BS ISO 6336-1:1996

Incorporating Corrigendum No. 1

Implementation of ISO 6336-1:1996

Calculation of load capacity of spur and helical gears —

Part 1: Basic principles, introduction and general influence factors

ICS 21.200

Committees responsible for this British Standard

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British Gear Association
British Horological Federation
British Horological Institute
Engineering Equipment and Materials Users' Association
Federation of Manufacturers of Construction Equipment and Cranes
Gauge and Tool Makers' Association
Institution of Mechanical Engineers
Lloyds Register of Shipping
London Underground Ltd.
Ministry of Defence
Power Generation Contractors' Association [PGCA (BEAMA Ltd.)]

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Contents

National foreword

This British Standard reproduces verbatim ISO 6336-1:1996 and implements it as the UK national standard.

This British Standard is published under the direction of the Engineering Sector Board whose Technical Committee MCE/5 has the responsibility to:

— aid enquirers to understand the text;

— present to the responsible international committee any enquiries on interpretation, or proposals for change, and keep UK interests informed;

— monitor related international and European developments and promulgate them in the UK.

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Summary of pages

This document comprises a front cover, an inside front cover, pages i and ii, the ISO title page, pages ii to iv, pages 1 to 90, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

INTERNATIONAL STANDARD

ISO 6336-1

First edition
1996-05-15

Calculation of load capacity of spur and helical gears —

Part 1:
Basic principles, introduction and general influence factors

Calcul de la capacité de charge des engrenages cylindriques à dentures droite et hélicoïdale —

Partie 1: Principes de base, introduction et facteurs généraux d'influence

Reference number
ISO 6336-1:1996(E)

Contents

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 6336-1 was prepared by Technical Committee ISO/TC 60, *Gears,* Subcommittee SC 2, *Gear capacity calculation.*

ISO 6336 consists of the following parts, under the general title *Calculation of load capacity of spur and helical gears:*

— *Part 1: Basic principles, introduction and general influence factors;*

— *Part 2: Calculation of surface durability (pitting);*

— *Part 3: Calculation of tooth bending strength;*

— *Part 5: Strength and quality of materials.*

Annex A to Annex D of this part of ISO 6336 are for information only.

Introduction

This part of ISO 6336 and parts 2, 3 and 5 provide the principles for a coherent system of procedures for the calculation of the load capacity of cylindrical involute gears with external or internal teeth. ISO 6336 is designed to facilitate the application of future knowledge and developments, also the exchange of information gained from experience.

Design considerations to prevent fractures emanating from stress raisers in the tooth flank, tip chipping and failures of the gear blank through the web or hub should be analyzed by general machine design methods.

Several methods for the calculation of load capacity, also for the calculation of various factors are permitted (see **4.1.8**). The directions in ISO 6336 are thus complex, but also flexible. As appropriate, the more detailed or simplified versions should be chosen for inclusion in application standards derived from this basic standard. Such application standards cover the following fields:

— industrial gears (detailed and simplified method);

— high-speed gears and gears of similar requirements;

— marine gears;

— vehicle gears.

These application standards feature clear, and to some extent simplified, rules for the calculations.

Included in the formulae are the major factors which are presently known to affect gear tooth pitting and fractures at the root fillet. The formulae are in a form that will permit the addition of new factors to reflect knowledge gained in the future.

1 Scope

1.1 Intended use

This part of ISO 6336, together with parts 2, 3 and 5, provides a method by which different gear designs can be compared. It is not intended to assure the performance of assembled drive gear systems. It is not intended for use by the general engineering public. Instead, it is intended for use by the experienced gear designer who is capable of selecting reasonable values for the factors in these formulae based on knowledge of similar designs and awareness of the effects of the items discussed.

The formulae in ISO 6336 are intended to establish a uniformly acceptable method for calculating the pitting resistance and bending strength capacity of cylindrical gears with straight or helical teeth.

ISO 6336 includes procedures based on testing and theoretical studies such as those of Hirt [1], Strasser [2], and Brossman [3]. The results of rating calculations made by following this method are in good agreement with previously accepted gear calculations methods (see references [4] through [8]), for normal working pressure angles up to 25° and reference helix angles up to 30°.

For larger pressure angles and larger helix angles the trends of products Y_F Y_S Y_β and respectively Z_H Z_ε Z_β are not the same as those of some earlier methods. The user of ISO 6336 is cautioned that when the methods in ISO 6336 are used for other helix angles and pressure angles, the calculated results should be confirmed by experience.

1.2 Exceptions

The formulae in ISO 6336 are not applicable when any of the following conditions exist:

— spur gears with transverse contact ratios less than 1,0;

— spur or helical gears with transverse contact ratios greater than 2,5;

— where interference exists between tooth tips and root fillets;

— when teeth are pointed;

— when backlash is zero.

The rating formulae in ISO 6336 are not applicable to other types of gear tooth deterioration such as plastic yielding, scuffing, case crushing, welding and wear, and are not applicable under vibratory conditions where there may be an unpredictable profile breakdown. The bending strength formulae are applicable to fractures at the tooth fillet, but are not applicable to fractures on the tooth working surfaces, failure of the gear rim, or failures of the gear blank through web and hub. ISO 6336 does not apply to teeth finished by forging or sintering. It is not applicable to gears which have a poor contact pattern.

The procedures in ISO 6336 provide for the calculation of load capacity, based on pitting and tooth-root breakage. At pitch line velocities below 1 m/s the gear load capacity is often limited by abrasive wear (see other literature for information on the calculation for this).

1.2.1 ***Scuffing***

Formulae for scuffing resistance on cylindrical gear teeth are not included in ISO 6336. At the present time, there is insufficient agreement concerning the method for designing cylindrical gears to resist scuffing failure.

1.2.2 ***Wear***

Very little attention and concern have been devoted to the study of gear tooth wear. This subject primarily concerns gear teeth with low surface hardness or gears with improper lubrication. No attempt has been made to cover the subject in ISO 6336.

1.2.3 ***Micropitting***

ISO 6336 does not cover micropitting, which is an additional type of surface distress that may occur on gear teeth.

1.2.4 ***Plastic yielding***

ISO 6336 does not extend to stress levels greater than those permissible at 10^3 cycles or less, since stresses in this range may exceed the elastic limit of the gear tooth in bending or in surface compressive stress. Depending on the material and the load imposed, a single stress cycle greater than the limit level at $< 10^3$ cycles could result in plastic yielding of the gear tooth.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this part of ISO 6336. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this part of ISO 6336 are encouraged to investigate the possibility of applying the most recent edition of the standards indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 53:1974, *Cylindrical gears for general and heavy engineering — Basic rack.*

ISO 468:1982, *Surface roughness — Parameters, their values and general rules for specifying requirements.*

ISO 701:1976, *International gear notation — Symbols for geometrical data.*

ISO 1122-1:1983, *Glossary of gear terms — Part 1: Geometrical definitions.*

ISO 1328-1:1995, *Cylindrical gears — ISO system of accuracy — Part 1: Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth.*

ISO 6336-2:1996, *Calculation of load capacity of spur and helical gears — Part 2: Calculation of surface durability (pitting).*

ISO 6336-3:1996, *Calculation of load capacity of spur and helical gears — Part 3: Calculation of tooth bending strength.*

ISO 6336-5:1996, *Calculation of load capacity of spur and helical gears — Part 5: Strength and quality of material.*

ISO/TR 10495:1996, *Calculation of cylindrical gears — Calculation of service life under variable load.*

3 Definitions, symbols and units

For the purposes of ISO 6336, the definitions given in ISO 1122-1 apply.

Symbols are based on and are extensions of the symbols given in ISO 701 and ISO 1328-1. Only symbols for quantities used for the calculation of the particular factors treated in ISO 6336 are given, together with preferred units. Table 1 list the symbols used in the calculations for all parts of ISO 6336.

Table 1 — Symbols and abbreviations used within ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5

Symbol	Description	Unit
Principal symbols and abbreviations		
a	center distance[a]	mm
b	facewidth	mm
c	constant	—
d	diameter (without subscript, reference diameter)	mm
e	auxiliary quantity	—
f	deviation, tooth deformation	µm
g	path of contact	mm
h	tooth depth (without subscript, root circle to tip circle)	mm
i	transmission ratio	—
k	addendum truncation factor	—
l	bearing span	mm
m	module, mass	mm kg
n	rotational speed	s^{-1} or min^{-1}
p	pitch, number of planet gears	mm —
q	auxiliary factor, flexibility of pair of meshing teeth, see clause **9** material allowance for finish machining, see clause **5** of ISO 6336-3	— (mm·µm)/N mm
r	radius (without subscript, reference radius)	mm
s	tooth thickness, distance between mid-plane of pinion and the middle of the bearing span	mm
u	gear ratio (z_2/z_1) $\geq$ 1[a]	—
v	tangential velocity (without subscript, at the reference circle ≈ tangential velocity at pitch circle)	m/s
w	specific load (per unit facewidth, F_t/b)	N/mm
x	profile shift coefficient	—
y	running-in allowance (only with subscript α or β	µm
z	number of teeth[a]	—
A, B, C, D, E	points on path of contact (pinion root to pinion tip, regardless of whether pinion or wheel drives, only for geometrical considerations)	—
B	total facewidth of double helical gear including gap width	mm
C	constant, coefficient, relief of tooth flank	— µm
D	diameter (design)	mm
E	modulus of elasticity	N/mm^2
Eh	material designation for case-hardening steel, case hardened	—
Eht	case depth, see ISO 6336-5	mm

[a] For external gearing a, z_1, and z_2 are positive; for internal gearing a and z_2 have a negative sign, z_1 has a positive.

Table 1 — Symbols and abbreviations used within ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5

Symbol	Description	Unit
F	composite and cumulative deviations, force or load	µm N
G	shear modulus	N/mm^2
GG	material designation for grey cast iron	—
GGG	material designation for cast iron (perlitic, bainitic, ferritic structure)	—
GTS	material designation for black malleable cast iron (perlitic structure)	—
HB	Brinell hardness	—
HRC	Rockwell hardness (C scale)	—
HR 30N	Rockwell hardness (30 N scale), see ISO 6336-5	—
HV	Vickers hardness	—
HV 1	Vickers hardness at load F = 9,81 N, see ISO 6336-5	—
HV 10	Vickers hardness at load F = 98,10 N, see ISO 6336-5	—
IF	material designation for steel and GGG, flame or induction hardened	—
J	Jominy hardenability, see ISO 6336-5	—
K	constant, factors concerning tooth load	—
L	lengths (design)	mm
M	moment of a force	Nm
MX ME MQ ML	symbols identifying material and heat-treatment requirements, see ISO 6336-5	—
N	number, exponent, number of load cycles, resonance ratio	—
NT	material designation for nitriding steel, nitrided	—
NV	material designation for through-hardening and case-hardening steel, nitrided (nitr.), nitrocarburized (nitrocar.)	—
P	transmitted power	kW
S	safety factor	—
St	material designation for steel (σ_B < 800 N/mm^2)	—
T	torque, tolerance	Nm µm
V	material designation for through-hardening steel, through-hardened ($\sigma_B \geq$ 800 N/mm^2)	—
W	weighing factor	—
Y	factor related to tooth-root stress	—
Z	factor related to contact stress	—
α	pressure angle (without subscript, at reference cylinder)	°
β	helix angle (without subscript, at reference cylinder)	°
γ	auxiliary angle, shear strain, pinion offset factor, see equations in clause **7**	° — µm

Table 1 — Symbols and abbreviations used within ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5

Symbol	Description	Unit
δ	deflection	μm
ε	contact ratio, overlap ratio, relative eccentricity (see clause **7**)	—
η	effective dynamic viscosity of the oil wedge at the mean temperature of wedge	mPa s
ϑ	temperature	°C
μ	coefficient of friction	—
ν	Poisson's ratio, kinematic viscosity of the oil	— mm²/s
ρ	radius of curvature, density (for steel, $\rho = 7{,}83 \times 10^{-6}$)	mm kg/mm³
σ	normal stress	N/mm²
τ	shear stress	N/mm²
χ	running-in factor	—
ψ	auxiliary angle, relative bearing clearance (see clause **7**)	° —
ω	angular velocity	rad/s
Γ	parameter on the line of action	—
Ψ	reduction of area on fracture	%
Subscripts to symbols		
—	reference values (without subscript)	
a	addendum, tooth tip	
ann	annulus gear	
b	base circle, facewidth	
be	bearing	
ca	case	
cal	calculated	
co	contact pattern	
dyn	dynamic	
e	outer limit of single pair tooth contact	
eff	effective value, real stress	
f	tooth-root, dedendum	
i	internal	
k	tooth truncation, values related to the notched test piece	
lim	value of reference strength	
m	mean or average value (mean section)	
ma	manufacturing	
max	maximum value	
min	minimum value	

Table 1 — Symbols and abbreviations used within ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5

Symbol	Description	Unit
n	normal plane, virtual spur gear of a helical gear, number of revolutions	
oil	oil	
p	pitch, values related to the smooth polished test piece	
par	parallel	
pla	planet gear	
r	radial	
red	reduced	
rel	relative	
s	tooth thickness, notch effect	
sh	shaft	
stat	static (load)	
sun	sun pinion, sun gear	
t	transverse plane	
th	theoretical	
v	velocity, losses	
w	working (this subscript may replace the apostrophe)	
y	running-in, any point on the tooth flank	
A	application, external shock loads	
C	pitch point, profile and helix modification	
D	speed transformation, reducing or increasing	
E	elasticity of material, resonance	
F	tooth-root stress	
G	geometry	
H	Hertzian stress (contact stress)	
L	lubrication	
M	material	
N	number (a specific number may be inserted after the subscript N in the life factor)	
P	permissible value, rack profile	
R	roughness, rows	
T	test gear, values related to the standard reference test gear	

Table 1 — Symbols and abbreviations used within ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5

Symbol	Description	Unit
W	pairing of materials	
X	dimension (absolute)	
z	sun	
α	transverse contact, profile	
β	helix, facewidth, crowning	
γ	total (total value)	
Δ	rough specimen	
ε	contact ratio	
0	basic value, tool	
1	pinion	
2	wheel	
1..9	general numbering	
I(II)	end relief, reference (nonreference) face	
′	single-flank (subscript w possible) single-pair tooth contact	
″	double-flank contact (simultaneous contact between working and non-working flanks)	
Combined symbols		
b_{be}	length of journal bearing	mm
b_{cal}	calculated facewidth (Figure 9)	mm
b_{c0}	length of tooth bearing pattern at no load (contact marking)	mm
b_{red}	reduced facewidth (facewidth minus end reliefs)	mm
b_s	web thickness	mm
b_B	facewidth of one helix on a double helical gear	mm
$b_{I(II)}$	length of end relief	mm
c_P	bottom clearance between basic rack profile and mating profile	mm
c_γ	mean value of mesh stiffness per unit facewidth	N/(mm·µm)
c'	maximum tooth stiffness per unit facewidth (single stiffness) of a tooth pair	N/(mm·µm)
c'_{th}	theoretical single stiffness	N/(mm·µm)
d_a	tip diameter	mm
d_b	base diameter	mm
d_e	diameter of circle through outer point of single pair tooth contact	mm
d_f	root diameter	mm
d_{f2}	root diameter of internal gear	mm
d_{sh}	external diameter of shaft, nominal for bending deflection	mm

Table 1 — Symbols and abbreviations used within ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5

Symbol	Description	Unit
d_{shi}	internal diameter of a hollow shaft	mm
d_w	pitch diameter	mm
d_B	ball diameter (ball bearing)	mm
$d_{1,2}$	reference diameter of pinion (or wheel)	mm
f_{be}	component of equivalent misalignment[a] due to bearing deformation	µm
f_{ca}	component of equivalent misalignment[a] due to case deformation	µm
$f_{f\alpha}$	profile form deviation (the value for the total profile deviation F_α may be used alternatively for this, if tolerances complying with ISO 1328-1 are used)	µm
f_{ma}	mesh misalignment[a] due to manufacturing deviations	µm
f_p	transverse single pitch deviation	µm
f_{par}	non-parallelism of pinion and wheel axes (manufacturing deviation)[a]	µm
f_{pb}	transverse base pitch deviation (the values of f_{pt} may be used for calculations in accordance with ISO 6336, using tolerances complying with ISO 1328-1)	µm
f_{sh}	component of equivalent misalignment[a] due to deformations of pinion and wheel shafts	µm
f_{shT}	component of misalignment due to shaft and pinion deformation measured at a partial load	µm
f_{sh0}	shaft deformation under specific load[a]	µm·mm/N
$f_{H\beta}$	helix slope deviation (the value for the total helix deviation F_β may be used alternatively for this, if tolerances complying with ISO 1328-1-are used)	µm
$f_{H\beta 6}$	tolerance on helix slope deviation for ISO accuracy grade 6	µm
g_α	length of path of contact	mm
h_{aP}	addendum of basic rack of cylindrical gears	mm
h_{a0}	tool addendum	mm
h_{fP}	dedendum of basic rack of cylindrical gears	mm
h_{f0}	tool dedendum	mm
h_{f2}	dedendum of tooth of an internal gear	mm
h_{min}	minimum lubricant film thickness	mm
h_F	bending moment arm for tooth-root stress	mm
h_{Fa}	bending moment arm relevant to load application at the tooth tip (defined by the contact point of the 30° tangents)	mm
h_{Fe}	bending moment relevant to load application at the outer point of single pair tooth contact	mm
l_a	effective length of roller (roller bearings)	mm
m^*	relative individual gear mass per unit facewidth referenced to line of action	kg/mm
m_n	normal module	mm

[a] The components in the plane of action are determinant.

Table 1 — Symbols and abbreviations used within ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5

Symbol	Description	Unit
m_{red}	reduced gear pair mass per unit facewidth referenced to the line of action	kg/mm
m_t	transverse module	mm
$n_{1,2}$	rotation speed of pinion (or wheel)	min^{-1} or s^{-1}
n_E	resonance speed	min^{-1}
p_{bn}	normal base pitch	mm
p_{bt}	transverse base pitch	mm
q'	minimum value for the flexibility of a pair of meshing teeth	(mm·µm)/N
q_{pr}	protuberance of the tool, see Figure 2 of ISO 6336-3	mm
q_s	notch parameter, $q_s = s_{Fn}/2\rho_F$	—
q_{sk}	notch parameter of the notched test piece	—
q_{sT}	notch parameter of the standard reference test gear, $q_{sT} = 2{,}5$	—
q_α	auxiliary factor	—
s_c	film thickness of marking compound used in contact pattern determination	µm
s_{pr}	residual fillet undercut, $s_{pr} = q_{pr} - q$	mm
s_{Fn}	tooth-root chord at the critical section	mm
s_R	rim thickness	mm
t_g	maximum depth of grinding notch	mm
w_m	mean specific load (per unit facewidth)	N/mm
w_t	tangential force per unit tooth width, including overload factors	N/mm
x_E	rack shift coefficient for adjustment of tooth thickness	—
$x_{1,2}$	addendum modification coefficient of pinion (or wheel)	—
y_α	running-in allowance for a gear pair	µm
y_β	running-in allowance (equivalent misalignment)	µm
z_n	virtual number of teeth of a helical gear	—
$z_{1,2}$	number of teeth of pinion (or wheel)[a], see page 3	—
B^*	constant, see equations in clause **7**	—
C_a	tip relief	µm
C_B	basic rack factor (same rack for pinion and wheel)	—
C_{B1}	basic rack factor (pinion), see **9.3.1.4**	—
C_{B2}	basic rack factor (wheel), see **9.3.1.4**	—
C_M	correction factor, see clause **9**	—
C_R	gear blank factor, see clause **9**	—
$C_{ZL, ZR, Zv}$	factors for determining lubricant film factors, see **11.2** of Part 2	—
C_β	crowning height	µm
$C_{I(II)}$	end relief	µm
D_{be}	bearing bore diameter (plain bearings)	mm
D_{sh}	journal diameter (plain bearings)	mm
$F_{be\ r}$	radial force on bearing	N

Table 1 — Symbols and abbreviations used within ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5

Symbol	Description	Unit
F_{bn}	(nominal) load, normal to the line of contact	N
F_{bt}	nominal transverse load in plane of action (base tangent plane)	N
F_m	mean transverse tangential load at the reference circle relevant to mesh calculations, $F_m = (F_t\ K_A\ K_v)$	N
$F_{m\ T}$	mean transverse tangential part load at reference circle	N
F_{max}	maximum tangential tooth load for the mesh calculated	N
F_t	(nominal) transverse tangential force at reference cylinder	N
F_{tH}	determinant tangential load in a transverse plane for $K_{H\alpha}$ and $K_{F\alpha}$, $F_{tH} = F_t\ K_A\ K_v\ K_{H\beta}$	N
F_α	total profile deviation	µm
F_β	total helix deviation	µm
$F_{\beta 6}$	tolerance on total helix deviation for ISO accuracy grade 6	µm
$F_{\beta x}$	initial equivalent misalignment (before running-in)	µm
$F_{\beta x\ cv}$	initial equivalent misalignment for the determination of the crowning height (estimate)	µm
$F_{\beta x\ T}$	equivalent misalignment measured under a partial load	µm
$F_{\beta y}$	effective equivalent misalignment (after running-in)	µm
K'	constant for the pinion position in relation to the torqued end	—
K_v	dynamic factor	—
K_A	application factor	—
$K_{F\alpha}$	transverse load factor (root stress)	—
$K_{F\beta}$	face load factor (root stress)	—
$K_{H\alpha}$	transverse load factor (contact stress)	—
$K_{H\beta}$	face load factor (contact stress)	—
K_γ	mesh load factor (takes into account the uneven distribution of the load between meshes for multiple transmission paths)	—
J^*	moment of inertia per unit facewidth	kg·mm²/mm
N_B	number of balls (or rollers) per row	—
N_F	exponent	—
N_L	number of load cycles	—
N_M	number of mesh contacts per revolution (normally 1, for idler 2)	—
N_R	number of rows per bearing	—
N_S	resonance ratio in the main resonance range	—
N_w	number of webs	—
R_a	arithmetic mean roughness value, $R_a = 1/6\ R_z$	µm
R_z	mean peak-to-valley roughness (as specified in ISO 468)	µm
R_{zk}	mean peak-to-valley roughness of the notched, rough test piece	µm
R_{zT}	mean peak-to-valley roughness of the standard reference test gear, $R_{zT} = 10$	µm
S_F	factor of safety from tooth breakage	—

Table 1 — Symbols and abbreviations used within ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5

Symbol	Description	Unit
S_H	factor of safety from pitting	—
So	Sommerfeld number	—
$T_{1,2}$	nominal torque at the pinion (or wheel)	N·m
Y_F	tooth form factor, for the influence on nominal tooth-root stress with load applied at the outer point of single pair tooth contact	—
Y_{Fa}	form factor, for the influence on nominal tooth-root stress with load applied at the tooth tip	—
Y_{FS}	tip factor, equal (Y_{Fa} Y_{Sa}), accounts for influences covered by Y_{Fa} and Y_{Sa}	—
Y_{Nk}	life factor for tooth-root stress, relevant to the notched test piece	—
Y_{Np}	life factor for tooth-root stress, relevant to the plain polished test piece	—
Y_{NT}	life factor for tooth-root stress for reference test conditions	—
Y_R	tooth-root surface factor (relevant to the plain polished test piece)	—
$Y_{R\ rel\ k}$	relative roughness factor, the quotient of the gear tooth-root surface factor of interest divided by the notch test piece factor, $Y_{R\ rel\ k} = Y_R/Y_{Rk}$	—
$Y_{R\ rel\ T}$	relative surface factor, the quotient of the gear tooth-root surface factor of interest divided by the tooth-root surface factor of the reference test gear, $Y_{R\ rel\ T} = Y_R/Y_{RT}$	—
Y_S	stress correction factor, for the conversion of the nominal bending stress, determined for application of load at the outer point of single pair tooth contact, to the local tooth-root stress	—
Y_{Sa}	stress correction factor, for the conversion of the nominal bending stress determined for load application at the tooth tip, to the local tooth-root stress	—
Y_{Sag}, Y_{Sg}	stress correction factors for teeth with grinding notches	—
Y_{Sk}	stress correction factor, relevant to the notched test piece	—
Y_{ST}	stress correction factor, relevant to the dimensions of the reference test gears	—
Y_X	size factor (tooth-root)	—
Y_β	helix angle factor (tooth-root)	—
Y_δ	notch sensitivity factor of the actual gear (relative to a polished test piece)	—
$Y_{\delta k}$	sensitivity factor of a notched test piece, relative to a smooth polished test piece	—
$Y_{\delta T}$	sensitivity factor of the standard reference test gear, relative to the smooth polished test piece	—
$Y_{\delta\ rel\ k}$	test relative notch sensitivity factor, the quotient of the gear notch sensitivity factor of interest divided by the notched test piece factor, $Y_{\delta\ rel\ k} = Y_\delta/Y_{\delta k}$	
$Y_{\delta\ rel\ T}$	relative notch sensitivity factor, the quotient of the gear notch sensitivity factor of interest divided by the standard test gear factor, $Y_{\delta\ rel\ T} = Y_\delta/Y_{\delta T}$	—
Y_ε	contact ratio factor (tooth-root)	—
Z_v	velocity factor	—
Z_B, Z_D	single pair tooth contact factors for the pinion, for the wheel	—
Z_E	elasticity factor	$\sqrt{N/mm^2}$
Z_H	zone factor	—
Z_L	lubricant factor	—
Z_N	life factor for contact stress	—

Table 1 — Symbols and abbreviations used within ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5

Symbol	Description	Unit
Z_{NT}	life factor for contact stress for reference test conditions	—
Z_R	roughness factor affecting surface durability	—
Z_W	work-hardening factor	—
Z_x	size factor (pitting)	—
Z_β	helix angle factor (pitting)	—
Z_ε	contact ratio factor (pitting)	—
α_{an}	tip pressure angle of a virtual spur gear	°
α_{en}	form-factor pressure angle, pressure angle at the outer point of single pair tooth contact of virtual spur gears	°
α_n	normal pressure angle	°
α_t	transverse pressure angle	°
α'_t or α_{wt}	pressure angle at the pitch cylinder	°
α_{Fan}	tip load angle, angle relevant to direction of application of load at the tooth tip of virtual spur gears	°
α_{Fen}	load direction angle, relevant to direction of application of load at the outer point of single pair tooth contact of virtual spur gears	°
α_{Pn}	normal pressure angle of the basic rack for cylindrical gears	°
β_a	tip helix angle (at the tip surface of a gear)	°
β_b	base helix angle	°
β_e	form-factor helix angle, helix angle at the outer point of single tooth contact	°
$\delta_{1,2}$	deformation of bearing (1, 2) in direction of load	µm, mm
$\delta_{b\ th}$	combined deflection of mating teeth assuming even load distribution over the facewidth	µm
δ_g	difference in feeler gauge thickness measurement of mesh misalignment f_{ma}	µm
δ_S	elongation on fracture	%
ε_α	transverse contact ratio	—
$\varepsilon_{\alpha n}$	virtual contact ratio, transverse contact ratio of a virtual spur gear	—
ε_β	overlap ratio	—
ε_γ	total contact ratio, $\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta$	—
ε_1	addendum contact ratio of the pinion, $\varepsilon_1 = CE/p_{bt}$	—
ε_2	addendum contact ratio of the wheel, $\varepsilon_2 = AC/p_{bt}$	—
ρ_{a0}	tip radius of the tool	mm
ρ_{fP}	root fillet radius of the basic rack for cylindrical gears	mm
ρ_g	radius of grinding notch	mm
ρ_{rel}	radius of relative curvature	mm
ρ_C	radius of relative curvature at the pitch surface	mm
ρ_F	tooth-root radius at the critical section	mm
ρ'	slip-layer thickness	mm
$\sigma_{k\ lim}$	nominal notched-bar stress number (bending)	N/mm^2

Table 1 — Symbols and abbreviations used within ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5

Symbol	Description	Unit
$\sigma_{p\ lim}$	nominal plain-bar stress number (bending)	N/mm²
σ_B	tensile strength	N/mm²
σ_F	tooth-root stress	N/mm²
$\sigma_{F\ lim}$	nominal stress number (bending)	N/mm²
σ_{FE}	allowable stress number (bending), $\sigma_{FE} = \sigma_{F\ lim}\, Y_{ST}$	N/mm²
σ_{FG}	tooth-root stress limit	N/mm²
σ_{FP}	permissible tooth-root stress	N/mm²
σ_{F0}	nominal tooth-root stress	N/mm²
σ_H	calculated contact stress	N/mm²
$\sigma_{H\ lim}$	allowable stress number (contact)	N/mm²
σ_{HG}	modified allowable stress number, $\sigma_{HG} = \sigma_{H\ lim}\, S_H$	N/mm²
σ_{HP}	permissible contact stress	N/mm²
σ_{H0}	nominal contact stress	N/mm²
σ_S	yield stress	N/mm²
$\sigma_{0,2}$	proof stress (0,2 % permanent set)	N/mm²
χ^*	relative stress gradient in the root of a notch	mm⁻¹
χ_β	factor characterizing the equivalent misalignment after running-in	—
χ^*_p	relative stress gradient in a smooth polished test piece	mm⁻¹
$\omega_{1,2}$	angular velocity of pinion (or wheel)	rad/s

4 Basic principles

4.1 Application

Refer to **1.1** for intended use.

4.1.1 *Particular categories*

Pitting resistance and bending strength rating systems for a particular category of cylindrical gearing may be established by selecting proper values for the factors used in these general formulae.

4.1.2 *Specific applications*

For the design of gears it is very important to recognize that requirements for different fields of application vary considerably. Use of ISO 6336 procedures for specific applications demands a realistic and knowledgeable appraisal of all applicable considerations, particularly of the:

— allowable stress of the material and the number of load repetitions;

— consequences of any percentage of failure (failure rate);

— appropriate factor of safety.

The following three application fields exemplify the requirements of the above mentioned characteristics.

4.1.2.1 Vehicle final drive gears, which are relatively low speed, coarse pitch teeth are chosen for adequate strength. As a consequence, pinions have small numbers of teeth (z_1 of about 14), whereas a value z_1 of about 28 would be chosen for a comparatively high speed gear of similar size. Thus, the tooth bending strength of the former would be about twice that of the latter.

The computed reliability of vehicle gears can be as low as 80 % to 90 % whereas that of high speed industrial gears should be at least 99 %.

In general, the material used in high volume vehicle gear production may be of more uniform quality than that used for gears produced in small numbers.

Comparison of applied gear designs has indicated that for about 10 000 cycles, the load transmitted by truck final drive gears is about four times greater than that transmitted by aircraft or space vehicle gears, where the material, the quality, the size and the design are the same.

For low speed vehicle gears which are intended to have short lives (less than 100 000 cycles), small amounts of plastic deformation, pitting and abrasive wear can usually be tolerated. Consequently, the levels of surface stress which are permissible are substantially higher than would be permissible for long life, high speed gears.

4.1.2.2 Main drive for aircraft and space vehicles, which are found in helicopter rotor drives and the main pump drives of space vehicle boosters, where gears of the highest quality material and accuracy are used. Such gears are extensively tested. For example, 10 to 20 transmissions of the same production series may be tested under operational conditions for the full design life. The tolerable wear rate is established on the basis of test results. Lubricant spray rate, position of injection points and direction of spray is optimized.

For these reasons, higher loading is permissible for a design life up to 100 times longer (in cycles of tooth loading), and speeds about 10 times greater than those of a typical vehicle transmission. The probability of damage in such cases shall not exceed 0,1 % to 1 %. Overall loading cannot be as high as for vehicle gears since neither surface wear nor minor damage can be tolerated.

4.1.2.3 Industrial turbine gears, where the pitch line velocities exceed 50 m/s, the pinions are usually designed with 30 or more teeth with the objective of minimizing the risk of scuffing and wear. A typical gear pair would consist of a pinion with 45 teeth and a wheel with 248.

Industrial turbine gearing should be better than 99 % reliable for a normal life of more than 10^{10} cycles. Extensive prototype testing is normally excluded because of the cost. As a consequence, the load capacity ratings of turbine gears tend to be conservative with relatively high safety factors.

4.1.3 ***Safety factors***

It is necessary to distinguish between the safety factor relative to pitting, S_H, and the safety factor relative to tooth breakage, S_F.

For a given application, adequate gear load capacity is demonstrated by the computed values of S_H and S_F being equal to or greater than the values $S_{H\,min}$ and $S_{F\,min}$, respectively.

Certain minimum values for safety factors shall be determined. Recommendations concerning these minimum values are made in ISO 6336, but values are not proposed.

An appropriate probability of failure and the safety factor shall be carefully chosen to meet the required reliability at a justifiable cost. If the performance of the gears can be accurately appraised through testing of the actual unit under actual load conditions, a lower safety factor and more economical manufacturing procedures may be permissible.

$$\text{Safety factor} = \frac{\text{Modified allowable stress number}}{\text{Calculated stress}}$$

Safety factors based on load are permitted. When they are based on load the safety factor equals the specific calculated load capacity divided by the specific operating load transmitted. When the factor is based on load, this shall be stated clearly.

NOTE 1 Safety factors based on load (power) relative to tooth bending are proportional to S_F. Safety factors based on load (power) relative to pitting are proportional to S_H^2.

In addition to the general requirements mentioned and the special requirements for surface durability, pitting, (ISO 6336-2) and tooth bending strength (ISO 6336-3), the safety factors shall be chosen after careful consideration of the following influences:

— reliability of material data (The allowable stress numbers used in the calculation are valid for a given probability of failure, the material values in ISO 6336-5 are valid for 1 % probability of damage. This risk of damage reduces with the increase of the safety factor and vice versa);

— reliability of load values used for calculation (If loads or the response of the system to vibration, are estimated rather than measured, a larger factor of safety should be used);

— variations in gear geometry due to manufacturing tolerances;

— variations in alignment;

— variations in material due to process variations in chemistry, cleanliness and microstructure (material quality and heat treatment);

— variations in lubrication and its maintenance over the service life of the gears.

Depending on the reliability of the assumptions on which the calculations are based (e.g. load assumptions) and according to the reliability requirements (consequences of damage occurrence), a corresponding safety factor is to be chosen.

Where gears are produced under a specification or a request for proposal (quotation), in which the gear supplier is to provide gears or assembled gear drives having specified calculated capacities (ratings) in accordance with ISO 6336, the value of the safety factor for each mode of failure (pitting, tooth breakage) is to be agreed upon between the parties.

4.1.4 ***Testing***

The most reliable known approach to the appraisal of overall system performance is that of testing a proposed new design. Where sufficient field or test experience is available, satisfactory results can be obtained by extrapolation of previous tests or field data.

When suitable test results or field data are not available, values for the rating factors should be chosen conservatively.

4.1.5 ***Manufacturing tolerances***

Evaluation of rating factors should be based on the minimum accuracy grade limits specified for the component parts in the manufacturing process.

4.1.6 ***Implied accuracy***

Where empirical values for rating factors are given by curves, curve fitting equations are provided to facilitate computer programming. The constants and coefficients used in curve fitting often have significant digits in excess of those appropriate to the reliability of the empirical data.

4.1.7 ***Other considerations***

In addition to the factors considered in ISO 6336 influencing pitting resistance and bending strength, other interrelated system factors can have a significant influence on overall transmission performance. The following factors are particularly significant.

4.1.7.1 Lubrication. The ratings determined by these formulae are valid only if the gear teeth are operated with a lubricant of proper viscosity and additives for the load, speed, and surface finish, and if there is a sufficient quantity of lubricant supplied to the gear teeth and bearings to lubricate and maintain an acceptable operating temperature.

4.1.7.2 Misalignment and deflection of foundations. Many gear systems depend on external supports such as machinery foundations to maintain alignment of the gear mesh. If these supports are poorly designed, initially misaligned, or become misaligned during operation through elastic or thermal deflection or other influences, overall gear system performance will be adversely affected.

4.1.7.3 Deflections of gear teeth, gear blanks, gear shafts, bearings and housings affect performance and distribution of total tooth load over meshing flanks. Since these deflections vary with load, it is impossible to obtain optimum tooth contact at different loads in those transmissions that encounter variable load. When gear tooth flanks are not modified, the face load factor increases with increasing deflection, thereby lowering rated capacity.

4.1.7.4 System dynamics. The method of analysis used in ISO 6336 provides a dynamic factor in the formulae by derating the gears for increased loads caused by gear tooth inaccuracies and for harmonic effects. In general, simplified values are given for easy application. The dynamic response of the system results in additional gear tooth loads due to the relative motions of the connected masses of the driver and the driven equipment. The application factor, K_a, is intended to account for the operating characteristics of the driving and driven equipment. It must be recognized, however, that if the operating roughness of the driver, gearbox, or driven equipment causes an excitation with a frequency that is near to one of the system's major natural frequencies, resonant vibrations may cause severe overloads which may be several times higher than the nominal load.

For critical service applications, it is recommended that a vibration analysis be performed. This analysis shall include the total system of driver, gearbox, driven equipment, couplings, mounting conditions, and sources of excitation. Natural frequencies, mode shapes, and the dynamic response amplitudes should be calculated. The resulting load spectrum cumulative fatigue effect calculation, if necessary or required, is given in ISO/TR 10495.

4.1.7.5 Contact pattern. The teeth of most cylindrical gears are modified in both profile and lengthwise directions during the manufacturing operation to accommodate deflection of the mountings. This results in a localized contact pattern during roll testing under light loads. Under design load, the contact should spread over the tooth flank without any concentration of the pattern at the edges. This influence shall be taken into account by the corresponding load distribution factor.

4.1.7.6 Corrosion of gear tooth surfaces can significantly reduce the bending strength and pitting resistance of the teeth. Quantifying the extent of these reductions is beyond the scope of ISO 6336.

4.1.8 ***Influence factors***

The influence factors presented in ISO 6336 are derived from results of research and field service. It is convenient to distinguish between the following:

a) Factors which are determined by gear geometry or which have been established by convention. They shall be calculated in accordance with the equations given in ISO 6336.

b) Factors which account for several influences and which are dealt with as independent of each other, but, which may nevertheless influence each other to a degree for which no numerical value can be assigned. These include the factors K_A, K_v, $K_{H\alpha}$, $K_{H\beta}$, or $K_{F\alpha}$ and the factors influencing allowable stress.

The factors K_v, $K_{H\beta}$, and $K_{H\alpha}$ also depend on the magnitudes of the profile and helix modifications. Profile and helix modifications are only effective if they are significantly larger than the manufacturing deviations. For this reason, the influence of the profile and helix modifications may only be taken into consideration if the gear manufacturing deviations do not exceed specific limit values. The minimum required gear manufacturing accuracy is stated, together with reference to ISO 1328-1, for each factor.

The influence factors can be determined by various methods. These are qualified, as necessary, by adding subscripts A through E to the symbols. Unless otherwise specified, e.g. in an application standard, the more accurate of the methods is to be preferred for important transmissions. In cases of dispute, when proof of accuracy and reliability is supplied, method A is superior to method B, and method B to method C, etc.

NOTE 2 It is recommended that supplementary subscripts be used whenever the method used for evaluation of a factor would not be readily identifiable.

In some applications it may be necessary to choose between factors which have been determined using alternative methods (e.g. the alternatives for the determination of the equivalent misalignment). When necessary, the relevant method can be indicated by extending the subscript, e.g. $K_{H\beta\text{-}B1}$.

ISO 6336 is primarily intended for verifying the load capacity of gears for which essential calculation data are available by way of detail drawings, or in similar form.

The data available at the primary design stage is usually restricted. It is therefore necessary, at this stage, to make use of approximations or empirical values for some factors.

For given fields of application or for rough calculations, it is often permissible to substitute unity or some other constant for some factors. In doing so, it is necessary to verify that a good margin of safety is assured. Otherwise, the safety factor shall be adequately increased.

More precise evaluation is possible when manufacture and inspection is completed, for then data obtained by direct measurement are available.

Contractual provisions relating to the nature of the calculation proof shall be agreed in advance between manufacturer and purchaser.

4.1.8.1 Method A factors are derived from the results of full scale load tests, precise measurements, or comprehensive mathematical analysis of the transmission system on the basis of proven operating experience, or any combination of these. All gear and loading data shall be available. In such cases the accuracy and reliability of the method used shall be demonstrated and the assumptions clearly stated.

In general, for the following reasons method A is seldom used:

— the relevant relationships have not been more extensively researched than those in methods B, C, D and E;

— details of the operating conditions are incomplete;

— suitable measuring equipment is not available;

— the costs of analysis and measurements exceed their value.

4.1.8.2 Method B factors are derived with sufficient accuracy for most applications. Assumptions involved in their determination are listed. In each case, it is necessary to assess whether or not these assumptions apply to the conditions of interest. Additional subscripts should be inserted when necessary, e.g. $K_{\text{v-B}}$.

4.1.8.3 Method C is where simplified approximations are specified for some factors. The assumptions under which they have been determined are listed. On each occasion an assessment should be made as to whether or not these assumptions apply to the existing conditions. The additional subscript C shall be used when necessary, e.g. $K_{\text{v-C}}$.

4.1.8.4 Methods D and E give additional special procedures for the determination of some factors. To some extent, these are applicable to a specific field of application or under special conditions, e.g. when an acceptance test is a requirement. When necessary, these factors are indicated by adding a subscript D or E to the symbol, e.g. $K_{\text{H}\beta\text{-D}}$ or $K_{\text{H}\beta\text{-E}}$.

4.1.9 ***Numerical equations***

It is necessary to apply the numerical equations specified in ISO 6336 with the stated units. Any exceptions are specially noted.

4.1.10 ***Succession of factors in the course of the calculation***

The factors K_v, $K_{\text{H}\beta}$ or $K_{\text{F}\beta}$ and $K_{\text{H}\alpha}$ or $K_{\text{F}\alpha}$ depend on a nominal tangential force. They are also to some extent interdependent and shall therefore be calculated successively as follows:

a) K_v with the force F_t K_A;

b) $K_{\text{H}\beta}$ or $K_{\text{F}\beta}$ with the force F_t K_A K_v;

c) $K_{\text{H}\alpha}$ or $K_{\text{F}\alpha}$ with the force F_t K_A K_v $K_{\text{H}\beta\ (\text{F}\beta)}$[1].

When a gear drives two or more mating gears, it is necessary to multiply by (K_A K_γ) instead of K_A; also see **4.2**.

4.1.11 ***Determination of the tolerance values***

The tolerance values shall be determined in accordance with ISO 1328-1.

4.2 Tangential load, torque and power

When assessing the load acting on gear teeth, all forces affecting the gearing shall be considered.

In the case of double helical gearing, it is assumed that the total tangential load is divided equally between the two helices. If this is not the case, for example as a consequence of externally applied axial forces, this shall be taken into consideration. The two halves of the helix should be treated as two helical gears arranged in parallel.

Concerning multiple-path transmissions, the total tangential load is not quite evenly distributed to the various load paths (irrespective of design, tangential velocity or accuracy of manufacture). Allowance is made for this by means of the mesh load factor K_γ (see also **4.1.10**). If possible, K_γ should preferably be determined by measurement; alternatively its value may be estimated from the literature.

If the operating speed is near to a resonance speed, a careful study is necessary. See clauses **5** and **6**.

4.2.1 ***Nominal tangential load, nominal torque and nominal power***

The nominal tangential load F_t is determined in the transverse plane at the reference cylinder. It is derived from the nominal torque or power transmitted by the gear pair.

The load capacity rating of gears is effectively based on the input torque to the driven machine. This is the torque corresponding to the heaviest regular working condition. Alternatively, the nominal torque of the prime mover can be used as a basis if it corresponds to the torque requirement of the driven machine, or some other suitable basis can be chosen.

[1] $K_{\text{H}\beta}$ is also to be used in the evaluation of $K_{\text{F}\alpha}$, since for tooth bending it is $K_{\text{H}\beta}$ which represents the determinant load due to uneven distribution of F_t over the facewidth; (see definition in **7.3.1**).

F_t is defined as the nominal tangential load per mesh, i.e. for the mesh under consideration. T and P are defined accordingly. In the following equations, $n_{1,2}$ is given in revolutions per minute.

$$F_t = \frac{2000\ T_{1,2}}{d_{1,2}} = \frac{19\,098 \times 1000\ P}{d_{1,2}\ n_{1,2}} = \frac{1000\ P}{v} \qquad ...(1)$$

$$T_{1,2} = \frac{F_t\ d_{1,2}}{2000} = \frac{1000\ P}{\omega_{1,2}} = \frac{9549\ P}{n_{1,2}} \qquad ...(2)$$

$$P = \frac{F_t\ v}{1000} = \frac{T_{1,2}\ \omega_{1,2}}{1000} = \frac{T_{1,2}\ n_{1,2}}{9549} \qquad ...(3)$$

$$v = \frac{d_{1,2}\ \omega_{1,2}}{2000} = \frac{d_{1,2}\ n_{1,2}}{19\,098} \qquad ...(4)$$

$$\omega_{1,2} = \frac{2000\ v}{d_{1,2}} = \frac{n_{1,2}}{9549} \qquad ...(5)$$

4.2.2 ***Equivalent tangential load, equivalent torque and equivalent power***

When the transmitted load is not uniform, consideration should be given not only to the peak load and its anticipated number of cycles, but also to intermediate loads and their numbers of cycles. This type of load is classed as a *duty cycle* and may be represented by a load spectrum. In such cases, the cumulative fatigue effect of the duty cycle is considered in rating the gear set. A method of calculating the effect of the loads under this condition is given in ISO/TR 10495.

4.2.3 ***Maximum tangential load, maximum torque and maximum power***

This is the maximum tangential load $F_{t\,max}$ (or corresponding torque T_{max}, corresponding power P_{max}) in the variable duty range. Its magnitude can be limited by a suitably responsive safety clutch. $F_{t\,max}$, T_{max} and P_{max} are required to determine the safety from pitting damage and from sudden tooth breakage due to loading corresponding to the static stress limit.

5 Application factor K_A

The factor K_A adjusts the nominal load F_t in order to compensate for incremental gear loads from external sources. These additional forces are largely dependent on the characteristics of the driving and driven machines, as well as the masses and stiffness of the system, including shafts and couplings used in service.

For applications such as marine gears, etc., which are subjected to cyclic peak torque (torsional vibrations) and are designed for infinite life, the application factor can be defined as the ratio between the peak cyclic torques and the nominal rated torque. The nominal rated torque is defined by the rated power and speed. It is the torque used in the load capacity calculations.

If the gear is subjected to a limited number of known loads in excess of the amount of the peak cyclic torques, this influence may be covered directly by means of cumulative fatigue or by means of an increased application factor, representing the influence of the load spectrum.

It is recommended that the purchaser and manufacturer/designer agree on the value of the application factor.

5.1 Method A — Factor $K_{A\text{-}A}$

K_A is determined in this method by means of careful measurements and a comprehensive analysis of the system, or on the basis of reliable operational experience in the field of application concerned. See **4.2.2**.

5.2 Method B — Factor $K_{A\text{-}B}$

If no reliable data, obtained as described in **5.1**, are available, even as early as the first design phase, it is possible to use the guideline values for K_A as described in Annex B.

6 Internal dynamic factor K_v

The internal dynamic factor makes allowance for the effects of gear tooth accuracy grade as related to speed and load. High accuracy gearing requires less derating than low accuracy gearing.

It is generally accepted that the internal dynamic load on the gear teeth is influenced by:

a) design;

b) manufacturing.

The dynamic factor relates the total tooth load including internal dynamic effects to the transmitted tangential tooth load:

$$K_v = \frac{\text{Internal effected dynamic load + Transmitted tangential tooth load}}{\text{Transmitted tangential tooth load}}$$

6.1 Parameters affecting internal dynamic load and calculations

6.1.1 *Design*

The design parameters include the following:

— pitchline velocity;

— tooth load;

— inertia and stiffness of the rotating elements;

— tooth stiffness variation;

— lubricant properties;

— stiffness of bearings and case structure;

— critical speeds and internal vibration within the gear itself.

6.1.2 *Manufacturing*

The manufacturing considerations include the following:

— pitch deviations;

— runout of reference surfaces with respect to the axis of rotation;

— tooth flank deviations;

— compatibility of mating gear tooth elements;

— balance of parts;

— bearing fit and preload.

6.1.3 *Transmission perturbance*

Even when the input torque and speed are constant, significant vibration of the gear masses, and resultant dynamic tooth forces, can exist. These forces result from the relative displacements between the mating gears as they vibrate in response to an excitation known as transmission error. The ideal kinematics of a gear pair require a constant ratio between the input and output rotations. Transmission error is defined as the departure from uniform relative angular motion of a pair of meshing gears. It is influenced by all deviations from the ideal gear tooth form and spacing due to the design and manufacture of the gears, and to the operational conditions under which the gears shall perform. The latter include the following.

a) Pitch line velocity: The frequencies of excitation depend on the pitch line velocity and module.

b) Gear mesh stiffness variations as the gear teeth pass through the meshing cycle: This source of excitation is especially pronounced in spur gears. Spur and helical gears with total contact ratios greater than 2,0 have less stiffness variation.

c) Transmitted tooth load: Since deflections are load dependent, gear tooth profile modifications can be designed to give uniform velocity ratio only for one magnitude of load. Loads different from the design load will give increased transmission error.

d) Dynamic unbalance of the gears and shafts.

e) Application environment: Excessive wear and plastic deformation of the gear tooth profiles increase the transmission error. Gears shall have a properly designed lubrication system, enclosure, and seals to maintain a safe operating temperature and a contamination-free environment.

f) Shaft alignment: Gear tooth alignment is influenced by load and thermal deformations of the gears, shafts, bearings, and housing.

g) Excitation induced by tooth friction.

6.1.4 *Dynamic response*

The effects of dynamic tooth forces are influenced by the following:

— mass of the gears, shafts, and other major internal components.

— stiffness of the gear teeth, gear blanks, shafts, bearings, and housings.

— damping: The principal sources of damping are the shaft bearings and seals. Other sources of damping include hysteresis of the gear shafts and viscous damping at sliding interfaces and shaft couplings.

6.1.5 *Resonance*

When excitation frequencies (such as tooth meshing frequency and its harmonics) coincide or nearly coincide with a natural frequency of vibration of the gearing system, the resonant forced vibration may cause high dynamic tooth loading. When the magnitude of internal dynamic load at a speed involving resonance becomes large, operation near this speed should be avoided.

a) gear blank resonance: The gear blanks of high-speed, lightweight gearing may have natural frequencies within the operating speed range. If the gear blank is excited by a frequency which is close to one of its natural frequencies, the resonant deflections may cause high dynamic tooth loads. Also, there is the possibility of plate or shell mode vibrations which can cause the gear blank to fail. The dynamic factors K_v (from the following methods B through E) do not take account of gear blank resonance.

b) system resonance: The gearbox is only one component of a system comprised of a power source, gearbox, driven equipment, and interconnecting shafts and couplings. The dynamic response of this system depends on the configuration of the system. In certain cases a system may possess a natural frequency close to the excitation frequency associated with an operating speed. Under such resonant conditions, the operation shall be worked out carefully as mentioned above. For critical drives, a detailed analysis of the entire system is recommended. This should also be taken into account when determining the effects on the application factor.

6.2 Principles and assumptions

In accordance with the specifications of **4.1.8**, methods for determining K_v are given in **6.3**, from method A ($K_{v\text{-}A}$) to method E ($K_{v\text{-}E}$).

Given optimum profile modification appropriate to the loading, a large overlap ratio, even distribution of load over the facewidth, highly accurate teeth and high specific tooth loading, the value of the dynamic factor approaches 1,0.

In the case of high specific loading, high values of $(K_A\, F_t)/b$, high values of $(v\, z_1 / 100)\, \sqrt{u^2/(1+u^2)}$ and having the corresponding accuracy of gear cutting, tooth tips and/or roots should be suitably relieved.

6.3 Methods for the determination of the dynamic factor

6.3.1 *Method A — Factor $K_{v\text{-}A}$*

The maximum tooth loads, including the internally generated dynamic additional loads and the uneven distribution of forces as described in clause **8**, are determined in method A by measurement or by a comprehensive dynamic analysis of the general system. Under these circumstances K_v (just as $K_{H\alpha}$ and $K_{F\alpha}$) is assumed to have a value of 1,0.

K_v can also be assessed by measuring the tooth-root stresses of the gears when transmitting load at working speed and at a lower speed, then comparing the results.

The factor K_v may be determined by a comprehensive analytical procedure which is supported by experience of similar designs. Guidance on procedures can be found in the literature.

Reliable values of the dynamic factor, K_v, can best be predicted by a mathematical model which has been satisfactorily verified by measurement.

6.3.2 *Method B — Factor $K_{v\text{-}B}$*

For this method the simplifying assumption is made that the gear pair consists of an elementary single mass and spring system comprising the combined masses of pinion and wheel, the stiffness being the mesh stiffness of the contacting teeth. It is also assumed that each gear pair functions as a single stage gear pair, i.e. the influence of other stages in a multiple-stage gear system is ignored. This assumption is permissible if the torsional stiffness of the shafts connecting the wheel of one stage with the pinion of the next is relatively low. See **6.4.2** and **6.4.7.2** for the procedure dealing with very stiff shafts.

In accordance with this assumption, forces due to torsional vibration of the shafts and coupled masses are not covered by K_v. These latter forces should be included with other externally applied forces (e.g. with the application factor).

It is further assumed, in the evaluation of dynamic factors by method B, that damping at the gear mesh has an average value. (Other sources of damping such as friction at component interfaces, hysteresis, bearings, couplings, etc. are not taken into consideration.) Because of these additional sources of damping, the actual dynamic tooth loads are normally somewhat smaller than those calculated by this method. This does not apply in the range of main resonance (see **6.4.4**).

In gear trains which include multiple mesh gears such as idler gears and epicyclic gearing, planet and sun gears, there are several natural frequencies. These can be higher or lower than the natural frequency of a single gear pair which has only one mesh. When such gears run in the supercritical range, resonances of higher order can occur which cannot be disclosed by analysis using the simplified method B.

Transverse vibrations of the shaft-gear systems will also influence the dynamic load. Since magnitudes of gear-shaft bending stiffness are usually considerable, the natural frequencies of transverse vibration are normally above the running speed range. If the natural frequency of transverse vibration of the shaft-gear system lies within the running speed range, this shall be taken into account in the evaluation of K_v.

Calculation of K_v by this method serves no useful purpose when the value $(v\, z_1 / 100)\sqrt{u^2/(1+u^2)}$ is less than 3 m/s. Method C is sufficiently accurate for all cases in this range.

6.3.3 *Method C — Factor $K_{v\text{-}C}$*

Method C is derived from method B, by introducing the following additional simplifying assumptions:

a) the running speed range is subcritical;

b) steel solid disc wheels;

c) the pressure angle $\alpha_t = 20°$; $f_{pb} = f_{pt} \cos 20°$ according to ISO/TR 10064-1;

d) helix angle $\beta = 20°$ for helical gearing (refers to c', c_γ);

e) total contact ratio $\varepsilon_\gamma = 2{,}5$ for helical gearing;

f) tooth stiffness:

For spur gears, $c' = 14$ N/(mm·µm), $c_\gamma = 20$ N/(mm·µm);

For helical gearing, $c' = 13{,}1$ N/(mm·µm), $c_\gamma = 18{,}7$ N/(mm·µm), see **9.1.3** and **9.4**;

g) tip relief $C_a = 0$ µm and tip relief after running-in $C_{ay} = 0$ µm;

h) effective deviation $f_{pb\,eff} = f_{f\alpha\,eff}$;

j) for assumed values for f_{pb}, y_p and $f_{pb\,eff}$, see equations (17) and (18) and Table 3.

The features described in **6.4.1** a) have not been taken into consideration in the application of method C. The influence of specific loading is taken into account.

6.3.4 *Method D — Factor $K_{v\text{-}D}$*

This method of approximation is derived from method C. In contrast to that method, the influence of variable specific loading has been neglected and $(F_t\, K_A)/b$ is to be made a constant value for a given application field. The method is suited to a rapid, approximate estimation of the dynamic tooth loads of industrial gears and transmissions with similar requirements, when the specific tooth load is not known.

6.3.5 *Method E — Factor $K_{v\text{-}E}$*

Similar to method D, the influence of variable specific loading is neglected and $(F_t\, K_A)/b$ is set at a constant value for a given application. This method (see **6.7**) gives dynamic factor values for K_v which have been empirically developed from a broad range of working applications, independent of specific tooth loading.

6.4 Determination of the dynamic factor using Method B: $K_{v\text{-}B}$

According to the preconditions and assumptions described in **6.3.2**, method B is suited for all types of transmission (spur and helical gearing with any basic rack profile and any gear accuracy grade) and, in principle, for all operating conditions. However, there are restrictions for certain fields of application and operation which will be noted in each case and should be taken into account.

The resonance ratio N (ratio of the running speed to the resonance speed) is determined as described in **6.4.2**)[2)]. The entire running speed range can be divided into three sectors — subcritical, main resonance and supercritical. Formulae are provided for calculating K_v in each sector.

NOTE 3 The dynamic factors calculated from the equations in **6.4.3** to **6.4.6** correspond to the experimentally determined mean dynamic tooth load values. In the subcritical and main resonance ranges, values of K_v derived from measurement data usually deviate from the calculated values by up to + 10 %.

Even greater deviations can occur when there are other natural frequencies in the gear and shaft system. See **6.4.1** a), **6.4.3**, and **6.4.4**.

6.4.1 *Running speed ranges*

a) Subcritical range (N < 1)[3)] In this sector resonances may exist if the tooth mesh frequency coincides with $N = 1/2$ and $N = 1/3$. Under such circumstances the dynamic forces can exceed the values calculated using equation (12). The risk of this is slight for precision helical or spur gears, if the latter have suitable profile modification (gears to accuracy grade 5 of ISO 1328-1 or better).

When the contact ratio of spur gears is small or if the accuracy is of low grade, K_v can be just as great as in the main resonance-speed range. If this occurs, the design or operating parameters should be altered.

Resonances at $N = 1/4, 1/5, ...$ are seldom troublesome because the associated vibration amplitudes are usually small.

For the case of gear pairs in which the stiffness of the driving and driven shafts are not equal, in the range $N \approx 0{,}2 \ldots 0{,}5$, the tooth contact frequency can excite natural frequencies if the torsional stiffness c, of the stiffer shaft referred to the line of action, is of the same order of magnitude as the tooth stiffness, i.e. if c/r_b^2 is of the order of magnitude of c_γ. When this is so, dynamic load increments can exceed values calculated using equation (12).

When the specific loading $(F_t\, K_A)/b$ < 50 N/mm, a particular risk of vibration exists (under some circumstances, with separation of working tooth flanks), above all for spur or helical gears of coarse accuracy grade running at higher speed.

b) Main resonance range (N = 1) Operation in this range should generally be avoided, especially for spur gears with unmodified tooth profiles, or helical gears of accuracy grade 6 or coarser as specified in ISO 1328-1. High accuracy grade helical gears with high total contact ratio can function satisfactorily in this sector. Spur gears of grade 5 or better as specified in ISO 1328-1 shall have suitable profile modification.

c) Supercritical range (N > 1) The same limitations on gear accuracy grade as in b) apply to gears operating in this speed range. Resonance peaks can occur at $N = 2, 3 \ldots$ in this range. However, in the majority of cases vibration amplitudes are small, since excitation forces with lower frequencies than meshing frequency are usually small.

For some gears in this speed range, it is also necessary to consider dynamic loads due to transverse vibration of the gear and shaft assemblies (see **6.3.2**). If the critical frequency is near to the frequency of rotation, the associated effective value of K_v can exceed the value calculated using equation (20) by up to 100 %. This condition should be avoided.

[2)] When it is known in advance that gears will operate in the supercritical sector, there is no need to evaluate the resonance speed. As a consequence, the dynamic factor can be directly determined in accordance with **6.4.5**.

[3)] For a definition of N see equation (9). In practice, the calculated resonance sector is broadened to ensure a safe margin. See equations (10) and (11a) and the preamble thereto.

6.4.2 ***Determination of the resonance running speed (main resonance) of a gear pair***[4)]

$$n_{E1} = \frac{30 \times 10^3}{\pi \; z_1} \sqrt{\frac{c_\gamma}{m_{red}}} \qquad \text{...(6)}$$

where

m_{red} is the relative mass of a gear pair, i.e. of the mass per unit facewidth of each gear, referred to its base radius or to the line of action.

$$m_{red} = \frac{m_1^* \; m_2^*}{m_1^* + m_2^*} = \frac{J_1^* \; J_2^*}{J_1^* \; r_{b2}^2 + J_2^* \; r_{b1}^2} \qquad \text{...(7)}$$

where

$$m_{1,2}^* = \frac{J_{1,2}^*}{r_{b1,2}^2} \qquad \text{...(8)}$$

See **6.4.8** for the method of calculating an approximate value of m_{red}. See clause **9** for the stiffness c_γ. Values of n_{E1} for steel gear pairs may be read from Figure 1.

Method A is to be preferred for less common transmission designs. A method for deriving approximate values is specified in **6.4.7** for the following cases:

a) pinion on large diameter shaft;

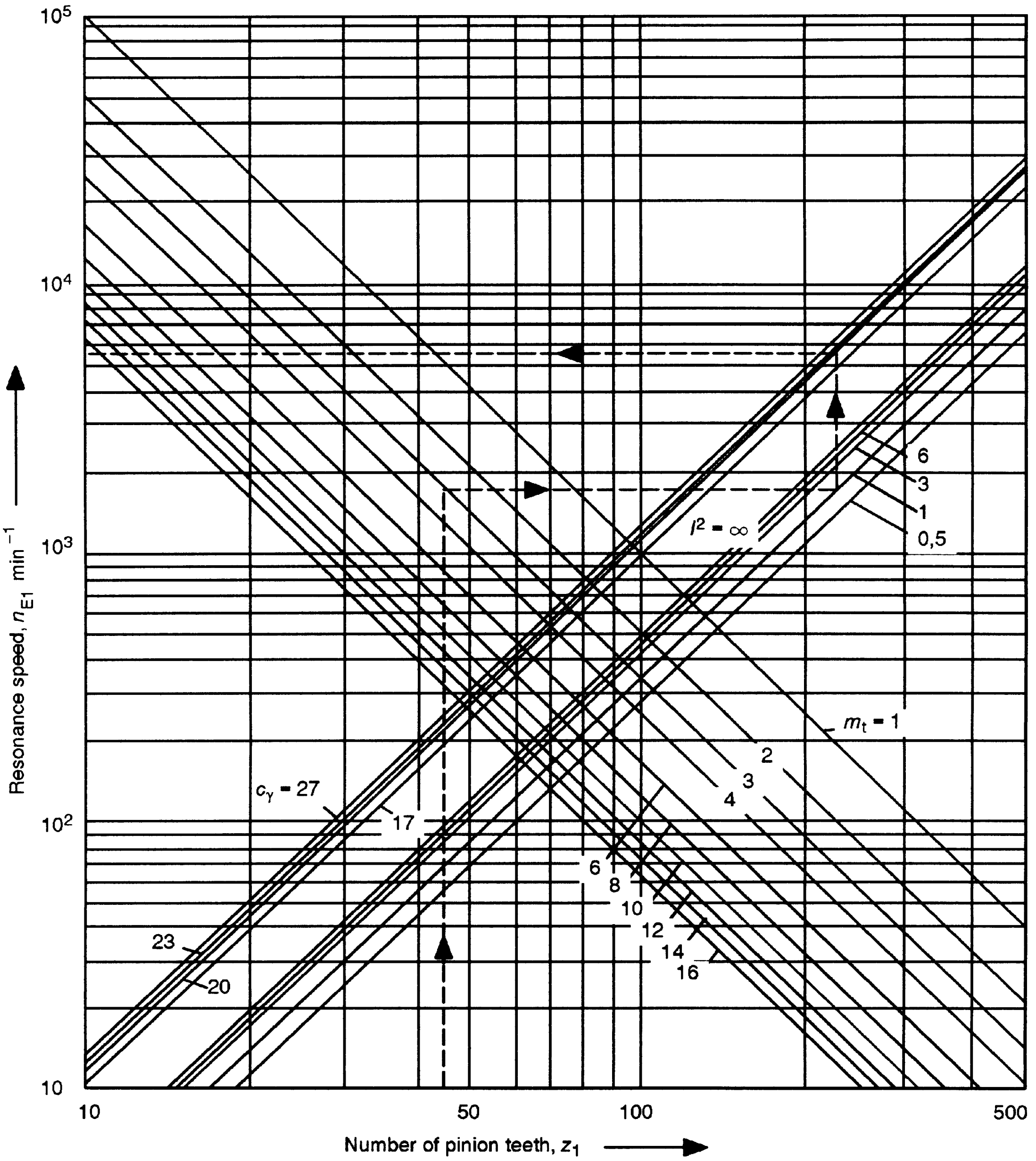
b) two neighbouring gears rigidly joined together;

c) one big wheel driven by two pinions;

d) simple planetary gears;

e) idler gears.

[4)] When it is known in advance that gears will operate in the supercritical sector, there is no need to evaluate the resonance speed. As a consequence, the dynamic factor can be directly determined in accordance with **6.4.5**.



NOTE The pinion being of solid construction with q_2 determined in accordance with equation (28) $I^2 = (1 - q_2^4)\ u^2$

Figure 1 — Nomogram for the determination of the resonance speed n_{E1} of steel gear pairs

The ratio of pinion speed to resonance speed is termed the "resonance ratio", N (n_1 is in revolutions per minute):

$$N = \frac{n_1}{n_{E1}} = \frac{n_1\ \pi\ z_1}{30\,000}\sqrt{\frac{m_{red}}{c_\gamma}} \qquad ...(9)$$

The resonance running speed n_{E1} may be above or below the running speed calculated from equation (6) because of stiffnesses which have not been included, e.g. the stiffness of shafts, bearings, housings, etc. and as a result of damping. For reasons of safety, the resonance ratio in the main resonance range, N_S, is defined by the following upper limit.

$$N_S < N \le 1{,}15 \qquad ...(10)$$

The lower limit of resonance ratio N_S is determined

at loads such that $(F_t\, K_A)/b$ is less than 100 N/mm, as

$$N_S = 0{,}5 + 0{,}35\sqrt{\frac{F_t\, K_A}{100\ b}} \qquad \text{...(11a)}$$

for loads where $(F_t\, K_A/b) \geq 100$ N/mm, as

$N_S = 0{,}85$ ...(11b)

For resonance ratio, N, in the main resonance range, see Figure 2.

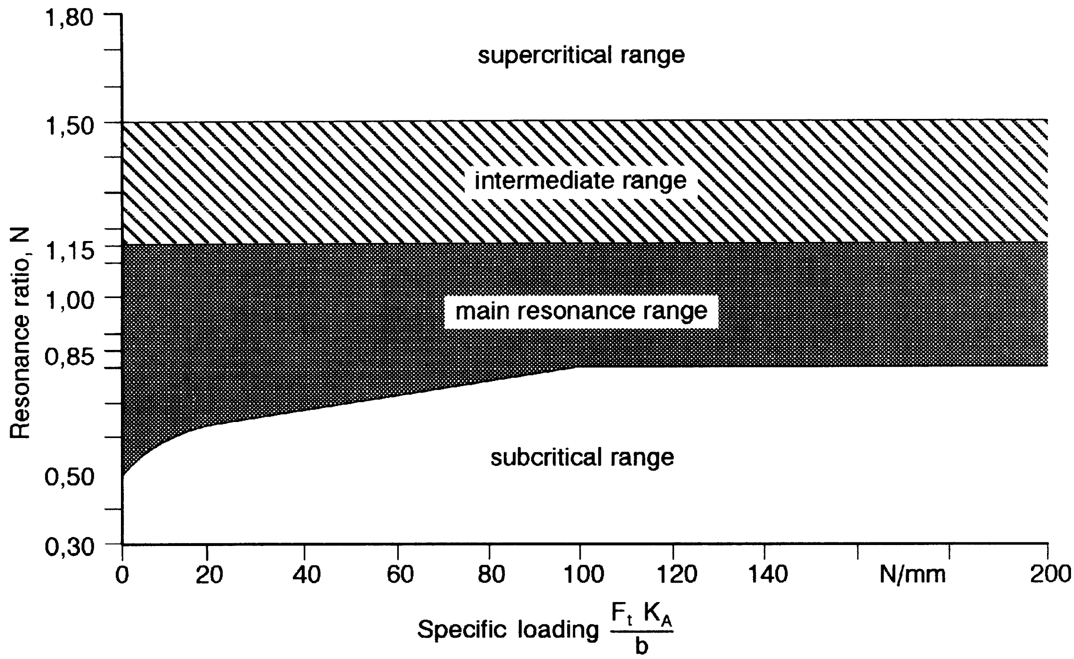


Figure 2 — Resonance range

Thus the following ranges result for the calculation of K_v:

a) subcritical range, $N \leq N_S$ (see **6.4.3**);

b) main resonance range, $N_S < N \leq 1{,}15$ (see **6.4.4**); this field should be avoided; refined analysis by method A is recommended for K_v;

c) intermediate range, $1{,}15 < N \leq 1{,}5$ (see **6.4.6**); refined analysis by method A is recommended;

d) supercritical range, $N \geq 1{,}5$ (see **6.4.5**).

6.4.3 *Dynamic factor in the subcritical range* ($N \leq N_S$)

See **6.4.1** a) for special features; the majority of industrial gears operate in this range.

$$K_v = (N\, K) + 1 \qquad \text{...(12)}$$

$$K = (C_{v1}\, B_p) + (C_{v2}\, B_f) + (C_{v3}\, B_k) \qquad \text{...(13)}$$

where

— C_{v1} allows for pitch deviation effects and is assumed to be constant at $C_{v1} = 0{,}32$ (see Figure 3);

— C_{v2} allows for tooth profile deviation effects and can be read from Figure 3 or determined in accordance with Table 2;

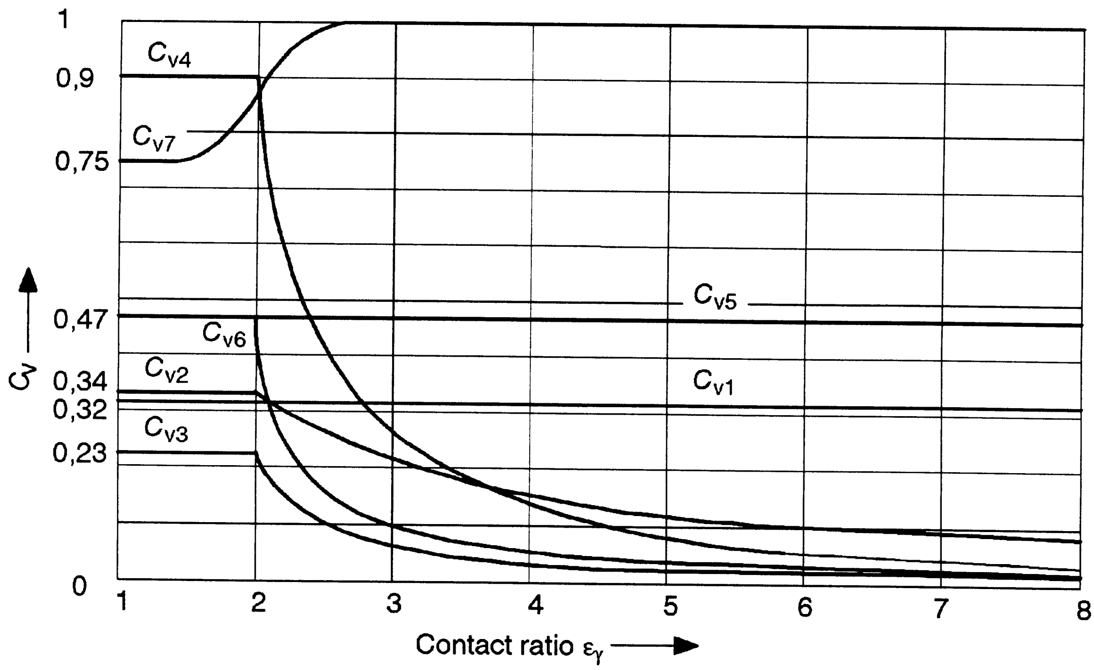
— C_{v3} allows for the cyclic variation effect in mesh stiffness and can be read from Figure 3 or determined in accordance with Table 2.

Table 2 — Equations for the calculation of factors C_{v1} to C_{v7} and C_{ay} for determination of $K_{v\text{-}B}$, method B; (the equations are connected with the curves in Figure 3 and Figure 4)

	$1 < \varepsilon_\gamma \leq 2$	$\varepsilon_\gamma > 2$	
C_{v1}	0,32	0,32	
C_{v2}	0,34	$\dfrac{0{,}57}{\varepsilon_\gamma - 0{,}3}$	
C_{v3}	0,23	$\dfrac{0{,}096}{\varepsilon_\gamma - 1{,}56}$	
C_{v4}	0,90	$\dfrac{0{,}57 - 0{,}05\,\varepsilon_\gamma}{\varepsilon_\gamma - 1{,}44}$	
C_{v5}	0,47	0,47	
C_{v6}	0,47	$\dfrac{0{,}12}{\varepsilon_\gamma - 1{,}74}$	
	$1 < \varepsilon_\gamma \leq 1{,}5$	$1{,}5 < \varepsilon_\gamma \leq 2{,}5$	$\varepsilon_\gamma > 2{,}5$
C_{v7}	0,75	$0{,}125 \sin[\pi(\varepsilon_\gamma - 2)] + 0{,}875$	1,0

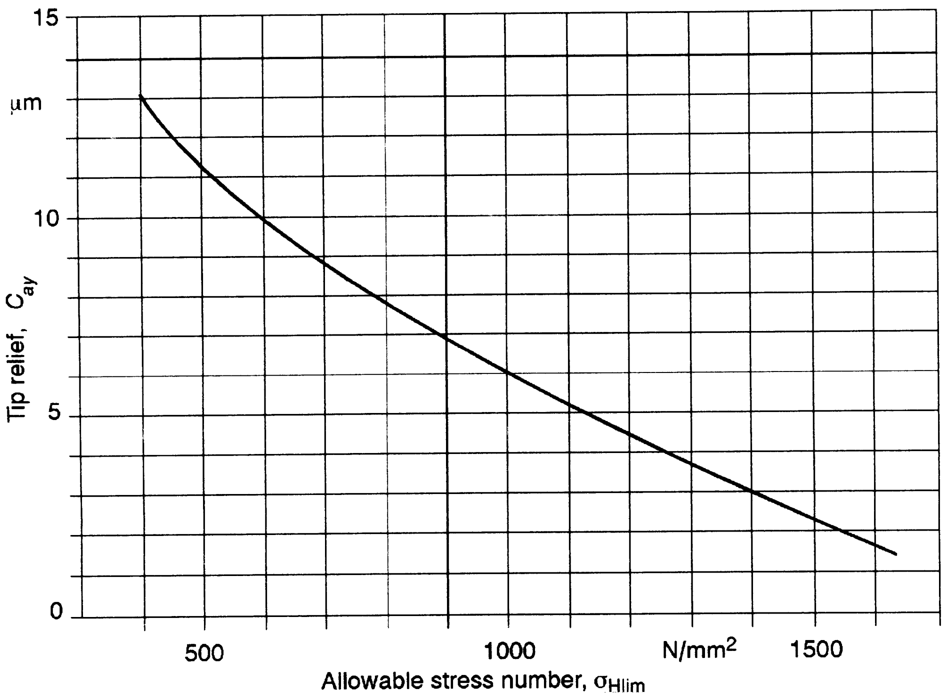
$$C_{ay} = \frac{1}{18}\left(\frac{\sigma_{H\,\mathrm{lim}}}{97} - 18{,}45\right)^2 + 1{,}5$$

NOTE When the material of the pinion (1) is different from that of the wheel (2), C_{ay1} and C_{ay2} are calculated separately; then $C_{ay} = 0{,}5\,(C_{ay1} + C_{ay2})$



NOTE For the equations used for calculation, see Table 2.

Figure 3 — Values of C_{v1} to C_{v7} for the determination of $K_{v\text{-}B}$ (method B)



NOTE When the pinion material (1) is different from the wheel (2) then $C_{ay} = 0.5\ (C_{ay1} + C_{ay2})$.

Figure 4 — Tip relief C_{ay} produced by running in (see Table 2 for calculation)

B_p, B_f and B_k are non-dimensional parameters to take into account the effect of tooth deviations and profile modifications on the dynamic load.[5)]

$$B_p = \frac{c'\ f_{pb\ \mathrm{eff}}}{K_A\ (F_t/b)} \quad \text{...(14)}$$

$$B_f = \frac{c'\ f_{f\alpha\ \mathrm{eff}}}{K_A\ (F_t/b)} \quad \text{...(15)}$$

$$B_k = \left| 1 - \frac{c'\ C_a}{K_A\ (F_t/b)} \right| \quad \text{...(16)}$$

The effective single pitch and profile deviations are those of the "run-in" pinion and wheel. Initial deviations are generally modified during early service (running-in). The values of $f_{pb\ \mathrm{eff}}$ and $f_{f\alpha\ \mathrm{eff}}$ are determined by deducting estimated running-in allowances (y_p and y_f) as follows:

$$f_{pb\ \mathrm{eff}} = f_{pb} - y_p \quad \text{...(17)}$$

$$f_{f\alpha\ \mathrm{eff}} = f_{f\alpha} - y_f \quad \text{...(18)}$$

Considerations of probability suggest that, in general, magnitudes of transmission deviation will not be greater than the allowable values of f_{pb} and $f_{f\alpha}$ for the wheel, which are the larger. They are therefore used in equations (17) and (18) respectively; these are usually the values for the largest wheel.

[5)] Equation (16) is not suitable for the determination of an "optimum" tip relief C_a. The amount C_a of tip relief may only be used in equation (16) for gears of quality grades in the range 0 to 6 as specified in ISO 1328-1. For gears in the range 7 to 12, B_k = 1,0. Also see **4.1.8**.

In the event that neither experimental nor service data on relevant material running-in characteristics are available (method A), it can be assumed that: $y_p = y_\alpha$ with y_α as defined in method C from Figure 27 or Figure 28 or **8.3.5.1**. y_f can be determined in the same way as y_α when the profile deviation $f_{f\alpha}$ is used instead of base pitch deviation f_{pb}.

C_a is the design amount for profile modification (tip relief at the beginning and end of tooth engagement).

A value C_{ay} resulting from running-in is to be substituted for C_a in equation (16) in the case of gears without a specified profile modification. The value of C_{ay} can be obtained from Figure 4 or calculated as indicated in Table 2.

See clause **9** for single tooth stiffness c'.

6.4.4 *Dynamic factor in the main resonance range* ($N_S < N \leq 1,15$)

Subject to restriction [see **6.4.1** b)], this factor is equal to:

$$K_v = (C_{v1} B_p) + (C_{v2} B_f) + (C_{v4} B_k) + 1 \qquad \text{...(19)}$$

See **6.4.3** for details regarding C_{v1}, C_{v2}, B_p, B_f and B_k.

C_{v4} takes into account resonant torsional oscillations of the gear pair, excited by cyclic variation of the mesh stiffness. Its value can be taken from Figure 3 or calculated as indicated in Table 2.

NOTE 4 The dynamic factor at this speed is strongly influenced by damping. The real value of the dynamic factor can deviate from the calculated value (see equation 19) by up to 40 %. This is especially true for spur gears with incorrectly designed profile modifications.

6.4.5 *Dynamic factor in the supercritical range* ($N \geq 1,5$)

Most high precision gears used in turbine and other high speed transmissions operate in this sector; see **6.4.1** c) for features.

$$K_v = (C_{v5} B_p) + (C_{v6} B_f) + C_{v7} \qquad \text{...(20)}$$

In this range the influences on K_v of C_{v5} and C_{v6} correspond to those of C_{v1} and C_{v2} on K_v in the subcritical range. See **6.4.3** for data on these factors and on B_p and B_f.

C_{v7} takes into account the component of force which, due to mesh stiffness variation, is derived from tooth bending deflections during substantially constant speed.

C_{v5}, C_{v6} and C_{v7} can be obtained from Figure 3 or calculated as indicated in Table 2.

6.4.6 *Dynamic factor in the intermediate range* ($1,15 < N < 1,5$)

In this range, the dynamic factor is determined by linear interpolation between K_v at $N = 1,15$ as specified in **6.4.4** and K_v at $N = 1,5$ as specified in **6.4.5**.

$$K_v = K_{v(N=1,5)} + \frac{K_{v(N=1,15)} - K_{v(N=1,5)}}{0,35}(1,5 - N) \qquad \text{...(21)}$$

See **6.4.4** and **6.4.5** for details and explanatory notes.

6.4.7 *Resonance speed determination for less common gear designs*

The resonance speed determination for less common gear designs should be made with the use of method A. However, other methods may be used to approximate the effects. Some examples are as follows.

6.4.7.1 Pinion shaft with diameter at mid-tooth depth, d_{m1}, about equal to the shaft diameter The resonance speed tends to decrease because the pinion mass is supplemented by the shaft mass. The high torsional stiffness of the pinion shaft has the opposite effect on the resonance speed. These two influence resonance speeds in the opposite sense and to a great extent compensate each other.

Thus the resonance speed can be calculated in the normal way, using the mass of the pinion (toothed portion) and the normal mesh stiffness c_γ.

6.4.7.2 Two rigidly connected, coaxial gears The mass of the larger of the connected gears is to be included. The mass of the smaller gear can often be ignored. This gives a useable approximation when the diameters of the connected gears are markedly different (see also **6.4.2**).

6.4.7.3 One large wheel driven by two pinions See also **6.3.2**. As the mass of the wheel is normally much greater than the masses of the pinions, each mesh can be considered separately, i.e.:

— as a pair comprising the first pinion and the wheel;

— as a pair comprising the second pinion and the wheel.

6.4.7.4 Planetary gears Because of the many transmission paths which include stiffnesses other than mesh stiffness, the vibratory behaviour of planetary gears is very complex. The calculation of dynamic load factors using simple formulae, such as method B, is generally quite inaccurate. Nevertheless, method B, modified as follows, can be used for a first estimate of K_v. This estimate should, if possible, be verified by means of a subsequent detailed theoretical or experimental analysis, or on the basis of operating experience. See also the comments in **6.3.2**.

a) Sun gear — planet gear: The reduced mass for the determination of the resonance speed, n_{E1}, of a sun gear is given by:

$$m_{red} = \frac{m^*_{pla}\, m^*_{sun}}{(p\, m^*_{pla}) + m^*_{sun}} \qquad \text{...(22)}$$

where

m^*_{sun} is the moment of inertia per unit facewidth of the sun gear, divided by $r^2_{b\ sun}$, where $r_{b\ sun} = d_{b\ sun}/2$;

m^*_{pla} is the moment of inertia per unit facewidth of a planet gear, divided by $r^2_{b\ pla}$, where $r_{b\ pla} = d_{b\ pla}/2$;

p is the number of planet gears in the gear stage under consideration.

The value of m_{red} determined from equation (22) is to be used in equation (6) or (9), where a mesh stiffness approximately equal to a single planetary gear is to be used for the mesh stiffness c_γ and the number of teeth on the sun gear is to be used for z_1.

Concerning planetary gears, it should be noted that F_t in equations (14) to (16) is equal to the total tangential load applied to the sun gear divided by the number of planet gears.

b) Planet gear — annulus gear rigidly connected to the gear case: In this case the mass of the annulus gear can be assumed to be infinite. Thus, the reduced mass becomes equal to the relative mass, m^*_{pla}, of the planet gear. This can be determined as:

$$m_{red} = m^*_{pla} = \frac{\pi}{8} \frac{d^4_{m\ pla}}{d^2_{b\ pla}} (1 - q^4_{pla})\, \rho_{pla} \qquad \text{...(23)}$$

in which the subscript "pla" refers to the planet gears and $d_{m\ pla}$ and q_{pla} are determined by analogy with equations (27) to (29).

c) Planet gear — rotating annulus gear: In this case the relative mass of the annulus gear can be determined as for an external wheel. The reduced mass of the planet gear is calculated in accordance with equation (26). The procedure described in **6.4.7.3** is to be used when the annulus gear meshes with several planet gears.

6.4.7.5 Idler gears See information in **6.3.2**. Approximate values can be obtained from the following equations when the driving and driven gears are roughly of the same size and the idler gear is also about the same size or a little larger:

a) reduced mass

$$m_{red} = \frac{2}{\left(\frac{1}{m^*_1} + \frac{2}{m^*_2} + \frac{1}{m^*_3}\right)} \qquad \text{...(24)}$$

b) mesh stiffness

$$c_\gamma = 0{,}5\,(c_{\gamma 1,2} + c_{\gamma 2,3}) \qquad \text{...(25)}$$

where

m^*_1, m^*_2, m^*_3 are the mass moments of inertia per millimeter facewidth of the pinion, the idler and the wheel respectively, referred to the lines of action;

$c_{\gamma\,1,2}$ is the mesh stiffness of the driver and idler gear pair;

$c_{\gamma\,2,3}$ is the mesh stiffness of the idler and driven gear pair (see clause **9** for the determination of c_γ). More accurate analysis is recommended if the reference speed is in the
range $0{,}6 < N < 1{,}5$.

If the idler is substantially larger than the driving and driven gears or, if the driving gear or driven gear is substantially smaller than the two other gears, K_v can be calculated separately for each meshing pair, i.e.:

— for the driver-idler gear combination, and

— for the idler-driven gear combination.

An accurate analysis is recommended for cases not mentioned here.

6.4.8 ***Calculation of the reduced mass of a gear pair with external teeth***

Approximate values of reduced mass which are sufficiently accurate can be derived from the following equations (see Figure 5 for symbols).

$$m_{red} = \frac{\pi}{8}\left(\frac{d_{m1}}{d_{b1}}\right)^2 \frac{d_{m1}^2}{\dfrac{1}{(1-q_1^4)\,\rho_1} + \dfrac{1}{(1-q_2^4)\,\rho_2\,u^2}} \quad ...(26)$$

where

$$d_{m1,\,m2} = \frac{d_{a1,\,a2} + d_{f1,\,f2}}{2} \quad ...(27)$$

$$q_1 = \frac{d_{i1}}{d_{m1}}; \quad q_2 = \frac{d_{i2}}{d_{m2}} \quad ...(28)$$

Equations (26) to (28) apply to external double helical, external single helical and external spur gears. They ignore the masses of web and hub because of their negligible influence on the moment of inertia.

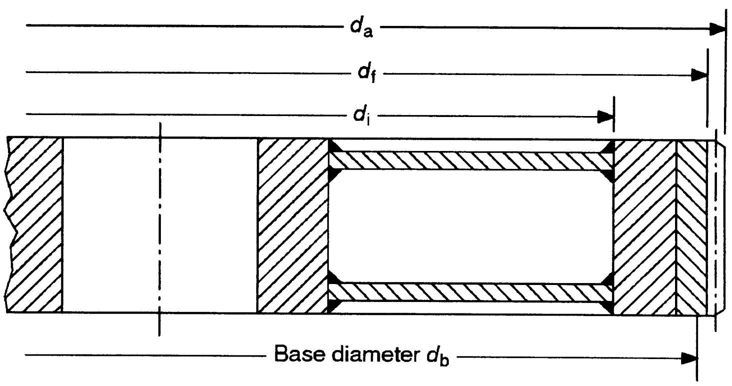


Figure 5 — Definitions of the various diameters

For pinions and wheels of solid construction:

$$1 - q_1^4 = 1; \; 1 - q_2^4 = 1 \quad ...(29)$$

The calculation of $(1 - q_1^4)$ or $(1 - q_2^4)$ for a gear rim whose rim width differs from the facewidth, is only valid where the masses of the rim are directly connected to the gear rim. More distant masses on the same shaft are ignored since the stiffness of the interconnecting shaft is generally of minor significance compared to tooth stiffness.

6.5 Determination of the dynamic factor using method C: $K_{v\text{-}C}$

In conjunction with the conditions and assumptions described in **6.3.3**, method C supplies average values which can be used for industrial transmissions and gear systems with similar requirements in the following fields of application:

a) subcritical running speed range, i.e.: $(v\, z_1 / 100)\sqrt{u^2/(1+u^2)} < 10$ m/s, the restrictions in **6.3.3** a) apply accordingly;

b) external and internal spur gears;

c) basic rack profile as specified in ISO 53;

d) straight and helical spur gears with $\beta \leq 30°$;

e) pinion with relatively low number of teeth: $z_1 < 50$;

f) solid disc wheels or heavy steel gear rim[6].

Method C can also generally be used, with restrictions for the following field of application:

g) all types of cylindrical gears, if $(v\, z_1 / 100)\sqrt{u^2/(1+u^2)} < 3$ m/s ;

h) lightweight gear rim[6];

i) helical gears where $\beta > 30°$[6].

K_v can be read from graphs (see **6.5.1**) or computed (see **6.5.2**). The method gives similar values.

6.5.1 *Graphical values of the dynamic factor using method C*

$$K_v = (f_F\, K_{350}\, N) + 1 \qquad ...(30)$$

f_F takes into account the influence of the load on the dynamic factor, K_{350}, the influence of the gear accuracy grade at the specific loading of 350 N/mm and N is the resonance ratio (see equation 9).

The curves for gear accuracy grade in Figure 6 and Figure 7 extend only to the value

$(v\, z_1 / 100)\sqrt{u^2/(1+u^2)}$, which is not generally exceeded for this accuracy grade.

a) For helical gears with overlap ratio $\varepsilon_\beta \geq 1$ (also approximately for $\varepsilon_\beta > 0{,}9$) The correction factor f_F shall be in accordance with Table 5 and $(K_{350}\, N)$ in accordance with Figure 6.

b) For spur gears The correction factor f_F shall be in accordance with Table 6 and $(K_{350}\, N)$ in accordance with Figure 7.

c) For helical gears with overlap ratio $\varepsilon_\beta < 1$ The value K_v is determined by linear interpolation between values in accordance with a) and b):

$$K_v = K_{v\alpha} - \varepsilon_\beta\,(K_{v\alpha} - K_{v\beta}) \qquad ...(31)$$

where

$K_{v\alpha}$ is the dynamic factor for spur gears using b);

$K_{v\beta}$ is the dynamic factor for helical gears using a).

[6] If the rim is very light or if helical gears have a very large overlap ratio, values obtained from Figure 6 or Figure 7 are too unfavourable. Thus calculated values tend to be safe. The same applies when gears are made of cast iron.

6.5.2 ***Determination by calculation of the dynamic factor using method C***

a) For spur gears and helical gears with overlap ratio $\varepsilon_\beta \geq 1$ (also approximately for $\varepsilon_\beta > 0,9$)

$$K_v = 1 + \left(\frac{K_1}{K_A \dfrac{F_t}{b}} + K_2 \right) \frac{v\, z_1}{100} \sqrt{\frac{u^2}{1 + u^2}} \qquad \text{...(32)}$$

Table 3 — Assumed mean values of the effective base pitch deviation, $f_{pb\ eff}$ for factor $K_{v\text{-}C}$

	Hardened nitrided ←			Accuracy grades as specified in ISO 1328-1					Tempered normalized →	
	3	4	5	6	7	8	9	10	11	12
$f_{pb\ eff}$	2,8	5,1	9,8	19,5	35	51	69	100	134	191

Table 4 — Values of the factors K_1 and K_2 for the calculation of $K_{v\text{-}C}$, by equation (32)

	K_1										K_2
	Accuracy grades as specified in ISO 1328-1										**All accuracy grades**
	3	4	5	6	7	8	9	10	11	12	
Spur gears	2,1	3,9	7,5	14,9	26,8	39,1	52,8	76,6	102,6	146,3	0,0193
Helical gears	1,9	3,5	6,7	13,3	23,9	34,8	47,0	68,2	91,4	130,3	0,0087

Numerical values for K_1 and K_2 shall be as specified in Table 4. If $(F_t\ K_A)/b$ is less than 100 N/mm, this value to be assumed equal to 100 N/mm. See **6.4.1** a) in this regard.

b) For helical gears with overlap ratio $\varepsilon_\beta < 1$ The value K_v is determined by linear interpolation between values determined for spur gears ($K_{v\alpha}$ and helical gears ($K_{v\beta}$) in accordance with **6.5.1** c). See equation (31).

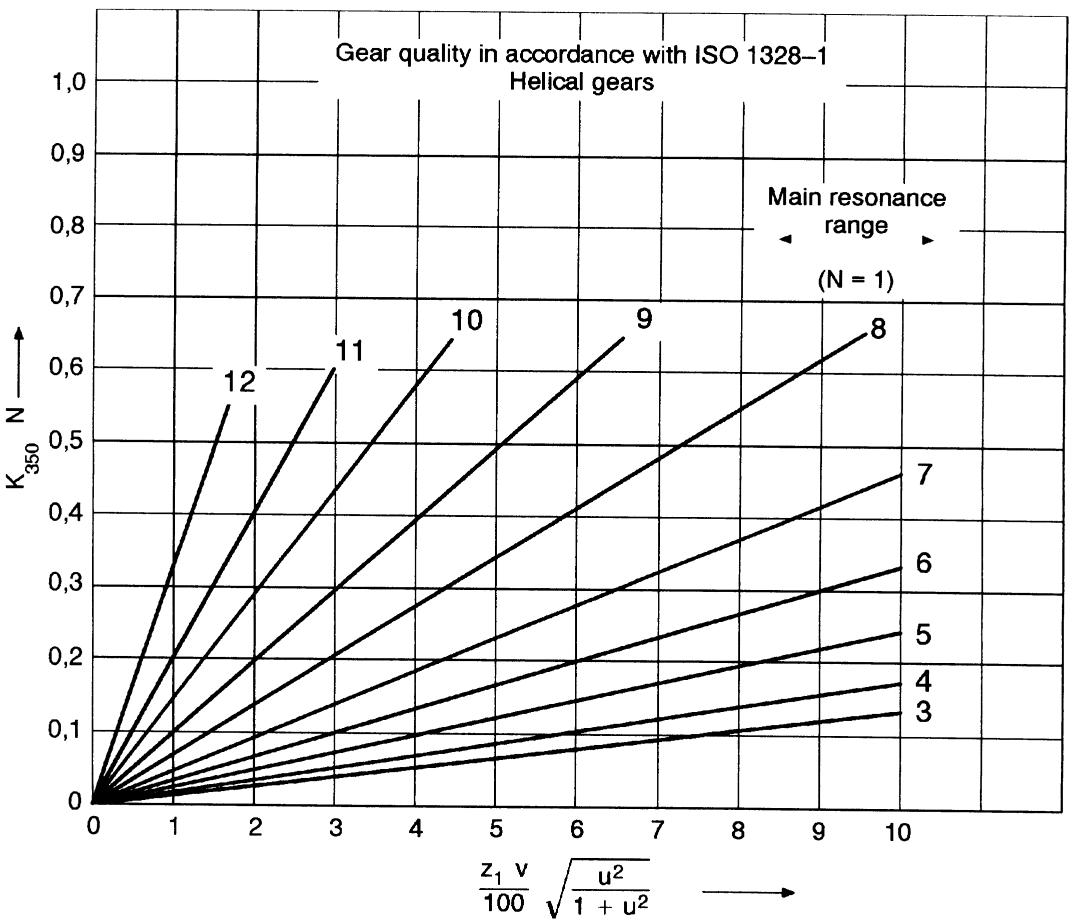


Figure 6 — Values of K_{350} N for helical gears with $\varepsilon_\beta \geq 1$

Table 5 — Load correction factor f_F for helical gears

Gear accuracy grade[a]	Load correction factor f_F							
	$(F_t\ K_A)/b$ in N/mm							
	≤ 100	200	350	500	800	1 200	1 500	2 000
3	1,96	1,29	1	0,88	0,78	0,73	0,70	0,68
4	2,21	1,36	1	0,85	0,73	0,66	0,62	0,60
5	2,56	1,47	1	0,81	0,65	0,56	0,52	0,48
6	2,82	1,55	1	0,78	0,59	0,48	0,44	0,39
7	3,03	1,61	1	0,76	0,54	0,42	0,37	0,33
8	3,19	1,66	1	0,74	0,51	0,38	0,33	0,28
9	3,27	1,68	1	0,73	0,49	0,36	0,30	0,25
10	3,35	1,70	1	0,72	0,47	0,33	0,28	0,22
11	3,39	1,72	1	0,71	0,46	0,32	0,27	0,21
12	3,43	1,73	1	0,71	0,45	0,31	0,25	0,20

NOTE Interpolate for intermediate values

[a] Gear accuracy grade in accordance with ISO 1328-1

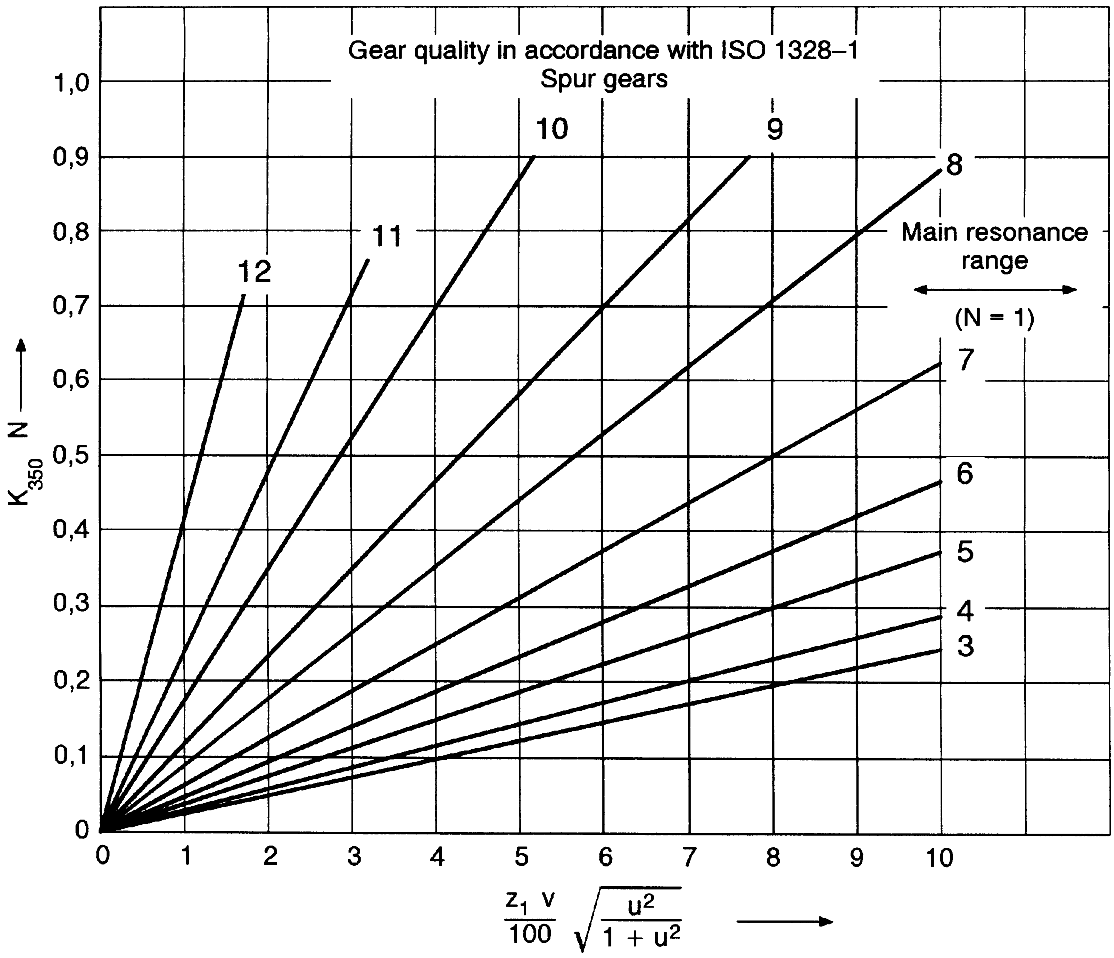


Figure 7 — Value of K_{350} N for spur gears

Table 6 — Load correction factor f_F for spur gears

Gear accuracy grade[a]	Load correction factor f_F							
	$(F_t\ K_A)/b$ in N/mm							
	≤ 100	200	350	500	800	1 200	1 500	2 000
3	1,61	1,18	1	0,93	0,86	0,83	0,81	0,80
4	1,81	1,24	1	0,90	0,82	0,77	0,75	0,73
5	2,15	1,34	1	0,86	0,74	0,67	0,65	0,62
6	2,45	1,43	1	0,83	0,67	0,59	0,55	0,51
7	2,73	1,52	1	0,79	0,61	0,51	0,47	0,43
8	2,95	1,59	1	0,77	0,56	0,45	0,40	0,35
9	3,09	1,63	1	0,75	0,53	0,41	0,36	0,31
10	3,22	1,67	1	0,73	0,50	0,37	0,32	0,27
11	3,30	1,69	1	0,72	0,48	0,35	0,30	0,24
12	3,37	1,71	1	0,72	0,47	0,33	0,27	0,22

NOTE Interpolate for intermediate values
[a] Gear accuracy grade in accordance with ISO 1328-1

6.6 Determination of the dynamic factor using Method D: $K_{v\text{-}D}$

The same simplifying assumptions applicable to method C apply to this method according to **6.3.4**. See **6.5** a) to f). A specific load $(F_t\ K_A)/b$, the magnitude varying depending on application, is, however, assumed in the simplest case to be 350 N/mm (see Table 5 and Table 6).

Consequently, for $K_{v\text{-}D}$, the calculation procedure for $K_{v\text{-}C}$ (see **6.5**) applies. In this the correction factor f_F in equation (30) has a constant value (in the simplest case f_F = 1), and in equation (32) the value $(F_t\ K_A)/b$ = constant (in the simplest case: 350 N/mm). For this case, $K_v = (K_{350}\ N) + 1$ and can be taken directly from Figure 6 or Figure 7. Contrary to method C, when the specific load exceeds 350 N/mm, calculation for the simplest case gives rather safe values (K_v values too large); when the specific load is less than 350 N/mm, values tend to be less than completely safe (K_v values too small).

6.7 Determination of the dynamic factor using Method E: $K_{v\text{-}E}$

Figure 8 shows dynamic factors which can be used in the absence of specific knowledge of the dynamic loads. The curves of Figure 8 and equations (33) to (36) are based on empirical data, and do not account for resonance (see **6.1**).

Due to the approximate nature of the empirical curves, selection of the dynamic factor curve should be based on experience with the manufacturing methods and operating considerations (see **6.1**). In most cases the contact pattern on the tooth flank (tooth bearing pattern) is a useful aid for comparison against previous experience.

Choice of curves, 6 through 12 and "very accurate gearing", should be based on transmission error (see **6.3.5**). When transmission error is not known, it is reasonable to refer to the contact pattern on the tooth flank. If the contact pattern on each tooth flank is not uniform, pitch accuracy (single pitch deviation) can be incorporated as a representative value to determine the dynamic factor. C is the transmission accuracy number. Due to the approximation mentioned above, slight deviations from the selected C value are not considered significant to the gearset rating.

6.7.1 *Very accurate gearing*

Where gearing is manufactured using process control which provide tooth accuracies which correspond to "very accurate gearing", or design and manufacturing techniques which ensure a low transmission error equivalent to this accuracy, values of K_v between 1,0 and 1,1 may be used, depending on the specifier's experience with similar applications and the degree of accuracy actually achieved. To use these values, the value of similar applications should first be verified by method A or B, and the gearing shall be maintained in accurate alignment and adequately lubricated so that its accuracy is maintained under the operation conditions.

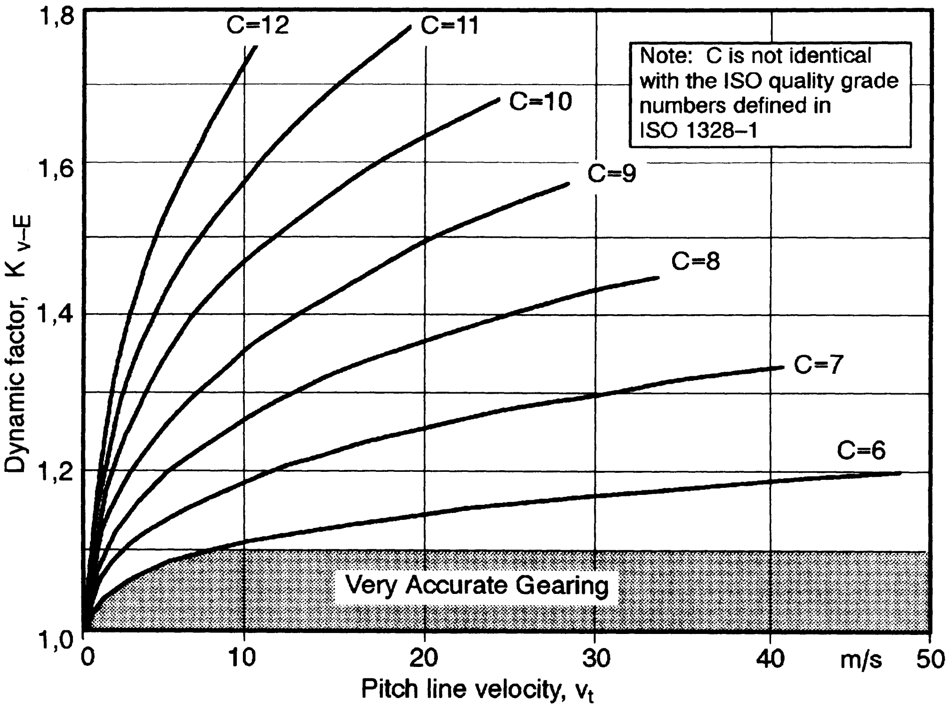


Figure 8 — Dynamic factor $K_{v\text{-}E}$

6.7.2 *Curves numbered C = 6 to 12*

The empirical curves of Figure 8 are generated by the following equations for values of C, such that:

$6 \leq C \leq 12$;

$6 \leq z \leq 1\,200$ or $10\,000/m_n$, whichever is less;

$1{,}25 \leq m_n \leq 50$.

Curves may be extrapolated beyond the end points shown in Figure 8, based on experience and careful consideration of the factors influencing dynamic load. For purposes of computer calculations, equation (37) defines the end points of the curves in Figure 8.

$$K_v = \left[\frac{A}{A + \sqrt{200\ v_t}}\right]^{-B} \qquad \text{...(33)}$$

where

$$\boldsymbol{A = 50 + 56\ (1{,}0 - B)} \qquad \text{...(34)}$$

For $6 \leq C \leq 12$

$$\boldsymbol{B = 0{,}25\ (C - 5{,}0)^{0{,}667}} \qquad \text{...(35)}$$

C is related to (but not identical to) the transmission accuracy grade number, obtained by rounding the calculated C value to the next higher integer:

$$\boldsymbol{C = -\ 0{,}5048\ \ln(z_{1,2}) - 1{,}144\ \ln(m_n) + 2{,}852\ \ln(f_{pt\ 1,2}) + 3{,}32} \qquad \text{...(36)}$$

The value of C shall first be calculated for the combination of z_1, $f_{\text{pt}\ 1}$ and also for the combination of z_2, $f_{\text{pt}\ 2}$. The larger resulting value shall be used.

The maximum recommended pitch line velocity $v_{t\,max}$ for a given grade C defined as follows is determined by

$$v_{t\,max} = \frac{[A + (14 - C)]^2}{200} \qquad \text{...(37)}$$

where

$v_{t\,max}$ is maximum pitch line velocity at the operating pitch diameter (end point of K_v curves in Figure 8), in metres per second.

7 Face load factors $K_{H\beta}$ and $K_{F\beta}$

7.1 Gear tooth load distribution

The face load factor takes into account the effects of the non-uniform distribution of load over the gear facewidth on the surface stress ($K_{H\beta}$) and on the tooth-root stress ($K_{F\beta}$).

See **7.3.1** for definitions of the face load factors.

The extent to which the load is unevenly distributed depends on the following influences:

a) the gear tooth manufacturing accuracy: lead, profile, and spacing;

b) alignment of the axes of rotation of the mating gear elements;

c) elastic deflections of gear unit elements: shafts, bearings, housings, and foundations which support the gear elements;

d) bearing clearances;

e) Hertzian contact and bending deformations at the tooth surface;

f) thermal deformations due to operating temperature (especially important for gears with large facewidths);

g) centrifugal deflections due to operating speed;

h) helix modifications including tooth crowning and end relief;

i) running-in effects;

j) total tangential tooth load (including increases due to application factor K_A and dynamic factor K_v; but see **7.3.3**);

k) additional shaft loads, (e.g. from belt or chain drives);

l) gear geometry.

7.2 Methods for the determination of the face load factor: principles, assumptions

Several methods in accordance with the specifications of **4.1.8** are given in **7.2.1** to **7.2.4** for the determination of the face load factors.

Careful analysis is recommended when the face/diameter ratio, b/d, of the pinion is greater than 1,5 for through hardened gears and greater than 1,2 for surface hardened gears.

When equivalent misalignments due to mechanical and thermal deformations are compensated for by helix modification (possibly varying over the facewidth), a nearly uniform load distribution over the facewidth may be achieved for a given operating condition, if there is a high degree of manufacturing accuracy. In this case the value of the face load factor approaches unity. See **7.6.1** and **7.6.2.1** for instructions on calculations. See Annex A for guidance data on face crowning and tip relief. See **4.1.8** for accuracy grade limitations.

7.2.1 *Method A — Factors $K_{H\beta\text{-}A}$ and $K_{F\beta\text{-}A}$*

By this method, the load distribution over the facewidth is determined by means of a comprehensive analysis of all influence factors. The load distribution over the facewidth of gears under load can be assessed from measured values of tooth-root strain, during operation at working temperature or, with limitations, by a critical examination of the tooth bearing pattern.

Data to be given in the delivery specification or drawing include:

a) maximum (permissible) face load factor, or

b) maximum permissible total mesh misalignment under operating load and temperature. The face load factor can be derived from this by using a precise calculation method; for this calculation it is also necessary that all other relevant influences be known.

7.2.2 ***Method B — Factors $K_{H\beta\text{-}B}$ and $K_{F\beta\text{-}B}$***

Concerning this method it is assumed that the initial equivalent misalignment $F_{\beta x}$ is known. This can be determined by measurement or by means of a correspondingly precise calculation method.

In this method, all deformations and displacements relevant to the distribution of load over the facewidth, including manufacturing deviations and modifications, are to be taken into consideration with the correct sign.

7.2.3 ***Method C — Factors $K_{H\beta\text{-}c}$ and $K_{F\beta\text{-}c}$***

By using this method, account is taken of those components of equivalent misalignment due to pinion and pinion-shaft deformations and also those due to manufacturing deviations. Means of evaluating approximate values of the variables include: calculation, measurement, experience, individually or in combination (see **7.6**).

7.2.3.1 *Method C1*

When the pinion is symmetrically positioned between bearings and the pinion shaft diameter is approximately equal to the root diameter, $K_{H\beta}$ can be calculated directly with sufficient precision, when the following conditions apply and the following assumptions are justified:

a) wheel and wheel shaft are sufficiently stiff that their deflections can be ignored;

b) deformations of the gear case and of the bearings, as well as bearing clearances are of low enough magnitude that they can be ignored. (If this is not permissible, the corresponding deformations and displacements are to be added with their correct sign to f_{ma});

c) the torsional and bending deflection of a pinion integral with a shaft can be determined with loading distributed evenly over the facewidth;

d) the bearings do not absorb any bending moments;

e) only the pinion shaft torque is used to determine tooth and bearing loads, with the bearing reactions on the pinion shaft;

f) the running-in allowance is estimated in accordance with **7.4.2**. It is further assumed that the running-in allowance y_β is proportional to the equivalent misalignment prior to running-in $F_{\beta x}$;

g) the mesh misalignments f_{ma} due to manufacturing are in accordance with **7.6.3**;

h) special features of rod mill pinion: mating pinion with $u = 1$; torsional deflection of each in opposite sense thus compensating for each other. The flexural components due to the load transmitted by the pinions are added together;

i) special features of simple planetary gear trains: as a rule, the diameters of sun gears and planet gears do not differ greatly, thus the deformations of both are taken into consideration. These include the torsional deflection of the sun gear due to the multiple meshes and, depending on the bearing arrangement, any bending deflections of planet gear assemblies due to meshing with sun and annulus gears. See annex **C.1** for information on calculations.

7.2.3.2 *Method C2*

This method is based on the following conditions and assumptions:

a) same as in **7.2.3.1** items a), b), c), d), e) f) and g);

b) pinion on solid shaft for single or two-stage gear trains is in accordance with Figure 16;

c) the torsional deflection is determined for load distributed uniformly over the facewidth, and the bending deflection is determined for point loading at the centre of the facewidth;

d) the assumed slope of the bending curve is tangent to a parabolic line of bending at $s/l = -0{,}24$ and 0 and + 0,24;

e) a factor of 1,5 to compensate for non-linear components of torsional and bending deflections and for converting the tangential load in the plane of action to the tangential load at the reference circles;

f) the constant 0,3 in equations (76) and (77) takes into account the fact that the torsional and bending deflections cannot fully compensate for each other;

g) shaft diameter d_{sh} is constant;

h) factor K' takes into account the stiffening effect of the pinion body;

i) shaft material is steel;

j) for gears with well designed crowning, 50 % of the elastic deflection component is included in equivalent misalignment; for gears with well designed end relief, 67 % is included; also see Annex A.

See annex **C.2** for further assumptions and explanatory notes regarding the derivation of $K_{H\beta}$ in accordance with method C2.

7.2.4 ***Method D — Factors $K_{H\beta\text{-}D}$ and $K_{F\beta\text{-}D}$***

For certain fields of application there is a simplified method of determining the face load factor for common accuracy grades. The values of $K_{H\beta\text{-D}}$ and $K_{F\beta\text{-D}}$ for each accuracy level can be read directly from a graph (or calculated using approximately corresponding equations); see **7.7**.

Method D is derived from method C2 for the special case of a pinion positioned symmetrically between the bearings. From experience it appears that calculated values give a margin of approximately 10 % additional safety over values in accordance with method C. The method involves the following conditions and assumptions:

a) Assumptions

1) wheel and wheel shaft are sufficiently stiff so that their deflections may be ignored;

2) the difference between the deflections of the two bearings in the loaded condition is negligible (see also **7.6.5**). Bearing clearance can be ignored;

3) the deflection of the case may be ignored (see also **7.6.4**);

4) the pinion is located symmetrically between the bearings (s/l = 0) and its bending deflections can be ignored;

5) helix modifications are not taken into consideration;

6) mesh stiffness c_γ = 20 N/(µm·mm);

7) the ratio of facewidth to tooth depth $b/h \leq 12$;

8) calculated contact pattern width $b_{cal}/b \geq 1$; i.e. $K_{H\beta} \leq 2$;

9) steel pinion and pinion shaft with helical gearing;

10) mesh misalignment due to manufacturing, f_{ma}, is as specified in **7.6.3.5** [see equation (95) or (96)];

11) lubrication is with mineral oil with or without customary additives.

b) Special features of structural steel, through hardened steel, and nodular cast iron gears

1) running-in allowance $y_\beta = 0,5\ F_{\beta x}$ for $\sigma_{H\ lim}$ = 640 N/mm^2 (corresponding to the maximum original equivalent misalignment $F_{\beta x}$; see Figure 13 and Figure 14);

2) specific loading F_m/b = 650 N/mm.

c) Specific features of surface hardened gears

1) ratio of pinion facewidth/pinion reference diameter: $b/d_1 < 1,5$;

2) running-in allowance y_β has the following values:

— when $K_{H\beta} \leq 1,34$, the value $y_\beta = 0,15\ F_{\beta x}$ applies for the running-in process (for proportionality range see **7.4.2**); corresponding to equivalent misalignment $F_{\beta x} \leq$ 40 µm;

— when $K_{H\beta} > 1,34$, the value y_β = 6 µm (as a constant);

3) specific loading F_m/b = 1 000 N/mm.

7.3 General principles for the determination of the face load factors $K_{H\beta}$ and $K_{F\beta}$

Uneven load distribution along the facewidth is caused by an equivalent mesh misalignment in the plane of action, comprising load-induced elastic deformation of gears and housing, and displacements of bearing journals; also by manufacturing deviations and thermal distortions.

When combined, the housing and gear manufacturing deviations, deflections of the housing and displacements of journal bearings, always result in a straight line deviation within the plane of action. Elastic deformations of shafts and gear bodies always result in non-linear deviations, as well as the deformations produced by thermal distortion resulting from uneven temperature distribution over the facewidth. The undulations and the flank form deviation are superimposed on the resulting mesh alignment. The unevenness of load distribution is reduced by running-in in accordance with the running-in effects characteristic of the material combination.

7.3.1 *Determination of the face load factor (for contact stress)* $K_{H\beta}$

$K_{H\beta}$ takes into account the effect of the load distribution over the facewidth on the contact stress and is defined as follows:

$$K_{H\beta} = \frac{\text{maximum load per unit facewidth}}{\text{average load per unit facewidth}} = \frac{(F/b)_{max}}{F_m/b} \quad ...(38)$$

The tangential loads at the reference cylinder are used for an approximate calculation, i.e. using the transverse specific loading [$F_m/b = (F_t\, K_A\, K_v)/b$] at the reference cylinder and the corresponding maximum local loading.

So as to simplify the procedures for the evaluation of $K_{H\beta}$ by methods C to D (excluding method C1), equivalent mesh misalignment due to elastic deflections is assumed to follow a straight line. Consequently, it is necessary to substitute a corrected value of the calculated deflection f_{sh} into the evaluation of $F_{\beta x}$ [see Annex C, Figure C.2 and, for example, equation (69)].

For the particular case of a pinion positioned symmetrically between bearings, $K_{H\beta\text{-}C1}$ can be calculated using a good approximation method without this simplification (see **7.2.3.1** and **7.6.1**).

With increasing curvature of the elastic deformation line, as occurs when gear pairs are heavily loaded or pinion facewidth to diameter ratios are large, or both, the assumption that the equivalent misalignment follows a straight line in the plane of action leads to increasing differences between calculated and actual distributions of load over the facewidth (see remark on limit values of b/d in **7.2**).

7.3.2 *Determination of face load factor (for tooth-root stress)* $K_{F\beta}$

$K_{F\beta}$ takes into account the effect of the load distribution over the facewidth on the stresses at the tooth-root. It depends on the variables which are determined for $K_{H\beta}$ and also on the facewidth to tooth depth ratio, b/h.

a) Determination by calculation

$$K_{F\beta} = K_{H\beta}^{N_F} \quad ...(39)$$

$$N_F = \frac{(b/h)^2}{1 + b/h + (b/h)^2} = \frac{1}{1 + h/b + (h/b)^2} \quad ...(40)$$

The smaller of the values b_1/h_1, b_2/h_2 is to be used as b/h. Boundary condition: when $b/h < 3$, substitute 3 for b/h. For double helical gears, b_B is to be used instead of b.

b) Graphical values

$K_{F\beta}$ can be read from Figure 11 when $K_{H\beta}$ and b/h have been determined. For a rough estimate, an average value in the range 3 < b/h 12 may be chosen. This results in a fixed relationship between $K_{H\beta}$ and $K_{F\beta}$ (see Figure 11). $K_{F\beta} = K_{H\beta}$ is used for $b/h \geq 12$. If the face load factor for surface stress $K_{H\beta}$ has been determined in accordance with method D, $K_{F\beta}$ can be read directly from the graphs for $K_{H\beta\text{-}D}$. See **7.2.4** a) for the assumptions made here.

7.4 Principles for the determination of the face load factor $K_{H\beta}$ used in Methods C and D

As explained in **7.3.1**, methods C2 and D involve the assumption that gear body elastic deflections produce, in the mesh, a linearly increasing separation over the facewidth of the working tooth flanks (see annex **C.2** for further information). That equivalent misalignment, inclusive of manufacturing deviations, involves similar separation of working flanks, which is implicit in this assumption.

Figure 9 and Figure 10 illustrate the influences of equivalent misalignment, according to these assumptions, and the tooth load, on the load distribution.

a) Method C2: Determination by calculation

As indicated in Figure 10, two conditions are identified, with F_m as shown. In the case of double helical gears, $b = 2b_B$. The smaller of the values for pinion or wheel shall be substituted for b or b_B, this being the width at the tooth roots excluding tooth end chamfering or rounding.

1) $b_{cal}/b \leq 1$ corresponding to $\dfrac{F_{\beta y}\, c_\gamma}{2\, F_m/b} \geq 1$:

$$K_{H\beta} = \sqrt{\frac{2\, F_{\beta y}\, c_\gamma}{F_m/b}} \geq 2 \qquad ...(41)$$

$$b_{cal}/b = \sqrt{\frac{2\, F_m/b}{F_{\beta y}\, c_\gamma}} \qquad ...(42)$$

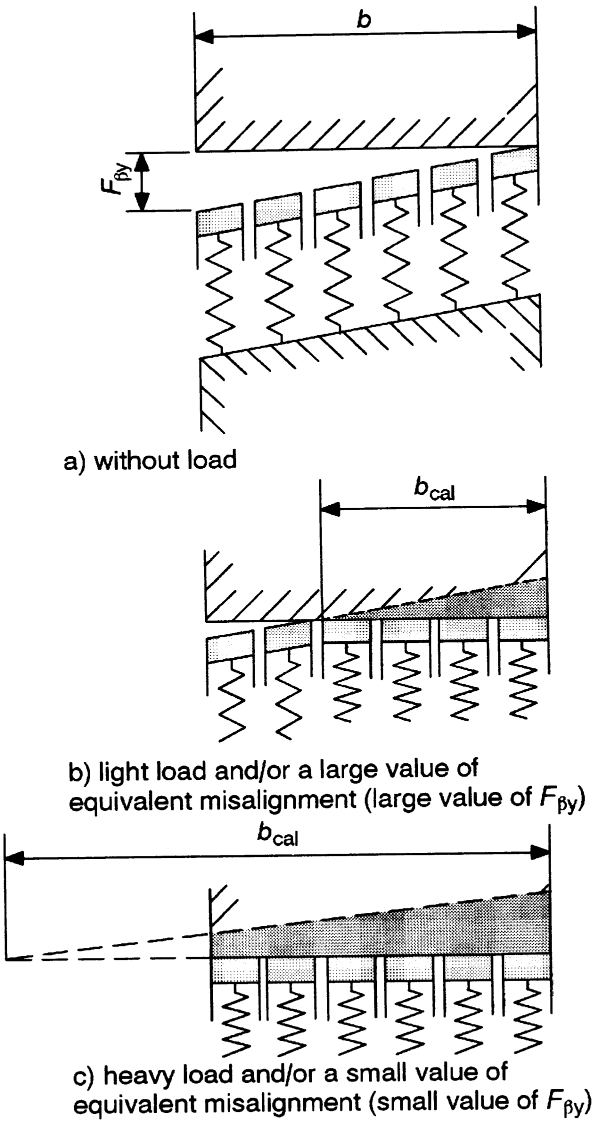
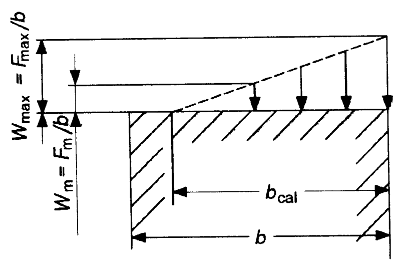
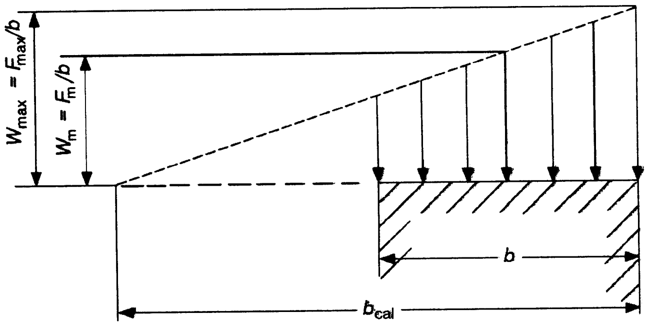


Figure 9 — Distribution of the load along facewidth with linear equivalent misalignment (illustration of principle)



a) low loading and/or large value of $F_{\beta y}$, $b_{cal}/b \leq 1$

$$W_{max} = \frac{F_{max}}{b} = \frac{2\,F_m}{b}\,\frac{b}{b_{cal}}$$



b) high loading and/or small value of $F_{\beta y}$, $b_{cal}/b > 1$

$$W_{max} = \frac{F_{max}}{b} = \frac{F_m}{b}\,\frac{b_{cal}}{b_{cal} - \frac{b}{2}}$$

Figure 10 — Calculation of the load per the unit facewidth $(F/b)_{max}$ with linear distribution of the load on the facewidth $F_m = (F_t\,K_A\,K_v)$

2) $b_{cal}/b > 1$ corresponding to $\dfrac{F_{\beta y}\,c_\gamma}{2\,F_m/b} < 1$:

$$K_{H\beta} = 1 + \frac{F_{\beta y}\,c_\gamma}{2\,F_m/b} \qquad ...(43)$$

$$b_{cal}/b = 0{,}5 + \frac{F_m/b}{F_{\beta y}\,c_\gamma} \qquad ...(44)$$

b) Method C2: Graphical values

The value of $K_{H\beta}$ can be read from Figure 11 when c_γ and $F_{\beta y}$ have been evaluated — see clause **9** for mesh stiffness c_γ.

c_γ recommended for method C is used for a rough estimate, i.e. c_γ = 20 N/(mm·µm). This value can also be used for helical gearing of which $\beta \leq 30°$; the effect of the helix angle in this range is practically negligible. Allowance is made for the relative mesh stiffness at low specific loading by replacing values of F_m/b < 100 N/mm with the value F_m/b = 100 N/mm. This enables the use of a simplified nomogram for the determination of face load factors (see Figure 12).

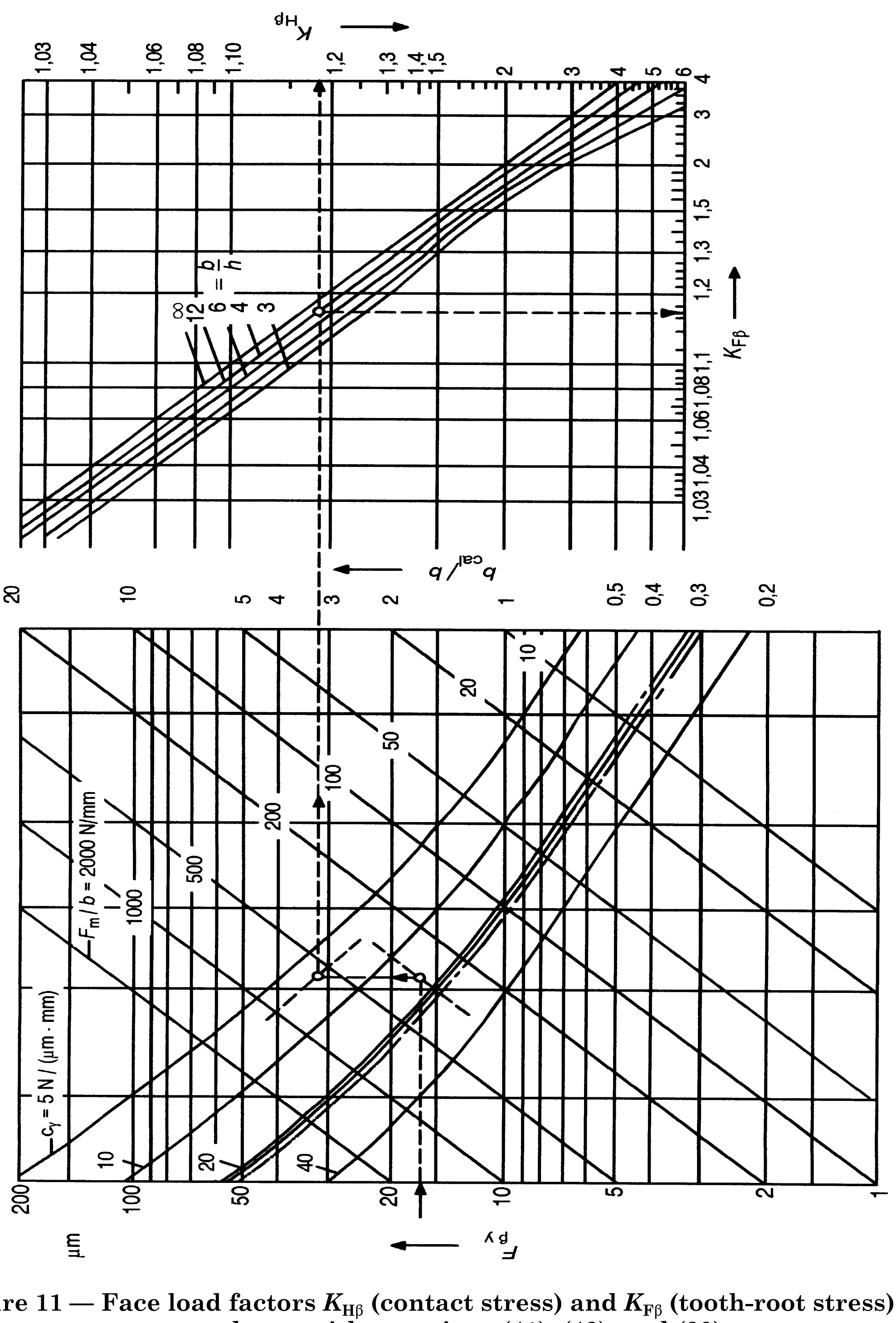


Figure 11 — Face load factors $K_{H\beta}$ (contact stress) and $K_{F\beta}$ (tooth-root stress), in accordance with equations (41), (43), and (39)

For values of $b/h \geq 12$, $K_{F\beta}$ shall be equated to $K_{H\beta}$. Values of $K_{H\beta}$ and $K_{F\beta}$ corresponding to values of F_m/b < 100 N/mm shall be made equal to $K_{H\beta}$ and $K_{F\beta}$ values appropriate to F_m/b = 100 N/mm, i.e. the grey zone in Figure 12 is thus not used. The hatched zone with interrupted lines shall be avoided.

c) Calculation of $K_{H\beta}$ using Methods C1 and D

With these methods, calculation of $K_{H\beta}$ is directly done using given equations; $F_{\beta y}$ does not need a separate determination.

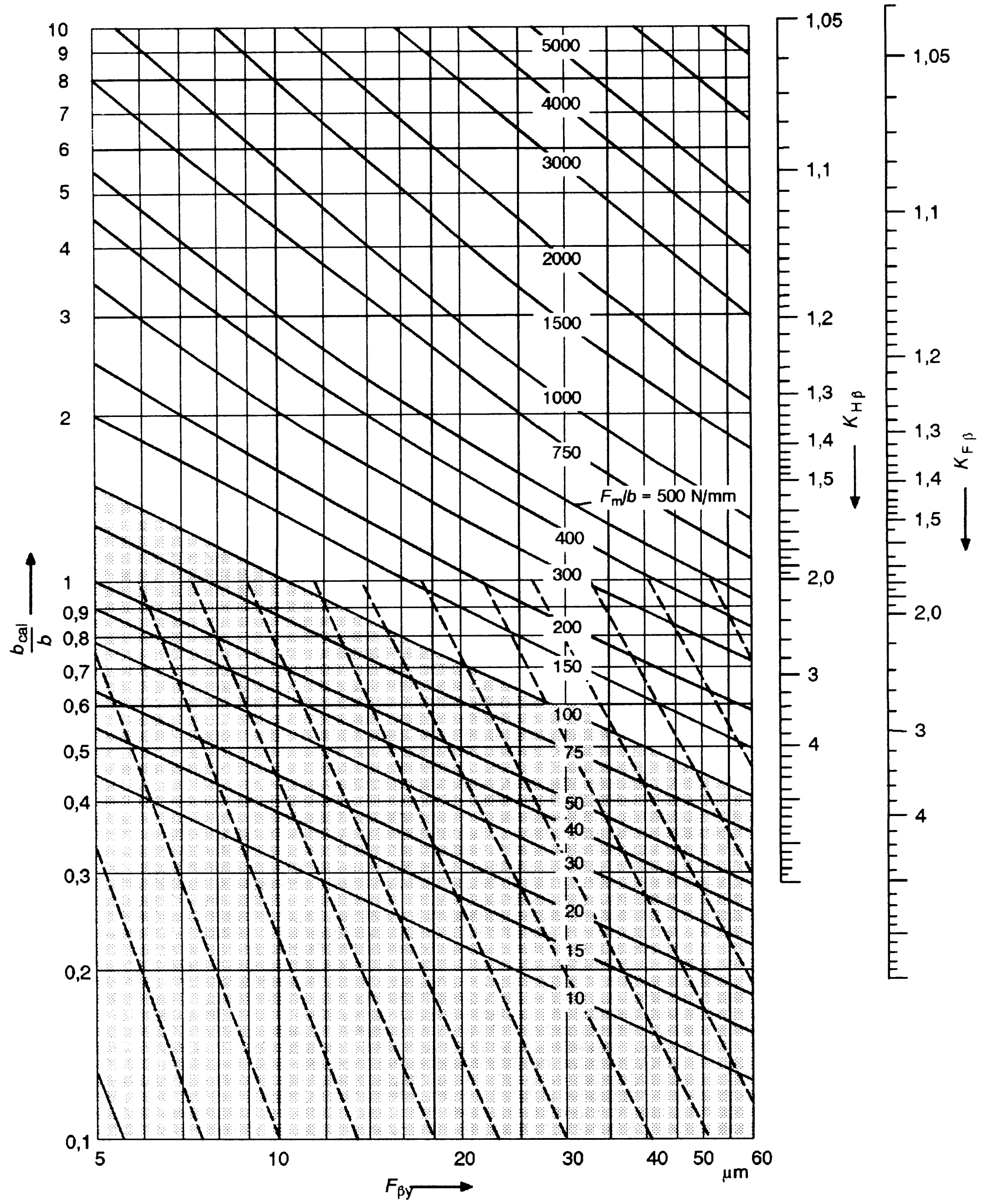


Figure 12 — Face load factors $K_{H\beta}$ (contact stress) and $K_{F\beta}$ (tooth-root stress) for values of b/h < 12 in accordance with method C2 (simplified method in which c_γ = 20 N/(mm µm)

7.4.1 *Effective equivalent misalignment $F_{\beta y}$*

The following equation can be used for common transmission designs:

$$F_{\beta y} = F_{\beta x} - y_\beta = F_{\beta x}\,\chi_\beta \qquad \text{...(45)}$$

where

$F_{\beta x}$ is the initial equivalent misalignment, i.e. the absolute value of the sum of deformations, displacements and manufacturing deviations of pinion and wheel, measured in the plane of action. $F_{\beta x}$ can be determined in accordance with method B or C2 (see **7.5** and **7.6.2.1**).

7.4.2 *Running-in allowance y_β and running-in factor* χ_β

y_β is the amount by which the initial equivalent misalignment is reduced by running-in since operation was commenced. χ_β is the factor characterizing the equivalent misalignment after running-in. It is convenient to use χ_β in calculations, only as long as y_β is proportional to $F_{\beta x}$. The important influences include:

— pinion and wheel material;

— surface hardness;

— rotational speed at the reference circle;

— type of lubricant;

— surface treatment;

— abrasive in the oil;

— initial equivalent misalignment $F_{\beta x}$ (as a result of deformations, displacements and manufacturing deviations).

y_β and χ_β do not take into account the effects of running-in operations which are obtained by manufacturing processes such as lapping. Removal of material by such means shall be taken into account in the value of f_{ma}.

In the absence of direct, assured data from experiment or operating experience (method A), y_β can be determined in accordance with method B given in **7.4.2.1** or **7.4.2.2**.

7.4.2.1 Determination of y_β and χ_β by calculation, method B The values from equations (46) to (54) reproduce the curves in Figure 13 and Figure 14.

a) For structural steel, through-hardened steel and pearlitic or bainitic nodular cast iron:

$$y_\beta = \frac{320}{\sigma_{H\,lim}} F_{\beta x} \quad \text{...(46)}$$

$$\chi_\beta = 1 - \frac{320}{\sigma_{H\,lim}} \quad \text{...(47)}$$

where $y_\beta \le F_{\beta x}$ and $\chi_\beta \ge 0$ ...(48)

For $v \le 5$ m/s:	no restriction
For 5 m/s < $v \le 10$ m/s:	the upper limit of y_β is 25 600/$\sigma_{H\,lim}$ corresponding to $F_{\beta x}$ = 80 µm
For v > 10 m/s:	the upper limit of y_β is 12 800/$\sigma_{H\,lim}$ corresponding to $F_{\beta x}$ = 40 µm

$\sigma_{H\,lim}$ is as specified in ISO 6336-5.

b) For grey cast iron and ferritic nodular cast iron:

$$y_\beta = 0{,}55\ F_{\beta x} \quad \text{...(49)}$$

$$\chi_\beta = 0{,}45 \quad \text{...(50)}$$

where

For $v \le 5$ m/s:	no restriction
For 5 m/s < $v \le 10$ m/s:	the upper limit of y_β is 45 µm corresponding to $F_{\beta x}$ = 80 µm
For v > 10 m/s:	the upper limit of y_β is 22 µm corresponding to $F_{\beta x}$ = 40 µm

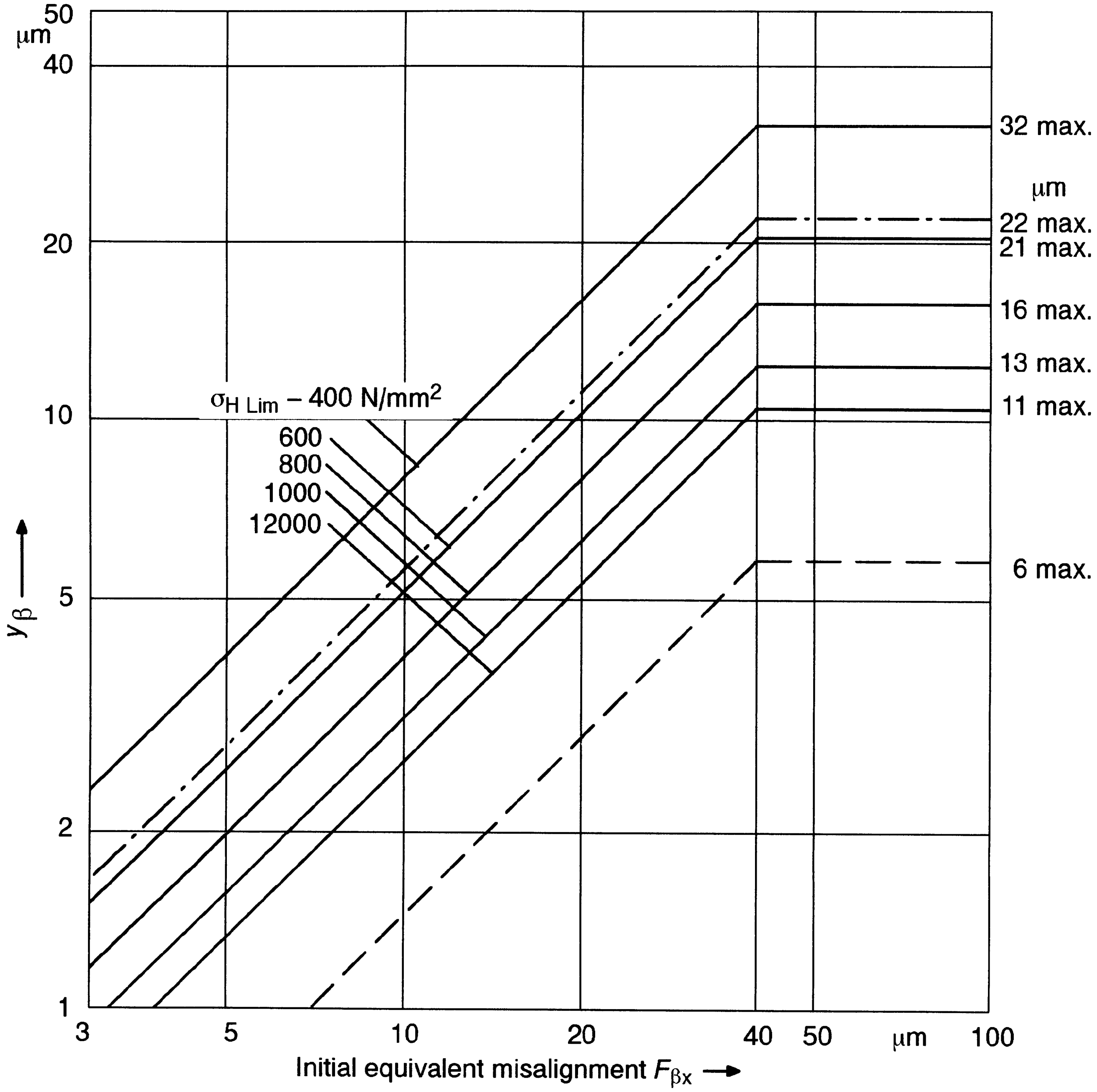
c) For case hardened, surface hardened, nitrided and nitrocarburized steel and surface hardened nodular cast iron:

$$y_\beta = 0{,}15\ F_{\beta x} \quad \text{...(51)}$$

$$\chi_\beta = 0{,}85 \quad \text{...(52)}$$

where

For all velocities: the upper limit of y_β is 6 µm corresponding to $F_{\beta x}$ = 40 µm



Through hardening steel, nodular cast iron, (pearlitic, bainitic) — Rotational velocity v > 10 ms

Grey cast iron, nodular cast iron (ferritic) — Rotational velocity v > 10 ms

Steel, case hardened, nitrided, surface hardened, or nitrocarburized
Surface hardened nodular cast iron — All speeds

(If the materials of the pinion and the wheel are different, y_β must be determined in accordance with equation (53))

Figure 13 — Running-in allowance for a gear pair, see also Figure 14

When the material of the pinion differs from that of the wheel, the values for the pinion ($y_{\beta 1}$ and $\chi_{\beta 1}$) and the values for the wheel ($y_{\beta 2}$ and $\chi_{\beta 2}$) are to be determined separately. Then the average values, of each (y_β and χ_β) from equations (53) and (54), are used for the calculations.

$$y_\beta = \frac{(y_{\beta 1} + y_{\beta 2})}{2} \qquad \text{...(53)}$$

$$\chi_\beta = \frac{(\chi_{\beta 1} + \chi_{\beta 2})}{2} \qquad \text{...(54)}$$

7.4.2.2 Graphical values of y_β, method B The value y_β can be read from Figure 13 and Figure 14 as a, function of the initial equivalent misalignment $F_{\beta x}$ and the value of $\sigma_{H\,lim}$ for the material.

7.5 Determination of the face load factor using method B: $K_{H\beta\text{-}B}$

Subject to the conditions and assumptions described in **7.2.2**, method B is suitable for all types of gear units and operating conditions.

Calculations are carried out in accordance with equations (41) to (45) with the running-in allowance Y_β calculated in accordance with **7.4.2**. The initial equivalent misalignment $F_{\beta x}$ is known from measurements, or from an accurate analysis of the deformations and displacements, or from transmissions of similar type which have already been constructed, and the relevant information is supplied for the calculation.
See also **7.2.2**.

7.6 Determination of the face load factor using method C: $K_{H\beta\text{-}C1}$, $K_{H\beta\text{-}C2}$

NOTE 5 If calculation by method C produces values of $K_{H\beta}$ greater than 1,5, then as a general rule, suitable steps should be taken to reduce such values (e.g. stiffer shafts etc.). This is not required when large values of $K_{H\beta}$ can be tolerated or if change of dimensions is inappropriate.

As explained in **7.2.3**, the components of the equivalent misalignment due to the deformations of pinion and pinion shaft and to manufacturing deviations measured in the plane of action, are taken into consideration in the calculation of $K_{H\beta}$. Concerning further variables see **7.6.2.1**, footnote 19 therein and also **7.6.4** and **7.6.5**.

7.6.1 ***Method C1, factor $K_{H\beta\text{-}C1}$***

Relative to the conditions and assumptions described in **7.2.3.1**, the use of method C is appropriate for gears having characteristics as in the following:

a) pinion is on a solid shaft, or hollow shaft where $d_{shi}/d_{sh} < 0{,}5$, positioned symmetrically between bearings ($s/l \leq 0{,}1$, see Figure 16). (An asymmetrically positioned pinion leads to an additional bending deformation which shall be evaluated and added to f_{ma});

b) pinion diameter is about equal to shaft diameter;

c) there are stiff wheel and case, stiff wheel shaft and stiff bearings (see also **7.6.4** and **7.6.5**);

d) the contact pattern, under load, extends over the entire facewidth;

e) there are no additional external loads acting on the pinion shaft (e.g. from shaft couplings);

f) running-in allowance $y_\beta \leq y_{\beta\,max}$ as is specified in **7.4.2**. A computed value for $F_{\beta x}$ may be verified using the equation:

$$F_{\beta x} = \frac{K_{H\beta} - 1}{\chi_\beta \left(\dfrac{c_\gamma / 2}{F_m / b} \right)} = \frac{2(K_{H\beta} - 1) F_m}{\chi_\beta \; c_\gamma \; b} \qquad \text{...(55)}$$

It is recommended that the values used for f_{ma} be verified by inspection checks, the contact pattern being in the working attitude.

See **7.6.1.3** for application to grooved roller mill pinions and **7.6.1.4** for application to planetary gears.

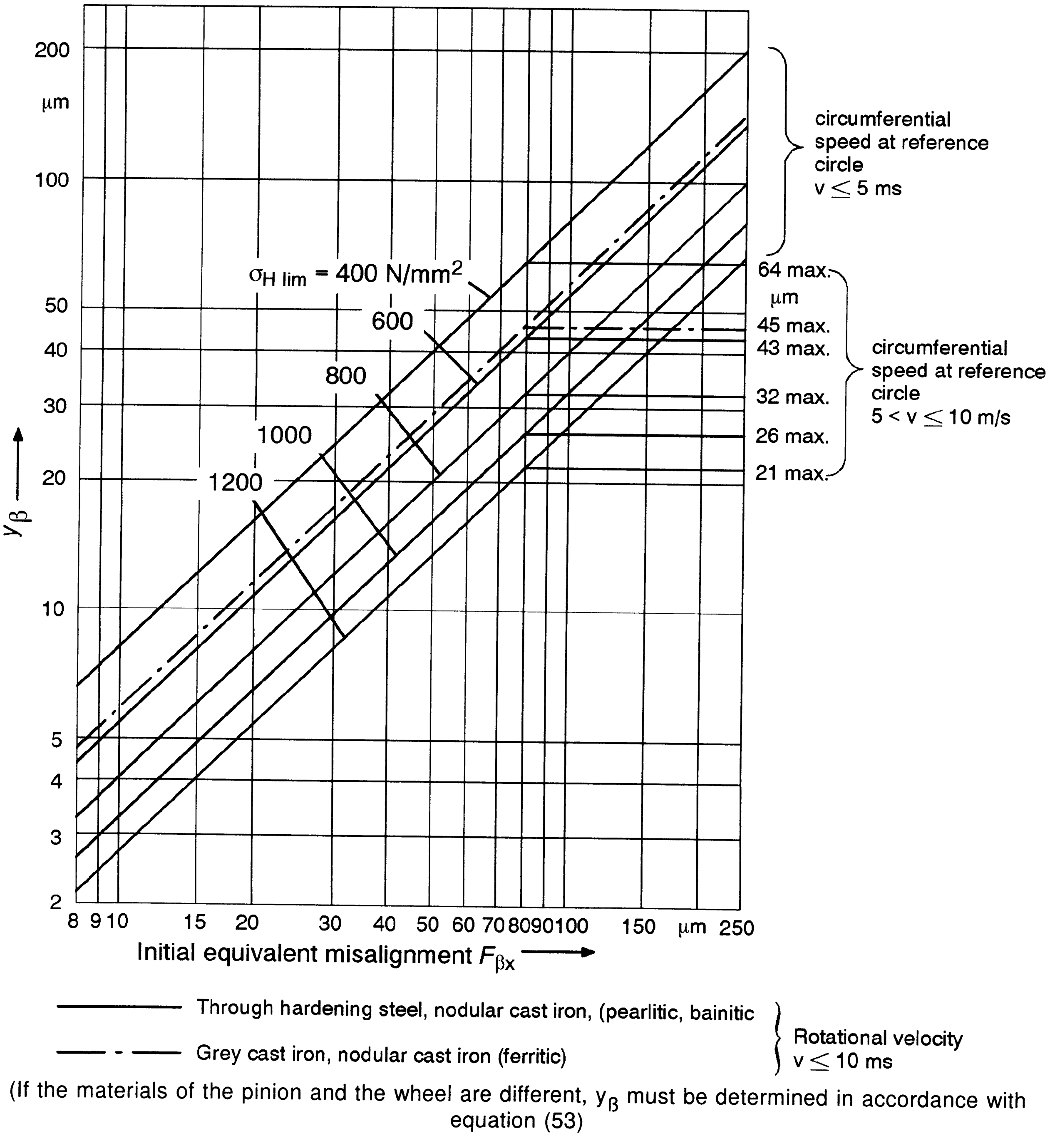


Figure 14 — Running-in allowance for a gear pair, see also Figure 13

7.6.1.1 *Gears with single pair tooth contact and unmodified helices*

a) Spur and single helical gears[7)]

$$K_{H\beta} = 1 + \frac{4000}{3\,\pi}\,\chi_\beta\,\frac{c_\gamma}{E}\left(\frac{b}{d_1}\right)^2\left[5{,}12 + \left(\frac{b}{d_1}\right)^2\left(\frac{l}{b} - \frac{7}{12}\right)\right] + \frac{\chi_\beta\,c_\gamma\,f_{ma}}{2\,F_m/b} \qquad \text{...(56)}$$

b) Double helical gearing[7), 8)]

$$K_{H\beta} = 1 + \frac{4000}{3\,\pi}\,\chi_\beta\,\frac{c_\gamma}{E}\left[3{,}2\left(\frac{2\,b_B}{d_1}\right)^2 + \left(\frac{B}{d_1}\right)^4\left(\frac{l}{B} - \frac{7}{12}\right)\right] + \frac{\chi_\beta\,c_\gamma\,f_{ma}}{F_m/b_B} \qquad \text{...(57)}$$

where

c_γ is the mesh stiffness of the pair (see clause **9**);
E is the modulus of elasticity of the pinion shaft;
d_1, b, l are the dimensions of pinion and pinion shaft (see Annex C, Figure C.1);
B is the total facewidth of double helical gears (including gap);
b_B is the width of one helix ($b = 2\,b_B$); for definition of b and b_B, see **7.4** a);
χ_β is the running-in factor [see equations (45) to (54) and condition f) in **7.6.1**].

See Annex C for the derivation of the equations.

7.6.1.2 *Gears with single pair tooth contact and with modified helices*

a) Spur and single helical gears[7)]

1) With partial helix modification[9)] (with compensation only for torsional deformation).

The face load factor is derived from the bending deflection and manufacturing deviation.

$$K_{H\beta} = 1 + \frac{4000}{3\,\pi}\,\chi_\beta\,\frac{c_\gamma}{E}\left(\frac{b}{d_1}\right)^4\left(\frac{l}{b} - \frac{7}{12}\right) + \frac{\chi_\beta\,c_\gamma\,f_{ma}}{2\,F_m/b} \qquad \text{...(58)}$$

2) With full helix modification (with compensation for torsional and bending deflections).

$$K_{H\beta} = 1 + \frac{\chi_\beta\,c_\gamma\,f_{ma}}{2\,F_m/b} \quad \text{and} \quad K_{H\beta} \geq 1{,}05 \qquad \text{...(59)}$$

[7)] It is assumed that the entire torque is input at one shaft end. If the torque is input at both shaft ends or in between helices of a double helical gear, a more accurate analysis is necessary.

[8)] Value of $K_{H\beta}$ is for the more severely stressed helix, which is that nearer to the torqued end of the pinion; tangential load is divided equally between the two helices; i.e. gap width small compared to the facewidth [$(B - 2b_B) \leq 0{,}5b_B$]. As for the calculation for $K_{H\beta}$, when half the tooth width (incorporating half the gap width) is used, the obtained values are large. Thus for double helical gears with large gap width, method C2 must be used for the calculation of $K_{H\beta}$.

[9)] Torsional deflection can be almost completely compensated for by means of a linear tooth trace or helix angle modification. In addition, crowning is necessary when compensation of bending deflection is required.

b) Double helical gearing[10), 11)]

With full helix modification[12)] (with compensation for torsional and bending deflections).

$$K_{H\beta} = 1 + \frac{\chi_\beta \, c_\gamma \, f_{ma}}{F_m / b_B} \quad \text{and} \quad K_{H\beta} \geq 1{,}05 \qquad \text{...(60)}$$

7.6.1.3 Grooved-roller mill pinions In the case of grooved-roller mill pinions (gear pair with $u = 1$ and branched power transmission paths) k % of the torque is absorbed by the driven pinion, $(100 - k)$ % is absorbed at the output end of the driving shaft. In keeping with **7.6.1**, it is assumed that the driving and driven pinions are mounted symmetrically between their bearings.

Equations (56) and (57) respectively are expanded in accordance with assumptions in **7.2.3.1**[10)]

NOTE 6 It is generally the case that calculation values tend to be safe when:
$(b/d_1)^2 \, (l/b - 7/12) < 5{,}12$; or when
$(B/d_1)^4 \, (l/b - 7/12) < 3{,}2 \, (2 \, b_B/d_1)^2$.

a) Spur and single helical gears without helix modification

$$K_{H\beta} = 1 + \frac{4000}{3\,\pi} \chi_\beta \frac{c_\gamma}{E} \left(\frac{b}{d_1}\right)^2 \left[5{,}12 + 7{,}68 \frac{100-k}{k} + \left(\frac{b}{d_1}\right)^2 \left(\frac{l}{b} - \frac{7}{12}\right)\right] + \frac{\chi_\beta \, c_\gamma \, f_{ma}}{2 \, F_m / b} \qquad \text{...(61)}$$

b) Spur and single helical gears with full helix modification Equation (59) applies in this case.

c) Double helical gearing without helix modification[10), 11), 13)]

$$K_{H\beta} = 1 + \frac{4000}{3\,\pi} \chi_\beta \frac{c_\gamma}{E} \left[\left(\frac{2\,b_B}{d_1}\right)^2 \left(1{,}28 + 1{,}92 \frac{100-k/2}{k/2}\right) + \left(\frac{B}{d_1}\right)^4 \left(\frac{l}{B} - \frac{7}{12}\right)\right] + \frac{\chi_\beta \, c_\gamma \, f_{ma}}{F_m / b_B} \qquad \text{...(62)}$$

Equation (60) applies for double helical gears with full helix modification.

7.6.1.4 Simple planetary gears According to **7.2.3.1** and **7.6.1**, method C1 is suitable for the gears of single planetary gearsets in which the following features are often found[14)].

— Either the sun or the planet carrier, and sometimes the annulus gear, are free to float; otherwise a comparable division of load between the individual planet gears is achieved by greater accuracy of manufacture and/or flexibility.

— Planet gears with single helical teeth will tilt under the influence of axial forces, to an extent corresponding to the bearing clearances. This inclination can be compensated for by helix modification and eventually by crowning.

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10) It is assumed that the entire torque is input at one shaft end. If the torque is input at both shaft ends or in between helices of a double helical gear, a more accurate analysis is necessary.

11) Value of $K_{H\beta}$ is for the more severely stressed helix, which is that nearer to the torqued end of the pinion; tangential load is divided equally between the two helices; i.e. gap width small compared to the facewidth $[(B - 2b_B) \leq 0{,}5b_B]$. As for the calculation for $K_{H\beta}$, when half the tooth width (incorporating half the gap width) is used, the obtained values are large. Thus for double helical gears with large gap width, method C2 must be used for the calculation of $K_{H\beta}$.

12) Full modification of both helices is necessary. Partial modification of the helix angle merely to compensate for torsional deflection is not appropriate for double helical gears which are symmetrically positioned between bearings. Torsional and bending deflections can be almost completely compensated for by means of helix angle modification. However, it is often sufficient if only the helix nearest the torque input end is modified; torsional and bending deflections of the other helix tend to compensate each other. This should be verified.

13) If $K_{H\beta} > 2$, it is likely that values are considerably overestimated and deviate on the safe side. Therefore, the design should be checked, preferably by a more accurate method (method A or B).

14) Restoring forces in toothed couplings are ignored. Restoring forces which lead to uneven distribution of load over the facewidth can occur when transmission elements are rigid and friction characteristics of flexible couplings are unsatisfactory.

— In planetary gears with ratios of $i > 4$, larger gears are commonly rotatably mounted with rolling bearings, on planet pins which are rigidly fixed to the planet carrier; thus the free bending length is very small. It is seldom possible to accommodate bearings in the bores of smaller planet gears for lower ratios, so these are equipped on each side with integral journals and are rotatably mounted between bearings fitted in the planet carrier. Since in such circumstances the diameters of sun and planet gears are of much the same size, elastic deflections of both are considered (sun gears: torsional deflection involving several meshes; planets: bending deflection influenced by balancing loads; annulus deformation being neglected). For such arrangements, approximate values of $K_{H\beta}$ can be calculated in accordance with the following procedures.

Determine:

— mesh misalignment due to manufacturing deviation f_{ma} in accordance with **7.6.3**;

— running-in factor χ_β in accordance with **7.4.2**;

— mesh stiffness c_γ in accordance with clause **9**.

Any unequal division of the total tangential load between the planet gears is covered by factor K_γ (see **4.2**). Thus for these gears $F_m = (F_t\, K_A\, K_\gamma\, K_v)$ and with F_t being the nominal tangential load transmitted per mesh, also the sum of the loads over both helices of double helical gears.

a) **Spur and single helical gears**[15)]

Gear pair **without** helix modification (e.g. sun gear/planet gear, mounted on a fixed, rigid planet pin)

$$K_{H\beta} = 1 + \frac{4000}{3\,\pi}\, p\, \chi_\beta\, \frac{c_\gamma}{E}\left(\frac{b}{d_{sun}}\right)^2 5{,}12 + \frac{\chi_\beta\, c_\gamma\, f_{ma}}{2\, F_m/b} \qquad ...(63)$$

3) For the same gear pair but **with** helix modification (with compensation only for torsional deflection): $K_{H\beta}$ is in accordance with equation (59) and $K_{H\beta} \geq 1{,}05$.

Gear pair **without** helix modification, (e.g. sun gear/planet gear with journals, mounted with bearings in the planet carrier)

$$K_{H\beta} = 1 + \frac{4000}{3\,\pi}\, \chi_\beta\, \frac{c_\gamma}{E}\left[5{,}12\, p\left(\frac{b}{d_{sun}}\right)^2 + 2\left(\frac{b}{d_{pla}}\right)^4\left(\frac{l_{pla}}{b} - \frac{7}{12}\right)\right] + \frac{\chi_\beta\, c_\gamma\, f_{ma}}{2\, F_m/b} \qquad ...(64)$$

4) For the same gear pair but **with** full helix modification (with full compensation for bending and torsional deflection): $K_{H\beta}$ is in accordance with equation (59) and $K_{H\beta} \geq 1{,}05$.

5) Gear pair **without** helix modification (e.g. annulus gear/planet gear with journals, mounted with bearings in the planet carrier)

$$K_{H\beta} = 1 + \frac{8000}{3\,\pi}\, \chi_\beta\, \frac{c_\gamma}{E}\left(\frac{b}{d_{pla}}\right)^4\left(\frac{l_{pla}}{b} - \frac{7}{12}\right) + \frac{\chi_\beta\, c_\gamma\, f_{ma}}{2\, F_m/b} \qquad ...(65)$$

6) For the same gear pair, but **with** helix modification (with compensation only for bending deflection): $K_{H\beta}$ is in accordance with equation (59) and $K_{H\beta} \geq 1{,}05$.

7) Gear pair **with or without** helix modification (e.g. annulus gear/planet gear mounted on a fixed, rigid planet pin): $K_{H\beta}$ is in accordance with equation (59) and $K_{H\beta} \geq 1{,}05$.

b) **Double helical gearing**[15), 16)]

1) Gear pair **without** helix modification (e.g. sun gear/planet gear mounted on fixed, rigid planet pin)

$$K_{H\beta} = 1 + \frac{4000}{3\,\pi}\, p\, \chi_\beta\, \frac{c_\gamma}{E}\left(\frac{2b_B}{d_{sun}}\right)^2 3{,}2 + \frac{\chi_\beta\, c_\gamma\, f_{ma}}{F_m/b_B} \qquad ...(66)$$

[15)] It is assumed that the entire torque is input at one shaft end. If the torque is input at both shaft ends or in between helices of a double helical gear, a more accurate analysis is necessary.

[16)] Value of $K_{H\beta}$ is for the more severely stressed helix, which is that nearer to the torqued end of the pinion; tangential load is divided equally between the two helices; i.e. gap width small compared to the facewidth $[(B - 2b_B) \leq 0{,}5b_B]$. As for the calculation for $K_{H\beta}$, when half the tooth width (incorporating half the gap width) is used, the obtained values are large. Thus for double helical gears with large gap width, method C2 must be used for the calculation of $K_{H\beta}$.

2) For the same gear pair but **with** helix modification (with compensation only for torsional deflection)[17]: $K_{H\beta}$ is in accordance with equation (60) and $K_{H\beta} \geq 1{,}05$.

3) Gear pair **without** helix modification (e.g. sun gear/planet gear with journals mounted with bearings in the planet carrier)

$$K_{H\beta} = 1 + \frac{4000}{3\,\pi}\chi_\beta \frac{c_\gamma}{E}\left[3{,}2\,p\left(\frac{2\,b_B}{d_{sun}}\right)^2 + 2\left(\frac{B}{d_{pla}}\right)^4\left(\frac{l_{pla}}{B} - \frac{7}{12}\right)\right] + \frac{\chi_\beta\, c_\gamma\, f_{ma}}{F_m/b_B} \quad ...(67)$$

4) For the same gear pair but **with** full helix modification (with full compensation for bending and torsional deflections)[17]: $K_{H\beta}$ is in accordance with equation (60) and $K_{H\beta} \geq 1{,}05$.

5) Gear pair **without** helix modification (e.g. annulus gear/planet gear with journals mounted with bearings in the planet carrier)

$$K_{H\beta} = 1 + \frac{8000}{3\,\pi}\chi_\beta \frac{c_\gamma}{E}\left(\frac{B}{d_{pla}}\right)^4\left(\frac{l_{pla}}{B} - \frac{7}{12}\right) + \frac{\chi_\beta\, c_\gamma\, f_{ma}}{F_m/b_B} \quad ...(68)$$

6) For the same gear pair but **with** helix modification (with compensation only for bending deflection): $K_{H\beta}$ is in accordance with equation (60) and $K_{H\beta} \geq 1{,}05$.

7) Gear pair **with or without** helix modification (e.g. annulus gear/planet gear mounted on a fixed, rigid planet pin): $K_{H\beta}$ is in accordance with equation (60) and $K_{H\beta} \geq 1{,}05$.

7.6.2 *Method C2, factor $K_{H\beta\text{-}C2}$* [18]

Subject to the conditions and assumptions described in **7.2.3.2** method C2 is suitable for gears having the following characteristics:

a) steel pinion on solid or hollow shaft with $d_{shi}/d_{sh} < 0{,}5$ for single stage or two stage transmissions complying with Figure 16;

b) constant or stepped diameter shafts, d_{sh} being the diameter of an equivalent shaft of constant diameter of which the bending deflection is the same as that of the actual shaft;

c) stiff wheel and gear case, stiff wheel shaft and stiff bearings (see also **7.6.4** and **7.6.5**);

d) there are no additional external loads acting on the pinion shaft (e.g. from shaft couplings);

e) the pinion is located on the shaft within, $0 \leq s/l \leq 0{,}3$ (see Figure 16). With suitable helix modification, restrictions on s/l are unnecessary.

It is recommended that the values used for f_{ma} be verified by checking the contact pattern in the working attitude.

Calculation is carried out in accordance with equations (41) to (45) with the running-in allowance y_β as defined in **7.4.2**. Of the components of deformations, displacement and deviation, only those in the plane of action are determinant for the calculation of $F_{\beta x}$.

7.6.2.1 Determination of the initial equivalent misalignment, $F_{\beta x}$ (see Annex C) The value $F_{\beta x}$ is the absolute value of the sum of manufacturing deviations and pinion and shaft deflections, measured in the plane of action [see caution in b)].

a) Gear pairs of which the size and suitability of the contact pattern is not proven and the bearing pattern under load is imperfect[19]:

See Annex C for an explanation of the factor 1,33 in equation (69).

$$F_{\beta x} = 1{,}33 f_{sh} + f_{ma};\ F_{\beta x} \geq F_{\beta x\,min} \quad ...(69)$$

[17] Value of $K_{H\beta}$ is for the more severely stressed helix, which is that nearer to the torqued end of the pinion; tangential load is divided equally between the two helices; i.e. gap width small compared to the facewidth $[(B - 2b_B) \leq 0{,}5b_B]$. As for the calculation for $K_{H\beta}$, when half the tooth width (incorporating half the gap width) is used, the obtained values are large. Thus for double helical gears with large gap width, method C2 must be used for the calculation of $K_{H\beta}$.

[18] Torsional deflection can be almost completely compensated for by means of a linear tooth trace or helix angle modification. In addition, crowning is necessary when compensation of bending deflection is required.

[19] Running clearances in rolling bearings should be very small under working conditions. Large clearances can contribute considerably to equivalent misalignment $F_{\beta x}$. When this is the case, a more accurate calculation using equation (71), or a contact pattern check under load is recommended.

Allowance should be made in f_{ma} for the effects of adjustment measures (lapping, running-in at part load), crowning or end-relief, similarly, for that of the position of the contact pattern.

b) Gear pairs with verification of the favourable position of the contact pattern (e.g. by modification of the teeth or adjustment of bearings)[20), 21)]

$$F_{\beta x} = |\,1{,}33\, f_{sh} - f_{H\beta 6}\,|\,;\ F_{\beta x} \geq F_{\beta x\,\min} \qquad \text{...(70)}$$

A check shall be made to ascertain which of the helices of double-helical gears has the larger equivalent misalignment and consequently, is determinate for $K_{H\beta}$.

By subtracting $f_{H\beta 6}$, which is the helix slope deviation tolerance for ISO quality 6 (see ISO 1328-1), allowance is made for the compensatory roles of elastic deformation and manufacturing deviations. See Annex C for explanatory notes to equation (70).

Subject to achieving the requisite contact patterns, $F_{\beta x}$ can be calculated using equation (70) for gears which have been processed by lapping, running-in at part load or other adjustment means, as well as for gears with carefully designed crowning or end-relief. For crowned gears, the contact pattern centre shall be suitably offset from the mid-face position. Concerning double helical gears, it is necessary to ascertain if the lesser deformed helix of the pinion has the largest value of $F_{\beta x}$.

CAUTION — When, apart from pinion body and pinion shaft deformations f_{shi}, those of the wheel/wheel shaft f_{sh2} and the gear case f_{ca}, and also the displacements of the bearings f_{be} are to be taken into consideration, equations (69) and (70) are to be extended to become equation (71) (see also **7.6.4** and **7.6.5**):

$$F_{\beta x} = 1{,}33 f_{sh} + f_{sh2} + f_{ma} + f_{ca} + f_{be};\ f_m \geq f_{H\beta 6} \qquad \text{...(71)}$$

The signs of f_{sh2}, f_{ca} and f_{be} shall be carefully heeded; if precise information is not available, it is essential that positive signs are chosen (so that calculated values tend to be safe). Only the bending deflection, if any, of the wheel shaft, is likely to be of consequence to f_{sh2}; previously this amount was taken as the wheel shaft misalignment component of f_{be}. Nevertheless, in general the approximations according to equations (69) and (70) are satisfactory.

The following influences shall be heeded: As a rule, the elastic deformations of "relatively flexible" spur gears tend to compensate for manufacturing misalignment. On the other hand, because of the axial component of F_m in single-helical gears, additional misalignment can be induced.

Special measures can be taken to secure even distribution of load over the facewidth. These include set-up bearings, lapping the gears, or running-in the gears as a specified process, in service. By way of a further example, a spur gear or a double-helical gear can be mounted directly on a spherical roller bearing and so be free to take up an attitude of mean, balanced alignment.

Uneven distribution of the body temperature of a large high-speed gear can cause deformation near mid-facewidth resulting in heavy local loading. Either allowance for this deformation shall be included in $K_{H\beta}$, or shall be compensated for by suitable helix modification.

Similar measures shall be taken when deformation is induced by a large centrifugal force.

Furthermore: the body temperature of a high-speed helical pinion is usually higher than that of the mating wheel. This creates additional misalignment which shall be accounted for in the calculations.

c) For gears having ideal contact pattern, full helix modification, under load (for both helices of double helical gears):

$$F_{\beta x} = F_{\beta x\,\min} \qquad \text{...(72)}$$

where $F_{\beta x\,\min}$ is the greater of the two values:

$$F_{\beta x\,\min} = (0{,}005\ \text{mm}\cdot\mu\text{m/N})\ \frac{F_m}{b}\,,\ \text{or}\ F_{\beta x\,\min} = 0{,}5\, f_{H\beta} \qquad \text{...(73)}$$

[20)] If $K_{H\beta} > 2$, it is likely that values are considerably overestimated and deviate on the safe side. Therefore, the design should be checked, preferably by a more accurate method (method A or B).

[21)] With a favourable position of the contact pattern, the elastic deformations and the manufacturing deviations compensate each other. See Figure 15 "Compensatory rules".

Helix modification is intended to compensate for the torsion and bending deflections of the pinion and wheel, also the deformations or displacements of other components under operating loads and, if known, the tooth alignment deviation of the mating wheel[22].

$F_{\beta y}$ would be equal to 0 at the design loading of gear pairs having optimum helix modification, i.e. the face load factor $K_{H\beta}$ would be equal to 1. However in the interest of safety, the minimum value in accordance with equations (72) and (73) is to be used as the equivalent misalignment.

Similarly, equation (70) can be used in designing suitable crowning.

See **7.6.2.2** for the determination of f_{sh}, the equivalent misalignment due to pinion and pinion shaft deflections. See **7.6.3** for the determination of mesh misalignment due to manufacturing deviations f_{ma}. See **7.4.2** for the determination of the running-in allowance y_β, i.e. the amount by which the equivalent misalignment is reduced.

For some common arrangements of gear pairs, guidance on the calculation of $F_{\beta x}$ is included in Figure 16 a) to Figure 16 e), in which particular regard is paid to the contact pattern position. A comprehensive analysis is recommended for other, more complex, arrangements.

7.6.2.2 Equivalent misalignment f_{sh} The value f_{sh} takes into account the components of equivalent misalignment resulting from bending and twisting of the pinion and pinion shaft, and its value may be determined as follows:

7.6.2.2.1 Approximate calculation of f_{sh} The following calculation is sufficiently accurate for many common designs:

$$f_{sh} = f_{sh0} \frac{F_m}{b} \qquad \text{...(74)}$$

See **7.3.1** for F_m/b. The deviation f_{sh0} represents the resultant deflections induced by unit loading. It takes into account the shaft and body deflections and, where appropriate, helix modification. f_{sh0} depends on the pinion dimensions and the location of the pinion relative to the bearings. This is taken into consideration by the pinion offset factor γ.

a) For spur and helical gears without crowning or end relief[23]

$$f_{sh0} = 0{,}023\,\gamma \qquad \text{...(75)}$$

with the pinion offset factor, γ, as defined in equation (76) or (77).

For both spur and single helical gears[24]:

$$\gamma = \left[\left|B^* + K' \frac{l\,s}{d_1^2}\left(\frac{d_1}{d_{sh}}\right)^4 - 0{,}3\right| + 0{,}3\right]\left(\frac{b}{d_1}\right)^2 \qquad \text{...(76)}$$

with $B^* = 1$, if the total power is transmitted through a single engagement.

For double helical gears [see **7.4** a) and footnotes 5 and 6]:

$$\gamma = 2\left[\left|B^* + K' \frac{l\,s}{d_1^2}\left(\frac{d_1}{d_{sh}}\right)^4 - 0{,}3\right| + 0{,}3\right]\left(\frac{b_B}{d_1}\right)^2 \qquad \text{...(77)}$$

[22] Helix angle modification is an alteration of the helix angle, a consequence of which the axial pitch is also modified. The latter concept is useful when dealing with gears having large overlap ratios and consideration of axial pitch is often necessary.

[23] The equations (75), (76), and (77) are based on the following conventions. The bending component is the product of the deflection at the middle point of the shaft, by point b, where the load F_m is assumed to be acting. The torsional component is calculated for a solid cylinder of diameter d_1, with the load distributed evenly over the facewidth. (In reality, a smaller diameter is determinant; also the load is not evenly distributed; however, the inaccuracies in the assumptions tend to balance each other out). The equation is valid when the elastic modulus and Poisson's ratio of the material are those of steel. Based on practical experience, equations (76) and (77) also include a constant empirical term. Concerning hollow shafts, the deflection component f_{sh} derived from either of these equations, is sufficiently accurate, provided that the bore diameter does not exceed 0,5 d_{sh}.

[24] It is assumed that the entire torque is input at one shaft end. If the torque is input at both shaft ends or in between helices of a double helical gear, a more accurate analysis is necessary.

with B^* equal to 1,5, if the total power is transferred by a single engagement.

If there is more than one power path, then only k % of input is through one gear mesh (e.g. as in the case of grooved-roller mill gears) and the following applies:

$B^* = 1 + 2(100 - k)/k$ for spur and single helical gears;

$B^* = 0{,}5 + (200 - k)/k$ for double helical gears.

See **7.6.1** for $K_{H\beta}$ for symmetrically mounted grooved-roller mill and planetary gear trains.

The constant K' makes allowances for the position of the pinion in relation to the torqued end. It can be taken from Figure 16.

A comprehensive analysis is recommended for other arrangements or where the values for s/l exceed those specified in Figure 16, and also where there are additional shaft loads, e.g. from belt pulleys or chain wheels.

Substitute the absolute value f_{sh} in equations (69) and (70). See Figure 15 and **7.6.2.1** for information on the compensation of f_{sh} by f_{ma}.

It is better to calculate in accordance with **7.6.1** when the pinion is positioned midway between the bearings ($s = 0$).

b) For crowned spur and helical gears

This type of helix modification is employed in order to compensate for manufacturing deviations and load induced deformations of the gears and in particular to relieve the tooth-endloading. Gears are usually crowned symmetrically about the mid-facewidth. See Annex C for recommendations on the extent of crowning.

$$f_{sh0} = 0{,}012\ \gamma \qquad \text{...(78)}$$

with γ as defined in equations (76) and (77).

If the height of the crowning is greater than that specified in Annex A, the reduced width $b_{(b)}$ is to replace the facewidth b in formulae used for calculating load capacity (See Annex A, Figure A.1). This is determined from values of $C_{\beta(b)}$ calculated in accordance with equation (A.1) or (A.2) in Annex A. It is to be assumed that the tooth ends outside $b_{(b)}$ are not bearing any load.

NOTE 7 In Figure 11, b_{cal}/b is not relevant to crowned gears.

c) For spur and helical gears with end relief

This type of helix modification is used to protect the tooth ends from the overloading caused by equivalent misalignment. Usually the relief applied is the same at both ends of the teeth. See Annex A for recommendation on the amount of end relief.

$$f_{sh0} = 0{,}016\ \gamma \qquad \text{...(79)}$$

with γ as defined in equations (76) or (77).

If the amount of end relief is greater than is specified in Annex A, a reduced width $b_{(b)}$ shall replace the facewidth b in formulae used for calculating load capacity (see Figure A.2). This is determined from values of $C_{I(II)(b)}$ calculated in accordance with equations (A.3) or (A.4). It is to be assumed that the tooth ends outside $b_{(b)}$ are not bearing any load.

NOTE 8 In Figure 11, b_{cal}/b is not relevant to gears with end relief.

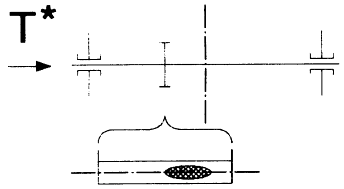
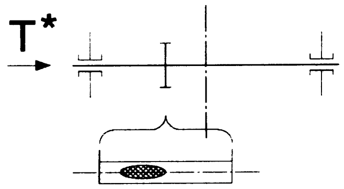
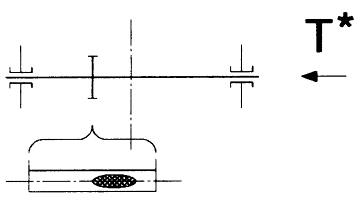
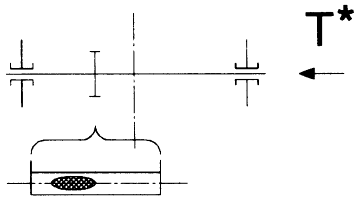
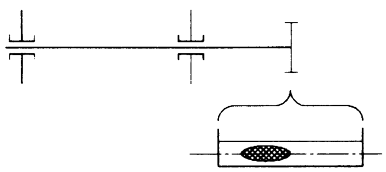
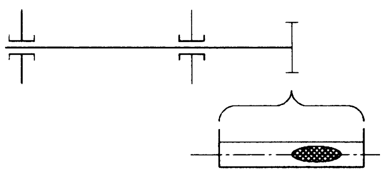
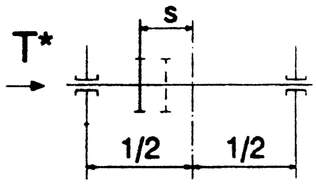
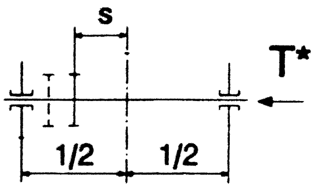
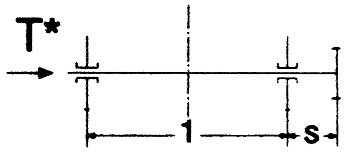
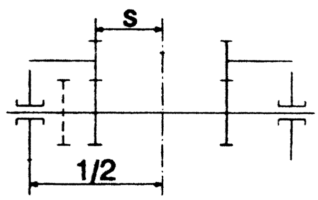
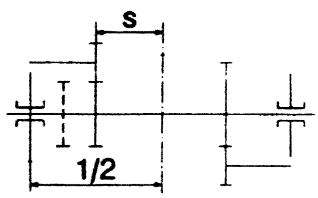
Figure	Position of contact pattern	Determination of $F_{\beta x}$
a)	Contact pattern lies towards mid bearing span 	$F_{\beta x}$ in accordance with equation (70) (compensatory)
b)	Contact pattern lies away from mid bearing span 	$F_{\beta x}$ in accordance with equation (69) (additive)
c)	Contact pattern lies towards mid bearing span 	$F_{\beta x}$ in accordance with equation (60) $\lvert K' \rvert \cdot l \cdot s/d_1^2\,(d_1/d_{sh})^4 \leq B^*$ (additive) $F_{\beta x}$ in accordance with equation (70) $\lvert K' \rvert \cdot l \cdot s/d_1^2\,(d_1/d_{sh})^4 > B^*$ (compensatory)
d)	Contact pattern lies away from mid bearing span 	$F_{\beta x}$ in accordance with equation (69) $\lvert K' \rvert \cdot l \cdot s/d_1^2\,(d_1/d_{sh})^4 \geq B^* - 0{,}3$ (additive) $F_{\beta x}$ in accordance with equation (70) $\lvert K' \rvert \cdot l \cdot s/d_1^2\,(d_1/d_{sh})^4 < B^* - 0{,}3$ (compensatory)
e)	Contact pattern lies towards the bearing 	$F_{\beta x}$ in accordance with equation (69) (additive)
f)	Contact pattern lies away from the bearing 	$F_{\beta x}$ in accordance with equation (70) (compensatory)
a) to d)	are the most common mounting arrangement with pinion between bearings.	
e) to f)	have overhung pinions.	
*T**	is the input or output torqued end, not dependent on direction of rotation.	
*B**	B^* = 1 for spur and single helical gears: B^* = 1,5 for double helical gears. The peak load intensity occurs on the helix near to the torqued end. See also **7.6.2.2.**	

Figure 15 — Rules for the determination of $F_{\beta x}$ with regard to the contact pattern position

Factor K' with stiffening[a]	Factor K' without stiffening[a]	Figure	Arrangement
0,48	0,8	a)	 with s/l < 0,3
– 0,48	– 0,8	b)	 with s/l < 0,3
1,33	1,33	c)	 with s/l < 0,3
– 0,36	– 0,6	d)	 with s/l < 0,3
– 0,6	– 1,0	e)	 with s/l < 0,3

T* is the input or output torqued end, not dependent on direction of rotation.

Dashed line indicates the less deformed helix of a double helical gear.

Determine f_{sh} from the diameter in the gaps of double helical gearing mounted centrally between bearings.

[a] When $d_1/d_{sh} \geq$ 1,15, stiffening is assumed; when d_1/d_{sh} < 1,15, there is no stiffening; furthermore, scarcely any or no stiffening at all is to be expected when a pinion slides on a shaft and feather key or a similar fitting, nor when normally shrink fitted.

Figure 16 — Constant K' for the calculation of the pinion offset factor γ

7.6.2.2.2 Specified maximum value for f_{sh} Sometimes experience with similar gear units enables the choice of an appropriate value of f_{sh} to be made.

Examples:

— $f_{sh} \approx 0$ in the case of a very rigid design; deformations are neglected.

— f_{sh} = 6 µm is occasionally specified as a maximum value for some turbine transmissions; the gears are to be designed accordingly.

When calculations are based on such assumptions, the assumptions shall be validated by computations or measurements.

7.6.2.2.3 The value f_{sh} corresponding to gear quality For certain gears, the value of f_{sh} is specified as a percentage of the allowable helix slope deviation. The gears are to be designed accordingly.

$$f_{sh} = 1{,}0\, f_{H\beta} \qquad \text{...(80)}$$

As for **7.6.2.2.2**, assumptions shall be validated by computations or measurements.

7.6.2.2.4 Determination of f_{sh} from the contact pattern Once the transmission has been assembled, the equivalent misalignment f_{sh} can be calculated for gears with or without helix modification from the width of the contact pattern without load and with part load. Equipment suitable for the application of part load shall be available.

Since the mesh stiffness falls off sharply at low specific loading, the specific loading at partial load should be at least 100 N/mm.

Care shall be taken to ensure that the pinion and wheel-shaft journals are in their working attitudes during contact pattern development (appropriate bearing clearances).

The procedure is as follows:

a) 1st step: determine the mesh misalignment f_{ma}, as described in **7.6.3.1**.

b) 2nd step: measure contact pattern length $b_{cal\,T}$ under partial load F_{mT} and calculate $b_{cal\,T}/b$.

It is necessary that the part load be chosen such that the contact pattern dimension b_{cal} is less than the facewidth ($b_{cal}/b < 1$); however, the smallest load should not be less than 10 % of the full load. The maximum length of contact pattern should not exceed 85 % of the facewidth ($b_{cal}/b < 0{,}85$), in order to ensure that the contact pattern width is less than the facewidth (the type of load distribution for $b_{cal} = b$ is not clearly defined, see Figure 7 and Figure 8).

c) 3rd step: determine the equivalent misalignment $F_{\beta x\,T}$ under partial load, (see clause **9** for tooth stiffness c_γ):

$$F_{\beta xT} = \frac{2\,F_{mT}}{\left[b \left(\frac{b_{cal\,T}}{b} \right)^2 c_\gamma \right]} \qquad \text{...(81)}$$

d) 4th step: calculate $f_{sh\,T}$ under partial load:

$$f_{sh\,T} = |\, F_{\beta xT} - f_{ma} \,| \qquad \text{...(82)}$$

e) 5th step: compute f_{sh} under full load (linear extrapolation):

$$f_{sh} = f_{sh\,T} \left(\frac{F_m}{F_{mT}} \right) \qquad \text{...(83)}$$

NOTE 9 Depending on the design, the accuracy of the method can be seriously impaired when the non-linear deflection components are induced at larger partial loads.

7.6.3 *Mesh misalignment f_{ma}*

f_{ma} is the maximum separation between the tooth flanks of the meshing teeth of mating gears, when the teeth are held in contact without significant load, the shaft journals being in their working attitudes.

f_{ma} depends on the way in which the deviations of individual components in the plane of action combine; i.e. whether, the helix slope deviation $f_{H\beta}$ of each gear and the alignment deviation of the shafts are additive or compensatory, or whether the alignment of the shafts is adjustable (e.g. by means of adjustable bearings).

For purposes of load capacity calculations in accordance with this part of ISO 6336, the methods given in **7.6.3.1** to **7.6.3.6** can be used for the determination of f_{ma}.

7.6.3.1 Determination of f_{ma} on the basis of the no-load contact pattern Under ideal conditions f_{ma} can be derived from:

$$f_{ma} = \left(\frac{b}{b_{c0}}\right) s_c \qquad \text{...(84)}$$

where b_{c0} is the length, at low loading, of the contact pattern of the assembled gears and s_c is the thickness of the coating of marking compound (see Figure 17)[25]. If gears are crowned or end-relieved, an accurate analysis is necessary.

The coating thickness of common marking compounds is in the range 2 µm to 20 µm. 6 µm can be used as a mean value consistent with good working practice.

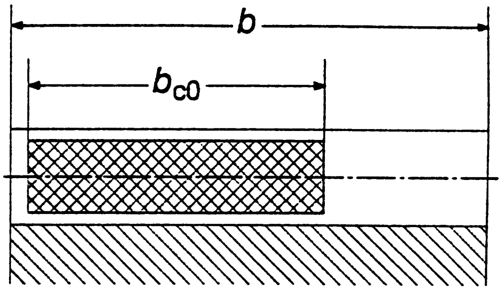


Figure 17 — Length of contact pattern b_{c0} and facewidth b

If the minimum length of contact pattern is stated on the drawing, it is convenient to determine the maximum permissible mesh misalignment.

$$f_{ma\,max} = \frac{b\, s_c}{b_{c0\,min}} \qquad \text{...(85)}$$

A mean value suitable for use in preliminary design calculations is:

$$f_{ma} = \frac{2}{3} f_{ma\,max} \qquad \text{...(86)}$$

After final assembly in the gear case, maximum and minimum values of mesh misalignment, $f_{ma\,max}$ and $f_{ma\,min}$ can be determined from the minimum or the maximum lengths of the contact pattern respectively. These values enable recalculation of preliminary, rated load capacity:

$$f_{ma} = 0{,}5\,(f_{ma\,max} + f_{ma\,min}) \qquad \text{...(87)}$$

$$f_{ma\,max} = \left(\frac{b}{b_{c0\,min}}\right) s_c \qquad \text{...(88)}$$

$$f_{ma\,min} = \left(\frac{b}{b_{c0\,max}}\right) s_c \qquad \text{...(89)}$$

The contact patterns shall be created with pinion and wheel shaft journals in their working attitudes.

7.6.3.2 Determination of f_{ma} from the length of the contact pattern under partial load and the theoretically determined deformations The following conditions are necessary for application:

[25] Precise knowledge of the coating thickness is of great importance. In case of doubt, the actual coating thickness should be determined.

The elastic deflections of pinion, wheel, shafts, gears and bearings: f_{sh}, f_{sh2}, f_{ca} and f_{be} (see **7.6.2.1**) are to be determined using an accurate calculation method. As a rule, method C2 (**7.6.2**) is not sufficiently accurate for the purpose. As indicated, the individual deflections shall be carefully considered.

The length of the contact pattern $b_{cal\,T}$ at partial loading F_{mT} [see **7.6.2.2.4** b)] is measured and the equivalent misalignment $F_{\beta xT}$ at partial loading is determined using equation (90).

$$F_{\beta xT} = \frac{2\,F_{mT}}{\left[b \left(\frac{b_{cal\,T}}{b} \right)^2 c_\gamma \right]} \qquad \text{...(90)}$$

When calculating mesh misalignment, it is necessary to distinguish between two cases:

a) Case 1: The elastic deflections augment the mesh misalignment (see, for example, Figure 15).

$$f_{ma} = F_{\beta xT} - |(f_{sh} + f_{sh2} + f_{ca} + f_{be})_T| \qquad \text{...(91)}$$

b) Case 2: The elastic deflections tend to compensate for the mesh misalignment (see, for example, Figure 15):

$$f_{ma} = F_{\beta xT} + |(f_{sh} + f_{sh2} + f_{ca} + f_{be})_T| \qquad \text{...(92)}$$

When gears are crowned or end relieved, an accurate analysis is necessary.

When the length of the contact pattern varies around the circumference, $f_{ma\,max}$ shall be derived from the minimum length, $f_{ma\,min}$ shall be derived from the maximum length, then f_{ma} shall be derived from equation (87).

7.6.3.3 Derivation of f_{ma} from the deviations of individual components (to be done after inspection and measurement of gears, bearings and gear-case)

The maximum mesh misalignment involves the most unfavourable combination of individual deviations:

$$f_{ma\,max} = \{|f_{par\,act} + f_{H\beta 1\,act} + f_{H\beta 2\,act}|\}_{max} \qquad \text{...(93)}$$

The minimum mesh misalignment from the most favorable combination:

$$f_{ma\,min} = \{|f_{par\,act} + f_{H\beta 1\,act} + f_{H\beta 2\,act}|\}_{min} \qquad \text{...(94)}$$

where $f_{H\beta 1\,act}$ and $f_{H\beta 2\,act}$ are the measured values of helix slope deviation of pinion and wheel (in accordance with ISO 1328-1). The values can vary in size and direction, around the circumference.

The combined effect of the helix slope deviation of pinion and wheel, i.e. $\Sigma f_{H\beta} = (f_{H\beta 1\,act} + f_{H\beta 2\,act})$ can be determined as follows.

The pinion and wheel, assembled on their shafts, are mounted on roller blocks which are aligned in parallel pairs and the contact patterns are generated. By moving one of the blocks, the tooth flanks are brought into contact over the entire facewidth. The $\Sigma f_{H\beta}$ can then be derived from the non-parallelism of the blocks.

$f_{par\,act}$ is the measured value of shaft misalignment, due to in-plane and out-of-plane deviations of either of the shafts. In the event of radial run-out of one or more journals, $f_{par\,act}$ can vary with the angle of rotation. Care shall be taken with the sign of each individual deviation.

A mean value derived from equation (87) is to used in gear load capacity calculations.

In this procedure the influence of bearing clearances is neglected. See **7.6.3.6**.

7.6.3.4 Specified maximum value of f_{ma} Sometimes permissible limits for the total manufacturing deviation f_{ma} [26]) are specified.

Examples:

$f_{ma\,max} \approx 0$ is sometimes demanded for accurate high speed transmissions; due to high precision of manufacturing, deviations can be neglected.

$f_{ma\,max} \approx 15$ µm can be a realistic value for certain industrial transmissions.

A mean value derived from equation (86) is to be used in gear load capacity calculations.

[26]) Appropriate control measures should be adopted to ensure that this value is maintained.

7.6.3.5 f_{ma} *for a given accuracy*

a) For assembly of gears without any modification or adjustment Inspection after assembly is recommended in this case. See also **7.6.3**.

If, according to ISO 1328-1 for a given gear quality grade, helix slope deviation tolerances are identical for pinion and wheel and if the alignment of axes tolerances are in keeping with these, the most unfavorable combination of deviations (pinion, wheel, case) would be around $f_{ma} = 3{,}0\ f_{H\beta}$.

Statistical studies show that there is a high probability that the deviations will combine to approximate values of 1,0 $f_{H\beta}$ (i.e. for one gear). A useful formulation is, thus:

$$f_{ma} = 1{,}0\ f_{H\beta} \qquad \text{...(95)}$$

b) For gear pairs with provision for adjustment (lapping or running-in under light load, adjustable bearings or appropriate helix angle modification) and gear pairs suitably crowned The no-load mesh misalignment can, to a great extent, be compensated for by means of adjustment measures such as reworking of bearings, bearing housings, etc. Satisfactory contact over the facewidth of the gears can often be achieved by these methods and by means of the other measures mentioned above. See Annex A for guide values for crowning.

If data from experience are not available, it may be assumed that properly effected adjustments will reduce the value of f_{ma} by 50 %, in which case equation (96) can be used.[27)]

$$f_{ma} = 0{,}5\ f_{H\beta} \qquad \text{...(96)}$$

c) For gear pairs with well designed end-relief In the absence of data from experience and subject to skilled execution, the following may be used:

$$f_{ma} = 0{,}7\ f_{H\beta} \qquad \text{...(97)}$$

Of a pair of gears, only the larger value of permissible helix slope deviation of one of the gears (see ISO 1328-1) is to be used in equations (95) to (97).

See Annex A for guide values for end relief.

7.6.3.6 Determination of f_{ma} with gears assembled in the gear-case After assembly in the gear-case, it may be possible to measure mesh misalignment directly with the top of the case removed. Values of $f_{ma\,max}$ and $f_{ma\,min}$ are determined from measurements made around the circumference, using feeler gauges; then f_{ma} is derived from equation (87).

For wide gears without helix modification mounted in journal bearings with relatively large clearances, the following procedure may be used. The shaft journals are supported in their working attitudes. The mating gear is clamped to prevent rotation. Bring the working faces into light contact, then insert feeler gauges between the faces at both ends of the mesh. The mesh misalignment f_{ma} is equal to the difference between the thicknesses of the gauges:

$$f_{ma} = \delta_g \left(\frac{b}{l}\right) \qquad \text{...(98)}$$

where

δ_g is the difference in the feeler gauge indications, b is the facewidth and l is the distance between the feeler gauges. When the helices of gears are modified, the amount is included in the difference δ_g. δ_g can also be determined as the difference in thickness of two lead wires which have been inserted between the flanks, where they are subjected to light load.

[27)] The equations (75), (76), and (77) are based on the following conventions. The bending component is the product of the deflection at the middle point of the shaft, by point b, where the load F_m is assumed to be acting. The torsional component is calculated for a solid cylinder of diameter d_1, with the load distributed evenly over the facewidth. (In reality, a smaller diameter is determinant; also the load is not evenly distributed; however, the inaccuracies in the assumptions tend to balance each other out). The equation is valid when the elastic modulus and Poisson's ratio of the material are those of steel. Based on practical experience, equations (76) and (77) also include a constant empirical term. Concerning hollow shafts, the deflection component f_{sh} derived from either of these equations, is sufficiently accurate, provided that the bore diameter does not exceed 0,5 d_{sh}.

7.6.4 *Component of mesh misalignment caused by case deformation: f_{ca}*

Case deformation may be ignored when the gears are assembled in rigid cases. The deflections of other cases f_{ca} may be determined by testing or, approximately, by using the finite element method.

7.6.5 *Component of mesh misalignment caused by shaft displacement: f_{be}*

In some cases the effects of bearing clearances and bearing deflections are greater than those of shaft and wheel blank deflections.

The components of misalignment in the plane of action as a result of bearing deflections, and journal displacements in bearings clearances, can usually be neglected when the pinion and wheel of spur or double helical gears are positioned midway between bearings of equal stiffness and clearance.

When gears are not positioned in this way, bearing deflections and displacements (clearances) can influence the distribution of load over the facewidth. This is also valid for single helical or overhung gears.

Since only the relative misalignments due to bearing deflections and displacements, of the common axis of the pinion bearings $f_{be\text{-}1}$ and the common axis of the wheel $f_{be\text{-}2}$, influence the equivalent misalignment, the directions and signs of the misalignments of bearings axes are to be given careful attention. The following equation is valid for the simplest arrangement of a mating pair, with each gear alone on a shaft;

$$f_{be} = f_{be1} + f_{be2} \quad \text{or} \quad f_{be} = f_{be1} - f_{be2} \qquad ...(99)$$

For a gear mounted between the bearings, see Figure 18:

$$f_{be} = \frac{b}{l}(\delta_1 - \delta_2) \qquad ...(100)$$

For an overhung gear, see Figure 19:

$$f_{be} = \frac{b}{l}(\delta_1 + \delta_2) \qquad ...(101)$$

where

δ_1 and δ_2 are the deflections of bearing 1 and bearing 2 parallel to the plane of action.

The effect of the tilting moment, due to the axial component of the tooth load, of single helical gears, shall be taken into account.

7.7 Determination of the face load factors using method D: $K_{H\beta\text{-}D}$ and $K_{F\beta\text{-}D}$

According to the conditions and assumptions described in **7.1.4**, method D is suited for transmission designs with stiff shafts and for preliminary estimates.

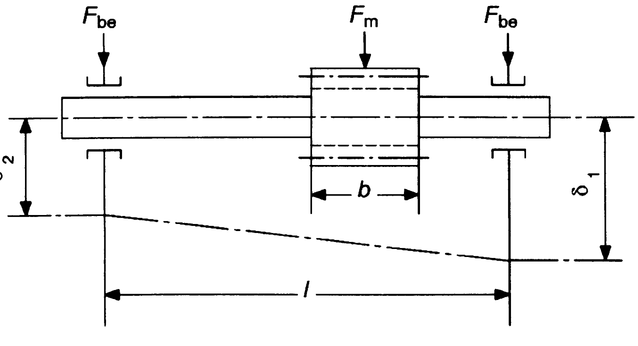


Figure 18 — Loading and deflections for a gear mounted between the bearings, see equation (100)

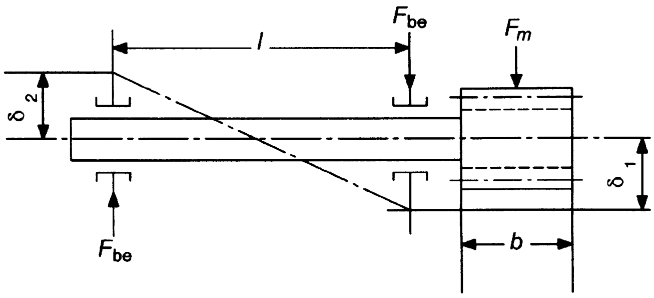


Figure 19 — Loading and deflections for an overhung gear, see equation (101)

7.7.1 ***Characteristics of gear units***

7.7.1.1 *General*

a) the centre of the pinion facewidth is located at the centre of the bearing span within ± 10 % of the bearing span length, i.e. $s/l < 0{,}1$; see Figure 16 a). Method D may also be used when $s/l < 0{,}3$, provided that the bending deflection does not add too much to the total deflection of the pinion;

b) gear case, wheel and shaft are stiff. Stiff bearings, having approximately identical displacements of the shaft journals in both bearings;

c) in general, no helix modification compensation for pinion and wheel deformations. In the event of optimum helix modification [see **4.1.8** b)], the values $K_{H\beta} = 1{,}2$ and $K_{H\beta} = 1{,}18$ can be used for preliminary estimates; then the limits of s/l, as set out in a), no longer apply;

d) mesh stiffness lies in the range 15 N/(µm·mm) < c_γ, < 25 N/(µm·mm);

e) facewidth lies in the range 50 mm < b < 400 mm;

f) the ratio b/h lies in the range $3 < b/h < 12$; see **7.3.2**;

g) the contact pattern ratio $b_{cal}/b \geq 1$ when $b/d_1 < 1{,}7$ (i.e. contact spreads over the entire facewidth), $b_{cal}/b \geq 0{,}9$ when b/d_1 is in the range $1{,}7 \leq b/d_1 < 2$;

h) lubrication is with mineral oil, with or without conventional additives.

7.7.1.2 *Features of structural steel, through hardened steel, and nodular cast iron gears*

a) gear quality grades are 5, 6, 7 or 8 as specified in ISO 1328-1;

b) the allowable stress number (contact) of the pinion and wheel materials is in the range 400 N/mm^2 $\leq \sigma_{H\,lim} \leq$ 1 000 N/mm^2;

c) the specific loading lies within the range 400 N/mm $\leq F_m/b \leq$ 1 000 N/mm;

d) the pinion facewidth/diameter ratio $b/d_1 < 2$.

7.7.1.3 *Features of surface-hardened gears*

a) gear quality grades are 5 or 6 as specified in ISO 1328-1;

b) the specific loading lies within the range 800 N/mm $\leq F_m/b \leq$ 1 500 N/mm;

c) the pinion facewidth/diameter ratio $b/d_1 < 1{,}5$.

7.7.2 ***Factors $K_{H\beta\text{-}D}$ and $K_{F\beta\text{-}D}$ for structural steel, through hardened steel, and nodular cast iron gears***

If $K_{H\beta} < 1{,}2$ according to equations (102) through (109), $K_{H\beta}$ should be made equal to 1,2.

7.7.2.1 *Determination by calculation of $K_{H\beta\text{-}D}$*

a) Case 1: No post-assembly adjustment, lapping of running-in.

1) For gear quality grade 5 according to ISO 1328-1

$$K_{H\beta\text{-}D} = 1{,}135 + 0{,}18\,(b/d_1)^2 + 0{,}23 \times 10^{-3}\,b \qquad \text{...(102)}$$

2) For gear quality grade 6 according to ISO 1328-1

$$K_{H\beta\text{-}D} = 1{,}15 + 0{,}18\,(b/d_1)^2 + 0{,}3 \times 10^{-3}\,b \qquad \text{...(103)}$$

3) For gear quality grade 7 according to ISO 1328-1

$$K_{H\beta\text{-}D} = 1{,}17 + 0{,}18\,(b/d_1)^2 + 0{,}47 \times 10^{-3}\,b \qquad \text{...(104)}$$

4) For gear quality grade 8 according to ISO 1328-1

$$K_{H\beta\text{-}D} = 1{,}23 + 0{,}18\,(b/d_1)^2 + 0{,}61 \times 10^{-3}\,b \qquad \text{...(105)}$$

b) **Case 2:** Gears adjusted after assembly in the gear-case, lapped or run-in as a specific manufacturing process:

1) For gear quality grade 5 according to ISO 1328-1

$$K_{H\beta\text{-}D} = 1{,}10 + 0{,}18\,(b/d_1)^2 + 0{,}115 \times 10^{-3}\,b \qquad \text{...(106)}$$

2) For gear quality grade 6 according to ISO 1328-1

$$K_{H\beta\text{-}D} = 1{,}11 + 0{,}18\,(b/d_1)^2 + 0{,}15 \times 10^{-3}\,b \qquad \text{...(107)}$$

3) For gear quality grade 7 according to ISO 1328-1

$$K_{H\beta\text{-}D} = 1{,}12 + 0{,}18\,(b/d_1)^2 + 0{,}23 \times 10^{-3}\,b \qquad \text{...(108)}$$

4) For gear quality grade 8 according to ISO 1328-1

$$K_{H\beta\text{-}D} = 1{,}15 + 0{,}18\,(b/d_1)^2 + 0{,}31 \times 10^{-3}\,b \qquad \text{...(109)}$$

7.7.2.2 Determination by calculation of $K_{F\beta\text{-}D}$ Calculation is as described in **7.3.2** a) using equations (39) and (40), with $b/h = 12$ and $K_{H\beta\text{-}D}$ from equations (102) through (109).

7.7.2.3 Graphical values of $K_{H\beta\text{-}D}$ and $K_{F\beta\text{-}D}$ The curves representing the face load factors in Figure 20 through Figure 23 are consistent with equations (102) through (109) for $K_{H\beta\text{-}D}$, and with **7.7.2.2** for $K_{F\beta\text{-}D}$.

7.7.3 *Factors $K_{H\beta\text{-}D}$ and $K_{F\beta}$ for surface hardened gears*

7.7.3.1 Determination by calculation of $K_{H\beta\text{-}D}$ First calculate a trial value $K_{H\beta\text{-}D}$ using one of the equations (110) through (117) for $K_{H\beta} \le 1{,}34$.

If $K_{H\beta\text{-}D} < 1{,}2$ then substitute 1,2 for $K_{H\beta}$.

If $1{,}2 \le K_{H\beta\text{-}D} \le 1{,}34$: then $K_{H\beta} = K_{H\beta\text{-}D}$.

If $K_{H\beta\text{-}D} > 1{,}34$, the value is recalculated using the equation for $K_{H\beta} > 1{,}34$ and substituted for $K_{H\beta}$.

This calculation procedure is in conformity with **7.2.4** c).

a) Case 1: No post-assembly adjustment, lapping or running-in.

1) For gear quality grade 5 according to ISO 1328-1

For $K_{H\beta} \le 1{,}34$:

$$K_{H\beta\text{-}D} = 1{,}09 + 0{,}26\,(b/d_1)^2 + 1{,}99 \times 10^{-4}\,b \qquad \text{...(110)}$$

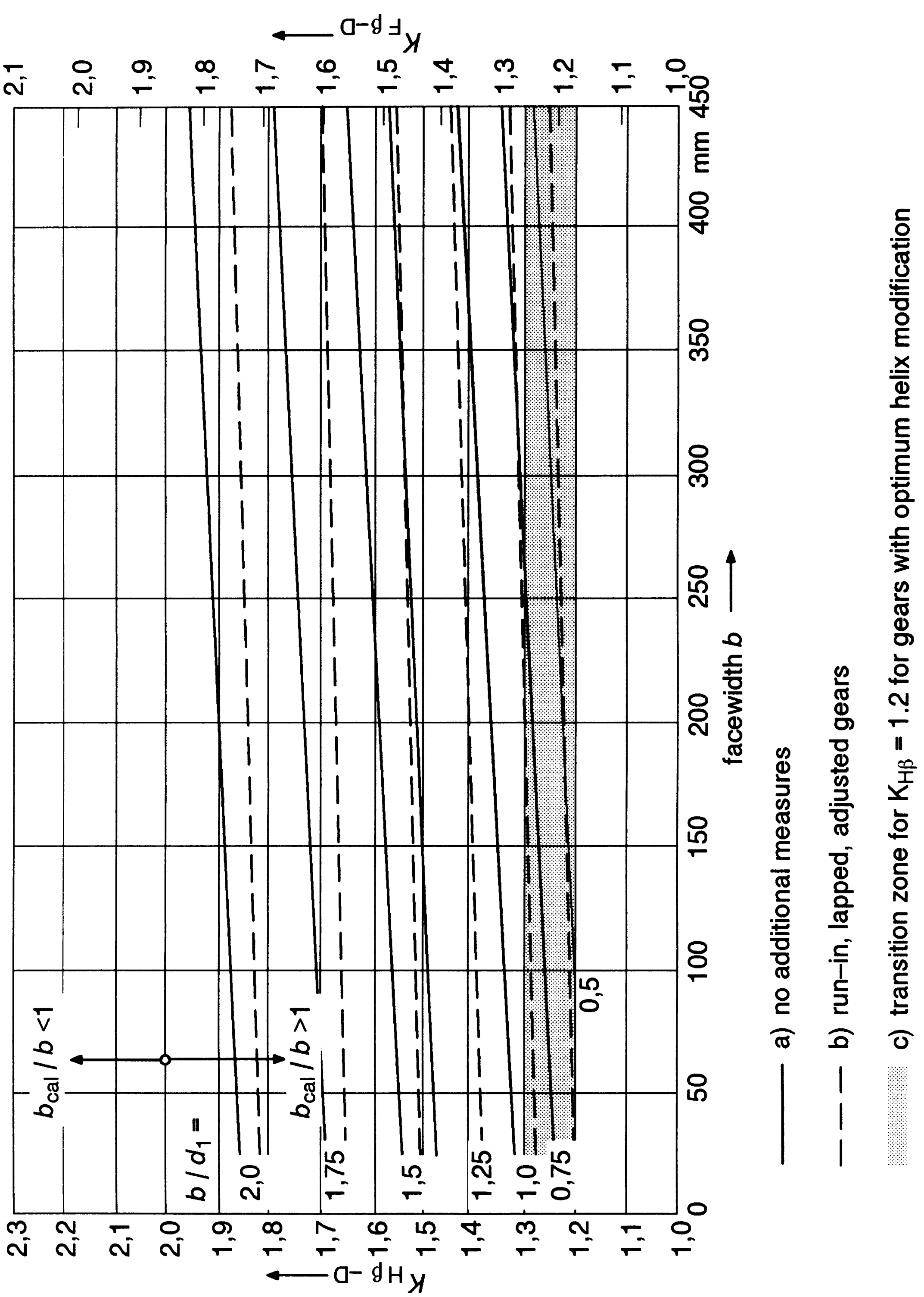


Figure 20 — Curves of face load factors $K_{H\beta\text{-}D} \geq 1{,}2$ and $K_{F\beta\text{-}D} \geq 1{,}18$ for cylindrical gears of quality grade 5 according to ISO 1328-1

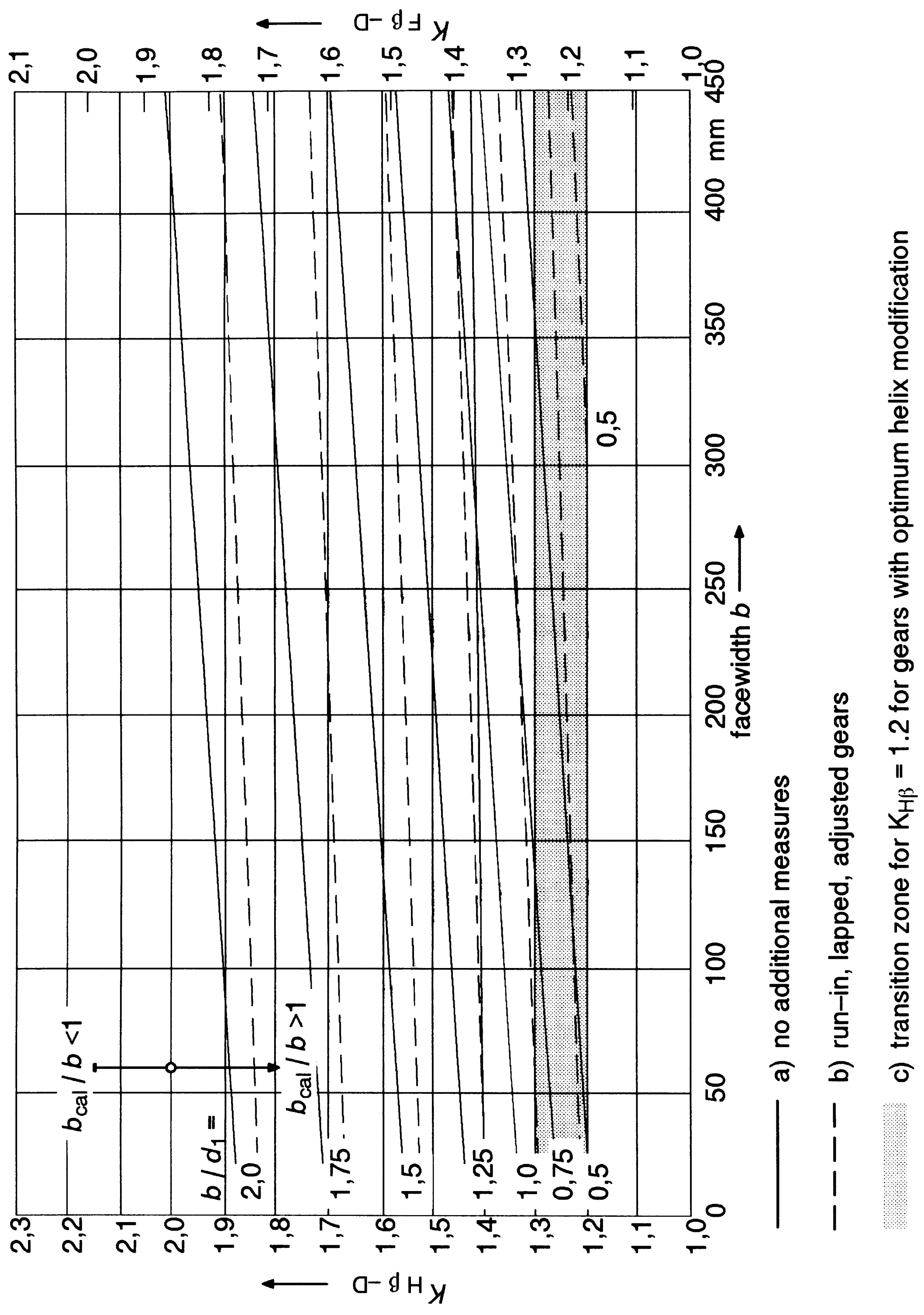


Figure 21 — Curves of face load factors $K_{H\beta\text{-}D}$ and $K_{F\beta\text{-}D}$ for unhardened cylindrical gears of quality grade 6 according to ISO 1328-1

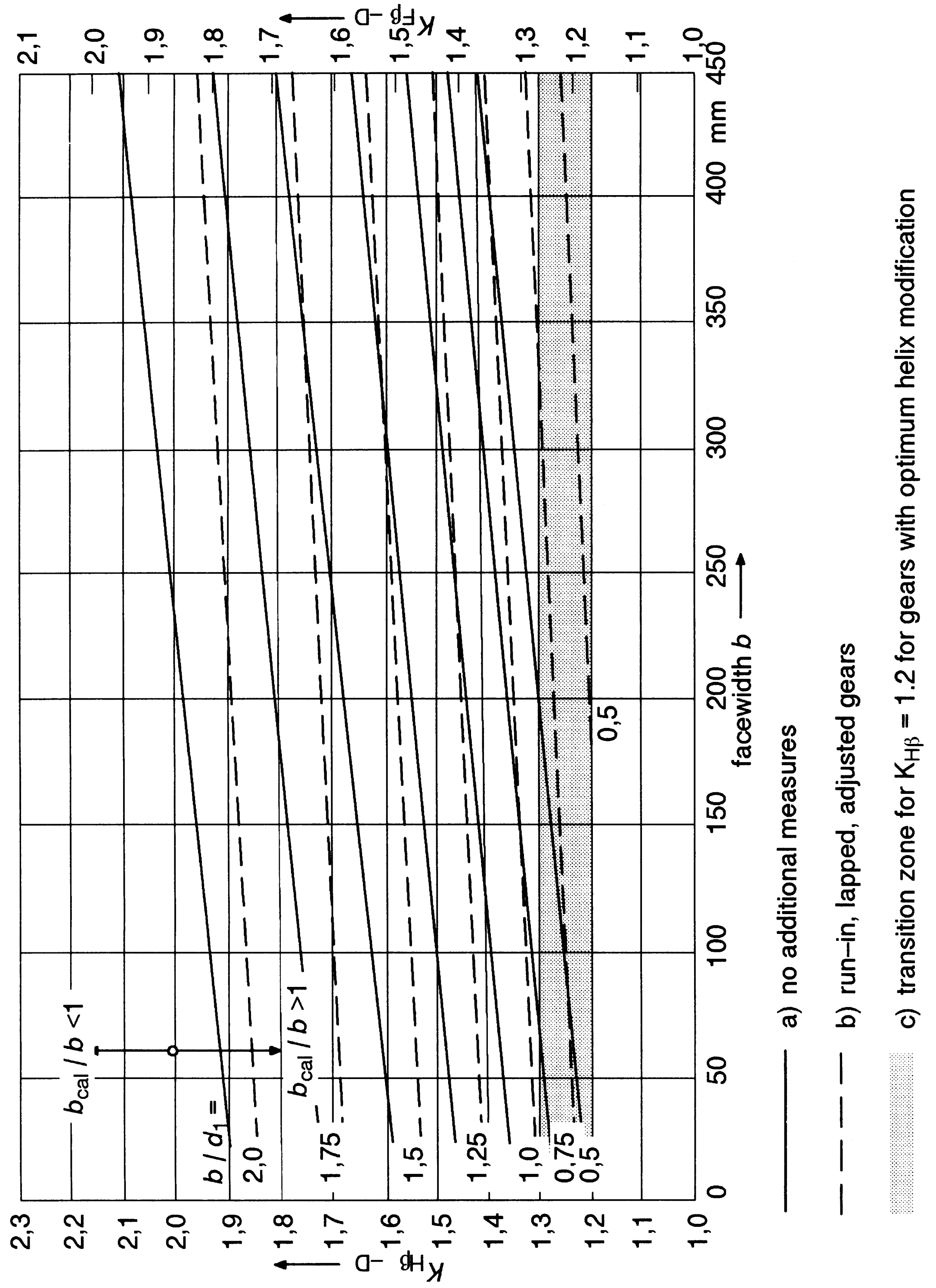


Figure 22 — Curves of face load factors $K_{H\beta-D}$ and $K_{F\beta-D}$ for unhardened cylindrical gears of quality grade 7 according to ISO 1328-1

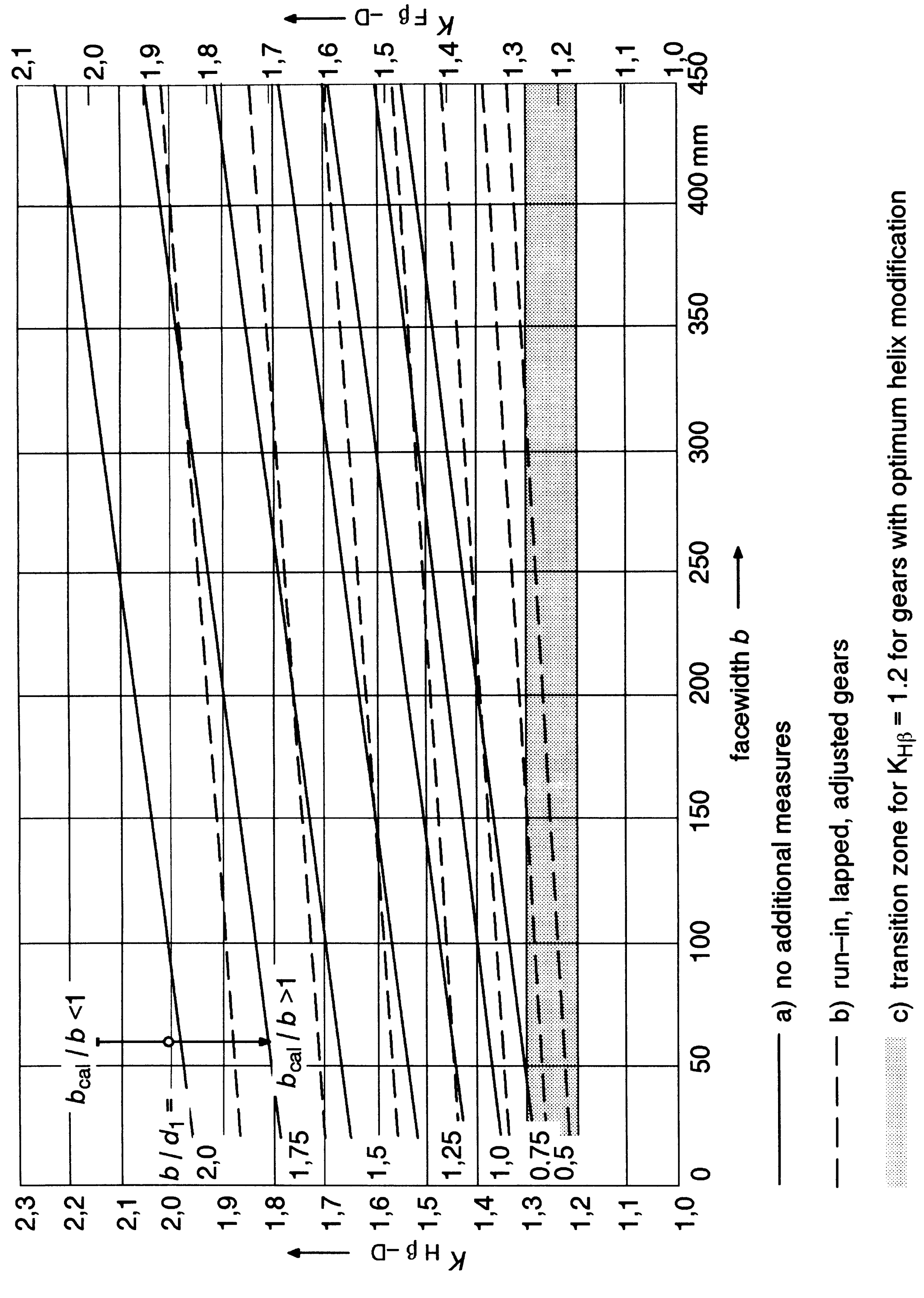


Figure 23 — Curves of face load factors $K_{H\beta\text{-}D}$ and $K_{F\beta\text{-}D}$ for unhardened cylindrical gears of quality grade 8 according to ISO 1328-1

For $K_{H\beta} > 1,34$:

$$K_{H\beta\text{-}D} = 1,05 + 0,31\ (b/d_1)^2 + 2,34 \times 10^{-4}\ b \qquad \text{...(111)}$$

2) For gear quality grade 6 according to ISO 1328-1

For $K_{H\beta} \leq 1,34$:

$$K_{H\beta\text{-}D} = 1,09 + 0,26\ (b/d_1)^2 + 3,3 \times 10^{-4}\ b \qquad \text{...(112)}$$

For $K_{H\beta} > 1,34$:

$$K_{H\beta\text{-}D} = 1,05 + 0,31\ (b/d_1)^2 + 3,8 \times 10^{-4}\ b \qquad \text{...(113)}$$

For $K_{H\beta} \leq 1,34$, y_β is proportional to $F_{\beta x}$; for $K_{H\beta} > 1,34$, y_β is constant.

b) Case 2: Gears adjusted after assembly in the gear-case, lapped or run-in as a specific manufacturing process

1) For gear quality grade 5 according to ISO 1328-1

For $K_{H\beta} \leq 1,34$:

$$K_{H\beta\text{-}D} = 1,05 + 0,26\ (b/d_1)^2 + 1,0 \times 10^{-4}\ b \qquad \text{...(114)}$$

For $K_{H\beta\text{-}D} > 1,34$:

$$K_{H\beta\text{-}D} = 0,99 + 0,31\ (b/d_1)^2 + 1,2 \times 10^{-4}\ b \qquad \text{...(115)}$$

2) For gear quality grade 6 according to ISO 1328-1

For $K_{H\beta} \leq 1,34$:

$$K_{H\beta\text{-}D} = 1,05 + 0,26\ (b/d_1)^2 + 1,6 \times 10^{-4}\ b \qquad \text{...(116)}$$

For $K_{H\beta} > 1,34$:

$$K_{H\beta\text{-}D} = 1,0 + 0,31\ (b/d_1)^2 + 1,9 \times 10^{-4}\ b \qquad \text{...(117)}$$

7.7.3.2 Determination by calculation of $K_{F\beta\text{-}D}$ Calculation is as described in **7.3.2** a), using equations (39) and (40), with $b/h = 12$ and $K_{H\beta\text{-}D}$ from equations (110) through (117).

7.7.3.3 Graphical values of $K_{H\beta\text{-}D}$ and $K_{F\beta\text{-}D}$ The curves representing the face load factors in Figure 24 and Figure 25 are consistent with equations (110) through (117) for $K_{H\beta\text{-}D}$, and with **7.7.3.2** for $K_{F\beta\text{-}D}$.

8 Transverse load factors $K_{H\alpha}$ and $K_{F\alpha}$

8.1 Transverse load distribution

The transverse load factors $K_{H\alpha}$ for surface stress and $K_{F\alpha}$ for tooth-root stress account for the effect of the non-uniform distribution of transverse load between several pairs of simultaneously contacting gear teeth as follows.

The transverse load factors are defined as the ratio of the maximum tooth load occurring in the mesh of a gear pair at near zero r/min, to the corresponding maximum tooth load of a similar gear pair which is free from inaccuracies. The main influences are:

a) deflections under load;

b) profile modifications;

c) tooth manufacturing accuracy;

d) running-in effects.

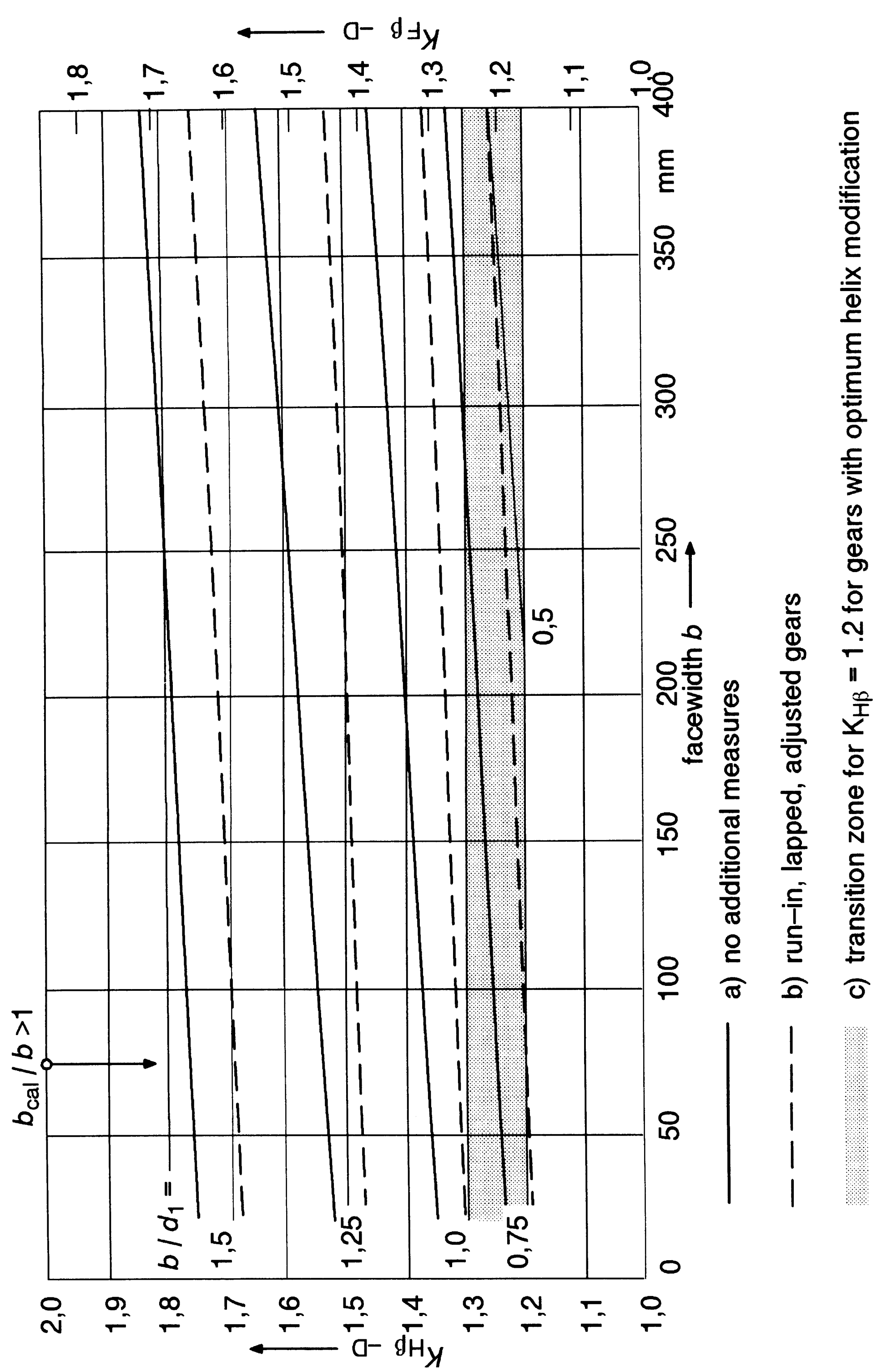


Figure 24 — Curves of face load factors $K_{H\beta-D}$ and $K_{F\beta-D}$ for surface hardened cylindrical gears of quality grade 5 according to ISO 1328-1

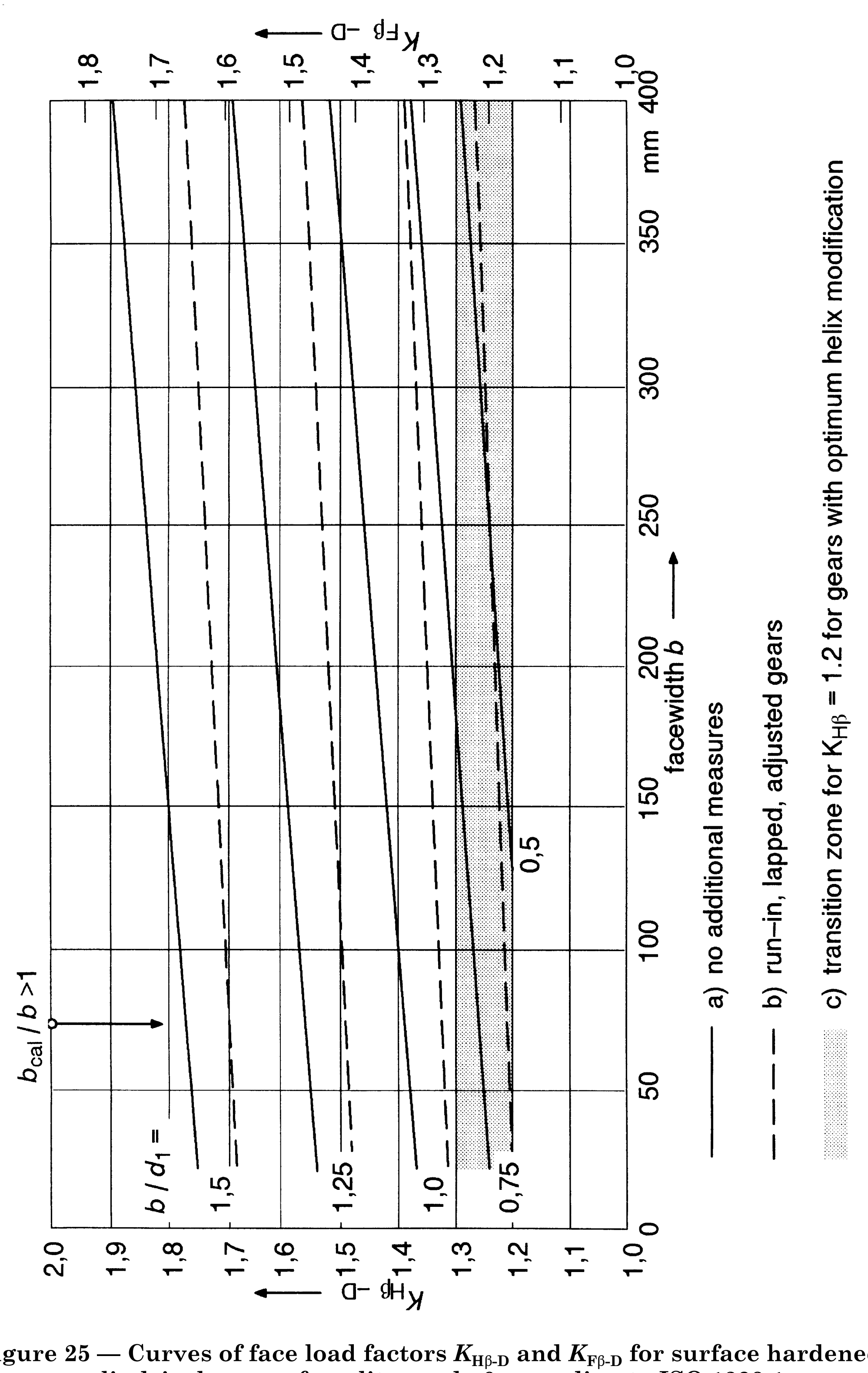


Figure 25 — Curves of face load factors $K_{H\beta\text{-}D}$ and $K_{F\beta\text{-}D}$ for surface hardened cylindrical gears of quality grade 6 according to ISO 1328-1

8.2 Determination methods for the transverse load factors: principles and assumptions

Several methods for the determination of transverse load factors in accordance with the specifications given in **4.1.8** are listed below.

With optimum profile modification appropriate to loading, high manufacture accuracy, even load distribution over the facewidth and a high specific loading level, the transverse load factor approaches unity.

8.2.1 *Method A — Factors $K_{H\alpha\text{-}A}$ and $K_{F\alpha\text{-}A}$*

As stated in **6.4.1**, the maximum tooth loads (including the inner dynamic tooth loads and the effect of uneven distribution of loading) can be determined directly by measurement or by a comprehensive mathematical analysis. $K_{H\alpha}$ and $K_{F\alpha}$ are then assumed to be unity (as is K_v).

The load distribution, in the tangential direction only, can also be determined by comprehensive analysis of all influence factors. The division of the total tangential load between simultaneously meshing tooth pairs, can be derived from strain gauge measurements, made at the tooth-roots of gears transmitting load at low speeds.

Information to be stated in the drawing or specification documents is as follow:

a) maximum (permissible) total tooth load, or

b) maximum (permissible) transverse load factor, or

c) all data (in particular, information relating to the effective difference of base pitch) necessary for making an accurate analysis.

8.2.2 *Method B — Factors $K_{H\alpha\text{-}B}$ and $K_{F\alpha\text{-}B}$*

This method involves the assumption that the average difference between the base pitches of the pinion and wheel is the major parameter in determining the distribution of load between several pairs of teeth in the mesh zone. See **7.6.2.1** b) and footnotes 14 and 19.

8.2.3 *Method C — Factors $K_{H\alpha\text{-}c}$ and $K_{F\alpha\text{-}c}$*

This simplified calculation method is based on method B. Calculation by method B for the transverse load factor for gears, with normally or heavily loaded teeth using equation (118) or (119), indicates that with increasing accuracy of manufacture the value approaches 1. Thus for $\varepsilon_\gamma < 2{,}0$ of ISO quality grade 6 or finer (according to ISO 1328-1), similarly for $\varepsilon_\gamma > 2{,}0$ of quality grade 5 or more accurate, the value 1 may be substituted for $K_{H\alpha}$ and $K_{F\alpha}$. Because of their better running-in characteristics, the same substitution can also be made for through hardened gears, which are one quality grade less accurate.

It is assumed that for spur gears of coarse quality grade, the transverse distribution of load is the least favourable. The implication that sometimes the entire tangential load is supported by a single pair of mating teeth, i.e. $b_{eff} = b$ for spur gears and $b_{eff} = b/\cos^2\beta_b$ for helical gears, is taken as the basis for the load bearing width, i.e. the smallest number of simultaneously loaded pairs of mating teeth.

Fixed numerical values for the transverse load factors can be allocated to gear quality grades, in the range between the limit values described above. Since this is a rough approximation, it is convenient to make $K_{H\alpha} = K_{F\alpha}$.

In accordance with the considerations discussed, the following additionally simplifying assumptions have been adopted for method C.

a) pressure angle $\alpha_t = 20°$, base pitch deviation $f_{pb} = f_p \cos 20°$;

b) helix angle $\beta = 20°$ for helical gearing (affects c_γ);

c) transverse contact ratio: spur gears, $\varepsilon_\alpha = 1{,}6$; for helical gears, $\varepsilon_\alpha = 1{,}4$ and $\varepsilon_\gamma = 2{,}5$;

d) mesh stiffness: $c_\gamma = 20$ N/(mm·µm) for spur gears and $c_\gamma = 18{,}7$ N/(mm·µm) for helical gears with the helix angle $\beta = 20°$ as specified in **9.3.1** and **9.4**;

e) f_{pb} is $f_{pb\,eff}$ as in Table 3 and y_α according to **8.3.5**, with ISO gear quality grades for $z_2 = 100$;

f) determinant specific loading: $F_{tH}/b = 350$ N/mm;

g) concerning gears of coarse quality grade, it is assumed that the determinant tangential load F_{tH} is correspondingly large.

These limit values are also assumed for specific loading $[(F_t\,K_A)/b] < 100$ N/mm, since then the deflections under this load are, as a general rule, small in comparison with base pitch deviations.

h) $K_{H\alpha}$, $K_{F\alpha}$ are assumed as constant for $[(F_t K_A)/b] \geq 100$ N/mm since c' and c_γ in this range can be considered to be proportional to the specific loading (see clause **9**).

8.3 Determination of the transverse load factors using method B: $K_{H\alpha\text{-B}}$ and $K_{F\alpha\text{-B}}$

According to the conditions and assumptions described in **8.2.2** and footnotes 25 and 26, method B is suitable for all types of gearing (spur or helical with any basic rack profile and any accuracy). Transverse load factors can be determined by calculation or graphically. The two methods give identical results.

8.3.1 *Determination of the transverse load factor by calculation*[28)]

a) Values $K_{H\alpha}$ and $K_{F\alpha}$ for gears with total contact ratio $\varepsilon_\gamma \leq 2$

$$K_{H\alpha} = K_{F\alpha} = \frac{\varepsilon_\gamma}{2}\left(0{,}9 + 0{,}4\,\frac{c_\gamma\,(f_{pb} - y_\alpha)}{F_{tH}/b}\right) \qquad \text{...(118)}$$

b) Values $K_{H\alpha}$ and $K_{F\alpha}$ for gears with total contact ratio $\varepsilon_\gamma > 2$

$$K_{H\alpha} = K_{F\alpha} = 0{,}9 + 0{,}4\sqrt{\frac{2\,(\varepsilon_\gamma - 1)}{\varepsilon_\gamma}\,\frac{c_\gamma\,(f_{pb} - y_\alpha)}{F_{tH}/b}} \qquad \text{...(119)}$$

where the following are to be determined:

c_γ mesh stiffness in accordance with clause **9**;

f_{pb} the larger of the base pitch deviations of pinion or wheel should be used; 50 % of this tolerance may be applied in the case of optimum profile modification adapted to the loading[a];

y_α running-in allowance as specified in **8.3.5**;

F_{tH} determinant tangential load in a transverse plane, $F_{tH} = F_t K_A K_v K_{H\beta}$

[a] The base pitch deviation f_{pb} accounts for the total effect of all gear tooth deviations which affect the transverse load factor. If, nevertheless, the profile form deviation $f_{f\alpha}$ is greater than the base pitch deviation, the profile form deviation is to be taken instead of the base pitch deviation.

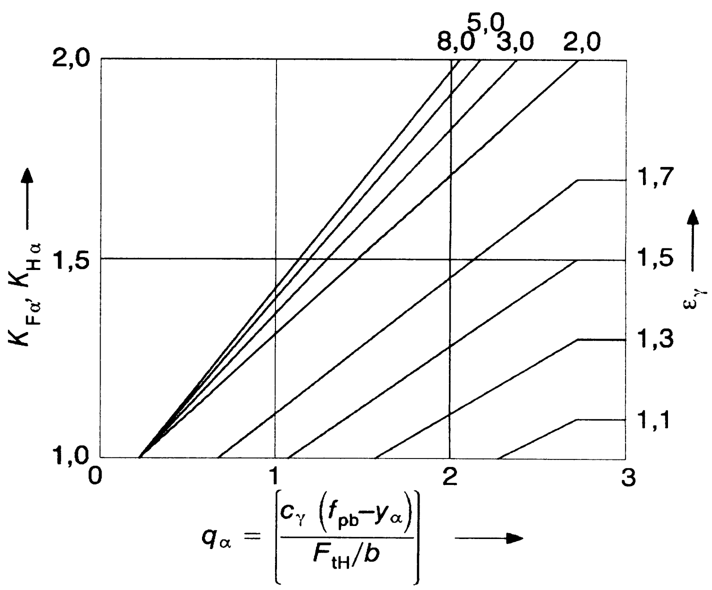


Figure 26 — Determination of the transverse load factors $K_{H\alpha}$ and $K_{F\alpha}$ by method B (see 8.3.3 and 8.3.4 for limiting conditions)

28) Equations (118) and (119) are based on the assumption that the base pitch deviations appropriate to the gear accuracy specified are distributed around the circumference of the pinion and wheel as is consistent with normal manufacturing practice. They do not apply when the gear teeth have some intentional deviation.

8.3.2 ***Load factors from graphs***

$K_{H\alpha}$ and $K_{F\alpha}$ can be read from Figure 26; the curves are consistent with equations (118) and (119).

8.3.3 ***Limiting conditions for $K_{H\alpha}$***

When, in accordance with equation (118) or (119)

$$K_{H\alpha} > \frac{\varepsilon_\gamma}{\varepsilon_\alpha Z_\varepsilon^2} \quad \text{...(120)}$$

then for $K_{H\alpha}$ substitute $\frac{\varepsilon_\gamma}{\varepsilon_\alpha Z_\varepsilon^2}$ and when $K_{H\alpha} < 1{,}0$ then for $K_{H\alpha}$ substitute as limit value 1,0.

See note 10 in **8.3.4**.

8.3.4 ***Limiting conditions for $K_{F\alpha}$***

If, in accordance with equation (118) or (119)

$$K_{F\alpha} > \frac{\varepsilon_\gamma}{\varepsilon_\alpha Y_\varepsilon} \quad \text{...(121)}$$

then for $K_{F\alpha}$ substitute $\frac{\varepsilon_\gamma}{\varepsilon_\alpha Y_\varepsilon}$ and when $K_{F\alpha} < 1{,}0$ then for $K_{F\alpha}$ substitute as limit value 1,0

NOTE 10 With limiting values in accordance with equations (120) and (121), the least favourable distribution of load is assumed, implying that the entire tangential load is transferred by only one pair of mating teeth. Furthermore it is recommended that the accuracy of helical gears is so chosen that $K_{H\alpha}$ and $K_{F\alpha}$ are not greater than ε_α. As a consequence it may be necessary to limit the base pitch deviation tolerances of gears of coarse quality grade.

8.3.5 ***Running-in allowance y_α***

The value y_α is the amount by which the initial base pitch deviation is reduced by running-in from the start of operation. See **7.4.2** for the main influences. y_α does not account for an allowance due to any extent of running-in as a controlled measure, being part of the production process, e.g. lapping. This adjustment is to be taken into consideration when considering the gear quality.

y_α may be determined in accordance with **8.3.5.1** or **8.3.5.2** (method B) where direct, verified values from experimentation or experience are lacking (method A).

The value for the base pitch deviation f_{pb} determined in accordance with **8.3.1.2** or **6.4.3** should be used in both methods. The equations and graphs should also be applied analogously for the profile form deviation $f_{f\alpha}$.

8.3.5.1 Determination by calculation:

The running-in allowance y_α may be calculated using equations (122) to (130). These are consistent with the curves in Figure 27 and Figure 28.

a) For structural steels, through hardened steels and nodular cast iron (pearlite; bainite):

$$y_\alpha = \frac{160}{\sigma_{H\,lim}} f_{pb} \quad \text{...(122)}$$

where

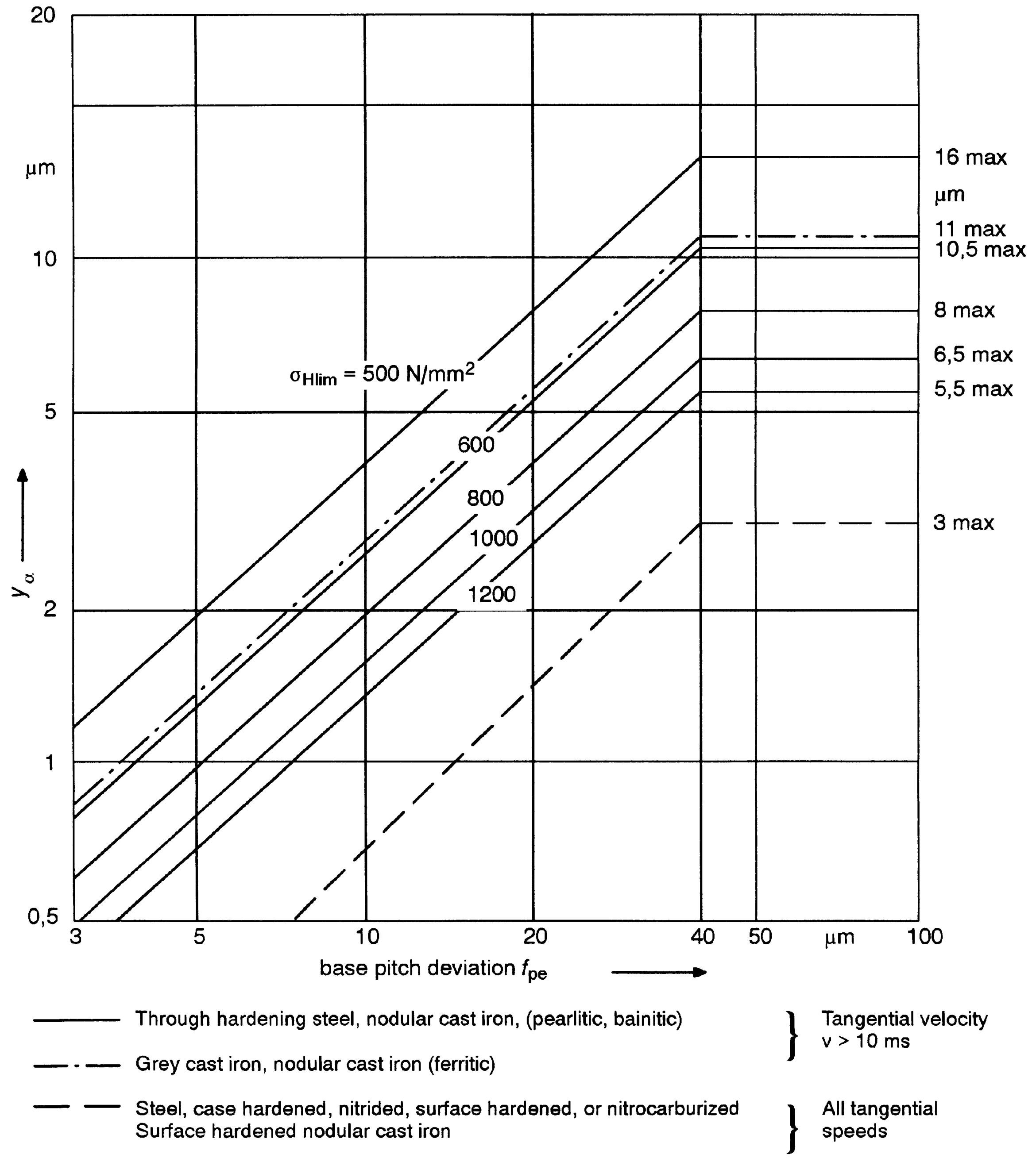
For $v \le 5$ m/s:	no restriction
For 5 m/s $< v \le 10$ m/s:	the upper limit of y_α is 12 800/$\sigma_{H\,lim}$ corresponding to f_{pb} = 80 μm
For $v > 10$ m/s:	the upper limit of y_α is 6400/$\sigma_{H\,lim}$ corresponding to f_{pb} = 40 μm

b) For grey cast iron and nodular cast iron (ferritic)

$y_\alpha = 0{,}275\, f_{pb}$ ...(123)

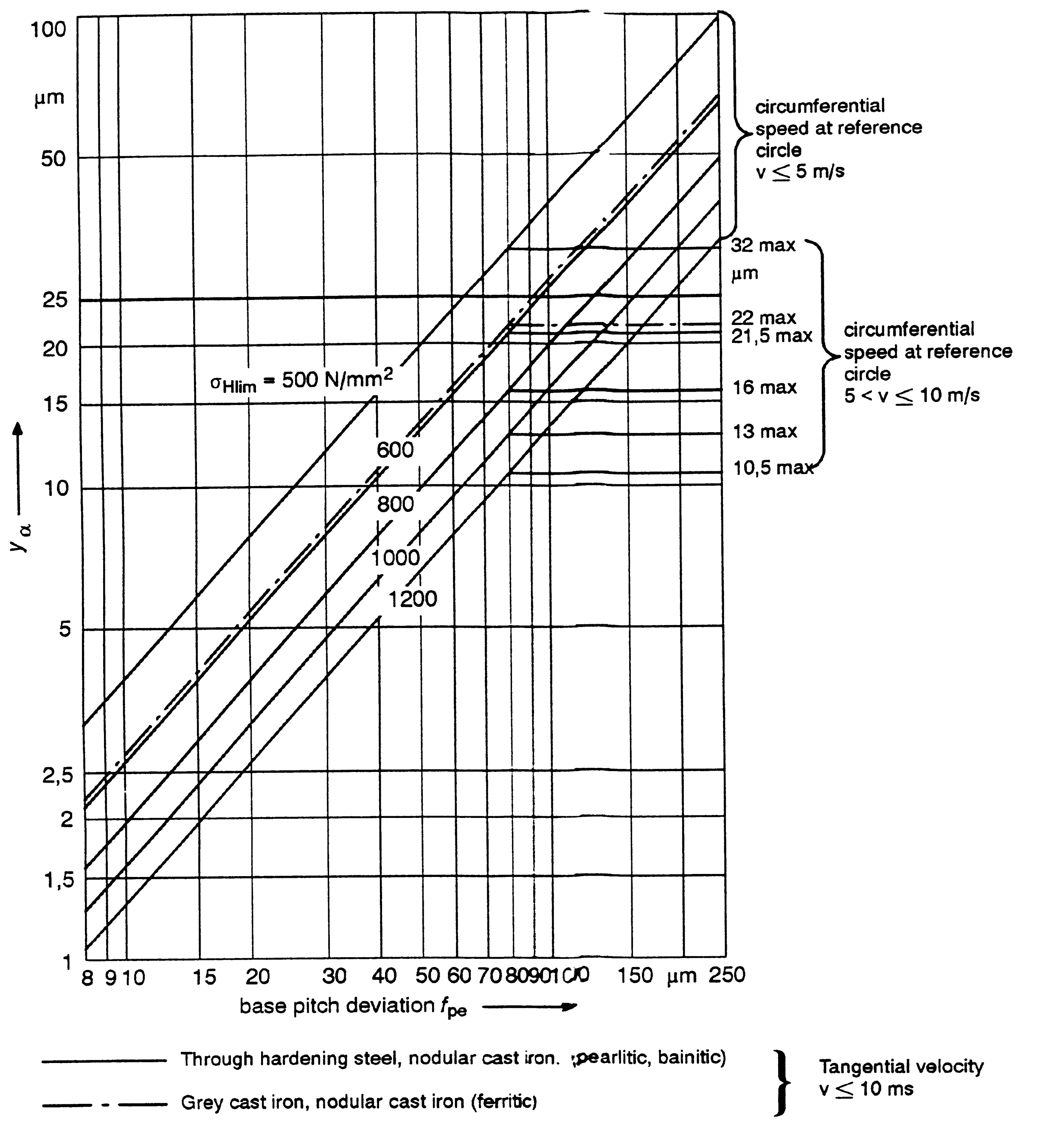
where

For $v \le 5$ m/s:	no restriction
For 5 m/s < v ≤ 10 m/s:	the upper limit of y_α is 22 µm corresponding to f_{pb} = 80 µm
For v > 10 m/s:	the upper limit of y_α is 11 µm corresponding to f_{pb} = 40 µm



NOTE This graph is derived from Figure 13. If the materials of the pinion and the wheel are different, y_α shall be determined in accordance with equation (125).

Figure 27 — Determination of running-in allowance y_α of a gear pair (see also Figure 28)



NOTE This graph is derived from Figure 14. If the materials of the pinion and the wheel are different, y_α shall be determined in accordance with equation (125).

Figure 28 — Determination of running-in allowance y_α of a gear pair (see also Figure 27)

c) For case hardened, nitrided or nitrocarburized steels

$$y_\alpha = 0,075\, f_{pb} \qquad ...(124)$$

for all velocities but with the restrictionthe upper limit of y_α is 3 µm corresponding to f_{pb} = 40 µm

When the materials differ, $y_{\alpha 1}$ should be determined for the pinion material and $y_{\alpha 2}$ for the wheel. The average value is used for the calculation:

$$y_\alpha = \frac{y_{\alpha 1} + y_{\alpha 2}}{2} \qquad \text{...(125)}$$

8.3.5.2 *Graphical values*

y_α may be read from Figure 27 and Figure 28 as a function of the base pitch deviation f_{pb} and the material value $\sigma_{H\,lim}$.

8.4 Determination of the transverse load factors using method C: $K_{H\alpha\text{-}c}$ and $K_{F\alpha\text{-}c}$

In accordance with the conditions and assumptions described in **8.2.3**, method C gives average values which can be used for industrial transmissions and transmissions with similar requirements which meet the following conditions:

a) external and internal gear teeth on any steel blank[29)];

b) basic rack profile complying with ISO 53;

c) spur and helical gears with $\beta \leq 30°$;

d) specific loading $F_{tH}/b \geq 350$ N/mm.

NOTE 11 When $F_{tH}/b < 350$ N/mm, the actual safety factor will be less than the calculated value since $K_{H\alpha}$ and $K_{F\alpha}$ are larger than the table values. When $F_{tH}/b \geq 350$ N/mm, calculated ratings tend to be safe.

Estimated values of $K_{H\alpha}$ and $K_{F\alpha}$, based on the assumptions in **8.2.3**, are listed in Table 6.

Table 7 — Transverse load factors $K_{H\alpha}$ and $K_{F\alpha}$ determined by method C

<table>
<tr><th colspan="3">Specific Loading (F_t K_A)/b</th><th colspan="7">> 100 N/mm</th><th>≤ 100 N/mm</th></tr>
<tr><td colspan="3">Gear quality grade
(conforming to ISO 1328-1)</td><td>6</td><td>7</td><td>8</td><td>9</td><td colspan="2">10</td><td>11 – 12</td><td>6
and lower</td></tr>
<tr><td rowspan="4">Case or surface hardened nitrided or nitro-carburized</td><td rowspan="2">Spur gearing</td><td>$K_{H\alpha}$</td><td colspan="2" rowspan="2">1,0</td><td rowspan="2">1,1</td><td rowspan="2">1,2</td><td colspan="4">$1/Z_\varepsilon^2 \geq 1,2$</td></tr>
<tr><td>$K_{F\alpha}$</td><td colspan="4">$1/Y_\varepsilon \geq 1,2$</td></tr>
<tr><td rowspan="2">Helical gearing</td><td>$K_{H\alpha}$</td><td rowspan="2">1,0</td><td rowspan="2">1,1[b]</td><td rowspan="2">1,2</td><td rowspan="2">1,4</td><td colspan="4" rowspan="2">$\varepsilon_\alpha/\cos^2\beta_b \geq 1,4$[a]</td></tr>
<tr><td>$K_{F\alpha}$</td></tr>
<tr><td rowspan="4">Not hardened nor nitrided nor carburized</td><td rowspan="2">Spur gearing</td><td>$K_{H\alpha}$</td><td colspan="3" rowspan="2">1,0</td><td rowspan="2">1,1</td><td rowspan="2">1,2</td><td colspan="3">$1/Z_\varepsilon^2 \geq 1,2$</td></tr>
<tr><td>$K_{F\alpha}$</td><td colspan="3">$1/Y_\varepsilon \geq 1,2$</td></tr>
<tr><td rowspan="2">Helical gearing</td><td>$K_{H\alpha}$</td><td colspan="2" rowspan="2">1,0</td><td rowspan="2">1,1</td><td rowspan="2">1,2</td><td rowspan="2">1,4</td><td colspan="3" rowspan="2">$\varepsilon_\alpha/\cos^2\beta_b \geq 1,4$[a]</td></tr>
<tr><td>$K_{F\alpha}$</td></tr>
<tr><td colspan="11">[a] See note 10 in 8.3.4
[b] For modified profiles which accounts for distortion caused with load $K_{H\alpha} = K_{F\alpha} = 1,0$</td></tr>
</table>

9 Tooth stiffness parameters c' and c_γ

9.1 Stiffness influences

A tooth stiffness parameter represents the requisite load over 1 mm facewidth, directed along the line of action[30)] to produce in line with the load, the deformation amounting to 1 µm, of one or more pairs of deviation-free teeth in contact. This deformation is equal to the base circle length of arc, corresponding to the load-induced rotation angle of one gear of the pair when the mating gear is held fast.

[29)] The transverse load factors determined in accordance with method C for grey cast iron or nodular cast iron, are on the safe side.

[30)] The tooth deflection can be determined approximately using F_t (F_m, F_{tH}, ...) instead of F_{bt}. Conversion from F_t to F_{bt} (load tangent to the base cylinder) is covered by the relevant factors, or the modifications resulting from this conversion can be ignored when compared with other uncertainties (e.g. tolerances on the measured values).

Single stiffness c' is the maximum stiffness of a single pair of a spur gear teeth. It is approximately equal to the maximum stiffness of a tooth pair in single pair contact[31]. The value c' for helical gears is the maximum stiffness normal to the helix of one tooth pair; c' is needed for the calculation of the dynamic factor K_v.

Mesh stiffness c_γ is the mean value of stiffness of all the teeth in a mesh. c_γ is needed for the calculation of the dynamic factor K_v, the face load factors $K_{H\beta}$ and $K_{F\beta}$, and transverse load factors $K_{H\alpha}$ and $K_{F\alpha}$.

The main influences affecting tooth stiffness are:

a) tooth data (number of teeth, basic rack profile, addendum modification, helix angle, transverse contact ratio);

b) blank design (rim thickness, web thickness);

c) specific load normal to the tooth flank;

d) shaft-hub connection;

e) roughness and waviness of the tooth surface;

f) mesh misalignment of the gear pair;

g) modulus of elasticity of the materials.

9.2 Determination methods for tooth stiffness parameters: principles and assumptions

Several methods of determining tooth stiffness parameters in accordance with the rules given in **4.1.8** are described in **9.2.1** to **9.2.3**. For methods B and C, these stiffness values apply for accurate gears; lower values can be expected for less accurate gears.

9.2.1 *Method A — tooth stiffness parameters c'_A and $c_{\gamma\text{-}A}$*

In this method the tooth stiffness is determined by comprehensive analysis including all influences. This can be done by making direct measurements on the gear pair of interest. Values based on the theory of elasticity can be calculated, or determined by finite element methods.

9.2.2 *Method B — tooth stiffness parameters c'_B and $c_{\gamma\text{-}B}$*

This method is based on studies of the elastic behaviour of solid disc spur gears. Based on this work, the curves in Figure 29 were calculated for the basic rack profile specified therein.

With the help of a series expansion, a sample expression was derived for cylindrical gears conjugate to a standard basic rack profile according to ISO 53; see note 13 in **9.3.1.1**. This was based on an assumed specific loading of F_t/b = 300 N/mm. Using this method, theoretical single stiffnesses c'_{th} are obtained.

Differences between these theoretical results and the results of measurements are adjusted by means of a correction factor C_M and extension section to adjust for low specific loading.

Additional correction factors, determined by measurement and theoretical means, allow this method to be applied to gears consisting of rims and webs (factor C_R), similar to gears conjugate to other basic rack profiles (factor C_B) and helical gears (factor cos β).

By superposition of the single stiffness of all tooth pairs simultaneously in contact, an expression for the calculation of c_γ was developed. Its accuracy was verified by measurement results.

9.2.3 *Method C — tooth stiffness parameters c'_C and $c_{\gamma\text{-}C}$*

This method is derived from method B. Gears with combinations of z_1/x_1, z_2/x_2 with basic rack profiles according to ISO 53 were used as a basis for the determination of constant average values specified for c' and c_γ (see Figure 29).

If, for other reasons (e.g. the dynamic factor), a distinction is to be made between spur and helical gearing, the fixed values in accordance with equations (137) and (138) can be recalculated to suit an average helix angle; see comments in **9.4**.

The mesh stiffness c_γ was calculated using equation (136) and a fixed value c' for an average transverse contact ratio ε_α = 1,6.

[31] c' at the outer limit of single pair tooth contact can be assumed to approximate the maximum value of single stiffness when $\varepsilon_\alpha > 1,2$.

9.3 Determination of the tooth stiffness parameters c' and c_γ according to method B

Subject to the conditions and assumptions described in **9.2.2**, c' and c_γ as determined by method B are, in general, sufficiently accurate for the calculation of the dynamic factor and face load factors as well as for the determination of profile and helix modifications for gears in accordance with the following:

a) external gears;

b) any basic rack profile;

c) spur and helical gears with $\beta \leq 45°$;

d) steel/steel gear pairs;

e) any design of gear blank;

f) shaft/hub fitting spreads the transfer of torque evenly around the circumference (pinion integral with shaft, interference fit or splined fitting);

g) specific loading $(F_t K_A)/b \geq 100$ N/mm

NOTE 12 The numbers of teeth of virtual spur gears in the normal section can be calculated approximately as:

$$z_{n1} \approx \frac{z_1}{\cos^3\beta} \quad \text{and} \quad z_{n2} \approx \frac{z_2}{\cos^3\beta}$$

Method B can also be used, either approximately or with further auxiliary factors, for gears in accordance with the following:

— internal gears;

— materials combination other than steel/steel;

— shaft-hub assembly other than under f), e.g. with fitted key;

— specific loading $(F_t K_A)/b < 100$ N/mm.

9.3.1 *Single stiffness c′*

For gears having features listed under **9.3** a) to g) the following equation provides acceptable average values:

$$c' = c'_{th} C_M C_R C_B \cos\beta \qquad \text{...(126)}$$

9.3.1.1 Theoretical single stiffness c'_{th} c'_{th} is appropriate to solid disc gears and to the specified standard basic rack tooth profile. c'_{th} for a helical gear is the theoretical single stiffness relevant to the appropriate virtual spur gear (see note 12).

a) Determination by calculation — Method B1 c'_{th} can be calculated for gear teeth having the basic rack profile specified in note 13 using equations (127) and (128).

$$c'_{th} = \frac{1}{q'} \qquad \text{...(127)}$$

where q' is the minimum value for the flexibility of a pair of teeth (compare with definition of c' in **9.1**):

$$q' = C_1 + C_2/z_{n1} + C_3/z_{n2} + C_4 x_1 + (C_5 x_1)/z_{n1} + C_6 x_2 + (C_7 x_2)/z_{n2} + C_8 x_1^2 + C_9 x_2^2 \qquad \text{...(128)}$$

See Table 7 for coefficients C_1 to C_9.

NOTE 13 Series progression in accordance with **9.2.2** for gears with basic rack profile: ($\alpha_P = 20°$, $h_{aP} = m_n$, $h_{fP} = 1{,}2\ m_n$, and $\rho_{fP} = 0{,}2\ m_n$. Equations (127) and (128) apply for the range $x_1 \geq x_2$; $-0{,}5 \leq x_1 + x_2 \leq 2{,}0$. Deviations of actual values from calculated values in range $100 \leq F_{bt}/b \leq 1\ 600$ N/mm are between + 5 % and – 8 %.

Table 7 — Coefficients for equation (128)

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
0,04723	0,15551	0,25791	– 0,00635	– 0,11654	– 0,00193	– 0,24188	0,00529	0,00182

b) Graphical values — Method B2 c'_{th} can be read from Figure 29 as a function of the virtual numbers of teeth z_n and the addendum modification coefficients of pinion x_1 and wheel x_2.

9.3.1.2 Correction factor C_M C_M accounts for the difference between the measured values and the theoretical calculated values for solid disc gears:

$C_M = 0{,}8$

9.3.1.3 Gear blank factor C_R C_R accounts for the flexibility of gear rims and webs. The following provides mean values of C_R, suitable for use when the mating gear body is equally stiff or stiffer.

For solid disc gears

$C_R = 1$

NOTE 14 The adoption of these average values is permissible considering the other uncertainties. Thus, for instance, the tooth stiffness of a gear of webbed design is not constant over the facewidth.

a) Determination by calculation C_R can be calculated using equation (129). It is consistent with curves in Figure 30, within – 1 % to + 7 %.

$$C_R = 1 + \frac{\ln (b_s/b)}{5\, e^{s_R/(5\, m_n)}} \qquad ...(129)$$

Boundary conditions: when $b_s/b < 0,2$ substitute $b_s/b = 0,2$
when $b_s/b > 1,2$ substitute $b_s/b = 1,2$
when $s_R/m_n < 1$ substitute $s_R/m_n = 1$

b) Graphical values C_R can be read from Figure 30 as a function of gear rim thickness s_R and central web thickness b_s.

9.3.1.4 Basic rack factor C_B C_B accounts for the deviations of the actual basic rack profile of the gear, from the standard basic rack profile (ISO 53).

a) When C'_{th} from Figure 29 is used as a basis, then:

$$C_B = [1 + 0,5(1,25 - h_{fP}/m_n)]\,[1 - 0,02(20° - \alpha_{Pn})] \qquad ...(130)$$

The influence of the radius of curvature of the fillet of the basic rack profile ρ_{fP} is negligibly small and therefore neglected[32)].

For gear tooth profiles according to the standard basic rack (ISO 53), with $\alpha_{Pn} = 20°$, $h_{aP} = m_n$, $h_{fP} = 1,25\ m_n$ and $\rho_{fP} = 0,25\ m_n$:

$C_B = 1$

b) When c'_{th} from equations (127) and (128) is used as a basis:

$$C_B = [1 + 0,5(1,2 - h_{fP}/m_n)]\,[1 - 0,02(20° - \alpha_{Pn})] \qquad ...(131)$$

For gear teeth according to basic rack profile $\alpha_{Pn} = 20°$, $h_{aP} = m_n$, $h_{fP} = 1,2\ m_n$, and $\rho_{fP} = 0,2\ m_n$:

$C_B = 1$

When the pinion basic rack dedendum is different from that of the wheel, the arithmetic mean of C_{B1} for a gear pair conjugate to the pinion basic rack and C_{B2} for a gear pair conjugate to the basic rack of the wheel is used:

$$C_B = 0,5(C_{B1} + C_{B2}) \qquad ...(132)$$

9.3.1.5 *Additional information*

a) Helical gearing The theoretical single stiffness of the teeth of virtual spur gears of a helical gear pair is transformed by the term cos β in equation (126) from the normal into the transverse theoretical single stiffness c'_{th} of the teeth of the helical gears.

b) Internal gearing Approximate values of the theoretical single stiffnesses of internal gear teeth can also be determined from Figure 29 or equations (127) and (128), by the substitution of infinity for z_{n2}.

c) Material combinations For material combinations other than steel with steel, the value of c' can be determined from the following equation:

$$c' = c'_{St/St} \left(\frac{E}{E_{St}} \right) \qquad ...(133)$$

[32)] These equations illustrate the trends. Since statements about tooth stiffness are subject to uncertainties, during calculations consideration of the influences represented by the factors C_B can be neglected.

where

$$E = \frac{2\,E_1\,E_2}{E_1 + E_2} \qquad \text{...(134)}$$

(E/E_{St}) = 0,74 for steel/grey cast iron, (E/E_{St}) = 0,59 for grey cast iron/grey cast iron.

d) Shaft and gear assembly If the pinion or the wheel or both are assembled on the shaft(s) with a fitted key, the single stiffness, under constant load, varies between maximum and minimum values twice per revolution.

The minimum value is approximately equal to the single stiffness with interference or spline fits.

When one gear of a pair is press fitted onto a shaft with a fitted key, and the mating gear is assembled with its shaft by means of an interference or splined fitting, the average value of single stiffness is about 5 % greater than the minimum. When both gears of a pair are push fitted onto shafts with fitted keys, the average single stiffness is about 10 % greater than the minimum.

e) Specific loading (F_t K_A/b) < 100 N/m At low specific loading, the single stiffness decreases with reduced load[33]. By way of approximation, when $(F_t\,K_A)/b$ < 100 N/mm:

$$c' = c'_{th}\,C_M\,C_R\,C_B \cos\beta\,[(F_t\,K_A/b)/100]^{0,25} \qquad \text{...(135)}$$

9.3.2 *Mesh stiffness c_γ*

Following the methods quoted in **9.2.2** for spur gears with $\varepsilon_\alpha \geq 1{,}2$ and helical gears with $\beta \leq 30°$, the mesh stiffness:

$$c_\gamma = c'\,(0{,}75\,\varepsilon_\alpha + 0{,}25) \qquad \text{...(136)}$$

with c' according to equation (126). The value c_γ can be up to 10 % less than values from equation (136) when for spur gears $\varepsilon_\alpha < 1{,}2$.

9.4 Determination of the stiffness parameters c' and c_γ according to method C

Subject to the conditions and assumptions described in **9.2.3** for method C, average values are given to be used, in general, for estimates, also for the determination of dynamic, face load and transverse load factors of industrial gears and gears of similar requirements with features as follows[34]:

a) external and internal cylindrical gears;

b) basic rack profile complying with ISO 53;

c) spur and helical gears with $\beta \leq 30°$;

d) gear pairs with transverse contact ratio, $1{,}2 < \varepsilon_\alpha < 1{,}9$;

e) gear pairs with steel/steel material combination;

f) solid disc gears;

g) any shaft-hub fitting;

h) specific loading $(F_t\,K_A)/b \geq 100$ N/mm.

single stiffness	c' = 14 N/(mm·µm)	...(137)
mesh stiffness	c_γ = 20 N/(mm·µm)	...(138)

With the aid of conversion factors or experimental data, the values obtained by (137) and (138) can be modified to be used in an extended range of application:

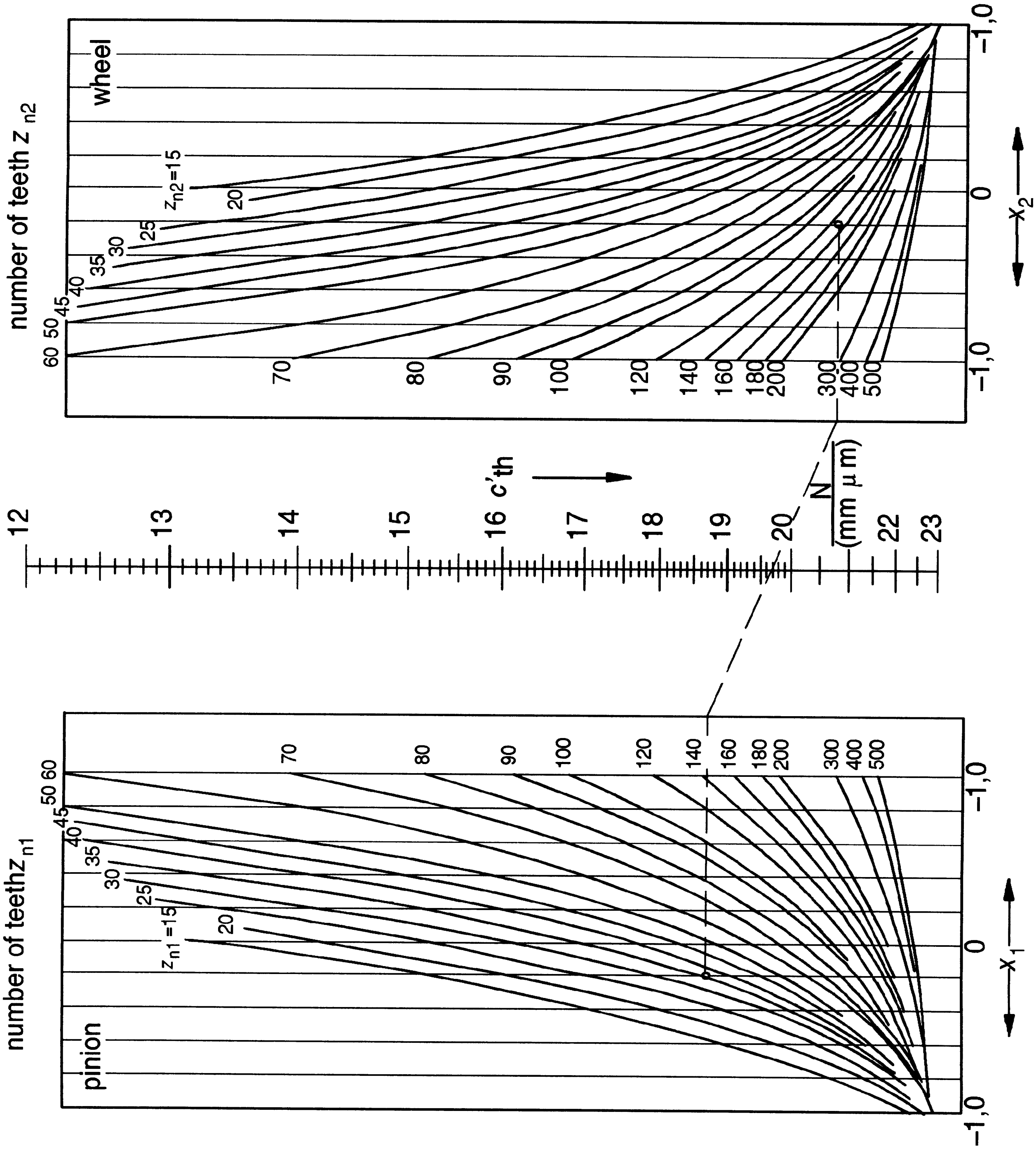
— conversion to suit other gear blank designs using equation (129);

— conversion to suit gears conjugate to different basic rack profiles using equation (131);

— conversion to suit other materials combinations using equations (133) and (134).

[33] When $(F_t\,K_A)/b$ > 100 N/mm, c' can be assumed to be constant.

[34] Values derived from equations (137) and (138) deviate from those derived from equation (126) by no more than ± 20 % if conditions a) to g) are met.



Example:
z_{n1} = 36; x_1 = + 0,2;
z_{n2} = 130; x_1 = − 0,2;
c'_{th} = 19,7 N/(mm·µm)

NOTE See note 12 in **9.3** for the calculation of z_{n1} and z_{n2}.

Figure 29 — Theoretical single stiffness of spur and virtual spur (of helical) gears, conjugate to a basic rack profile, ISO 53: $\alpha_P = 20°$, $h_{aP} = m_n$, $h_{fP} = 1{,}25\ m_n$, $\rho_{fP} = 0{,}25\ m_n$; steel/steel; method B2 (see 9.3.1.1)

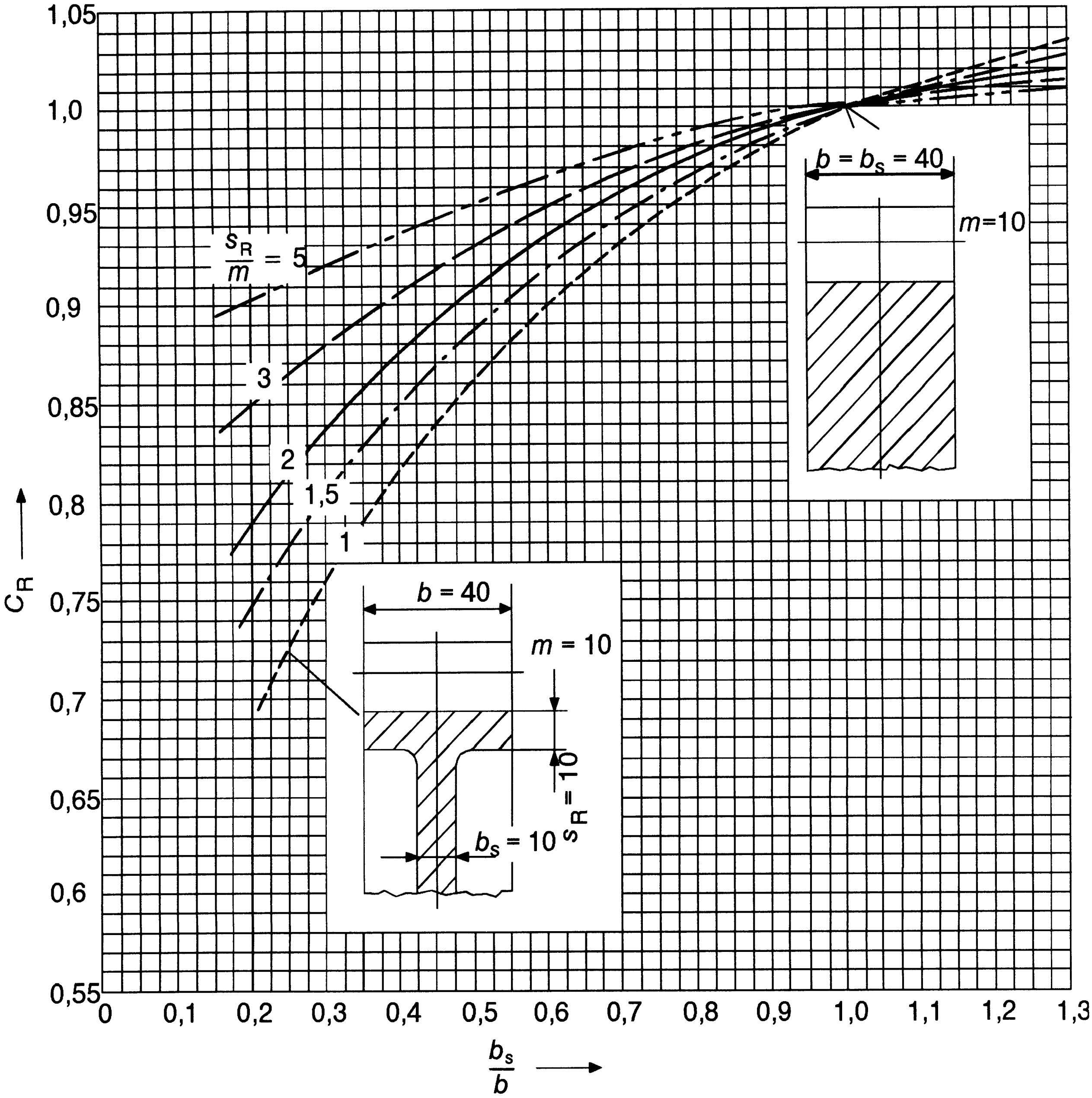


Figure 30 — Wheel blank factor C_R: mean values for mating gears of similar or stiffer wheel blank design

Annex A (informative)
Guide values for crowning and end relief of teeth of cylindrical gears

Well designed crowning and end relief have a beneficial influence on the distribution of load over the facewidth of a gear (see clause **7**). Design details should be based on a careful estimate of the deformations and manufacturing deviations of the gearing of interest. If deformations are considerable, helix angle modification might be superposed over crowning or end relief, but well designed helix modification is preferable.

A.1 Amount of crowning C_β

The following non-mandatory rule is drawn from experience; the amount of crowning which is necessary to obtain acceptable distribution of load can be determined as follows:

Subject to the limitations 10 µm ≤ C_β ≤ 40 µm plus a manufacturing tolerance of 5 µm to 10 µm, and that the value b_{cal}/b would have been greater than 1 had the gears not been crowned, $C_\beta \approx 0{,}5\ F_{\beta x\ cv}$ [see Figure 10b)].

The initial equivalent misalignment $F_{\beta x\ cv}$ shall be calculated as though the gears were not crowned, using a modified version of equation (69) in which 1,0 f_{sh} is substituted for 1,33 f_{sh}; see equation (A.1).

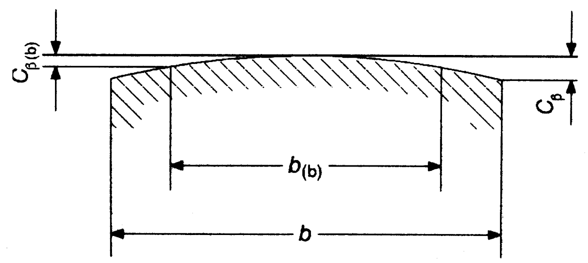


Figure A.1 — Amount of crowning $C_{\beta(b)}$ and width $b_{(b)}$ (see 7.6.2.2)

Furthermore, f_{sh} shall be determined as though the gears were not crowned in accordance with **7.6.2.2**.

So as to avoid excessive loading of tooth ends, instead of deriving f_{ma} from **7.6.3**, the value shall be calculated as:

$$f_{ma\ c} = 1{,}5\ f_{H\beta}$$

Thus the crowning amount:

$$C_\beta = 0{,}5(f_{sh} + 1{,}5\ f_{H\beta}) \qquad \text{...(A.1)}$$

When the gears are of such stiff construction that f_{sh} can for all practical purposes be neglected, or when the helices have been modified to compensate for deformation at mid-facewidth, the following value can be substituted:

$$C_\beta = f_{H\beta} \qquad \text{...(A.2)}$$

Subject to the restriction 10 µm ≤ C_β ≤ 25 µm plus a manufacturing tolerance of about 5 µm, 60 % to 70 % of the above values are adequate for extremely accurate and reliable high speed gears.

A.2 Amount $C_{I(II)}$ and width $b_{I(II)}$ of end relief

Method A.2.1: This method is based on an assumed value for the equivalent misalignment of the gear pair, without end relief and on the recommendations for the amount of gear crowning.

a) Amount of end relief

For through hardened gears: $C_{I(II)} \approx F_{\beta x\ cv}$ plus a manufacturing tolerance of 5 to 10 µm.

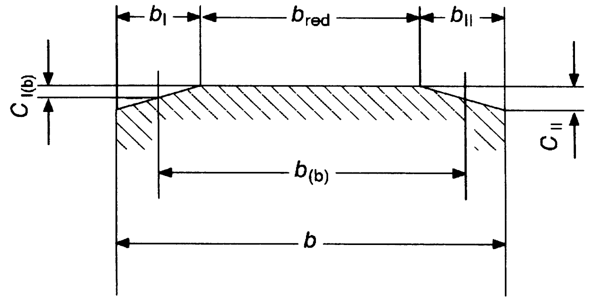


Figure A.2 — Amount $C_{I(II)\ (b)}$ and width $b_{(b)}$ of end relief (see 7.6.2.2)

Thus, by analogy, with $F_{\beta x\ cv}$ in **A.1**, $C_{I(II)}$ should be approximately:

$$C_{I(II)} = f_{sh} + 1{,}5\ f_{H\beta} \quad \text{...(A.3)}$$

For surface hardened and nitrided gears: $C_{I(II)} \approx 0{,}5\ F_{\beta x\ cv}$ plus a manufacturing tolerance of 5 to 10 µm.

Thus, by analogy with $F_{\beta x\ cv}$ in **A.1**, $C_{I(II)}$ should be approximately:

$$C_{I(II)} = 0{,}5(f_{sh} + 1{,}5\ f_{H\beta}) \quad \text{...(A.4)}$$

When the gears are of such stiff construction that f_{sh} can for all practical purposes be neglected, or when the helices have been modified to compensate deformation, proceed in accordance with equation (A.2).

60 % to 70 % of the above values is appropriate for very accurate and reliable gears with high tangential velocities.

b) Width of end relief

For approximately constant loading and higher tangential velocities:

$b_{I(II)}$ is the smaller of the values (0,1 b) or (1,0 m)

The following is appropriate for variable loading, low and average speeds:

$$b_{red} = (0{,}5 \text{ to } 0{,}7)b \quad \text{...(A.5)}$$

Method A.2.2: This method is based on the deflection of gear pairs assuming uniform distribution of load over the facewidth:

$$\delta_{bth} = F_m/(b\ c_\gamma), \text{ where } F_m = F_t\ K_A\ K_v \quad \text{...(A.6)}$$

For highly accurate and reliable gears with high tangential velocities, the following are appropriate:

$$C_{I(II)} = (2 \text{ to } 3)\delta_{bth} \quad \text{...(A.7)}$$

$$b_{red} = (0{,}8 \text{ to } 0{,}9)b \quad \text{...(A.8)}$$

For similar gears of lesser accuracy:

$$C_{I(II)} = (3 \text{ to } 4)\delta_{bth} \quad \text{...(A.9)}$$

$$b_{red} = (0{,}7 \text{ to } 0{,}8)b \quad \text{...(A.10)}$$

Annex B (informative)
Guide values for the application factor K_A

The application factor K_A is used to modify the value of F_t to take into account loads additional to nominal loads which are imposed on the gears from external sources. The empirical guidance values in Table B.1 can be used (for industry gears and high speed gears).

Table B.1 — Application factor K_A

Working characteristics of the driving machine	Working characteristics of the driven machine			
	Uniform	Light shocks	Moderate shocks	Heavy shocks
Uniform	1,00	1,25	1,50	1,75
Light shocks	1,10	1,35	1,60	1,85
Moderate shocks	1,25	1,50	1,75	2,00
Heavy shocks	1,50	1,75	2,00	2,25 or higher

The values apply to the nominal torque of the machine under consideration, or alternatively to the nominal torque of the driving motor, as long as this corresponds to the torque demand of the driving machine (see **4.2.1**).

The values only apply to transmissions which operate outside the resonance speed range under relatively steady loading. If operating conditions involve unusually heavy loading, motors with high starting torques, intermittent service or heavy repeated shock loading, service brakes with a torque greater than the driving-motor, the safety of the static and limited life load capacity of the gears shall be verified (see also ISO 6336, parts 2 and 3).

Examples:

a) Turbine/generator In this system short circuit torques of up to 6 times the nominal torque can occur. Such overloads can be shed by means of safety couplings.

b) Electric motor/compressor If pump and torsional natural frequencies coincide, considerable alternating stresses can occur.

c) Heavy plate and billet rolling mills Initial pass-shock-torques up to 6 times the rolling torque shall be taken into account.

d) Drives with synchronous motors Alternating torques up to 5 times the nominal torque can occur briefly (approximately 10 amplitudes) on starting; however, hazardous alternating torques can often be completely avoided by the appropriate detuning measures.

Information and numerical values provided here cannot be generally applied. The magnitude of the peak torque depends on the mass spring system, the forcing term, safety precautions (safety coupling, protection for unsynchronized switching of electrical machines) etc.

Thus in critical cases careful analysis should be demanded. It is then recommended that agreement is reached on suitable actions.

If special application factors are required for specific purposes, these shall be applied (e.g. because of a variable duty list specified in the purchase order, for marine gears according to the rules of a classification authority).

Where there are additional inertial masses, torques resulting from the flywheel effect are to be taken into consideration. Occasionally braking torque provides the maximum loading and thus influences calculation of load capacity.

It is assumed the gear materials used should have adequate overload capacity. When materials used have only marginal overload capacity, designs should be laid out for endurance at peak loading.

The K_A value for moderate, average, and heavy shocks can be reduced by using hydraulic couplings or torque matched elastic couplings, and especially vibration-attenuating couplings when the characteristics of the couplings so permit.

Table B.2 — Examples for driving machines with various working characteristics

Working characteristics	Driving machine
Uniform	Electric motor (e.g. DC motor), steam or gas turbine with uniform operation[a] and small rarely occurring starting torques[b]
Light shocks	Steam turbine, gas turbine, hydraulic or electric motor (large, frequently occurring starting torques)[b]
Moderate shocks	Multiple cylinder internal combustion engines
Heavy shocks	Single cylinder internal combustion engines

[a] Based on vibration tests or on experience gained from similar installations.

[b] See service life graphs Z_{NT} Y_{NT} for the material in ISO 6336, parts 2 and 3. For consideration of momentarily acting overload torques, see examples under Table B.1.

Table B.3 — Industrial gears: Examples of working characteristics of driven machine

Working characteristics	Driven machines
Uniform	Steady load current generator; uniformly loaded conveyor belt or platform conveyor; worm conveyor; light lifts; packing machinery; feed drives for machine tools; ventilators; light-weight centrifuges; centrifugal pumps; agitators and mixers for light liquids or uniform density materials; shears; presses, stamping machines[a]; vertical gear, running gear[b].
Light shocks	Non-uniformly (i.e. with piece or batched components) loaded conveyor belts or platform conveyors; machine tool main drives; heavy lifts; crane slewing gear; industrial and mine ventilator; heavy centrifuges; centrifugal pumps; agitators and mixers for viscous liquids or substances of non-uniform density; multi-cylinder piston pumps, distribution pumps; extruders (general); calenders; rotating kilns; rolling mill stands[c] (continuous zinc and aluminium strip mills, wire and bar mills).
Moderate shocks	Rubber extruders; continuously operating mixers for rubber and plastics; ball mills (light); wood-working machine (gang saws, lathes); billet rolling mills[c, d]; lifting gear; single cylinder piston pumps.
Heavy shocks	Excavators (bucket wheel drives), bucket chain drives; sieve drives; power shovels, ball mills (heavy); rubber kneaders, crushers (stone, ore); foundry machines; heavy distribution pumps; rotary drills; brick presses; de-barking mills; peeling machines; cold strip;[c, e] briquette presses; breaker mills.

[a] Nominal torque = maximum cutting, pressing or stamping torque.
[b] Nominal torque = maximum starting torque.
[c] Nominal torque = maximum rolling torque.
[d] Torque from current limitation.
[e] K_A up to 2,0 because of frequent strip cracking.

Table B.4 — High speed gears and gears of similar requirement: Examples of the working characteristics of driven machines

Working characteristics	Driven machine
Uniform	Centrifugal compressors for air conditioning installation, for process gas; dynamometer — test rig; base or steady load generator and exciter; paper machinery main drives.
Moderate shocks	Centrifugal compressors for air or pipelines; axial compressors; centrifugal fans; peak load generators and exciters; centrifugal pumps (all types other than those listed below); axial-flow rotary pumps; paper industry; Jordan or refining machines, machine auxiliary drives, stampers.
Medium shocks	Rotary-cam blower; rotary-cam compressor with radial flow; piston compressor (3 or more cylinders); ventilator suction-fans, mining and industrial (large, frequent start-up cycles); centrifugal boiler-feed pumps; rotary cam pumps, piston pumps (3 or more cylinders).
Heavy shocks	Piston compressor (2 cylinders); centrifugal pump (with water tank); sludge pump; piston pump (2 cylinders).

Annex C (informative)
Derivations and explanatory notes

NOTE 15 The notes in this annex are intended to assist the user's understanding of formulae used in this part of ISO 6336.

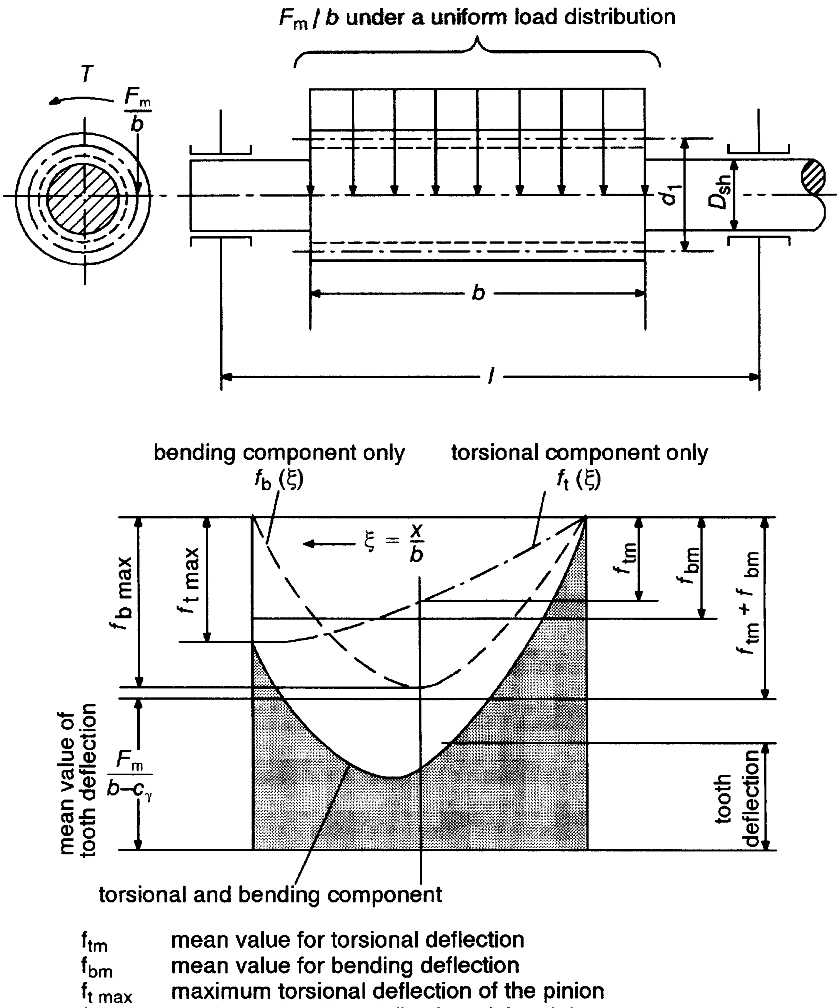
C.1 Derivation of equation (56) from the elastic torsional and bending deflections of the pinion

Figure C.1 shows the deformation of a pinion due to bending and torsion when the load is distributed uniformly. The following is the equation of torsional deflection under uniform load distribution.

$$f_t(\xi) = \frac{8}{\pi}\,\frac{F_m/b}{0{,}39\ E}\left(\frac{b}{d_1}\right)^2 \xi\left(1 - \frac{\xi}{2}\right) \qquad \text{...(C.1)}$$

The maximum value of f_t occurs at $\xi = 1$ and is

$$f_{t\,max} = \frac{4}{\pi}\,\frac{F_m/b}{0{,}39\ E}\left(\frac{b}{d_1}\right)^2 \qquad \text{...(C.2)}$$



f_{tm} mean value for torsional deflection
f_{bm} mean value for bending deflection
$f_{t\,max}$ maximum torsional deflection of the pinion
$f_{b\,max}$ maximum bending deflection of the pinion

Figure C.1 — Deflection of pinion shaft and pinion teeth

Mean value

$$f_{tm} = \int_0^1 f_t(\xi)\,\mathrm{d}\xi = \frac{2}{3} f_{t\,\max} \qquad \text{...(C.3)}$$

The following is the equation of the bending deflection when the load is evenly distributed across the facewidth.

$$f_b = \frac{8}{3\,\pi} \frac{F_m/b}{E} \left(\frac{b}{d_1}\right)^4 \left[\xi^4 - 2\,\xi^3 + 3\left(1 - \frac{l}{b}\right)\xi^2 + 2\left(\frac{3}{2}\,\frac{l}{b} - 1\right)\xi\right] \qquad \text{...(C.4)}$$

The maximum value of f_b occurs at $\xi = 1/2$ and is

$$f_{b\,\max} = \frac{2}{\pi} \frac{F_m/b}{E} \left(\frac{b}{d_1}\right)^4 \left(\frac{l}{b} - \frac{7}{12}\right) \qquad \text{...(C.5)}$$

Mean value

$$f_{bm} = \frac{4}{3\,\pi} \frac{F_m/b}{E} \left(\frac{b}{d_1}\right)^4 \left(\frac{l}{b} - \frac{3}{5}\right) \qquad \text{...(C.6)}$$

From which follows as an approximation

$$f_{bm} = \frac{2}{3} f_{b\,\max} \qquad \text{...(C.7)}$$

The total deformation component of equivalent misalignment is the sum of the mean values of torsional and bending deflections.

$$\frac{1}{2} F_{\beta x} = f_{bm} + f_{tm} = \frac{2}{3}\left(f_{b\,\max} + f_{t\,\max}\right) \qquad \text{...(C.8)}$$

To obtain the deformation component of $F_{\beta y}$ inclusive of a proportional amount of the running-in allowance, it is necessary to multiply the deformation component of equivalent misalignment by the factor χ_B.
The face load factor $K_{H\beta}$ is as defined in **7.3.1**, equation (38):

$$K_{H\beta} = \frac{(F/b)_{\max}}{F_m/b} \qquad \text{...(C.9)}$$

If the deflections calculated above are introduced into equation (38), the following is obtained:

$$\begin{aligned} K_{H\beta} &= \frac{c_\gamma \left[\dfrac{F_m}{b\,c_\gamma} + \left(f_{tm} + f_{bm} - y_\beta\right) 1000\right]}{c_\gamma \dfrac{F_m/b}{c_\gamma}} \\ &= 1 + \frac{c_\gamma\,\chi_\beta\,\left(f_{tm} + f_{bm}\right) 1000}{F_m/b} \\ &= 1 + \frac{2}{3}\,\frac{c_\gamma}{F_m/b}\,\chi_\beta\,\left(f_{t\,\max} + f_{b\,\max}\right) 1000 \end{aligned} \qquad \text{...(C.10)}$$

from which follows equation (56).

C.2 Explanatory notes to equations (69) and (70)

The factor 1,33 in equations (69), (70), and (71), corrects the error arising from the assumption that the elastic deformation f_{sh} is linear. Using the linear deformation formulation with 1,33 f_{sh}, the same value of $K_{H\beta}$ is calculated, as with the actual parabolic deformation and 1,0 f_{sh} (see Figure C.2).

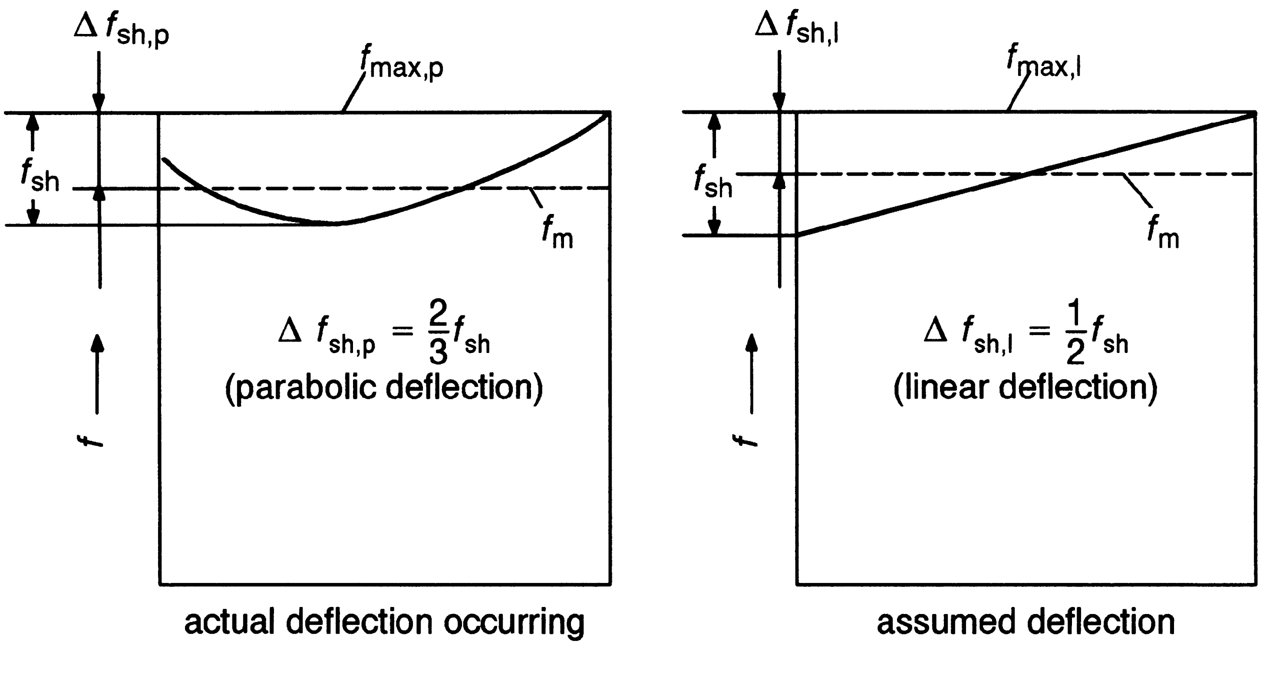
The following applies to equation (70):

When correct patterns which are suitable in both size and position are obtained (see Figure 15), one or more of the following is implied:

a) components have been correctly manufactured and assembled in accordance with an adequate design specification;

b) manufacturing deviations of the assembled components partially cancel each other and the deviations may be less than the permissible values according to ISO 1328-1;

c) manufacturing component f_{ma} and the deformation component f_{sb} of mesh alignment are mutually compensatory.



$$K_{H\beta} = \frac{F_{max}/b}{F_m/b} = \frac{f_{max,\,p}}{f_m} = 1 + \frac{\Delta f_{sh,\,p}}{f_m} = 1 + \frac{1{,}33\ \Delta f_{sh,\,l}}{f_m}$$

where f ($f = w/c_\gamma$) is the tooth deflection.

Figure C.2 — Elastic deflection of the pinion f_{sh} (principle). Comparison of actual and assumed progression

Annex D (informative) Bibliography

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