g}{l}\sqrt{\frac{5}{2}}\tanh\left(\sqrt{\frac{5}{2}}\frac{h}{l}\right)$
M_1	$\frac{M}{3}\sqrt{\frac{5}{2}}\frac{l}{h}\tanh\left(\sqrt{\frac{5}{2}}\frac{h}{l}\right)$	$\frac{M}{3}\sqrt{\frac{5}{2}}\frac{l}{h}\tanh\left(\sqrt{\frac{5}{2}}\frac{h}{l}\right)$
h_1	$h\left(1-\frac{\cosh\left(\sqrt{\frac{5}{2}}\frac{h}{l}\right)-\frac{135}{88}}{\sqrt{\frac{5}{2}}\frac{h}{l}\sinh\left(\sqrt{\frac{5}{2}}\frac{h}{l}\right)}\right)$	$h\left(1-\frac{\cosh\left(\sqrt{\frac{5}{2}}\frac{h}{l}\right)-\frac{135}{88}}{\sqrt{\frac{5}{2}}\frac{h}{l}\sinh\left(\sqrt{\frac{5}{2}}\frac{h}{l}\right)}\right)$
K_1	$\omega_1^2 M_1$	$\omega_1^2 M_1$

Comments:

1. For horizontally cylindrical tank with flat/hemispherical head, a rectangular tank equivalent to the horizontal tank in terms of the liquid volume needs to be found to analyze the sloshing effects and the width of the rectangular tank is D if the wet height of the liquid in the horizontally cylindrical tank is half of the diameter.
2. The formulae listed in the table are for the case in which the height of the liquid is equal to R
3. In the case $\frac{h}{l} > 1.5$, the procedure to deal with the rectangular tank in the calculation of M_0 and h_0 as shown in the comments of Table 1 is applicable for the equivalent rectangular tank.
4. In calculating the impulsive force or convective force, the results from the above formulae is only for the equivalent rectangular tank with unit thickness and they needs to be multiplied by the width of the equivalent rectangular tank.

When local stresses are of concern, the masses and associated sloshing mode horizontal springs shall be distributed over part of the basin wall height as shown in Fig. 9. The impulsive mass M_0 may be uniformly distributed over a height equal to twice the distance from the bottom of the basin to the center of mass (Note: It could be the fictitious bottom in the case of $\frac{h}{R}$ or $\frac{h}{L_e} > 1.5$). The horizontal springs for the sloshing effect shall be distributed over a height from the top of the water surface to the center of mass. The sloshing mass, M_1, shall be attached, through a rigid link, to the distributed springs at the position h_1 below the surface of the liquid. The impulsive mass M_0 and sloshing mass M_1 are calculated according to the formulae listed in the previous tables. The procedure for distributing the mass is as follows:

1. Calculate the center of the mass and then get the height over which the impulsive mass M_0 shall be uniformly distributed.
2. Distribute uniformly the spring stiffness K_1 to horizontal springs over the height l_r which denotes the center of the liquid mass and the springs are connected through a rigid link to which the sloshing mass located at the position of h_1 is attached.

In the case of $\frac{h}{R}$ or $\frac{h}{L_e} > 1.5$, the part of the liquid below the critical depth can be regarded as a solid mass which shall be uniformly distributed over the height between the actual bottom of the basin and the fictitious bottom. For the part of the liquid between the water surface and the fictitious bottom, repeat the previous procedure to add the impulsive mass and sloshing mass.

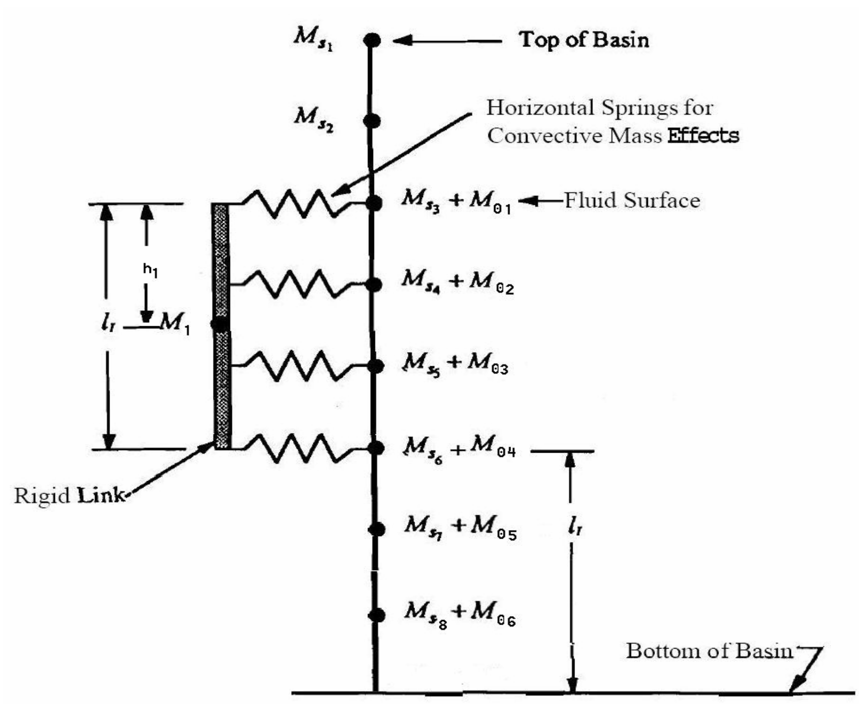
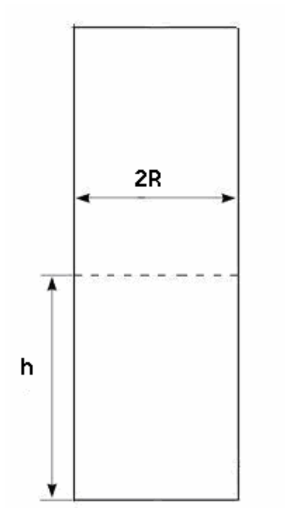


Figure 9. Distribution of fluid mass for horizontal seismic response analysis of basins with local stress problems.

Examples

1.

A vertically cylindrical tank of $R = 24.384$ m. radius and $h = 7.62$ m. depth of water as shown below. Assume the tank to be given a horizontal acceleration in the x direction of amplitude $a = 0.01g$ and let the frequency of the acceleration be 6.41 Hz. which is the first natural sloshing frequency of the tank. This acceleration persists for 20 sec.



Based on the equivalent system to the sloshing liquid as shown in Fig. 8 and the corresponding formula listed in Table 1 for the cylindrical tank, it is easily to get the vibration equation for single degree-of-freedom without damping.

$$\ddot{x} + \omega_1^2 x = a\sin(\varpi t)$$

with the following solution

$$x(t) = \frac{a}{\omega_1^2 - \varpi^2}\left(\sin(\varpi t) - \frac{\varpi}{\omega_1^2}\sin(\omega_1 t)\right)$$

In the case of resonance, the solution is expressed in the following format:

$$x(t) = \frac{a}{2\omega_1^2}\left(\sin(\omega_1 t) - \omega_1 t\cos(\omega_1 t)\right)$$

and the base shear from the sloshing is

$$P_s(t) = K_1 x(t) = M_1\frac{a}{2}\left(\sin(\omega_1 t) - \omega_1 t\cos(\omega_1 t)\right)$$

Then it is obvious that the total base shear has the form of

$$P(t) = M_0 a\sin(\varpi t) + P_s(t)$$

Next, we take one slice of the liquid along the diameter whose direction coincides with the ground acceleration, regard it as a rectangular tank with unit thickness and apply the formula listed in

Table 1 for the rectangular tank to calculate the shear force at the base of the tank. As a way of comparison, we also run the same model on **Splash** and the results are illustrated in Fig.10. It can be observed that there is good agreement between the two types of calculations as far as the amplitudes of base shear are concerned.

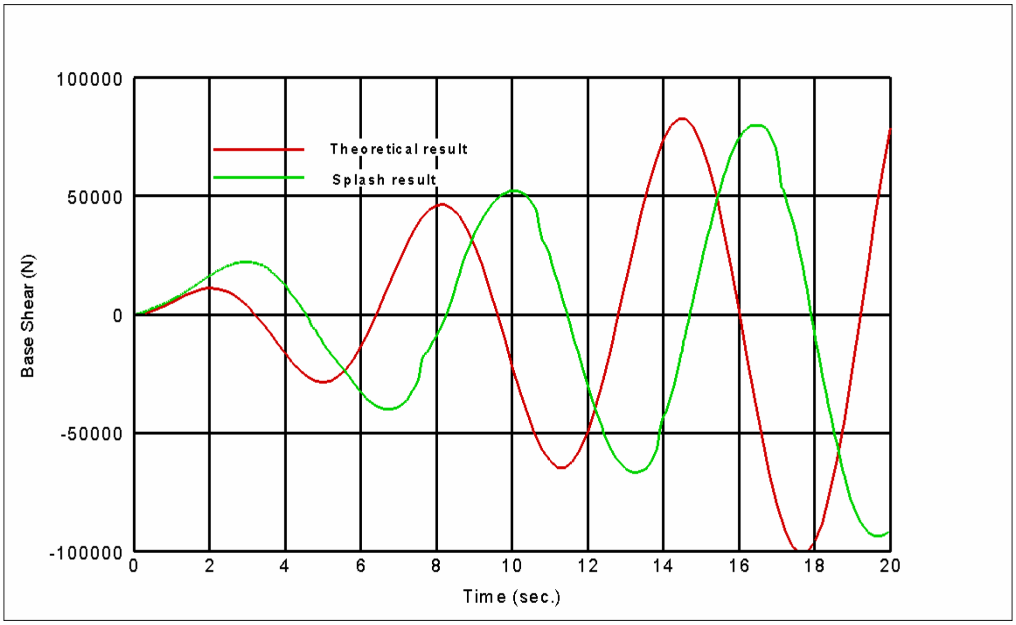


Figure 10. Comparison of time history of base shear from **Splash** and the theoretical calculation for rectangular tank with unit thickness.

The above results are two-dimensional ones. It can be seen that $\frac{h}{R} = 0.625$ and from Fig. 4 the ratio of converting two-dimensional result into three-dimensional case for $\frac{h}{R} = 0.625$ is about 0.78, which means that the base shear from the equivalent rectangular tank or **Splash** needs to be multiplied by factor $0.78D$ to get the base shear for vertically cylindrical tank. Fig. 11 shows the time histories of base shear from analytical solution and the equivalent rectangular tank which match each other well.

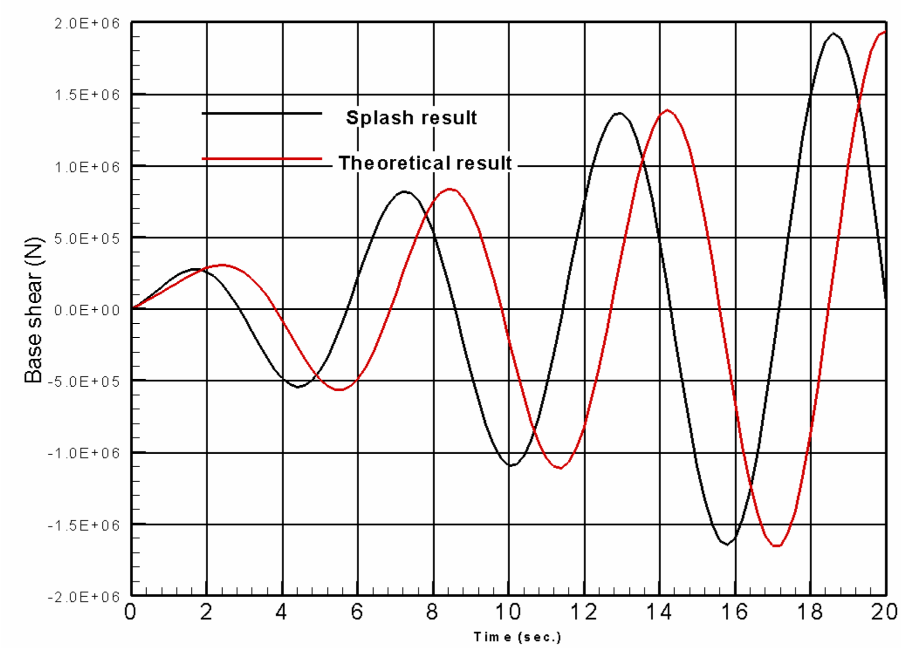
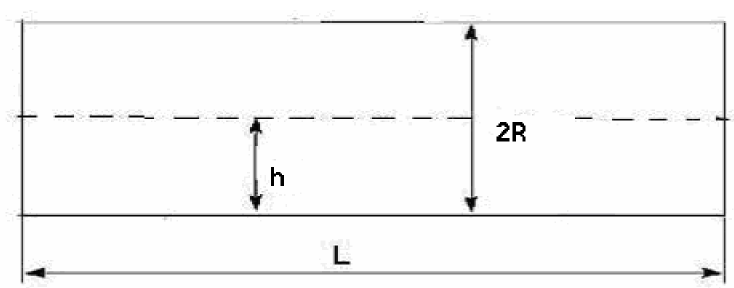


Figure 11. Comparison of base shear for vertically cylindrical tank with flat bottom resulting from analytical theory and **Splash**.

2.

A flat-head horizontally cylindrical tank of $R = 0.235\,\text{m}$. radius, $h = 0.235\,\text{m}$. depth of water and $L = 0.94\,\text{m}$. length of water. Assume the tank to be given a displacement in the axial direction of amplitude $dx = 0.005\,\text{m}$. and let the frequency of the acceleration be 0.74Hz. which is the first natural sloshing frequency of the tank. This acceleration persists for 10 sec.



For simplicity, we skip the procedure to calculate the theoretical result since it is illustrated in the example 1. If one slice of the tank is taken along the axial direction with unit thickness, we can consider it as a rectangular tank and immediately all the formulae for the equivalent system to the rectangular tank can be applied. Fig 12. is the comparison of the time history in base shear from the calculation of rectangular tank and software **Splash**.

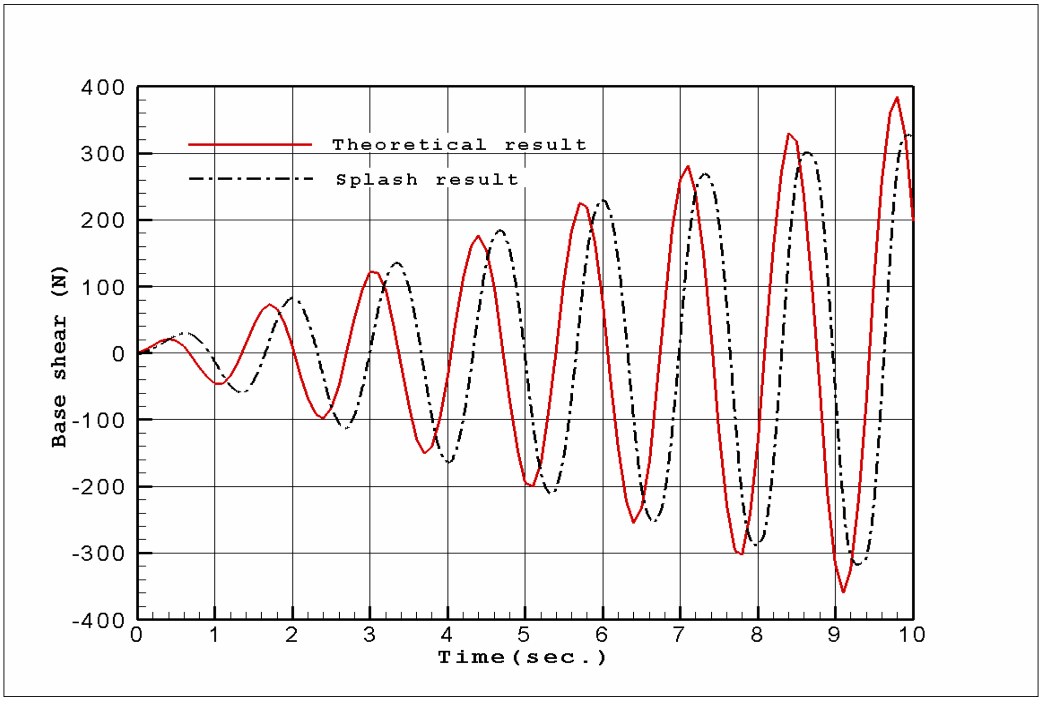


Figure 12. Comparison of time history of base shear from **Splash** and the theoretical calculation for rectangular tank with unit thickness.

It can be seen that $\frac{h}{l} = 0.5 \; (l = \frac{L}{2})$ and from Fig. 4 the ratio of converting two-dimensional result into three-dimensional case for $\frac{h}{l} = 0.5$ is about 0.64, which means that the base shear from the equivalent rectangular tank or **Splash** needs to be multiplied by factor $0.64D$ to get the base shear for horizontally cylindrical tank. Fig. 13 shows the time history of base shear from analytical solution and the equivalent rectangular tank which match each other well.

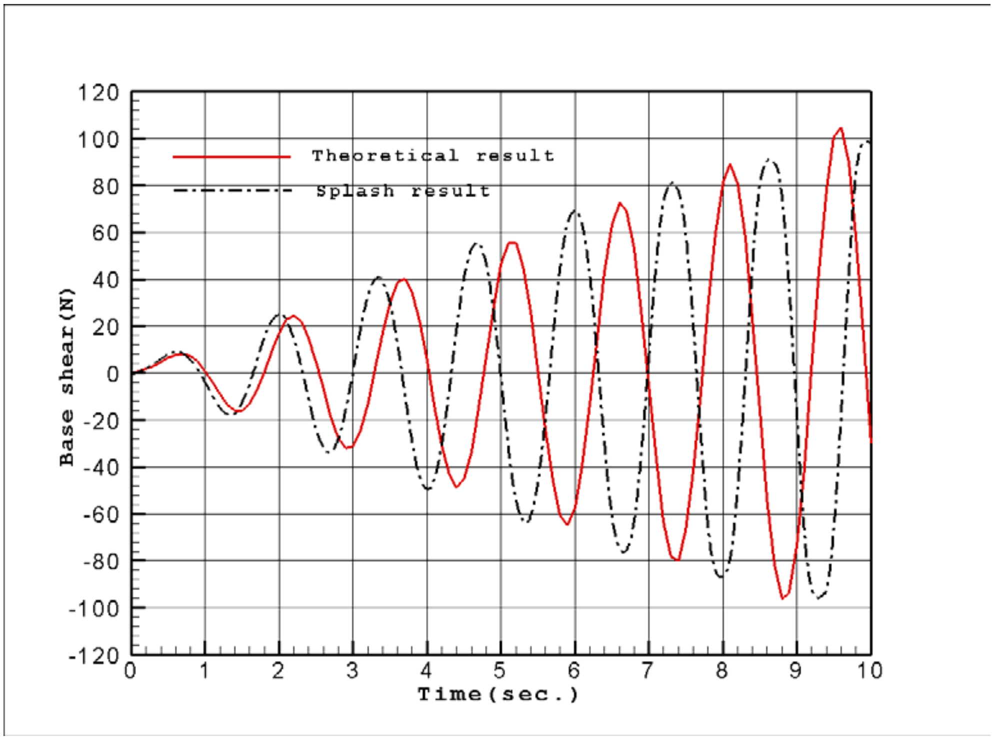


Figure 13. Comparison of base shear for horizontally cylindrical tank with flat head resulting from analytical theory and **Splash**.