*see page 25

Checking, $\quad 1.0 \times a = \dfrac{44.38 \times P \times a \times L^2}{12 \times E_o I_o \times 64}$ $\quad$ and $\quad P_{cr} = \dfrac{17.4\ E_o I_o}{L^2}$

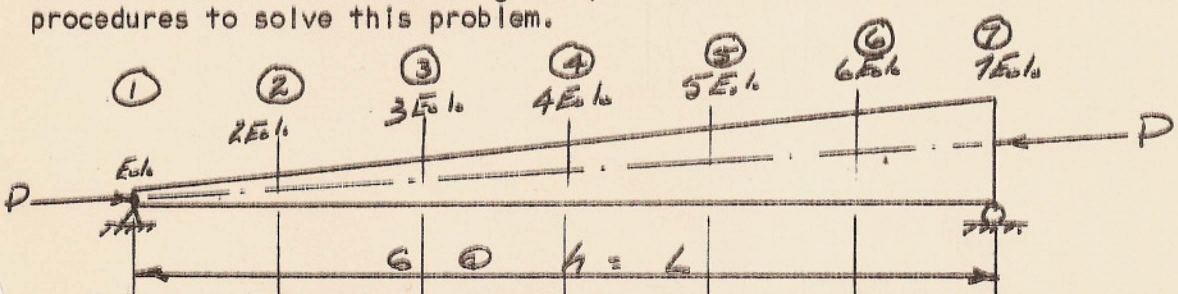
This value of P_{cr} is within 6% of actual value.

Checking at center, $\quad P_{cr} = \dfrac{16.2\ E_o I_o}{L^2}$.

From this example it is seen that when a cover plate is placed over center 50% column length, it is almost as effective as having cover plates over full length. In fact this type of column is about 85% as efficient as if the cover plates are extended over the full length.

(3-2) Columns with Uniformly Varying Moment of Inertia.

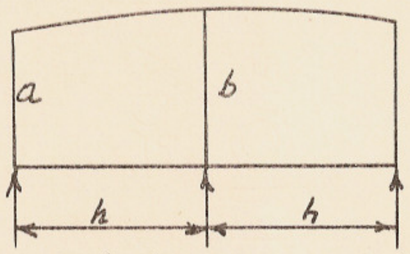
In columns with uniformly varying moment of inertia the numerical procedures show their advantages since columns of this type cannot be solved by the use of Dinnick Tables. The following example will demonstrate the use of the numerical procedures to solve this problem.



	①	②	③	④	⑤	⑥	⑦	
Δ_1	0	+1	+2	+2	+1.4	+0.8	0	$\times a$
M_1	0	+1	+2	+2	+1.4	+0.8	0	$\times Pa$
α	0	-0.5	-0.67	-0.5	-0.28	-0.133	0	$\times \frac{Pa}{E_oI_o}$
$\bar{\alpha}$	$\times$	-5.67	-7.70	-5.95	-3.43	-1.61	$\times$	$\times \frac{Pa}{E_oI_o} \times \frac{h}{12}$
ϕ	Assume +16	+10.33	+2.63	-3.32	-6.25	-8.36		$\times \frac{Pah}{12E_oI_o}$
y_1	0	+16	+26.33	+28.96	+25.64	+18.89	+10.53	$\times \frac{Pah^2}{12E_oI_o}$
y_a	0	-1.76	-3.52	-5.28	-7.04	-8.80	-10.53	$\times \frac{Pah^2}{12E_oI_o}$
y	0	+14.24	+22.81	+23.68	+18.60	+10.09	0	$\times \frac{Pah^2}{12E_oI_o}$
Δ_2	0	+1	+1.60	+1.66	+1.30	+0.71	0	$\times a$

The problem is to find the load that is critical in buckling for the beam shown which has a moment of inertia varying uniformly from I_o at one end to $7\ I_o$ at the other. Much work can often be saved if the buckled shape of the column is visualized and an intelligent guess is made as to the deflections Δ_1 . These deflections are then guessed and all multiplied by an arbitrary factor "a" which could be feet, inches, etc. The line M is then calculated by $M = P\Delta_1$. The line α is found by dividing the M at each panel point by the EI at that point. E.G. at point (3) $\alpha = \dfrac{2Pa}{3\ E_oI_o} = 0.67 \times Pa/E_oI_o$

The values of $\bar{\alpha}$ are found by the use of equivalent concentrations and Simpson's Rule. i.e.



$\frac{h}{24}(7a+6b-c)$ $\frac{h}{12}(a+10b+c)$ $\frac{h}{24}(7c+6b-a)$

The values for α at the various panel points are the values used for a, b and c . It is necessary to assume value for the slope so a value of +16 is used between panel points (1) and (2). Then on this basis the other slopes are calculated. For example, the slope between panel points (2) and (3) is $16 + \bar{\alpha}$ at (2) = $16 - 5.67 = 10.33\ Pa.h/12\ E_oI_o$. The deflections y_1 , based on the assumed values of $\emptyset$ calculated by $\Sigma\emptyset$ up to the point concerned times h . It is known that the deflection at point (1) = 0 so this is the starting point. E.G. to find y_1 at point (4)

$$\Sigma\ \emptyset \text{ up to point (4)} = 16.00 + 10.33 + 2.63 = 28.96$$

Therefore y_1 at (4) $= 28.96\ Pa.h^2/12\ E_oI_o$.

However, in this way y_1 at point (7) turns out to be $10.53\ Pa.h^2/12\ E_oI_o$. Since the support at (7) is non-yielding the deflection there must be 0. This error is due to the fact that the values of $\emptyset$ were merely assumed ones and necessitates the application of a linear correction which will be called y_c . Since the deflection at (7) = 0, $y_c = -y_1 = -10.53$. The values of y_c at the other points are found by simple ratio. The final deflection y is found by $y = y_1 + y_c$.

If these final deflections (y) are equal to the ones originally assumed (Δ_1), then the original assumptions were correct and $\Delta_1\ a = y(Pa.h^2/12\ E_oI_o)$ at every point on the column. Checking this at point (2)

$$1 \times a = \frac{12.24\ Pa.h^2}{12\ E_oI_o} \quad \text{and} \quad h = \frac{L}{6}$$

$$P = \frac{12\ E_oI_o \times 36}{14.24\ \ L^2} = \frac{30.3\ E_oI_o}{L^2}$$

At point (3)

$$2a = \frac{22.81\ Pa_1h^2}{12\ E_oI_o}$$

$$P = \frac{2 \times 12 \times 36\ E_oI_o}{22.81\ \ L^2} = \frac{37.8\ E_oI_o}{L^2}$$

In order to get a more accurate value for P the procedure is repeated using new values for Δ which will be called Δ_2. These values are found by ratio as follows:

At point (1) let $\Delta_2 = +1$

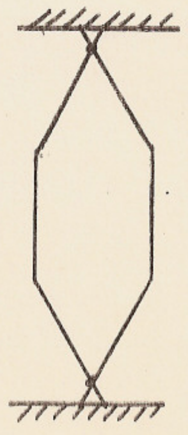
At point (2) $\Delta_2 = \frac{y\ \text{at}\ (2)}{y\ \text{at}\ (1)} = \frac{22.81}{14.24} = +1.60$

At point (3) $\Delta_2 = \frac{y\ \text{at}\ (3)}{y\ \text{at}\ (1)} = \frac{23.68}{14.24} = +1.66$

and so forth. The remainder of this problem will be left as an exercise for the reader. The value of P obtained by this second trial will be considerably more accurate than the value obtained by the first trial and P may be found to any degree of accuracy depending on how many trials are made.

Let us assume that $P = \frac{30.3\ E_oI_o}{L^2}$, although this is being conservative. A column with a moment of inertia of I_o would have a critical load of $P_{cr} = \pi^2 E_oI_o/L^2 = 9.87\ E_oI_o/L^2$. It is interesting to note that a column with moment of inertia varying from I_o to $7\ I_o$ has a critical load more than three times as great as the column with moment of inertia I_o. Thus, in calculating the critical load it would be more correct to use the average I than to use the minimum I. It can easily be seen from this problem and the previous one where a cover plate was put over the centre section of the column that the moment of inertia at the centre of the column is the important thing.

One problem which arises when a column of varying EI is used is the determination of the allowable stress on the column. Formulae for allowable stresses on columns usually give the allowable stress as a function of the L/r ratio. However, the expression L/r begins to lose its meaning when a column of the shape shown is encountered.



One possible solution to this problem is to find the critical load of the column shown and then to find the column with the same length but of uniform EI which will have the same critical load as the column shown.

The L/r ratio of this column with uniform EI is then used as the L/r ratio of the column shown for the purpose of calculating the allowable stress.

This discussion is not meant to be a course in numerical procedures but is meant to outline the basic principles used and to indicate some of the problems to which these procedures may be applied. Since many of these simpler problems may be solved more easily by other means, the numerical procedures do not show their true advantage. However, they are extremely useful in many more complicated problems, e.g. design of buildings to withstand earthquake vibrations.

*Diagram for top of page 22

					Sym ℄	
Δ_z	0	1.0	1.67	2.02	2.15	$\times a$
M_z	0	1.0	1.67	2.02	2.15	$\times Pa$
α_z	0	-1.0	-1.67	-2.02	-2.15	$\times \frac{Pa}{E_0 I_0}$
Σ_z	×	-11.67	-8.85 -5.43	-12.02	-12.82	$\times \frac{Pah}{12 E_0 I_0}$
ϕ_z		+44.38	+32.71	+18.43	+6.41	$\times \frac{Pah}{12 E_0 I_0}$
y_z	0	+44.38	+77.09	+95.52	+101.95	$\times \frac{Pah^2}{12 E_0 I_0}$