CHAPTER 15

WHIRLING OF SHAFTS

15.1 Whirling of shafts. Let the eccentricity of the centre of gravity G of a disc of weight W attached to a shaft be e, measured from the disc centre O, Fig. 15.1, and let y be the deflection of the shaft axis at the disc, measured from the static deflection position, when rotating at a speed ω rad/s.

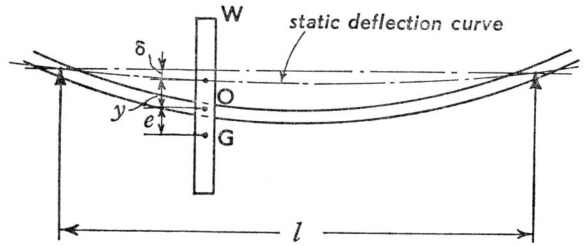

FIG. 15.1

Then centrifugal force on the shaft $= \dfrac{W}{g}\,\omega^2(y+e)$ and the inward pull exerted by the shaft $= \dfrac{kEI}{l^3}\,.\,y$, where k depends upon the position of the load and the end fixing conditions.

Equating these forces, $\dfrac{W}{g}\,\omega^2(y+e) = \dfrac{kEI}{l^3}\,y$

$$\therefore\ y = \frac{e}{\dfrac{kEIg}{W\omega^2 l^3} - 1} \qquad . \quad . \quad (15.1)$$

When $\dfrac{kEIg}{W\omega^2 l^3} = 1$, the deflection y becomes infinite and whirling takes place. The whirling or critical speed is therefore

$$\omega_c = \sqrt{\frac{kEIg}{Wl^3}} = \sqrt{\frac{g}{\delta}}\ \text{rad/s}$$

where δ denotes the static deflection of the shaft at the disc,

or $$N_c = \frac{60}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{187{\cdot}8}{\sqrt{\delta}}\ \text{rev/min}, \quad \text{where } \delta \text{ is in inches} \qquad (15.2)$$

This is the same as the frequency of transverse vibration of the same shaft, and similarly, the whirling speed of the shaft for any other system of loading is identical with the frequency of transverse vibration of the same shaft and loading.

If ω_c^2 be substituted for $\dfrac{kEIg}{Wl^3}$ in equation (15.1), this becomes

$$y = \frac{\omega^2 e}{\omega_c^2 - \omega^2} \qquad . \quad . \quad . \quad (15.3)$$

This gives the deflection of the shaft from the static deflection position at any speed ω in terms of the critical speed.

When $\omega < \omega_c$, y and e have the same sign, i.e. G lies to the outside of O. When $\omega > \omega_c$, y and e are of opposite sign, i.e. G lies between the centre of the rotating shaft and the static deflection curve. As the speed increases beyond ω_c, $y \to -e$, which means that G finally coincides with the static deflection curve.* Fig. 15.2 shows the posi-

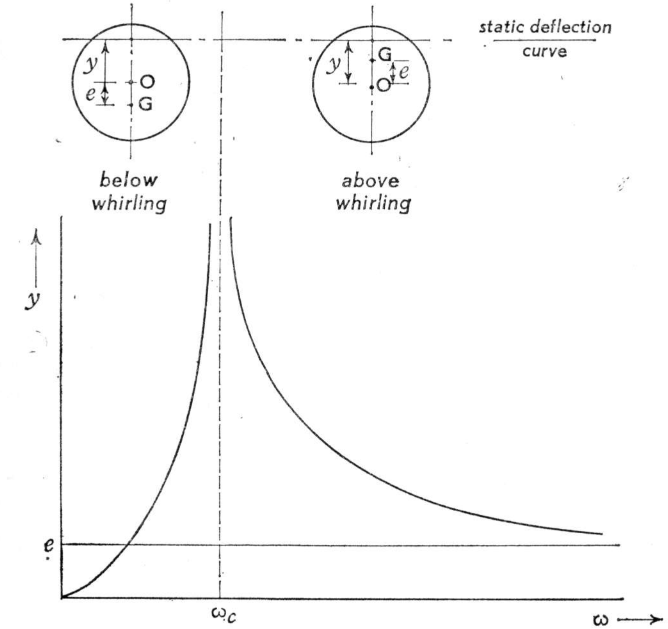

FIG. 15.2

tions of O and G relative to the static deflection curve and also the variation in the *numerical* value of y as ω varies from zero to above the whirling speed.

* A full treatment of the problem shows that at the whirling speed the radii to the centre of gravity of the disc and the centre of the rotating shaft are perpendicular and, as the speed increases beyond ω_c, the centre of gravity of the disc moves through a further 90° until it is again in the plane of bending. See *Theory of Vibrations*, E. B. Cole, p. 316, or *Applied Mechanics for Engineers*, Prof. Sir Charles Inglis, p. 325.