### 1.20 Calendar to Julian Date Conversion

## Introduction

This document provides the correct conversion from a Julian or Gregorian calendar date and fraction of a day to the Julian Date. The day of the week corresponding to a given calendar date is also calculated.

Astronomical days beginning at Greenwich Mean noon (12 UT) are numbered consecutively from an epoch far in the past. The ordinal number assigned to these days is the Julian Day Number which is defined to be 0 for the day starting at Greenwich mean noon on 1 January 4713 B.C. Astronomers and historians disagree on counting the years preceding year 1 (A.D. 1). In astronomy the year preceding year +1 is 0 , while in history, 1 B.C. Thus, for our purposes, 4713 B.C. corresponds to the year -4712.

The Julian Date (JD) corresponding to any instant is the Julian Day Number followed by the decimal fraction of the day elapsed since noon. The unit of day is the mean solar day.

The Julian Date can be expressed in Universal Time (UT) or Dynamical Time (DT). The term Julian Ephemeris Date (JED) was used prior to 1984 when the Julian Date was based upon Ephemeris Time (ET). Since then ET has been replaced with Terrestrial Dynamical Time (TDT). We will use the notation JED when either ET or TDT is used to express the time.

For many astronomical applications, in order to keep the numbers small, a Modified Julian Date (MJD) is used. MJ(E)D $=J(E) D-2400000.5$. Note that a day MJD begins at midnight of the civil day. Confusing, eh?

The Julian calendar (not to be confused with the Julian Date, which is named after Julius Scaliger, the father of the sixteenth century chronologist Joseph Justus Scaliger) was introduced by Julius Caesar in -45 (46 B.C.). That year is known as the "year of confusion". This calendar served as a standard, for better or worse, for European civilization until Pope Gregory's reform of +1582. Beginning 15 October 1582 (JD 2299160.5) the Gregorian calendar ("our" calendar) was used.

Besides its usefulness in astronomy by providing a continuous count of days, the Julian Date is useful for calculating the elapsed time between two calendar dates.

## Method

Following Meeus (op. cit.), the conversion of a calendar date to the Julian Date (JD) proceeds from the method below.
We write the calendar date as YYYY.MMDDdddddd
where

- YYYY = the year
- $M M=$ the month ( 1 for January, 2 for February, etc.)
- $\mathrm{DD}=$ the day of the month
- dddddd $=$ the decimal fraction of the day

Thus $08^{\text {h }} 22^{\mathrm{m}} 30^{\text {s }}$ on 2 January 1983 is written as
1983.0102348958

Note: January 0.0 is the same as December 31 of the preceding year.

Let

$$
\begin{array}{lr}
\mathrm{y}=\mathrm{YYYY}, & \text { if } \mathrm{M}>2 \\
\mathrm{y}=\mathrm{YYYY}-1, & \text { if } \mathrm{M} £ 2 \\
\mathrm{~m}=\mathrm{MM}, & \text { if } \mathrm{M}>2 \\
\mathrm{~m}=\mathrm{MM}+12, & \text { if } M M £ 2
\end{array}
$$

For Julian calendar dates ( YYYY.MMDD < 1582.1015)

```
JD = floor(365.25y) + floor(30.6001(m + 1)) + DD.dddddd + 1720994.5
```

where $\operatorname{floor}(\mathrm{x})$ is the integer part of the number $\mathbf{x}$.
For Gregorian calendar dates ( YYYY.MMDD ${ }^{3} 1582.1015$ )
$J D=$ floor(365.25y) + floor(30.6001(m + 1)) + DD.dddddd + 1720994.5 + b
where

$$
\begin{aligned}
& b=2-a+\operatorname{floor}(a / 4) \\
& a=\operatorname{floor}(y / 100)
\end{aligned}
$$

The relationship between the Julian Date and the start of the Besselian solar year (B) is given by

$$
J D=2433282.423+365.2422(B-1950.0)
$$

The relationship between the Julian Date and a Julian epoch $\mathbf{J}$ ( e.g., $\mathbf{J}=2000.0$ for $\mathbf{J} 2000.0$ ) is given by

$$
J D=365.25 \mathrm{~J}+1721045.0
$$

The day of the week corresponding to the date is obtained by

$$
w=\bmod (J D(Y Y Y Y . M M D D)+1.5,7)
$$

where $\bmod (x, y)$ is the remainder on dividing $x$ by $y$.
If $w=0$, it is Sunday, 1, Monday, 2 Tuesday, etc.

## Enter Data

Year: $\quad Y:=1985$

Month: $\quad \mathbf{M}:=2$
Day of the Month: $\mathbf{D}:=17$
Time of Day (UT;ET or TDT for JED):

$$
\left(\begin{array}{lll}
\mathbf{d}_{\mathbf{h r}} & \mathbf{d}_{\min } & \mathbf{d}_{\mathbf{s e c}}
\end{array}\right):=\left(\begin{array}{lll}
6 & 0 & 0
\end{array}\right)
$$

## Calculations

## Intermediate Variables

Convert to Decimal Notation
The functions necessary to convert between sexagesimal and decimal notations are defined past the right margin of the page.

$$
\mathbf{d}:=\operatorname{sexa} 2 \operatorname{dec}\left(\mathbf{d}_{\mathbf{h r}}, \mathbf{d}_{\min }, \mathbf{d}_{\mathbf{s e c}}\right)
$$

$$
\mathbf{p}:=\mathbf{Y}+\frac{\mathbf{M}}{100}+\frac{\mathbf{D}}{10000}
$$

If CAL = 1, Julian calendar, otherwise, Gregorian.

$$
\begin{aligned}
& \mathbf{C A L}:=\text { if }(\mathbf{p}<1582.1015,1,2) \\
& \mathbf{y}:=\text { if }(\mathbf{M}>2, \mathbf{Y}, \mathbf{Y}-1) \\
& \mathbf{m}:=\text { if }(\mathbf{M}>2, \mathbf{M}, \mathbf{M}+12) \\
& \mathbf{j d}:=\text { floor }(365.25 \cdot \mathbf{y})+\text { floor }[30.6001 \cdot(\mathbf{m}+1)]+\mathbf{D}+\frac{\mathbf{d}}{24}+1720994.5 \\
& \mathbf{a}:=\text { floor }\left(\frac{\mathbf{y}}{100}\right) \\
& \mathbf{b}:=2-\mathbf{a}+\mathbf{f l o o r}\left(\frac{\mathbf{a}}{4}\right)
\end{aligned}
$$

## Results

## Julian Dates

Julian date:

Note: If Ephemeris Time or Terrestrial Dynamical Time used then this is the JED.

$$
\begin{aligned}
& \text { JD }:=\text { if }(\mathbf{C A L}<2, \text { jd }, j d+b) \\
& \text { JD }=2446113.75
\end{aligned}
$$

Modified Julian date:

```
MJD := JD - 2400000.5
MJD = 46113.25
```

The day of the week:

|  | w | day of week |
| :--- | :---: | :--- |
| $\mathbf{w}:=\bmod \left(J D-\frac{d}{24}+1.5,7\right)$ | Monday |  |
|  | $\mathbf{1}$ | Tuesday |
| 2 | Wednesday |  |
| $\mathbf{w}=0.0000$ | 4 | Thursday |
|  | 5 | Friday |
|  | 6 | Saturday |
|  | 0 | Sunday |

Julian Date for the start of a Besselian solar year:

Besselian solar year: $\quad B:=2000.0$

$$
\begin{aligned}
& \text { JD }:=2433282.423+365.2422 \cdot(B-1950.0) \\
& \text { JD }=2451544.533
\end{aligned}
$$

Julian Date for a given Julian Epoch:
Julian epoch: $\quad \mathrm{J}:=2000.0$
$\mathrm{JD}:=365.25 \cdot \mathbf{J}+1721045.0$
$J D=2451545$

Sexagesimal (angles or time) to decimal
The conversion function is given by

$$
\boldsymbol{\operatorname { s e x } a 2 d e c}(\mathbf{a}, \mathbf{b}, \mathbf{c}):=\mathbf{a}+\frac{\mathbf{b}}{60}+\frac{\mathbf{c}}{3600}
$$

where
$a=$ integral number of degrees or hours
$b=$ integral number of arc minutes or minutes of time
$c=$ integral number of arc seconds or seconds of time

Decimal (angles or time) to sexagesimal
We must first define some intermediate functions:

$$
\begin{aligned}
& \text { hr_deg }\left(\mathbf{d} \_\mathbf{h}\right):=\text { floor }\left(\left|\mathbf{d} \_\mathbf{h}\right|\right) \\
& \min _{\text {man }}(\mathbf{d} \mathbf{h}):=\text { floor[(|d_h|-hr_deg(d_h)).60] } \\
& \sec (\mathbf{d} \mathbf{h}):=\left(\left|\mathbf{d} \_\mathbf{h}\right|-\mathbf{h r} \text { _deg }\left(\mathbf{d} \_\mathbf{h}\right)-\frac{\boldsymbol{\operatorname { m i n }}(\mathbf{d} \mathbf{-} \mathbf{h})}{60}\right) \cdot 3600
\end{aligned}
$$

where $d_{-} h$ is the angle or time in decimal notation.
The conversion function is then given by

$$
\left.\operatorname{dec} 2 \operatorname{sexa}\left(d_{-} h\right):=\left(\text { hr_deg }_{\left(d \_h\right.}\right) \quad \min \left(d_{-} h\right) \quad \sec \left(d_{-} h\right)\right) \cdot \frac{d_{-} h}{\left|d_{-} h\right|}
$$

## Unit Definitions:

```
hours \equiv1
degrees \equiv1
seconds =1
arc_seconds \equiv1
centuries =1
```

