Homework

**Given:**

(1)
$$f(x) = \mathrm{Sin}(2x)$$
$$g(x, s_i) = 1 + \mathrm{Sin}(2x + s_i)$$

subject to two conditions being satisfied simultaneously

(2)
$$f(x) = g(x, s_i)$$
$$f'(x) = g'(x, s_i)$$

Find $s_i$ …

**Solution:**

Combining (1) and (2) we have

(1)
$$\mathrm{Sin}(x) = 1 + \mathrm{Sin}(2x + s_i)$$
$$\mathrm{Cos}(x) = 2\,\mathrm{Cos}(2x + s_i)$$

but $\mathrm{Sin}^2\phi + \mathrm{Cos}^2\phi = 1$, so this can be expressed as

(2)
$$\mathrm{Sin}^2(2x + s_i) - \frac{2}{3}\mathrm{Sin}(2x + s_i) - \frac{4}{3} = 0$$

With a change of variable $\xi_i = \mathrm{Sin}(2x + s_i)$, we now have a quadratic equation and a solution given by

(3)
$$\xi_i = \frac{1}{3}(1 \pm \sqrt{13})$$

In other words

(4)
$$\mathrm{Sin}(2x + s_i) = \frac{1}{3}(1 \pm \sqrt{13})$$

Returning to the original equation (1), we now have

Homework

(5)

$$f(x) = \mathrm{Sin}(2x)$$
$$g(x, s_i) = 1 + \frac{1}{3}(1 \pm \sqrt{13})$$

since this is subject to $f(x) = g(x, s_i)$ we can solve for $x$ to locate the points satisfying the constraints in (2). QED