
surcharge_wall

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```
In [1]: %pylab inline
        from scipy.integrate import trapz,quad,romberg,dblquad
        Populating the interactive namespace from numpy and matplotlib
```

1 Navfac DM7.02

The following formula is the classic one from the Navfac DM7.02 manual for the calculation of the horizontal stress on a wall caused by a line load. The second function integrates the first one to provide a formula for the area load.

$$\sigma_x = \begin{cases} \frac{P}{H} \cdot \frac{0.2 \cdot n}{(0.16 + n^2)^2} & m \leq 0.4 \\ \frac{P}{H} \cdot \frac{0.28 \cdot m^2 \cdot n}{(m^2 + n^2)^2} & m > 0.4 \end{cases}$$

$$m = \frac{x}{h} \quad n = \frac{z}{h}$$

```
In [2]: def LL_navfac(P, x, H, z):
        m = x / H
        n = z / H
        if m <= 0.4:
            return (P/H) * 0.20 * n / (0.16 + n**2)**2
        else:
            return (P/H) * 1.28 * m**2 * n / (m**2 + n**2)**2
```

```
In [3]: def SL_navfac(q, x1, x2, H, z):
        return quad(lambda x: LL_navfac(q, x, H, z), x1, x2)[0]
```

2 Boussinesq $\nu = 0.5$

The same principle as above but with the classical Boussinesq formula:

$$\sigma_x = \frac{2P}{\pi} \frac{x^2 z}{(x^2 + z^2)^2}$$

```
In [4]: def LL_boussinesq(P, x, H, z):
        return (2.0 * P / pi) * x**2 * z / (x**2 + z**2)**2
```

```
In [5]: def SL_boussinesq(q, x1, x2, H, z):
        return quad(lambda x: LL_boussinesq(q, x, H, z), x1, x2)[0]
```

3 Original boussinesq with ν parameter

The original Boussinesq equation with poisson coefficient.

$$\sigma_x = \frac{Q}{2 \cdot \pi} \cdot \left(\frac{3 \cdot x^2 \cdot z}{R^5} - (1 - 2 \cdot \nu) \cdot \left(\frac{x^2 - y^2}{R \cdot r^2 \cdot (R + z)} + \frac{y^2 \cdot z}{R^3 \cdot r^2} \right) \right)$$

```
In [15]: def b(Q, x, y, z, nu):
          R = sqrt(x**2+y**2+z**2)
          r = sqrt(x**2+y**2)
          return Q/(2*pi)*(3*x**2*z/R**5-(1-2*nu)*((x**2-y**2)/(R*r**2*(R+z))+y**2*z/(R**3*r**2)))

          def LL_boussinesq_orgin(P,x,H,z,nu):
              return quad(lambda y: b(P, x, y, z, nu), -100.0,100.0)[0]

          #NOTE: function is way to slow
          def SL_boussinesq_orgin(q,x1,x2,H,z,nu):
              #return quad(lambda x: LL_boussinesq_orgin(q,x,H,z,nu),x1,x2)[0]
              return dblquad(lambda y, x: b(q, x, y, z, nu), x1, x2, \
                                  lambda x: -50.0, lambda x: 50.0)[0]
```

4 Example calculation: Point load

Wall height: 6.0 m Line load: 50 kN/m located 5 m behind the wall.

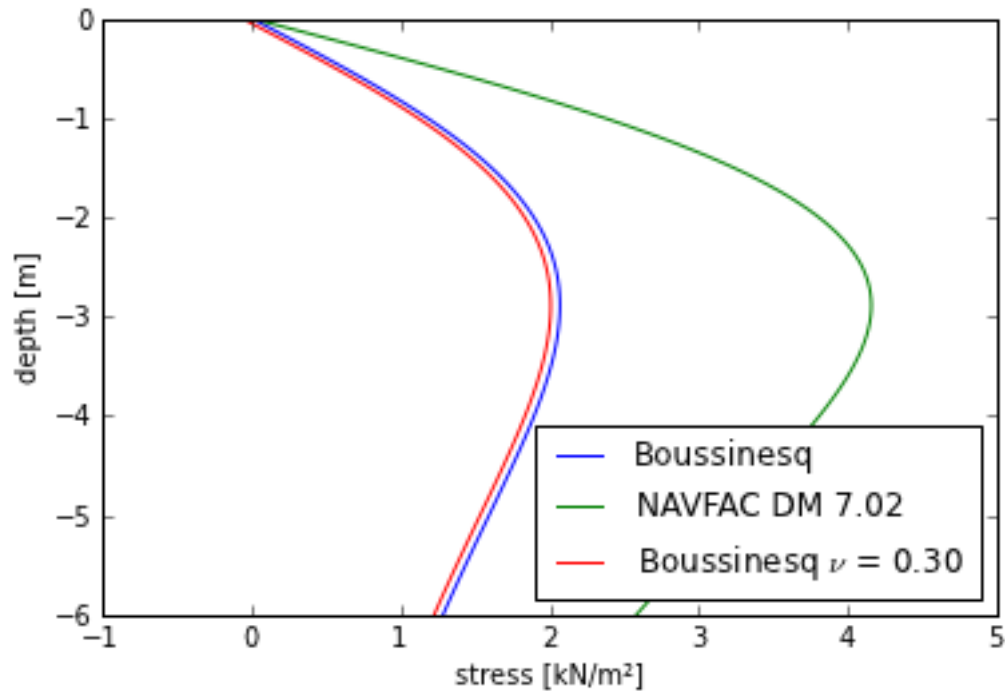
```
In [7]: P = 50.0 #load 50 kN/m
          x = 5.0 # 5 m from wall
          H = 6.0 # 6 m heigth wal
          nu =0.3
          z = np.arange(0.0,H+H/100.0, H/100.0)
          sigma1 = [LL_boussinesq(P,x,H,i) for i in z]
          sigma2 = [LL_navfac(P,x,H,i) for i in z]
          sigma3 = [LL_boussinesq_orgin(P,x,H,i,nu) for i in z]

          xlabel(u"stress [kN/m2"] )
          ylabel("depth [m]")

          plot(sigma1,-z,label="Boussinesq")
          plot(sigma2,-z,label="NAVFAC DM 7.02")
          plot(sigma3,-z,label="Boussinesq $\nu$ = %.2f" % nu)

          legend(loc=4)
          <matplotlib.legend.Legend at 0x3341350>
```

Out [7]:



The NAVFAC equations clearly uses a factor 2 for the mirror effect which was suggested by Midlin. (if $m > 0.4$) The results from the NAVFAC formula are closer to the Boussinesq ones for a load closer to the wall:

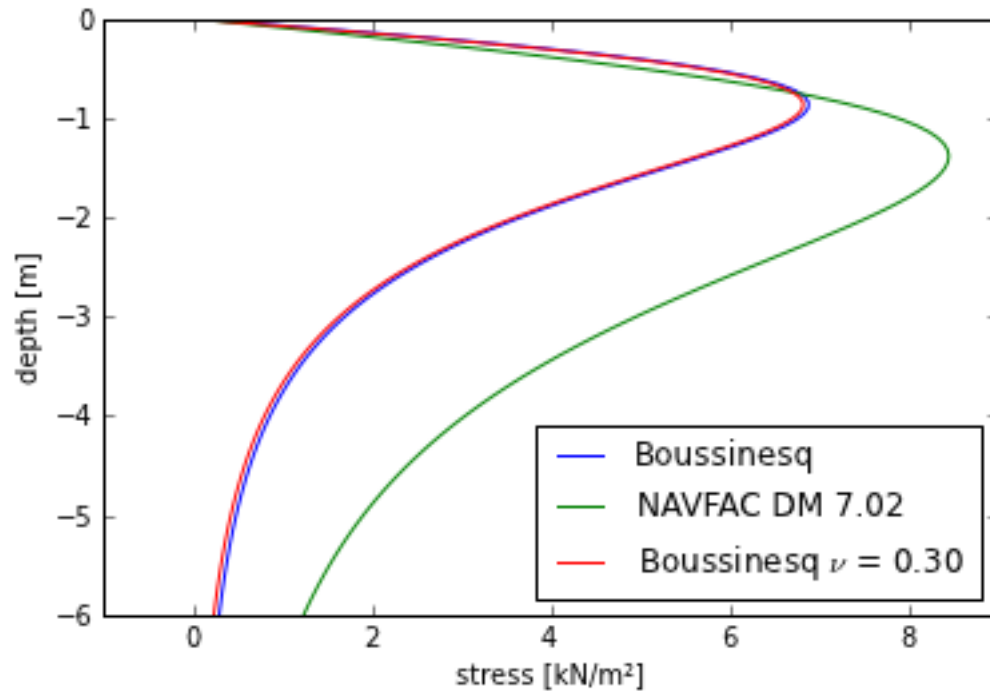
```
In [10]: P = 50.0 #load 50 kN/m
x = 1.5 # 1.5 m from wall
H = 6.0 # 6 m height wal
nu = 0.3
z = np.arange(0.0, H+H/100.0, H/100.0)
sigma1 = [LL_boussinesq(P,x,H,i) for i in z]
sigma2 = [LL_navfac(P,x,H,i) for i in z]
sigma3 = [LL_boussinesq_ordin(P,x,H,i,nu) for i in z]

xlabel(u"stress [kN/m²]")
ylabel("depth [m]")

plot(sigma1,-z,label="Boussinesq")
plot(sigma2,-z,label="NAVFAC DM 7.02")
plot(sigma3,-z,label="Boussinesq $\nu$ = %.2f" % nu)

legend(loc=4)
<matplotlib.legend.Legend at 0x4067690>
```

Out [10]:



5 Area load

The same comparison is done for an uniform area load, infinitely extended behind the wall. For example if we have a load of 50 kN/m² and a soil with $\phi = 30$ we would expect the following at rest pressure on the wall:

$$\phi = 30 \cdot \frac{\pi}{180} = 0.524$$

$$K_0 = 1 - \sin(\phi) = 0.500$$

$$\sigma_x = 50.0 \cdot K_0 = 25.000$$

We can do the same with the Boussinesq and the NAVFAC formula:

```
In [16]: Q = 50.0 #load 50 kN/m2
H = 6.0
x1 = 0.0 # load starting behind the wall
x2 = 50*H # 'infinitely' extended
nu = 0.50
z = np.arange(0.0, H+H/100.0, H/100.0)
sigma1 = [SL_boussinesq(Q, x1, x2, H, i) for i in z]
sigma2 = [SL_navfac(Q, x1, x2, H, i) for i in z]
sigma3 = [SL_boussinesq_origin(Q, x1, x2, H, i, nu) for i in z]

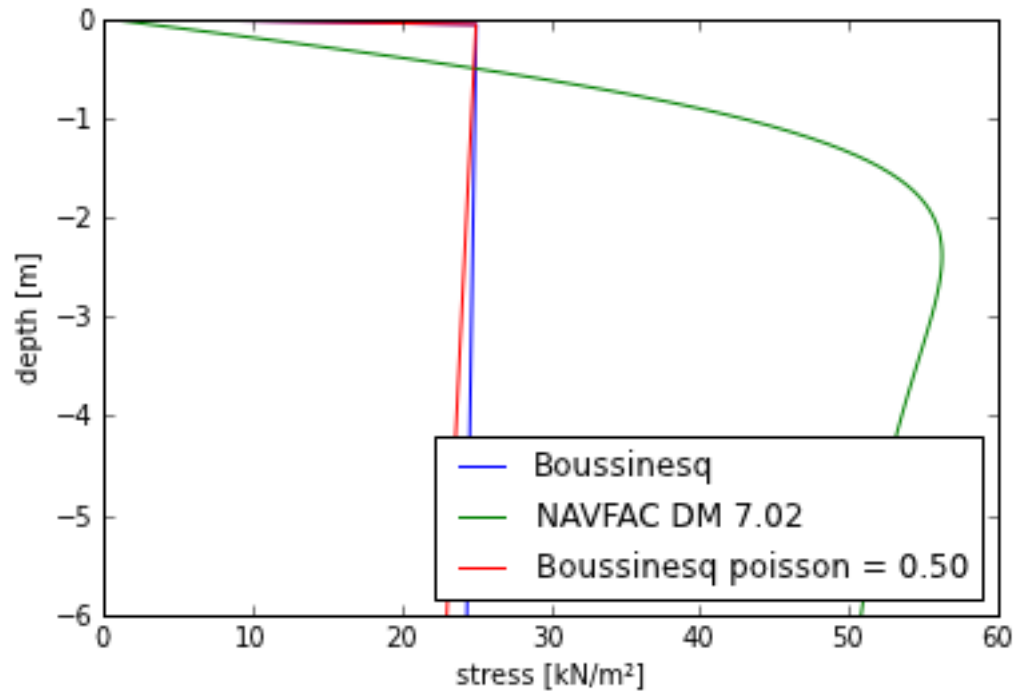
xlabel(u"stress [kN/m2]")
ylabel("depth [m]")

plot(sigma1, -z, label="Boussinesq")
plot(sigma2, -z, label="NAVFAC DM 7.02")
plot(sigma3, -z, label="Boussinesq poisson = %.2f" % nu)

legend(loc=4)
```

<matplotlib.legend.Legend at 0x7f5530b40dd0>

Out [16]:



Conclusion: The classical Boussinesq formula gives 25 kN/m^2 lateral pressure whilst the NAVFAC formula gives more than the double value.