

For the transverse reinforcement in the bottom of the footing, assuming a tension-controlled section, the reinforcement ratio required is

$$\rho = \frac{0.85f'_c \left(1 - \sqrt{1 - \frac{M_u}{0.383bd^2f'_c}} \right)}{f_y}$$

$$= \frac{(0.85) \left(5 \frac{\text{kips}}{\text{in}^2} \right)}{60 \frac{\text{kips}}{\text{in}^2}} \times \left(1 - \sqrt{1 - \frac{(502 \text{ ft-kips}) \left(12 \frac{\text{in}}{\text{ft}} \right) \times \left(1000 \frac{\text{lbf}}{\text{kip}} \right)}{(0.383)(228 \text{ in})(23.5 \text{ in})^2 \times \left(5000 \frac{\text{lbf}}{\text{in}^2} \right)}} \right)$$

$$= 0.00089$$

The minimum permissible reinforcement area governs, and the reinforcement area required in both the top and bottom of the footing transversely is

$$A_s = \frac{0.0018bh}{2} = \frac{(0.0018) \left(12 \frac{\text{in}}{\text{ft}} \right) (27 \text{ in})}{2}$$

$$= 0.29 \text{ in}^2/\text{ft}$$

Providing no. 4 bars at 8 in centers gives an area of $0.30 \text{ in}^2/\text{ft}$ (satisfactory).

5. STRAP FOOTING

Nomenclature

A_1	base area of pad footing no. 1	in^2
A_2	base area of pad footing no. 2	in^2
B_S	length of short side of strap	ft
B_1	length of short side of pad footing no. 1	ft
B_2	length of short side of pad footing no. 2	ft
h_S	depth of strap	in
h_1	depth of pad footing no. 1	in
h_2	depth of pad footing no. 2	in
l	distance between column centers	ft
l_R	distance between soil reactions	ft
L_S	length of long side of strap	ft
L_1	length of long side of pad footing no. 1	ft
L_2	length of long side of pad footing no. 2	ft
R_1	soil reaction under pad footing no. 1	kips
R_2	soil reaction under pad footing no. 2	kips
w_c	unit weight of concrete	lbf/ft^3
W_S	weight of strap beam	kips
W_1	weight of pad footing no. 1	kips
W_2	weight of pad footing no. 2	kips

Pressure Distribution

The strap footing shown in Fig. 2.9 has the strap beam, which connects the two pad footings, underlaid by a layer of Styrofoam™ so that the soil pressure under the strap may be considered negligible. Because of the stiffness of the strap beam, the strap and pad footings act as a rigid body producing uniform soil pressure under the pad footings. The base areas of the two pad footings may be adjusted to produce equal soil pressure q under both footings.

The total service load acting is

$$\sum P = P_1 + P_2 + W_1 + W_2 + W_S$$

$$q = \frac{\sum P}{A_1 + A_2}$$

The soil reactions act at the center of the pad footings and are given by

$$R_1 = qA_1$$

$$R_2 = qA_2$$

$$A_1 = B_1 L_1$$

$$A_2 = B_2 L_2$$

Pad footing no. 2 is located symmetrically with respect to column no. 2 so that the lines of action of P_2 and R_2 are coincident.

$$l_R = l + \frac{c_1}{2} - \frac{B_1}{2}$$

$$L_S = l_R - \frac{B_1 + B_2}{2}$$

Equating vertical forces gives

$$R_2 = \sum P - R_1 \quad [\text{equilibrium equation no. 1}]$$

Taking moments about the center of pad footing no. 2 gives

$$R_1 = \frac{P_1 l + W_1 l_R + \frac{W_S(L_S + B_2)}{2}}{l_R}$$

[equilibrium equation no. 2]

To determine suitable dimensions that will give a soil bearing pressure equal to the allowable pressure q , suitable values are selected for h_1 , h_2 , h_S , B_1 , B_2 , and B_S . l_R and L_S are determined, and

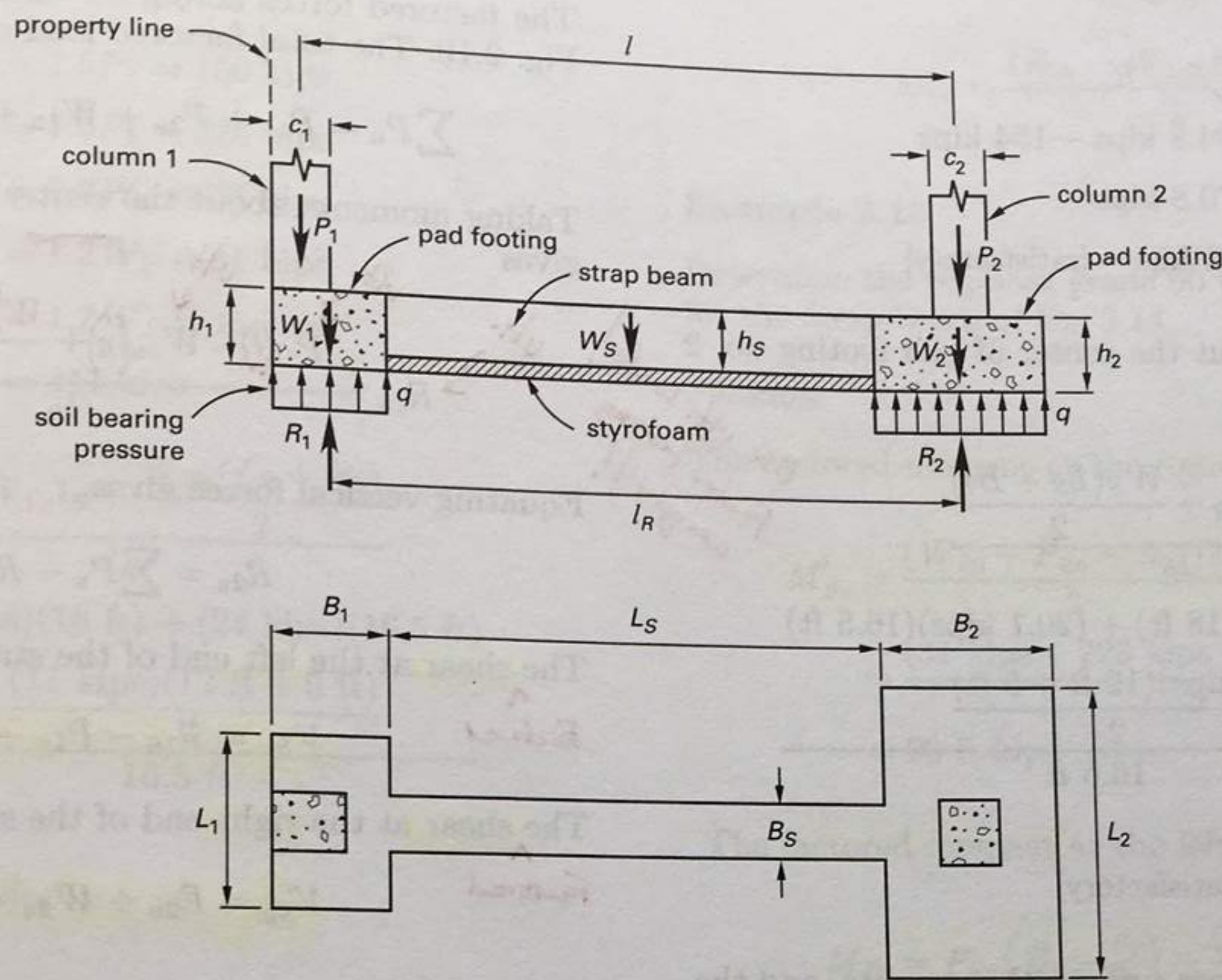
$$W_S = w_c L_S B_S h_S$$

An initial estimate is made of R_1 , and

$$A_1 = \frac{R_1}{q}$$

$$W_1 = w_c A_1 h_1$$

Figure 2.9 Strap Footing with Applied Service Loads



An initial estimate is made of R_2 , and

$$A_2 = \frac{R_2}{q}$$

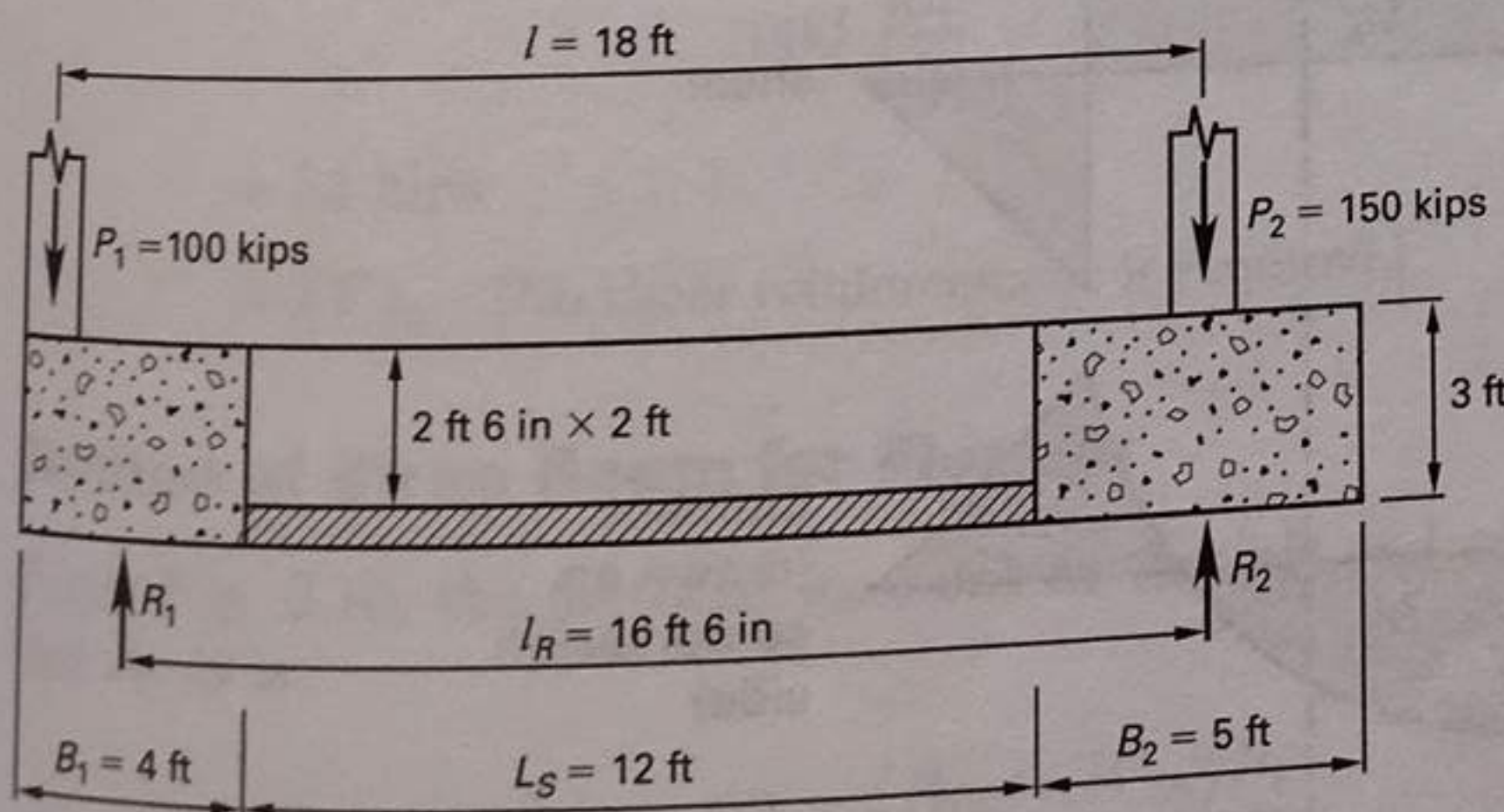
$$W_2 = w_c A_2 h_2$$

$$\sum P = P_1 + P_2 + W_1 + W_2 + W_s$$

Substituting in the two equilibrium equations provides revised estimates of R_1 and R_2 , and the process is repeated until convergence is reached.

Example 2.13

Determine the plan dimensions required for the strap footing shown in the following illustration to provide a uniform bearing pressure of 3000 lbf/ft² under both pad footings for the service loads indicated.



Solution

From the dimensions indicated in the illustration,

$$W_s = w_c L_s B_s h_s$$

$$= \left(0.15 \frac{\text{kip}}{\text{ft}^3}\right) (12 \text{ ft}) (2 \text{ ft}) (2.5 \text{ ft})$$

$$= 9 \text{ kips}$$

Assuming that $R_1 = 134$ kips, then

$$A_1 = \frac{R_1}{q} = \frac{134 \text{ kips}}{3 \frac{\text{kips}}{\text{ft}^2}} = 44.67 \text{ ft}^2$$

$$W_1 = w_c A_1 h_1 = \left(0.15 \frac{\text{kip}}{\text{ft}^3}\right) (44.67 \text{ ft}^2) (3 \text{ ft})$$

$$= 20.1 \text{ kips}$$

Assuming that $R_2 = 171$ kips, then

$$A_2 = \frac{R_2}{q} = \frac{171 \text{ kips}}{3 \frac{\text{kips}}{\text{ft}^2}} = 57 \text{ ft}^2$$

$$W_2 = w_c A_2 h_2 = \left(0.15 \frac{\text{kip}}{\text{ft}^3}\right) (57 \text{ ft}^2) (3 \text{ ft})$$

$$= 25.7 \text{ kips}$$

$$\sum P = P_1 + P_2 + W_1 + W_2 + W_s$$

$$= 100 \text{ kips} + 150 \text{ kips} + 20.1 \text{ kips}$$

$$+ 25.7 \text{ kips} + 9 \text{ kips}$$

$$= 304.8 \text{ kips}$$

Equating vertical forces gives

$$\begin{aligned} R_2 &= \sum P - R_1 \\ &= 304.8 \text{ kips} - 134 \text{ kips} \\ &= 170.8 \text{ kips} \\ &\approx 171 \text{ kips} \quad [\text{satisfactory}] \end{aligned}$$

Taking moments about the center of pad footing no. 2 gives

$$\begin{aligned} R_1 &= \frac{P_1 l + W_1 l_R + \frac{W_S(L_S + B_2)}{2}}{l_R} \\ &= \frac{(100 \text{ kips})(18 \text{ ft}) + (20.1 \text{ kips})(16.5 \text{ ft}) + \frac{(9 \text{ kips})(12 \text{ ft} + 5 \text{ ft})}{2}}{16.5 \text{ ft}} \\ &= 133.8 \text{ kips} \\ &\approx 134 \text{ kips} \quad [\text{satisfactory}] \end{aligned}$$

The initial estimates were sufficiently accurate, and the required pad footing areas are

$$A_1 = 44.67 \text{ ft}^2$$

$$A_2 = 57 \text{ ft}^2$$

Design of Strap Beam for Shear

The factored forces acting on the footing are shown in Fig. 2.10. The total factored load on the footing is

$$\sum P_u = P_{1u} + P_{2u} + W_{1u} + W_{2u} + W_{Su}$$

Taking moments about the center of pad footing no. 2 gives

$$R_{1u} = \frac{P_{1u}l + W_{1u}l_R + \frac{W_{Su}(L_S + B_2)}{2}}{l_R}$$

Equating vertical forces gives

$$R_{2u} = \sum P_u - R_{1u}$$

The shear at the left end of the strap is

$$V_{Su} = R_{1u} - P_{1u} - W_{1u}$$

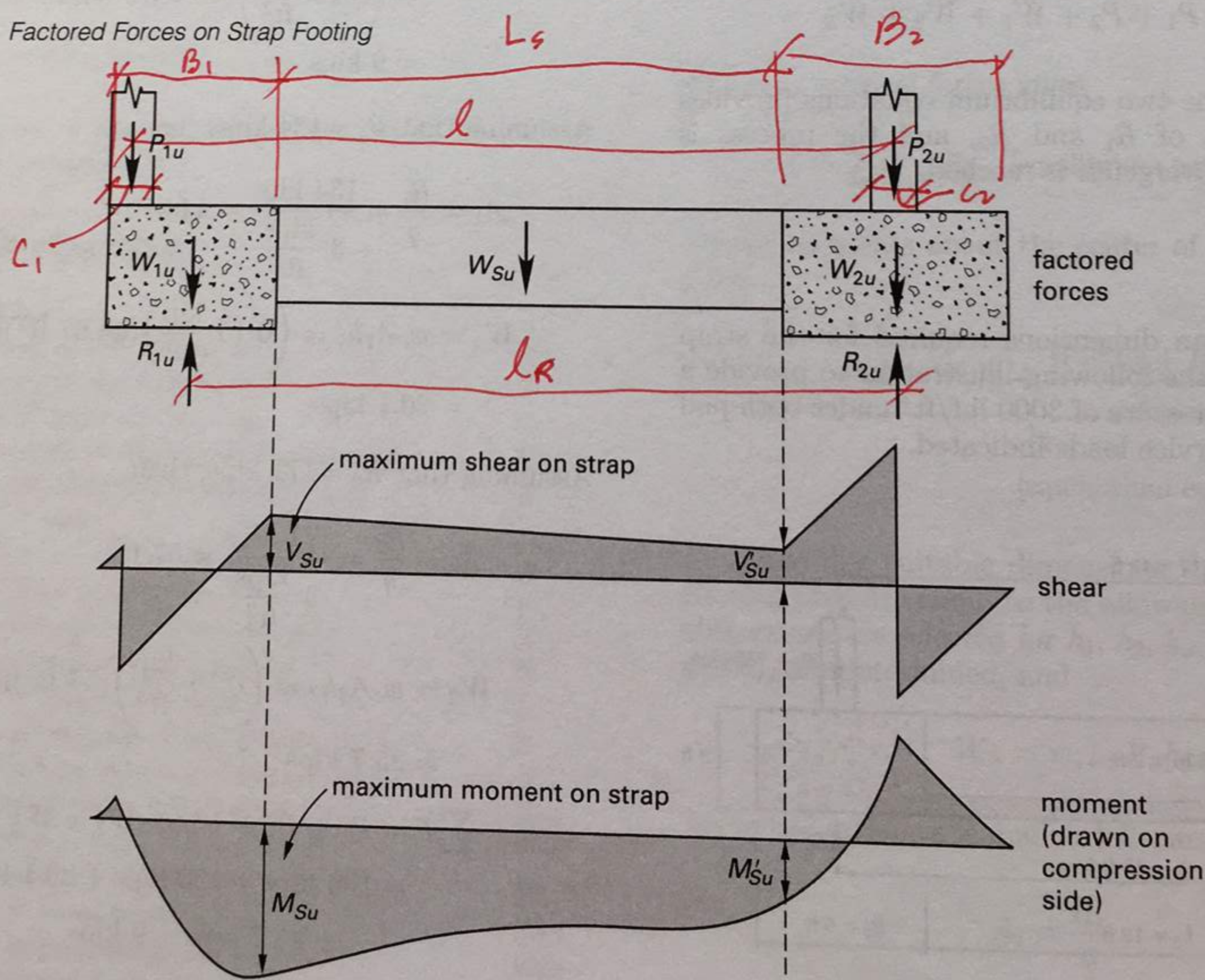
The shear at the right end of the strap is

$$V'_{Su} = P_{2u} + W_{2u} - R_{2u}$$

Example 2.14

The strap footing of normal weight concrete for Ex. 2.13 has a concrete strength of 3000 lbf/in² and a factored load on each column that is 1.5 times the service load. The strap beam has an effective depth of 27.5 in. Determine whether the shear capacity is adequate.

Figure 2.10 Factored Forces on Strap Footing



Solution

The factored forces are

$$P_{1u} = 1.5P_1 = 150 \text{ kips}$$

$$P_{2u} = 1.5P_2 = 225 \text{ kips}$$

$$W_{1u} = 1.2W_1 = 24 \text{ kips}$$

$$W_{2u} = 1.2W_2 = 31 \text{ kips}$$

$$W_{Su} = 1.2W_S = 11 \text{ kips}$$

$$\sum P_u = 441 \text{ kips}$$

$$R_{1u} = \frac{P_{1u}l + W_{1u}l_R + \frac{W_{Su}(L_S + B_2)}{2}}{l_R}$$

$$= \frac{(150 \text{ kips})(18 \text{ ft}) + (24 \text{ kips})(16.5 \text{ ft}) + \frac{(11 \text{ kips})(12 \text{ ft} + 5 \text{ ft})}{2}}{16.5 \text{ ft}}$$

$$= 193 \text{ kips}$$

$$R_{2u} = \sum P_u - R_{1u}$$

$$= 248 \text{ kips}$$

The shear at the right end of the strap is

$$V'_{Su} = P_{2u} + W_{2u} - R_{2u}$$

$$= 225 \text{ kips} + 31 \text{ kips} - 248 \text{ kips}$$

$$= 8 \text{ kips}$$

The shear at the left end of the strap is

$$V_{Su} = R_{1u} - P_{1u} - W_{1u}$$

$$= 193 \text{ kips} - 150 \text{ kips} - 24 \text{ kips}$$

$$= 19 \text{ kips} \quad [\text{governs}]$$

The design shear capacity of the strap beam is given by ACI Eq. 11-3 as

$$\phi V_c = 2\phi bd\lambda\sqrt{f'_c}$$

$$= \frac{(2)(0.75)(24 \text{ in})(27.5 \text{ in})(1.0)\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}}$$

$$= 54 \text{ kips}$$

$$> 2V_{Su} \quad [\text{No shear reinforcement is required.}]$$

Design of Strap Beam for Flexure

From Fig. 2.10, the factored moment at the left end of the strap is

$$M_{Su} = P_{1u}\left(B_1 - \frac{c_1}{2}\right) - \frac{(R_{1u} - W_{1u})B_1}{2}$$

The factored moment at the right end of the strap is

$$M'_{Su} = \frac{(R_{2u} - W_{2u} - P_{2u})B_2}{2}$$

Example 2.15

Determine the required grade 60 flexural reinforcement for the strap beam of Ex. 2.14.

Solution

The factored moment at the right end of the strap is

$$M'_{Su} = \frac{(W_{2u} + P_{2u} - R_{2u})B_2}{2}$$

$$= \frac{(31 \text{ kips} + 225 \text{ kips} - 248 \text{ kips})(5 \text{ ft})}{2}$$

$$= 20 \text{ ft-kips}$$

The factored moment at the left end of the strap is

$$M_{Su} = P_{1u}\left(B_1 - \frac{c_1}{2}\right) - \frac{(R_{1u} - W_{1u})B_1}{2}$$

$$= (150 \text{ kips})\left(4 \text{ ft} - \frac{1 \text{ ft}}{2}\right) - \left(\frac{(193 \text{ kips} - 24 \text{ kips})(4 \text{ ft})}{2}\right)$$

$$= 187 \text{ ft-kips} \quad [\text{governs}]$$

Assuming a tension-controlled section, the required reinforcement ratio is

$$\rho = \frac{0.85f'_c\left(1 - \sqrt{1 - \frac{M_{Su}}{0.383bd^2f'_c}}\right)}{f_y}$$

$$= \frac{(0.85)\left(3 \frac{\text{kips}}{\text{in}^2}\right)}{60 \frac{\text{kips}}{\text{in}^2}} \times \left(1 - \sqrt{1 - \frac{(187 \text{ ft-kips})\left(12 \frac{\text{in}}{\text{ft}}\right) \times \left(1000 \frac{\text{lbf}}{\text{kip}}\right)}{(0.383)(24 \text{ in})(27.5 \text{ in})^2 \times \left(3000 \frac{\text{lbf}}{\text{in}^2}\right)}}\right)$$

$$= 0.0023$$

The controlling minimum reinforcement ratio is given by ACI Sec. 10.5.1 and Sec. 10.5.3 as the lesser of the following results.

$$\rho_{\min} = \frac{200}{f_y} = \frac{200 \frac{\text{lbf}}{\text{in}^2}}{60,000 \frac{\text{lbf}}{\text{in}^2}} = 0.0033$$

$$\rho_{\min} = \left(\frac{4}{3}\right)(0.0023) = 0.0031 \quad [\text{governs}]$$

The reinforcement required in the top of the strap beam is

$$\begin{aligned} A_s &= bd\rho_{\min} \\ &= (24 \text{ in})(27.5 \text{ in})(0.0031) \\ &= 2.05 \text{ in}^2 \end{aligned}$$

Providing four no. 7 bars gives an area of 2.4 in² (satisfactory).

6. CANTILEVER RETAINING WALL

Nomenclature

F	frictional force at underside of base	kips
\bar{h}	equivalent additional height of fill, w/γ_s	ft
h_B	depth of base	ft
h_K	height of shear key	ft
h_T	total height of retaining wall, $h_B + L_W$	ft
h_W	stem thickness	ft
H_A	total active earth pressure behind wall	kips
H_L	total pressure behind wall due to live load surcharge	kips
H_P	total passive earth pressure in front of wall	kips
K_A	Rankine coefficient of active earth pressure $(1 - \sin \phi)/(1 + \sin \phi)$	—
K_P	Rankine coefficient of passive earth pressure $(1 + \sin \phi)/(1 - \sin \phi)$	—
L_B	length of base	ft
L_H	length of heel	ft
L_T	length of toe	ft
L_W	height of stem	ft
p_A	active lateral pressure due to a fluid of specific weight γ_A , $K_A\gamma_s$	lbf/ft ²
p_L	lateral pressure due to live load surcharge, wK_A	lbf/ft ²
p_P	passive lateral pressure due to a fluid of specific weight γ_P , $K_P\gamma_s$	lbf/ft ²
q	earth pressure under the base	lbf/ft ²
w	live load surcharge	lbf/ft ²
W_B	weight of base	lbf/ft ²
W_K	weight of key	kips
W_L	weight of surcharge	kips
W_S	weight of backfill	kips
W_W	weight of stem	kips

Symbols

γ_s	specific weight of backfill	lbf/ft ³
μ	coefficient of friction	—
ϕ	angle of internal friction	degree

Pressure Distribution

Figure 2.11 shows the forces acting on a retaining wall. The total active earth pressure behind the wall is given by Rankine's theory as

$$\begin{aligned} H_A &= \frac{p_A h_T^2}{2} = \frac{K_A \gamma_s h_T^2}{2} = \frac{\left(\frac{1 - \sin \phi}{1 + \sin \phi}\right) \gamma_s h_T^2}{2} \\ &= \frac{30 h_T^2}{2} \quad [\text{for } \gamma_s = 110 \text{ lbf/ft}^3 \text{ and } \phi = 35^\circ] \\ &= \text{pressure exerted by a fluid of density } 30 \text{ lbf/ft}^3 \end{aligned}$$

The total active earth pressure acts at a height above the base.

The total surcharge pressure behind the wall due to live load surcharge of w is

$$H_L = p_L h_T = w K_A h_T = \frac{w p_A h_T}{\gamma_s}$$

The surcharge may be represented by an equivalent height of fill given by

$$\bar{h} = \frac{w}{\gamma_s}$$

$$H_L = p_A \bar{h} h_T$$

The total surcharge pressure acts at a height above the base.

The total passive earth pressure in front of the wall

$$\begin{aligned} H_P &= \frac{p_P h_K^2}{2} = \frac{K_P \gamma_s h_K^2}{2} = \frac{\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right) \gamma_s h_K^2}{2} \\ &= \frac{400 h_K^2}{2} \quad [\text{for } \gamma_s = 110 \text{ lbf/ft}^3 \text{ and } \phi = 35^\circ] \\ &= \text{pressure exerted by a fluid of density } 400 \text{ lbf/ft}^3 \end{aligned}$$

The total passive earth pressure acts at a height above the bottom of the key. The frictional force on the underside of the base is given by

$$F = \mu \sum W$$

$\sum W$ is the total weight of the retaining wall plus fill plus live load surcharge.