CONVENZIONI DI SEGNO

Nella compilazione delle tabelle e nella risoluzione degli esercizi sono state usate le seguenti convenzioni di segno e i seguenti accorgimenti:

1) metodo delle forze

a) Positive le rotazioni se orarie.

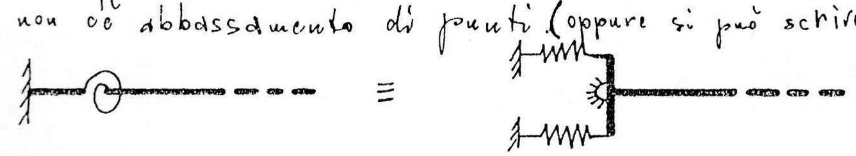
b) Rotazioni relative: $\varphi_{destra} - \varphi_{sinistra} = \varphi_{ik}$

c) Le molle vengono sempre isolate.

d) Se un momento M è applicato proprio nel nodo si risolve schematizzando come segue:

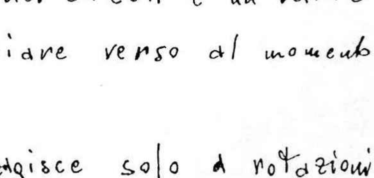
e) I momenti iperstatici "X" sono disegnati con la freccia rivolta verso il centro della trave; se il risultato dei calcoli è un valore negativo, significa solo che bisogna cambiare verso al momento stesso.

d) Quando c'è una molla "torsionale" questa reagisce solo a rotazioni con un momento $M = \mu\vartheta$ e non genera alcuna forza perché non c'è abbassamento di punti (oppure si può scrivere $\vartheta = kM$)



Viceversa una molla "lineare" produce solo delle forze dovute all'abbassamento del punto, ma non dà momenti dovuti a rotazione. ($\eta = kR$)

e) Quando c'è un nodo con tre aste i momenti iperstatici si mettono in evidenza come a lato:



2) metodo delle deformazioni

a) I momenti vengono considerati sulle estremità delle aste e sono considerati positivi se orari.

b) L'equilibrio dei nodi viene fatto considerando i momenti appli cati alle aste (è una forma di scrittura equivalente perchè cambiano di segno tutti i termini). Quindi se un momento è applicato proprio sul nodo, lo si introduce nell'equazione di equilibrio del nodo col segno cambiato.

c) Quando ci sono dei tratti di trave a sezione variabile, si pone il nodo nel punto in cui cambia la sezione:

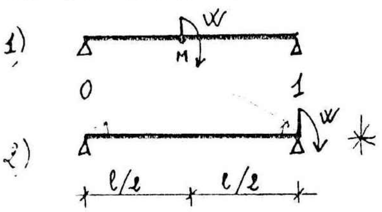
d) Quando ci sono dei cedimenti elastici schematizzati con molle "lineari", nell'equilibrio alla traslazione del nodo sono positive le reazioni se dirette verso l'alto e positive le forze se dirette verso il basso.

NB: dove compare l'espressione R_i^*, significa che, a meno di EJ, rappresenta la rotazione dell'estremo dell'asta quando ci fossero due appoggi, invece dei due incastri: $R_i^*/EJ = \varphi_i$

e) Quando la struttura è a nodi mobili, si introduce una biella fittizia che ne impedisca il movimento e poi si scrive una equazione di equilibrio alla traslazione ponendo $=0$ la reazione della biella fittizia.

Quando invece, nella direzione dello spostamento, c'è una molla, si scrive ancora l'equazione di equilibrio alla traslazione come si era fatto per la bielletta, ma in questo caso, però, non dobbiamo più porla $=0$ ma $=K\eta$ dove "η" è lo spostamento incognito.

TABELLE DEI CASI FONDAMENTALI



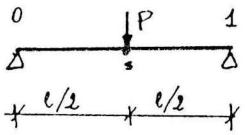
1) $\eta_M = 0$;

$\varphi_M = \dfrac{-W l}{16 EJ}$

$\varphi_0 = \varphi_1 = -\dfrac{W l}{24 EJ}$

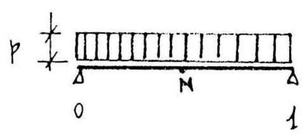
2) $\varphi_0 = \dfrac{-W l}{6 EJ}$

$\varphi_1 = \dfrac{+W l}{3 EJ}$



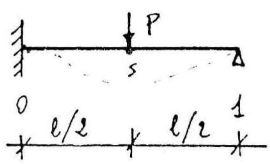
$\eta_s = \dfrac{P l^3}{48 EJ}$

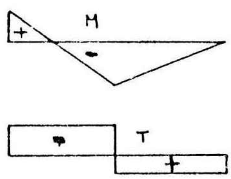
$\varphi_0 = -\varphi_1 = +\dfrac{P l^2}{16 EJ}$



$\eta_M = \dfrac{5 p l^4}{384 EJ}$

$\varphi_0 = -\varphi_1 = +\dfrac{p l^3}{24 EJ}$





$M_0 = -\dfrac{3}{16} P l$

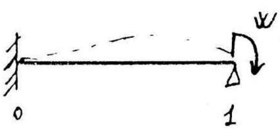
$M_s = \dfrac{5}{32} P l$

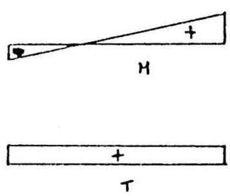
$T_0 = \dfrac{11}{16} P$

$T_1 = -\dfrac{5}{16} P$

$\varphi_0 = 0$

$\varphi_1 = -\dfrac{P l^2}{32 EJ}$





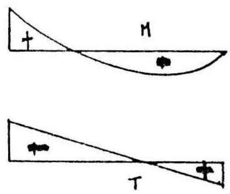
$M_0 = +\dfrac{W}{2}$

$M_1 = + W$

$T_0 = T_1 = -\dfrac{3 W}{2 l}$

$\varphi_0 = 0$

$\varphi_1 = +\dfrac{l}{4 EJ} W$



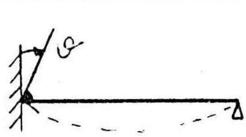
$M_0 = -\dfrac{p l^2}{8}$

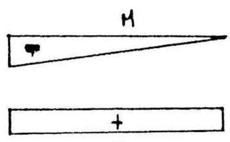
$T_0 = \dfrac{5 p l}{8}$

$T_1 = -\dfrac{3 p l}{8}$

$\varphi_0 = 0$

$\varphi_1 = -\dfrac{p l^3}{48 EJ}$





$M_0 = +\dfrac{3 EJ}{l} \vartheta$

$T_0 = T_1 = -\dfrac{3 EJ}{l^2} \vartheta$

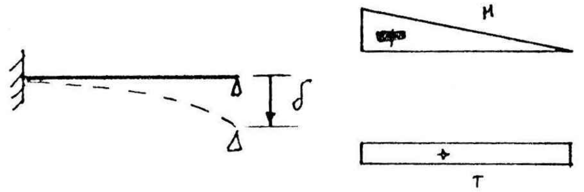
$\varphi_0 = +\vartheta$

$\varphi_1 = -\dfrac{\vartheta}{2}$

$\eta_1 = \dfrac{W l^2}{2 EJ}$

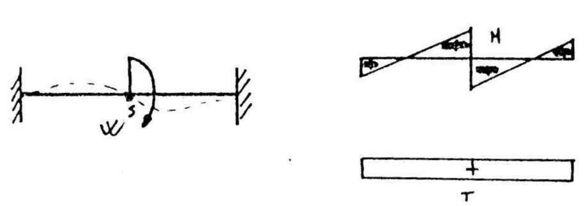
$\varphi_0 = 0$

$\varphi_1 = \dfrac{+W l}{EJ}$



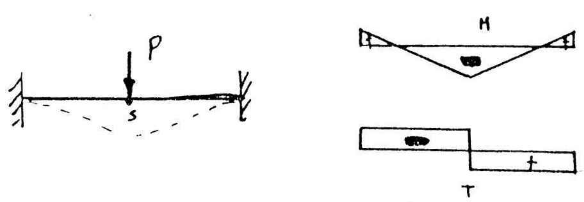
$M_0 = -\dfrac{3EJ}{\ell^2}\,\delta$ $\qquad \varphi_0 = 0$

$T_0 = T_1 = \dfrac{3EJ}{\ell^3}\,\delta$ $\qquad \varphi_1 = \dfrac{+3\,\delta}{2\,\ell}$



$+M_0 = M_1 = \dfrac{+W}{4}$ $\qquad \varphi_0 = \varphi_1 = 0$

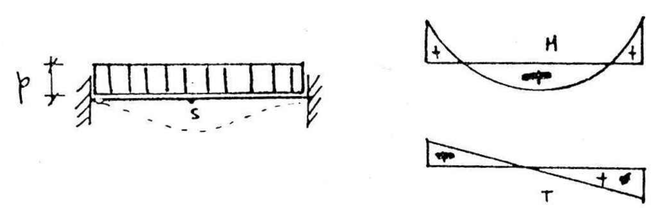
$T_0 = T_1 = -\dfrac{3\,W}{2\,\ell}$



$-M_0 = +M_1 = +\dfrac{P\ell}{8}$ $\qquad \varphi_0 = \varphi_1 = 0$

$M_s = \dfrac{P\ell}{8}$

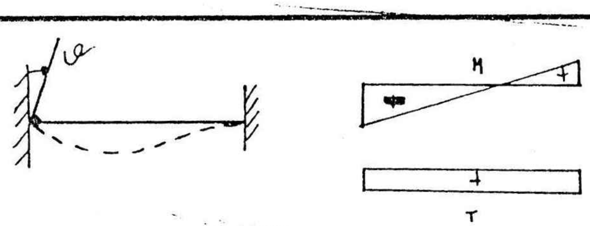
$T_0 = -T_1 = P/2$



$M_0 = -M_1 = -\dfrac{p\ell^2}{12}$ $\qquad \varphi_0 = \varphi_1 = 0$

$M_s = \dfrac{p\ell^2}{24}$

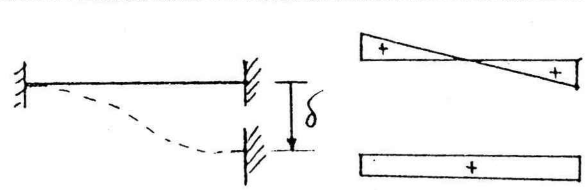
$T_0 = -T_1 = \dfrac{p\ell}{2}$



$M_0 = \dfrac{+4EJ}{\ell}\,\vartheta$ $\qquad \varphi_0 = -\vartheta$

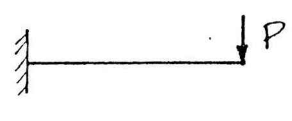
$M_1 = \dfrac{+2EJ}{\ell}\,\vartheta$ $\qquad \varphi_1 = 0$

$T_0 = T_1 = -\dfrac{6EJ}{\ell^2}\,\vartheta$



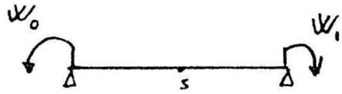
$M_0 = +M_1 = -\dfrac{6EJ}{\ell^2}\,\delta$ $\qquad \varphi_0 = \varphi_1 = 0$

$T_0 = T_1 = \dfrac{12\,EJ}{\ell^3}\,\delta$



$\eta_1 = \dfrac{P\ell^3}{3EJ}$ $\qquad \varphi_0 = 0$

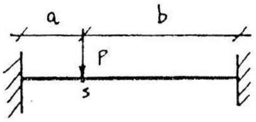
$\varphi_1 = \dfrac{+P\ell^2}{2\,EJ}$



se $W_0 = W_1$ $\qquad \varphi_0 = \dfrac{-\ell}{6EJ}\left(2W_0 + W_1\right)$

$\eta_s = \dfrac{-W\ell^2}{8EJ}$

$\varphi_1 = \dfrac{+\ell}{6EJ}\left(W_0 + 2W_1\right)$



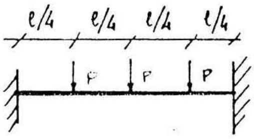
$$R_0^* = \frac{Pb(l^2 - b^2)}{6l}$$

$$R_1^* = \frac{-Pa(l^2 - a^2)}{6l}$$

$\varphi_0 = \varphi_1 = 0$

$$M_0 = \frac{-Pab^2}{l^2}$$

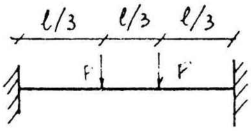
$$M_1 = +\frac{Pa^2 b}{l^2}$$



$\varphi_0 = \varphi_1 = 0$

$$R_0^* = -R_1^* = \frac{5Pl^2}{32}$$

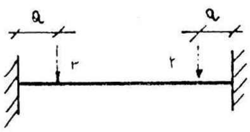
$$-M_0 = +M_1 = \frac{5Pl}{16}$$



$\varphi_0 = \varphi_1 = 0$

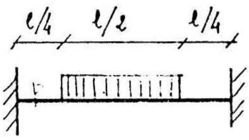
$$R_0^* = R_1^* = \frac{Pl^2}{9}$$

$$-M_0 = +M_1 = \frac{2Pl}{9}$$



$\varphi_0 = \varphi_1 = 0$

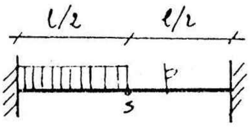
$$-M_0 = +M_1 = \frac{Pa(l-a)}{l}$$



$\varphi_0 = \varphi_1 = 0$

$$R_0^* = -R_1^* = \frac{11pl^3}{384}$$

$$-M_0 = +M_1 = \frac{11pl^2}{192}$$



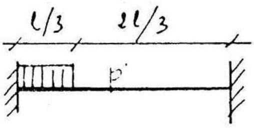
$$R_0^* = \frac{9pl^3}{384}$$

$$R_1^* = -\frac{7pl^3}{384}$$

$$M_s = \frac{pl^2}{48}$$

$$M_0 = \frac{-11pl^2}{192}$$

$$M_1 = +\frac{5pl^2}{192}$$

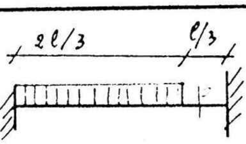


$$R_0^* = \frac{25pl^3}{1944}$$

$$R_1^* = -\frac{17pl^3}{1944}$$

$$M_0 = \frac{-11pl^2}{324}$$

$$M_1 = +\frac{3pl^2}{324}$$

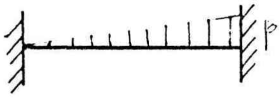


$$R_0^* = \frac{8pl^3}{243}$$

$$R_1^* = -\frac{7pl^3}{243}$$

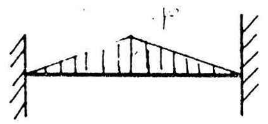
$$M_0 = \frac{-6pl^2}{81}$$

$$M_1 = +\frac{4pl^2}{81}$$



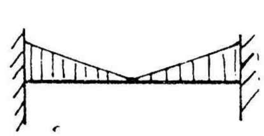
$$M_0 = \frac{-pl^2}{30} \qquad R_0^* = \frac{7Ql^2}{180}$$

$$M_1 = +\frac{pl^2}{20} \qquad R_1^* = -\frac{8Ql^2}{180}$$



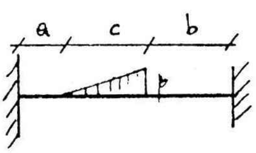
$$-M_0 = +M_1 = \frac{5pl^2}{96} \qquad R_0^* = \frac{5pl^3}{192}$$

$$R_1^* = -R_0^*$$



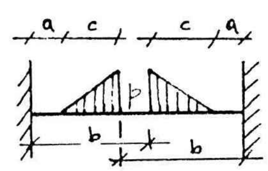
$$-M_0 = +M_1 = \frac{pl^2}{32}$$

$$R_0^* = -R_1^* = \frac{pl^3}{64}$$

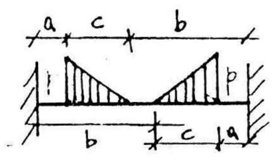


$$M_0 = \frac{-pc}{60l^2}\left[20bc(a+b) + 5c^2(a+2b) + 30ab^2 + 2c^3\right]$$

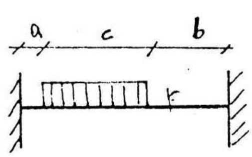
$$M_1 = +\frac{pc}{60l^2}\left[10ac(a+c) + 15b(2a^2+c^2) + 40abc + 3c^3\right]$$



$$-M_0 = +M_1 = \frac{pc}{12l}\left(6ab + 4bc + 2ac + c^2\right)$$

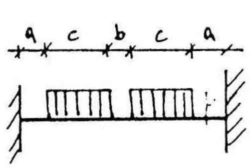


$$-M_0 = +M_1 = \frac{pc}{12l}\left(6ab + 4ac + 2bc + c^2\right)$$

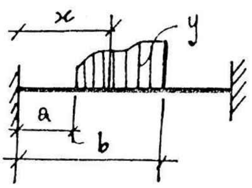


$$M_0 = \frac{-pc}{12l^2}\left[12ab(b+c) + 6b^2c + 4c^2(a+b) + c^3\right]$$

$$M_1 = +\frac{pc}{12l^2}\left[12ab(a+c) + 6a^2c + 4c^2(a+b) + c^3\right]$$

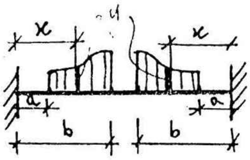


$$-M_0 = +M_1 = \frac{pc}{6l}\left[6a(l-a) + 3bc + 4c^2\right]$$

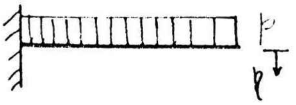


$$M_0 = \frac{-1}{l^2}\int_a^b y\,x\,(l-x)^2\,dx$$

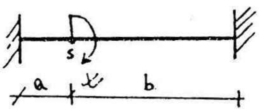
$$M_1 = +\frac{1}{l^2}\int_a^b y\,x^2\,(l-x)\,dx$$



$$-M_0 = +M_1 = \frac{1}{l}\int_a^b y\,x\,(l-x)\,dx$$

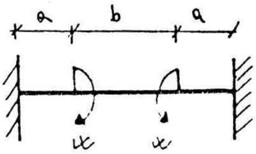


$$\eta = \frac{pl^4}{8EJ}$$



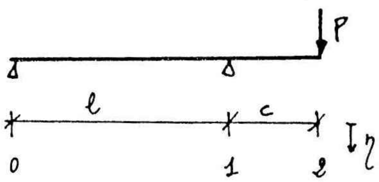
$$M_0 = \frac{Wb\,(2l-3b)}{l^2} \qquad R_0^+ = -\frac{W\,(l^2-3b^2)}{6l}$$

$$M_1 = \frac{Ma\,(2l-3a)}{l^2} \qquad R_1^+ = \frac{W\,(l^2-3a^2)}{6l}$$



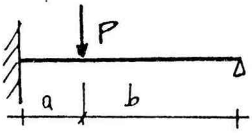
$$M_0 = \frac{Wb}{l}$$

$$M_1 = -\frac{Wb}{l}$$



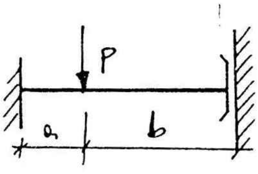
$$\eta_2 = \frac{Pc^2(l+c)}{3EJ} \qquad \varphi_0 = \frac{-Pcl}{6EJ}$$

$$\varphi_2 = -\frac{Pc}{6EJ}\,(2l+3c) \qquad \varphi_1 = +\frac{Pcl}{3EJ}$$

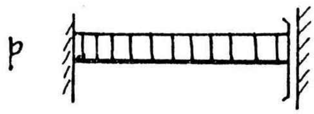


$$M_0 = \frac{-Pab}{l^2}\left(\frac{a}{2}+b\right)$$

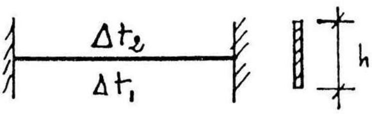
$$M_1 = 0$$



$$M_0 = \frac{-Pa\,(2b+a)}{2l}$$



$$M_0 = \frac{-p\,l^2}{3}$$



posto:

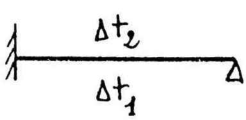
$\Delta t_1 > \Delta t_2$

e

$\Delta = \Delta t_1 - \Delta t_2$

$$M_0 = -\frac{\alpha\cdot\Delta\cdot EJ}{h}$$

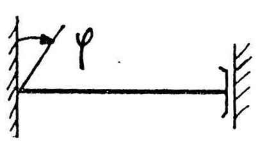
$$M_1 = +\frac{\alpha\Delta\cdot EJ}{h}$$



idem

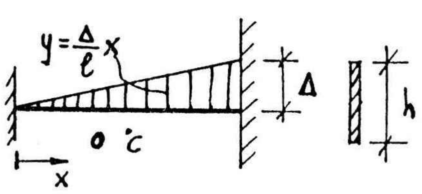
$$M_0 = -1{,}5\,\frac{\alpha\Delta\cdot EJ}{h}$$

$$M_1 = 0$$



$$M_0 = +\frac{EJ}{l}\varphi$$

$$M_1 = 0$$



$$M_0 = +\frac{1}{3}\,\frac{\alpha\Delta\cdot EJ}{h}$$

$$M_1 = -\frac{2}{3}\,\frac{\alpha\Delta\cdot EJ}{h}$$

BIBLIOGRAFIA :

Belluzzi Voll. I, II

F. Takabeya - Rahmentafeln - Berlin - Springer 1930

Manuale Colombo - 80° edizione