

or introducing the value of h from Eq. (4-4),

$$h_1 = \left(\frac{1}{C_v^2} - 1 \right) \frac{v^2}{2g} \quad (4-12)$$

Path of Jet. Figure 4-3 illustrates a jet discharging from a vertical orifice under a head h . x and y are, respectively, abscissa and ordinate of any point m in the path of the jet. If v is the velocity in the vena contracta, at the end of time t ,

$$x = vt \quad (4-13)$$

From the law of falling bodies,

$$y = \frac{1}{2}gt^2 \quad (4-14)$$

and eliminating t from Eqs. (4-13) and (4-14),

$$x^2 = \frac{2v^2}{g} y \quad (4-15a)$$

Fig. 4-3. Path of jet.

which is the equation of a parabola with its vertex at the orifice. Since

$$v = C_v \sqrt{2gh} \quad (4-7)$$

Eq. (4-15a) can also be written

$$x^2 = 4(C_v^2 h) y \quad (4-15b)$$

Orifices under Low Heads. In deriving Eq. (4-10), the head producing discharge was assumed to be the head on the center of the orifice. Where the head on a vertical orifice is small in comparison with the height of the orifice, there is an appreciable difference between the true theoretical discharge and the discharge given by Eq. (4-10).

Figure 4-4 shows a rectangular orifice of width L and height M . h_1 and h_2 are the respective heads on the upper and lower edges of the orifice. Neglecting velocity of approach, the theoretical discharge through any elementary strip of area $L dy$,

discharging under a head y , is

$$dQ_t = L \sqrt{2gy} dy$$

which, integrated between the limits h_2 and h_1 , gives

$$Q_t = \frac{2}{3} L \sqrt{2g} (h_1^{3/2} - h_2^{3/2}) \quad (4-16)$$

When h_1 is zero

$$Q_t = \frac{2}{3} L \sqrt{2g} h_2^{3/2} \quad (4-17)$$

which is the theoretical formula, without velocity-of-approach correction, for discharge over a weir.

Equation (4-16) gives the theoretical discharge for rectangular orifices. A similar but more complicated formula could be derived for circular orifices. For $h_1 = M$, Eq. (4-10) gives results about 1 per cent greater than Eq. (4-16), and for $h_1 = 2M$, about 0.3 per cent greater.

Equation (4-10) is generally employed for all orifices, including those discharging under low heads, deviation from the theoretical form of the formula being corrected for in the coefficient.

Discharge under Falling Head. Figure 4-5 shows a vessel filled with water to a depth h_1 . The time required to lower the water surface to a depth h_2 is required. a is the area of orifice, and A is the area of water surface for a depth y . C is the coefficient of discharge. The increment of time dt required to lower the water the infinitesimal distance dy is

$$dt = \frac{A dy}{Ca \sqrt{2gy}} \quad (4-18)$$

From (4-18), if A can be expressed in terms of y , by integrating between limits h_1 and h_2 , the time needed to lower the water surface the distance $h_1 - h_2$ can be gotten. Placing $h_2 = 0$ gives the time of emptying the vessel. Equation (4-18) applies to horizontal or inclined orifices provided the water surface does not fall below the top

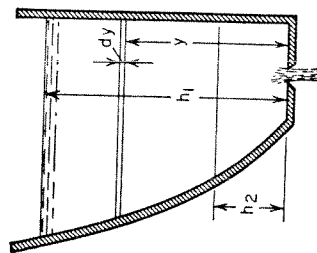


Fig. 4-5. Discharge under falling head.

of the orifice. For a cylinder or prism with vertical axis, A is constant, and Eq. (4-18), after integration, becomes

$$t = \frac{2.4}{Ca\sqrt{2g}} (\sqrt{h_1} - \sqrt{h_2}) \quad (4-19)$$

Orifice Coefficients. One of the earliest experimenters on sharp-edged orifices was Hamilton Smith, Jr.¹ His values of the coefficient of discharge for round and square orifices are given in Table 4-3. There have been many subsequent investigations of circular orifices, not all of which are in agreement.

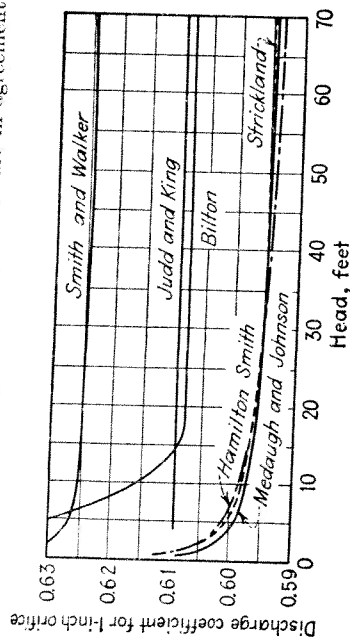


FIG. 4-6. Orifice coefficients.

Investigations by Medaugh and Johnson² check Smith's coefficients for orifices larger than $1/4$ in. in diameter within $1/3$ of 1 per cent. Values of the coefficient of discharge for a 1-in. orifice as determined by various investigators and plotted by Medaugh and Johnson are shown in Fig. 4-6. The differences between the values are undoubtedly not entirely due to experimental errors. Many other factors may contribute, as, for instance, the ratio of the orifice diameter to the dimensions of the tank wall, the sharpness of the edge of the orifice, the roughness of the inner surface, the orifice plate, and the temperature of the water. The effect of having the tank wall approach the orifice is to suppress contraction and therefore to make C approach the value of C_v .

¹ "Hydraulics," 1886.

² Medaugh and Johnson, Investigation of the Discharge and Coefficient of Small Circular Orifices, *Civil Eng.* (N.Y.), July, 1940, pp. 422-424.

Smith and Walker¹ found values of C_v to vary from 0.954 to 0.991 for orifices varying in diameter from 0.75 to 2.5 in., respectively. They also found a small variation with head, the above values being averages for heads varying from 1 to 60 ft.

Values of C_c for circular sharp-edged orifices were found to vary from approximately 0.67 for $3/4$ -in. orifices to 0.614 for 2.5-in. orifices when the head is 2 ft or more. Values are slightly larger for lower heads.

If there is suppression of contraction on one side and opportunity for complete contraction on the other sides, more water will approach with velocity components parallel to the face of the orifice on these sides and cause increased contraction. This to a large extent will compensate for loss of contraction on the other side. Williams² found, for rectangular orifices 30 in. wide and 2 to 4 in. high with full contraction at the top and completely suppressed contraction on the two sides and bottom, that the average coefficient of discharge was 0.607. This value corresponds closely to the coefficient for orifices with complete contraction. For orifices having full contraction at the top, one side a sharp edge 6 in. from the side of the channel, and contraction suppressed at one side and the bottom, Williams secured an average coefficient of 0.611. With the above orifices, except that the top was beveled to an angle of 45° , he obtained values of C of 0.776 and 0.755, respectively. Table 4-4, from results compiled by Smith,³ indicates the effect of "suppression of contraction" for small orifices. In this table, "suppressed contraction" means that the edge of the orifice coincides with the side of the channel, and "partly suppressed" means that the distance of the edge of the orifice from the side of the channel was 0.066 ft. A special case of suppressed contraction is the pipe orifice, which will be discussed later.

When the inner edge of the orifice is rounded, as in Fig. 4-2, contraction is suppressed, C_c approaches 1, and C approaches the value of C_v . Values of C_c for such orifices are approximately the same as for sharp-edged orifices.

Roughening the inner surface of the orifice plate retards the

¹ D. Smith and W. J. Walker, Orifice Flow, *Proc. Inst. Mech. Engrs.* (London), 1923, pp. 23-36.

² Unpublished experiments performed at the University of Michigan in 1928.

³ *Op. cit.*, pp. 65-67.