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THE PORE-PRESSURE COEFFICIENTS *A* AND *B*

by

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SYNOPSIS

In a number of problems involving the undrained shear strength of soils (especially in the design of earth dams) the change in pore pressure Δu occurring under changes in total stresses must be known. The equation $\Delta u = \boldsymbol{B}[\Delta\sigma_3 + \boldsymbol{A}(\Delta\sigma_1 - \Delta\sigma_3)]$ is derived, and some typical values of the experimentally determined pore-pressure coefficients $\boldsymbol{A}$ and $\boldsymbol{B}$ are given. Some practical applications of these coefficients have been outlined by Bishop (1954).

Pour un certain nombre de problèmes comportant la résistance au cisaillement à teneur en eau constante des sols (en particulier, pour le calcul des barrages en terre), il est nécessaire de connaître les changements dans la pression interstitielle Δu qui se produisent lors des changements dans les contraintes totales. L'équation $\Delta u = \boldsymbol{B}[\Delta\sigma_3 + \boldsymbol{A}(\Delta\sigma_1 - \Delta\sigma_3)]$ est dérivée et certaines valeurs typiques des coefficients $\boldsymbol{A}$ et $\boldsymbol{B}$ de pression interstitielle obtenues expérimentalement sont données dans cet article. Certaines applications pratiques de ces coefficients ont été exposées par Bishop (1954).

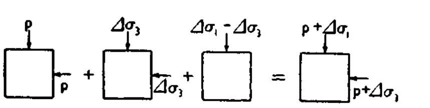
INTRODUCTION

In problems concerning the undrained shear strength of soils, it has been found convenient to express the pore-pressure change Δu, which occurs under changes in the principal stresses $\Delta\sigma_1$ and $\Delta\sigma_3$, by the following equation:

$$\Delta u = \boldsymbol{B}[\Delta\sigma_3 + \boldsymbol{A}(\Delta\sigma_1 - \Delta\sigma_3)],$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are "pore-pressure coefficients." These coefficients are measured experimentally in the undrained triaxial test, and the values of $\Delta\sigma_1$ and $\Delta\sigma_3$ are, in general, chosen to represent the changes in principal stress occurring in the practical problem under consideration.

If the sample in the test, or if an element of soil in the ground or in an earth dam, is originally in equilibrium under an all-round * effective pressure p (which may in certain cases be close to zero), then the application of the stresses $\Delta\sigma_1$ and $\Delta\sigma_3$ can be considered as taking place in two stages (see Fig. 1). Firstly, the element is subjected to an equal all-round increment $\Delta\sigma_3$ and, secondly, it is subjected to a deviator stress $(\Delta\sigma_1 - \Delta\sigma_3)$. Corresponding to each of these stages there will be pore-pressure changes Δu_a and Δu_d, where:



Undrained test	Δu_a	Δu_d	$\Delta u = \Delta u_a + \Delta u$
Drained test	ΔV_a	ΔV_d	$\Delta V = \Delta V_a + \Delta V$

Fig. 1

$$\Delta u = \Delta u_a + \Delta u_d.$$

THE COEFFICIENT *B*

The relation between Δu_a and $\Delta\sigma_3$ for a typical test on a partially saturated soil is shown in Fig. 2 (*a*). The increase in effective stress in the test is:

$$\Delta\sigma' = \Delta\sigma_3 - \Delta u_a$$

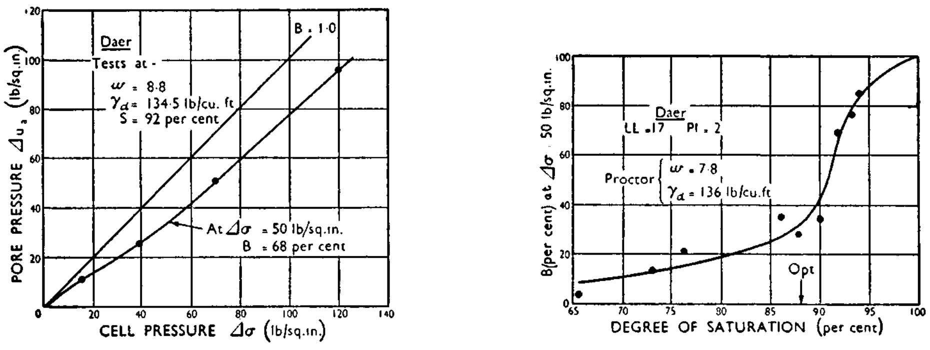
and, if C_c is the compressibility of the soil structure, then the volume change is:

$$\Delta V_c = -C_c \,.\, V(\Delta\sigma_3 - \Delta u_a),$$

where V is the original volume of the sample. And, if C_v is the compressibility of the fluid (air and water) in the voids and if n is the porosity of the soil, then the change in volume in the void space is:

$$\Delta V_v = -C_v \,.\, nV \,.\, \Delta u_a.$$

* The all-round pressure condition is assumed for simplicity of presentation. The case of an element consolidated under p and Kp can also be treated by the pore-pressure coefficients.



(a) (b)

Fig. 2

But these two changes in volume are identical and, hence,

$$\frac{\Delta u_a}{\Delta \sigma_3} = \boldsymbol{B} = \frac{1}{1 + \dfrac{nC_v}{C_c}}.$$

Now, in saturated soils (zero air voids), C_v/C_c is approximately equal to zero, since the compressibility of water is negligible compared with that of the soil structure. Consequently, for such soils,

$$\boldsymbol{B} = 1, \text{ when the degree of saturation } = 1.$$

An experimental confirmation of this result, for a saturated clay, is given below in Table 1.

Table 1

$\Delta\sigma_3$	Δu_a	$\boldsymbol{B}$
0	0	—
15 lb/sq. in.	14·7 lb/sq. in.	0·980
30 ,, ,,	29·5 ,, ,,	0·984
45 ,, ,,	45·0 ,, ,,	1·000
60 ,, ,,	59·8 ,, ,,	0·996

If, in contrast, the soil is dry, then C_v/C_c approaches infinity, since the compressibility of air is far greater than that of the soil structure. Hence, for dry soils,

$$\boldsymbol{B} = 0, \text{ when the degree of saturation } = 0.$$

For partially saturated soils, $0 < \boldsymbol{B} < 1$ and, at the Proctor optimum water content and density, the values of $\boldsymbol{B}$ range typically from about 0·1 to 0·5. The relation between $\boldsymbol{B}$ and the degree of saturation, for a clay gravel, is shown in Fig. 2 (*b*).

THE COEFFICIENT *A*

The changes in pore pressure during the application of a deviator stress are shown, for two compacted clay soils, in Fig. 3. At any time when the increment of deviator stress is $(\Delta\sigma_1 - \Delta\sigma_3)$, the pore pressure due to this increment is Δu_d and the corresponding changes in the principal effective stresses are :

$$\Delta\sigma_1' = (\Delta\sigma_1 - \Delta\sigma_3) - \Delta u_d$$

and

$$\Delta\sigma_3' = -\Delta u_d.$$

If, for the moment, it is assumed that the soil behaves in accordance with elastic theory, the volume change of the soil structure under the increment of deviator stress is :

$$\Delta V_c = -C_c \,.\, V \,.\, \tfrac{1}{3}(\Delta\sigma_1' + 2\Delta\sigma_3')$$

or

$$\Delta V_c = -C_c \,.\, V \,.\, \tfrac{1}{3}[(\Delta\sigma_1 - \Delta\sigma_3) - 3\Delta u_d].$$

And the volume change in the void space is :

$$\Delta V_v = -C_v \,.\, nV \,.\, \Delta u_d.$$

But, as before, these two volume changes are identical and, hence,

$$\Delta u_d = \frac{1}{1 + \dfrac{nC_v}{C_c}} \,.\, \tfrac{1}{3}(\Delta\sigma_1 - \Delta\sigma_3)$$

or

$$\Delta u_d = \boldsymbol{B} \,.\, \tfrac{1}{3}(\Delta\sigma_1 - \Delta\sigma_3).$$

In general, however, the behaviour of soils is by no means in accordance with elastic theory and the above expression must be written in the form :

$$\Delta u_d = \boldsymbol{B} \,.\, \boldsymbol{A}(\Delta\sigma_1 - \Delta\sigma_3),$$

where $\boldsymbol{A}$ is a coefficient to be determined experimentally.

Combining the expressions for the two components of pore pressure, we have :

$$\Delta u = \boldsymbol{B}[\Delta\sigma_3 + \boldsymbol{A}(\Delta\sigma_1 - \Delta\sigma_3)],$$

which is the equation given at the beginning of the Paper. It may be noted that for the important particular case of fully saturated soils, where $\boldsymbol{B} = 1$, the equation becomes :

$$\Delta u = \Delta\sigma_3 + \boldsymbol{A}(\Delta\sigma_1 - \Delta\sigma_3).$$

This expression was given by the author in 1948. Test results for a saturated clay are plotted in Fig. 4.

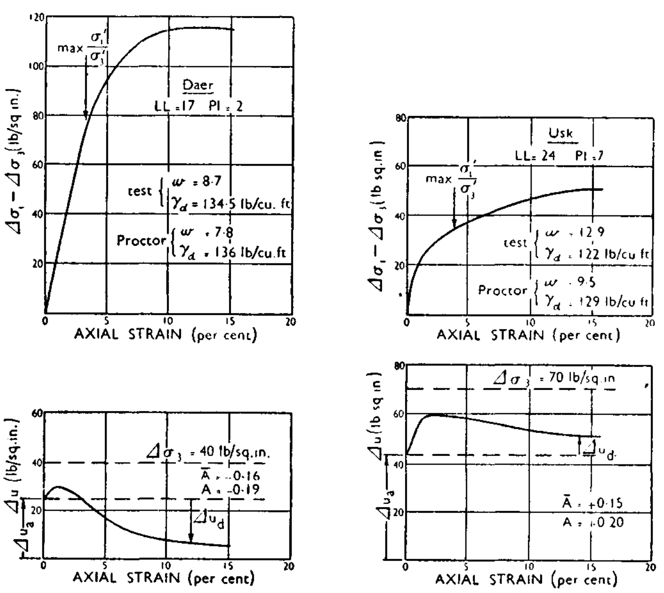


Fig. 3. Undrained triaxial tests on two compacted clay-gravels

For any given soil, the coefficient A varies with the stresses and strains. Its value may be quoted at failure (maximum deviator stress), at maximum effective principal stress ratio, or at any other required point. At failure, the values of A for various clay soils, with positive total stress increments, may be summarized approximately as in Table 2. With decreasing total stresses, A will have different values in general, but the data for this case are scanty.

Table 2

Type of Clay	A
Clays of high sensitivity	$+\frac{3}{4}$ to $+1\frac{1}{2}$
Normally consolidated clays	$+\frac{1}{2}$ to $+1$
Compacted sandy clays	$+\frac{1}{4}$ to $+\frac{3}{4}$
Lightly over-consolidated clays ..	0 to $+\frac{1}{2}$
Compacted clay-gravels	$-\frac{1}{4}$ to $+\frac{1}{4}$
Heavily over-consolidated clays ..	$-\frac{1}{2}$ to 0

ALTERNATIVE FORMS OF THE PORE-PRESSURE EQUATION

The pore-pressure equation :

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

may be written in several alternative forms, each of which has some particular advantages. In the normal laboratory undrained test, the pore pressures under $\Delta\sigma_3$ and under $(\Delta\sigma_1 - \Delta\sigma_3)$ are measured and, hence, the coefficients determined directly from the test are those in the following equation :

$$\Delta u = B . \Delta\sigma_3 + \bar{A}(\Delta\sigma_1 - \Delta\sigma_3).$$

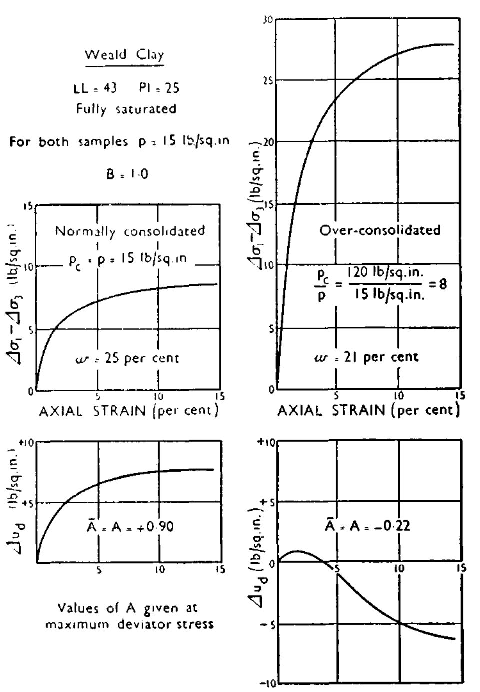


Fig. 4. Undrained triaxial tests on two samples of remoulded saturated clay

In evaluating A from $\bar{A}$, care must be taken to use a value of B appropriate to the pressure range in the deviator part of the test.

For earth-dam problems, it is convenient to write the basic equation in the forms :

$$\Delta u = B[\Delta\sigma_1 - (1 - A)(\Delta\sigma_1 - \Delta\sigma_3)]$$

and

$$\frac{\Delta u}{\Delta\sigma_1} = \bar{B} = B\left[1 - (1 - A)\left(1 - \frac{\Delta\sigma_3}{\Delta\sigma_1}\right)\right].$$

The " overall " coefficient $\bar{B}$ is a useful parameter, especially in stability calculations involving rapid draw-down, and it can be measured directly in the laboratory for the relevant values of stress-change in any particular problem.

From a physical point of view, the pore-pressure equation is best written in the form :

$$\Delta u = B\left[\tfrac{1}{3}(\Delta\sigma_1 + 2\Delta\sigma_3) + \frac{3A - 1}{3}(\Delta\sigma_1 - \Delta\sigma_3)\right],$$

since this shows that, for a material behaving in accordance with elastic theory, with $A = \frac{1}{3}$, the pore pressure depends solely on the mean principal stress, whereas in soils with $A \neq \frac{1}{3}$ the pure shear stress has a marked influence on the pore pressures.

APPLICATIONS

During the past few years a number of practical problems have been encountered in which the pore-pressure coefficients have proved to be helpful. Bishop (1954) has described briefly some of these applications.

ACKNOWLEDGEMENT

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REFERENCES

BISHOP, A. W., 1954. The Use of Pore-Pressure Coefficients in Practice. *Géotechnique*, 4 : 4 : 148.
SKEMPTON, A. W., 1948. The Effective Stresses in Saturated Clays Strained at Constant Volume. *Proc. 7th Int. Congr. App. Mech.*, 1 : 378.