M ATARAZZO

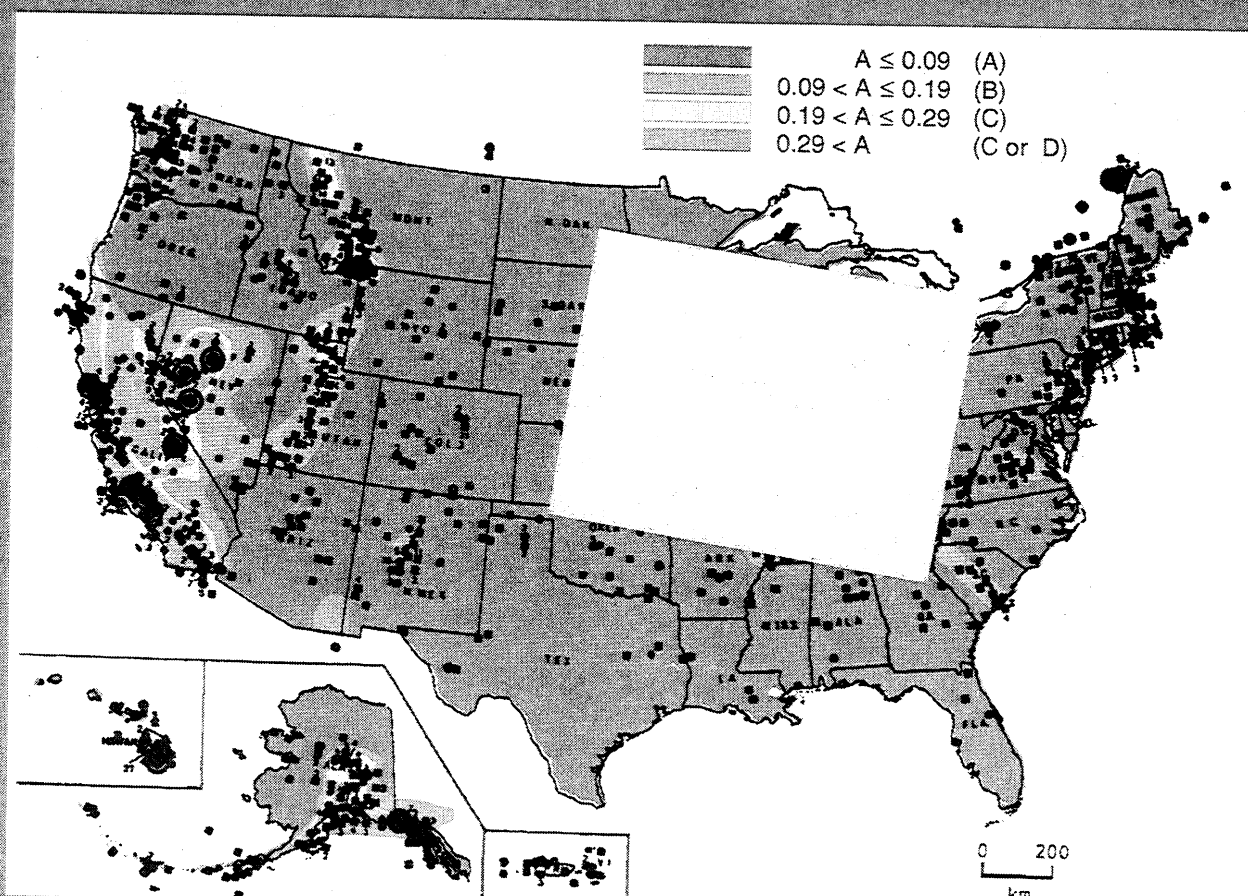
U.S. Department
of Transportation

Federal Highway
Administration

October 1996

Seismic Design of Bridges Design Example No. 7

Twelve-Span Viaduct AASHTO Precast Concrete Bridge



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16. Abstract

This document describes one of seven seismic design examples that illustrate "how" to apply AASHTO's seismic analysis and design requirements on actual different bridge types across the United States. Each provides a complete set of "designer's notes" covering the seismic analysis, design, and details for that particular bridge including flow charts, references to applicable AASHTO requirements, and thorough commentary that explains each step. In addition, each example highlights separate issues (skew effects, wall piers, elastomeric bearings, pile foundations, etc.).

The **first example** is a 242' reinforced concrete box girder two span overcrossing with spread footing foundations, SPC-C & A = 0.28g. The **second example** is a 400' 3-span skewed steel plate girder bridge over a river in New England with spread footing foundations, SPC-B & A = 0.15g. The **third example** is a skewed 70' single span prestressed concrete girder bridge with tall-closed seat-type abutments on spread footings, SPC-C & A = 0.36g. The **fourth example** is a 320' reinforced concrete box girder 3-span skewed bridge in the western United States with spread footing foundations, SPC-C & A = 0.30g. The **fifth example** is a 1488' steel plate girder bridge in the inland Pacific Northwest with pile foundations, SPC-B & A = 0.15g. It has nine spans and consists of two units: a four-span tangent (Unit 1) and a five-span with a 1300-foot radius curve (Unit 2). The **sixth example** is a 290' sharply curved (104 degrees) 3-span concrete box girder bridge in the Northwestern United States with pile abutment foundations and drilled shaft pier foundations, SPC-C & A = 0.20g. The **seventh example** is a 717' 10-span prestressed girder bridge with open pile bents, SPC-B & A = 0.10g. The superstructure consists of three continuous span units arranged in a 3-4-3 span series.

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Seismic Design Course

Design Example No. 7

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PLEASE NOTE

Data, specifications, suggested practices, and drawings presented herein are based on the best available information, are delineated in accord with recognized professional engineering principles and practices, and are provided for general information only. None of the procedures suggested or discussed should be used without first securing competent advice regarding their suitability for any given application.

This document was prepared with the help and advice of FHWA, State, academic, and private engineers. The intent of this document is to aid practicing engineers in the application of the AASHTO seismic design specification. BERGER/ABAM and the United States Government assume no liability for its contents or use thereof.

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Section I
Introduction

PURPOSE OF DESIGN EXAMPLE

This is the seventh in a series of seismic design examples developed for the FHWA. A different bridge configuration is used in each example. The bridges are in either Seismic Performance Category B or C sites. Each example emphasizes different features that must be considered in the seismic analysis and design process. The matrix below is a summary of the features of the first seven examples.

DESIGN EXAMPLE NO.	DESIGN EXAMPLE DESCRIPTION	SEISMIC CATEGORY	PLAN GEOMETRY	SUPER-STRUCTURE TYPE	PIER TYPE	ABUTMENT TYPE	FOUNDATION TYPE	CONNECTIONS AND JOINTS
1	Two-Span Continuous	SPC - C	Tangent Square	CIP Concrete Box	Three-Column Integral Bent	Seat Stub Base	Spread Footings	Monolithic Joint at Pier Expansion Bearing at Abutment
2	Three-Span Continuous	SPC - B	Tangent Skewed	Steel Girder	Wall Type Pier	Tall Seat	Spread Footings	Elastomeric Bearing Pads (Piers and Abutments)
3	Single-Span	SPC - C	Tangent Square	AASHTO Precast Concrete Girders	(N/A)	Tall Seat (Closed-In)	Spread Footings	Elastomeric Bearing Pads
4	Three-Span Continuous	SPC - C	Tangent Skewed	CIP Concrete	Two-Column Integral Bent	Seat	Spread Footings	Monolithic at Col. Tops Pinned Column at Base Expansion Bearings at Abutments
5	Nine-Span Viaduct with Four-Span and Five-Span Continuous Structs.	SPC - B	Curved Square	Steel Girder	Single-Column (Variable Heights)	Seat	Steel H-Piles	Conventional Steel Pins and PTFE Sliding Bearings
6	Three-Span Continuous	SPC - C	Sharply-Curved Square	CIP Concrete Box	Single Column	Monolithic	Drilled Shaft at Piers, Steel Piles at Abutments	Monolithic Concrete Joints
7	12-Span Viaduct with (3) Four-Span Structures	SPC - B	Tangent Square	AASHTO Precast Concrete Girders	Pile Bents (Battered and Plumb)	Seat	Concrete Piles and Steel Piles	Pinned and Expansion Bearings

REFERENCE AASHTO SPECIFICATIONS

The examples conform to the following specifications.

AASHTO Division I (herein referred to as "Division I")

Standard Specifications for Highway Bridges, American Association of State Highway and Transportation Officials, Inc., 15th Edition, as amended by the Interim Specifications-Bridges-1993 through 1995.

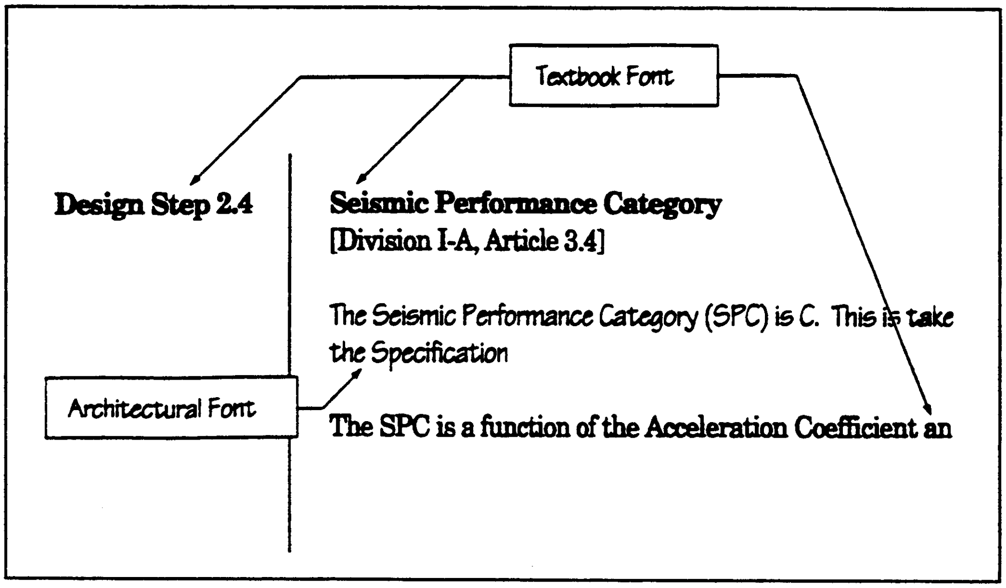
AASHTO Division I-A (herein referred to as "Division I-A" or the "Specification")

Standard Specifications for Highway Bridges, Division I-A, Seismic Design, American Association of State Highway and Transportation Officials, Inc., 15th Edition, as amended by the Interim Specifications-Bridges-1995.

USE OF DIFFERENT TYPE FONTS

In the example, two primary type fonts have been used. One font, similar to the type used for textbooks, is used for all section headings and for commentary. The other, an architectural font that appears hand printed, is used for all primary calculations. The material in the architectural font is the essential calculation material and essential results.

An example of the use of the fonts is shown below.

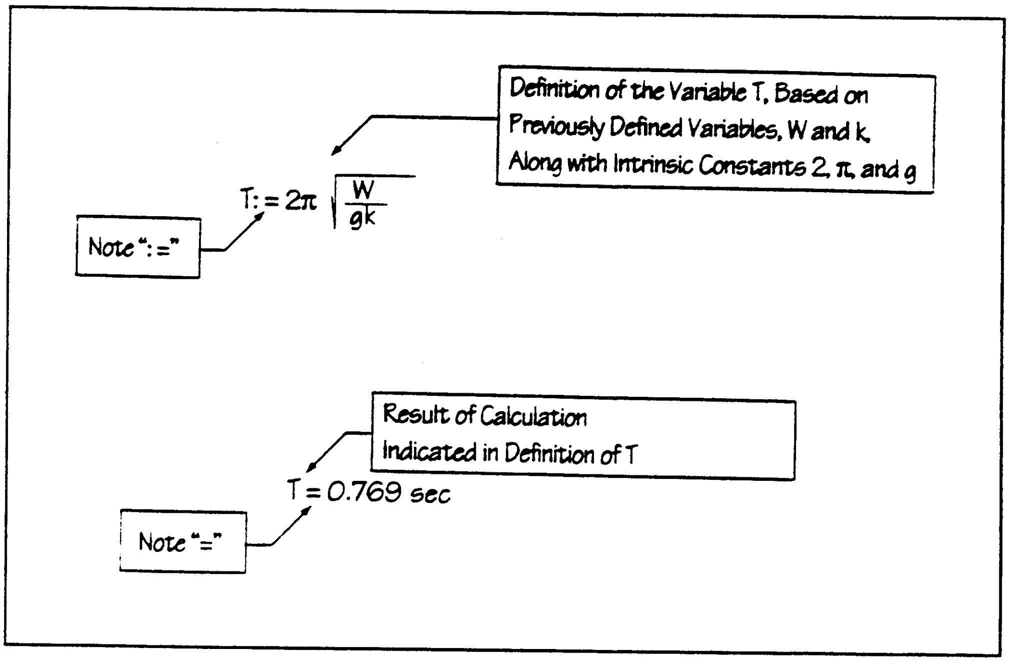


USE OF MATHCAD®

To provide consistent results and quality control, all calculations have been performed using the program Mathcad®.

The variables used in equations calculated by the program are defined before the equation, and the **definition** of either a variable or an equation is distinguished by a ':=' symbol. The **echo** of a variable or the result of a calculation is distinguished by a '=' symbol, i.e., no colon is used.

An example is shown below.



Note that Mathcad® carries the full precision of the variables throughout the calculations, even though the listed result of a calculation is rounded off. Thus, hand-calculated checks made using intermediate rounded results may not yield the same result as the number being checked.

Also, Mathcad® does not allow the superscript " ′ " to be used in a variable name. Therefore, the specified compressive strength of concrete is defined as f_c in this example (not f'_c).

Section II
Overview

SECTION II — OVERVIEW OF EXAMPLE NO. 7

INTRODUCTION

Introduction and Summary of Preliminary Design of Open Bent Pile-Supported Substructure — The bridge in Example No. 7 consists of a multispan concrete superstructure, with an open pile bent substructure. The bridge has a total of ten spans, consisting of three continuous span bridges, arranged in a 3-4-3 span series. The example will focus on the middle four-span structure, referred to as "Unit 2."

Figure 1 shows the layout for "Bridge Option No. 1." As will be shown in Design Step 3, this pile layout results in unacceptably large axial loads in the batter piles under seismic loading. The remainder of the example, rather than focusing on the final analysis procedures or the detailed design of the various elements, will investigate the impact of using different pile layouts. Both plumb and batter piles are considered.

A total of six different options are investigated, three using concrete piles (Design Steps 3 to 5) and three using steel piles (Design Steps 6 to 9). See Figure 1 for a summary sketch of each option. Each layout is assigned a unique option number. For each option, the seismic analysis is done using hand calculations once the basic formulas for stiffness are developed. The goal is to demonstrate how various options can be quickly evaluated without using a computer.

The emphasis of this example is the preliminary seismic analysis of the pile-supported substructure. A "roadmap" of the example is presented below with a brief description of what is included. The reader is encouraged to refer to this roadmap frequently when reading the example.

GENERAL

Design Step 1 — Bridge Layout

These figures can be referred to throughout the example for general information on the bridge, except for the pile layout. The particular pile layout for each bridge option is given later, as the example progresses. This design step also contains general information on site data, basic unit weights, and soil capacities for different pile options.

GENERAL
(continued)

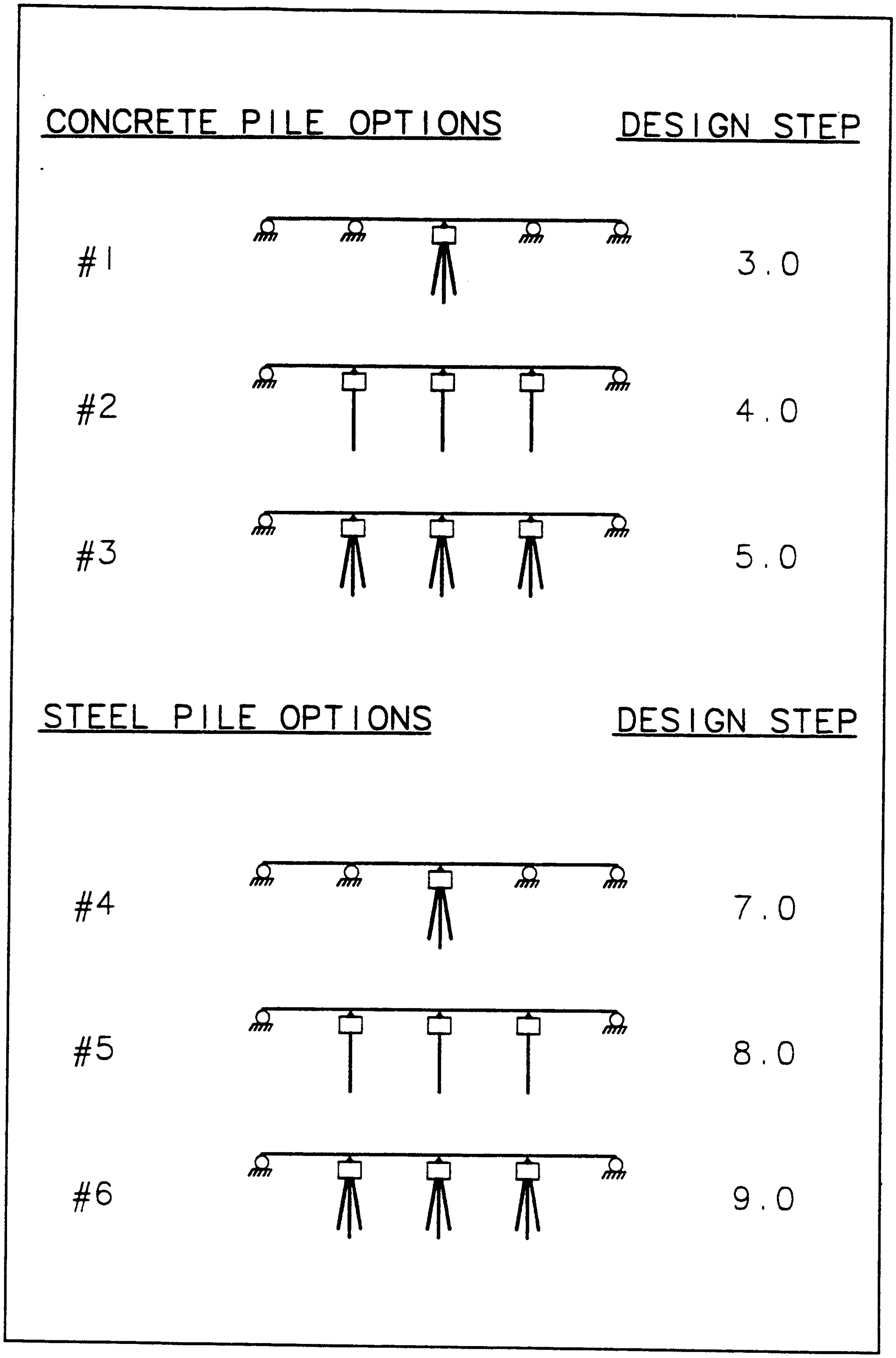


Figure 1 — Summary of Pile Layout Options

OPTIONS USING CONCRETE PILES

Design Step 2 — Horizontal Stiffness of a Concrete Pile

The horizontal stiffness of a single concrete pile is developed for different batters. Both the axial and flexural contributions to the horizontal stiffness of a concrete pile are addressed. These data will be used in Design Steps 3, 4, and 5 for the design of concrete piles.

Design Step 3 — Bridge Option No. 1 (Concrete Piles)

This option, common in existing bridges of this type, has only one bent (Bent 5) pinned to resist longitudinal movement. Bent 5 consists of three plumb and four 2:12 concrete batter piles.

Design Step 4 — Bridge Option No. 2 (Concrete Piles)

This option has three bents (Bents 4, 5, and 6) pinned to resist longitudinal movement. All the bents consist of concrete plumb piles.

Design Step 5 — Bridge Option No. 3 (Concrete Piles)

This option also has three bents (Bents 4, 5, and 6) pinned to resist longitudinal movement. But, each of these bents consists of three plumb and four 2:12 concrete batter piles.

OPTIONS USING STEEL PILES

Design Step 6 — Horizontal Stiffness of a Steel Pile

The horizontal stiffness of a single steel pile is developed for different batters. Both the axial and flexural contributions to the horizontal stiffness of a steel pile are addressed. Data will be used in Design Steps 7, 8, and 9, which use steel piles.

Design Step 7 — Bridge Option No. 4 (Steel Piles)

This option has only one bent (Bent 5) pinned to resist longitudinal movement. Bent 5 consists of three plumb and four 2:12 steel batter piles.

Design Step 8 — Bridge Option No. 5 (Steel Piles)

This option has three bents (Bents 4, 5 and 6) pinned to resist longitudinal movement. All the bents consist of steel plumb piles.

OPTIONS USING STEEL PILES (continued)

Design Step 9 — Bridge Option No. 6 (Steel Piles)

This option also has three bents (Bents 4, 5, and 6) pinned to resist longitudinal movement. But, each of these bents consists of three plumb and four 2:12 steel batter piles.

SUMMARY

Design Step 10

This step summarizes the results of the design example.

Section III
Analysis and Design

SECTION III ANALYSIS AND DESIGN

DATA

The configuration of the bridge consists of a multispan superstructure using AASHTO girders and a CIP deck. The substructure consists of open pile bents. Expansion joints are provided at both ends of each continuous structure. Refer to Figure 1 for details of the configuration.

The bridge is underlain by a deep deposit (>200 feet) of cohesionless soils. The groundwater table is at the ground surface.

The alignment of the roadway on the bridge is straight and there is no vertical curve or skew.

The bridge is to be built in a seismic zone with

- Acceleration Coefficient, A = 0.10g
- Importance Classification, IC = II
- Seismic Performance Category, SPC = B
- Site Coefficient, S = 1.2

REQUIRED

Design the bridge for seismic loading using the *Standard Specifications for Highway Bridges*, Division I-A: Seismic Design, American Association of State Highway and Transportation Officials, Inc., 15th Edition, as amended by the Interim Specification-Bridges-1995.

FEATURES

ISSUES EMPHASIZED IN THIS EXAMPLE

- Preliminary Analysis Using Hand Calculations
- Open Pile Bent Concepts
- Plumb Versus Batter Piles
- Concrete and Steel Piles

Design Step 1 — Bridge Layout

Design Step 1.1 — Layout for Bridge Option No. 1

See Figure 1 (a to f).

BRIDGE DATA
(continued)

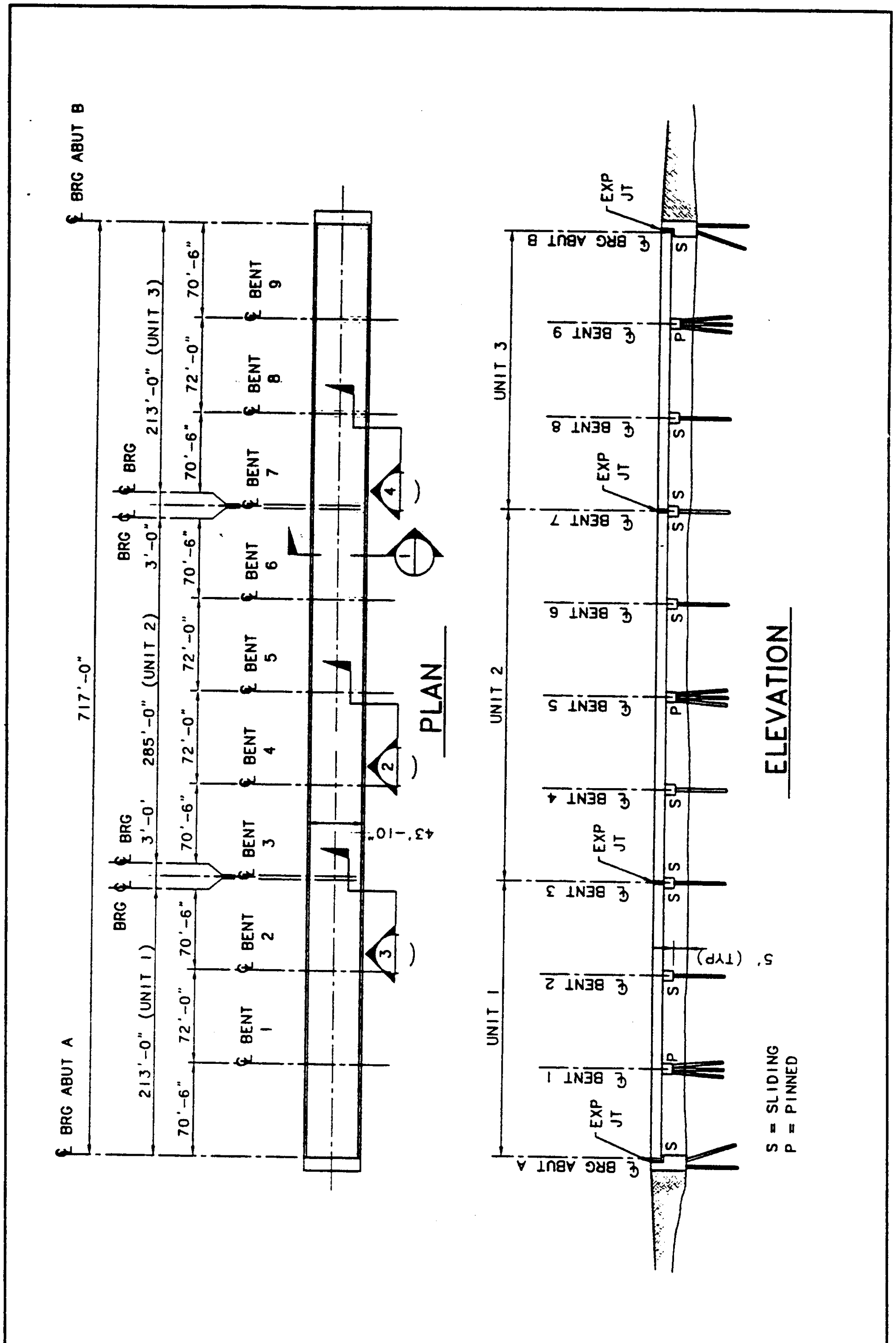


Figure 2a — Bridge No. 7 - Plan and Elevation

BRIDGE DATA
(continued)

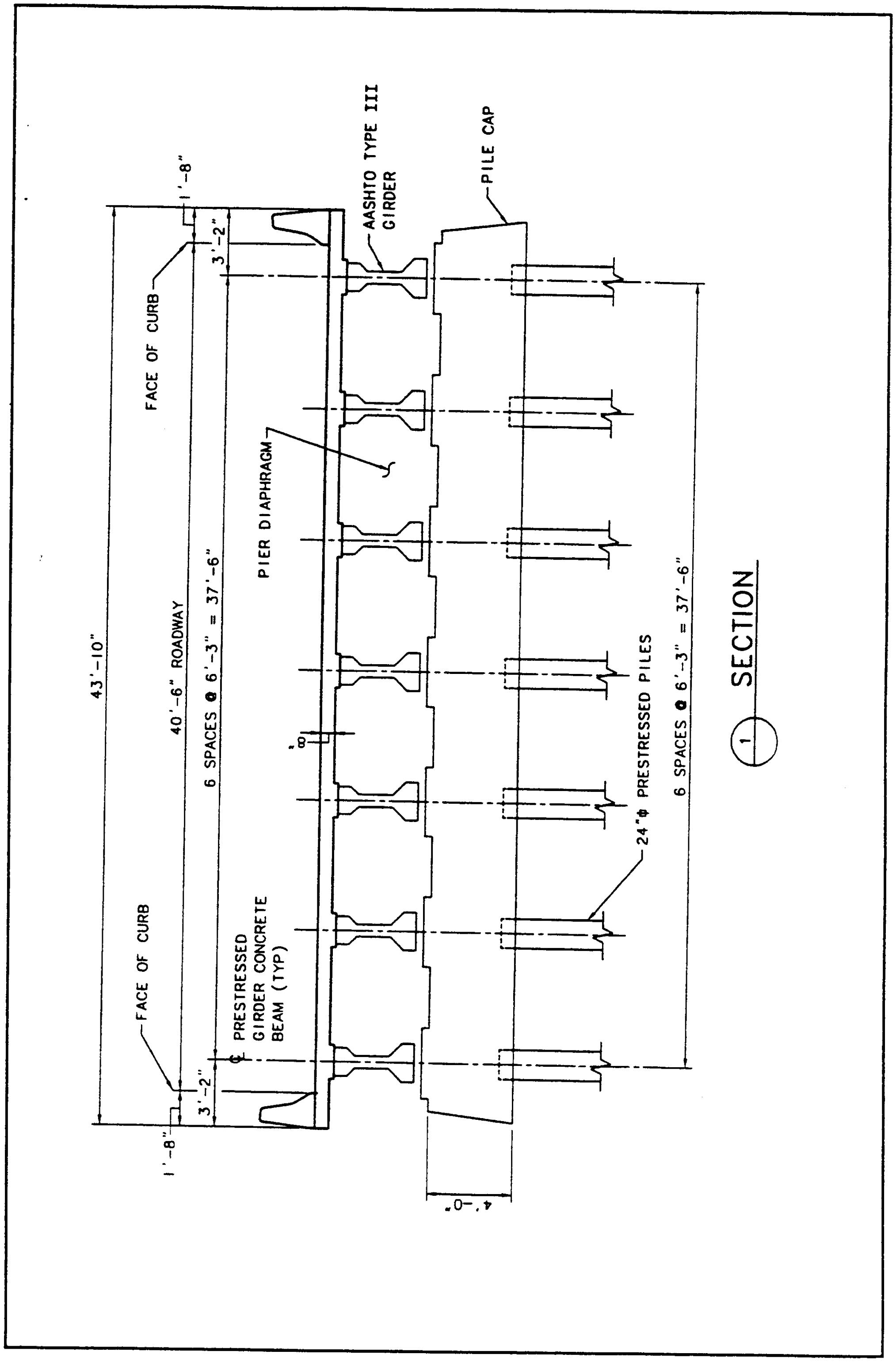


Figure 2b — Bridge No. 7 - Typical Section

BRIDGE DATA
(continued)

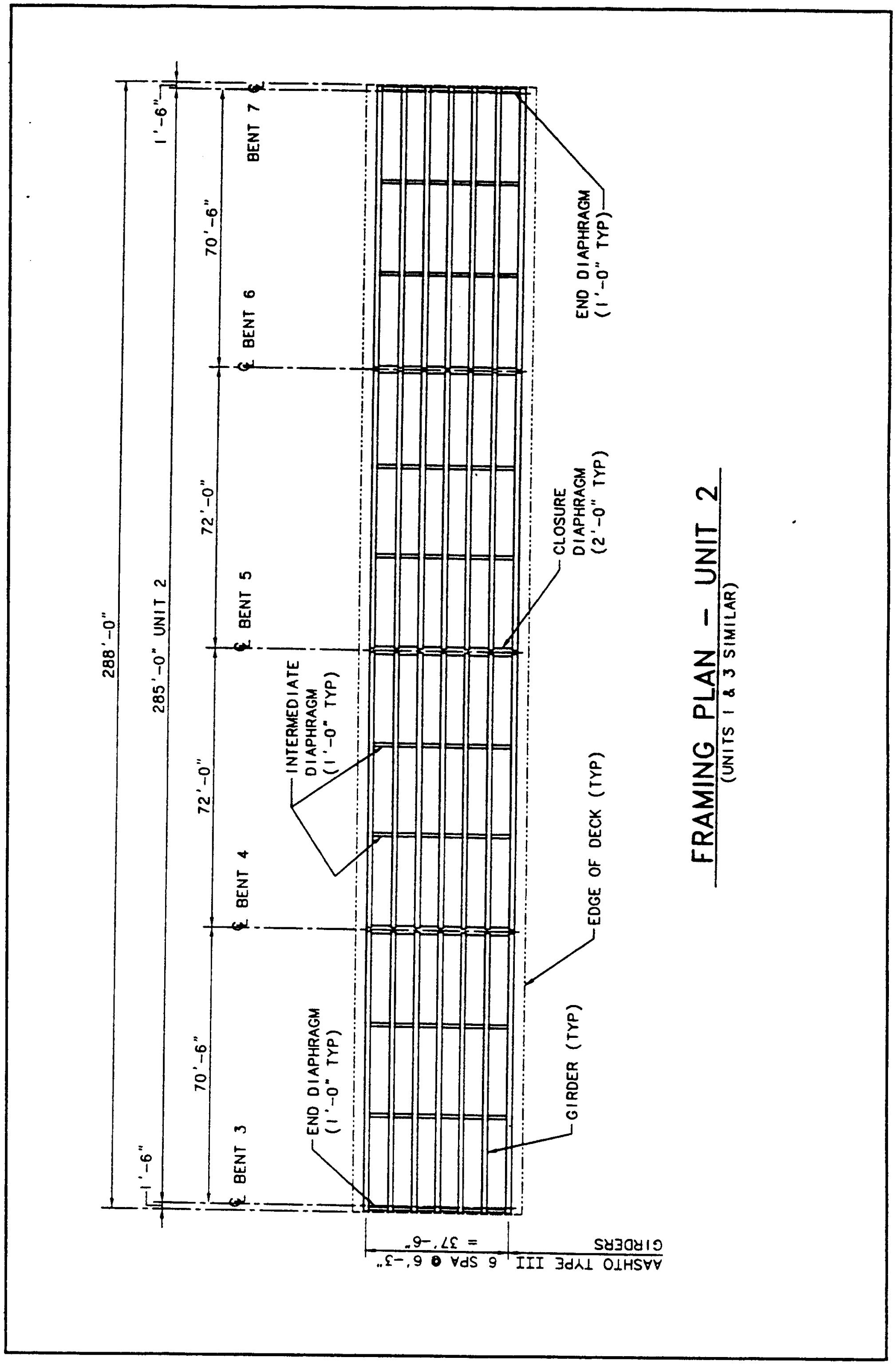


Figure 2c — Bridge No. 7 - Framing Plan

BRIDGE DATA
(continued)

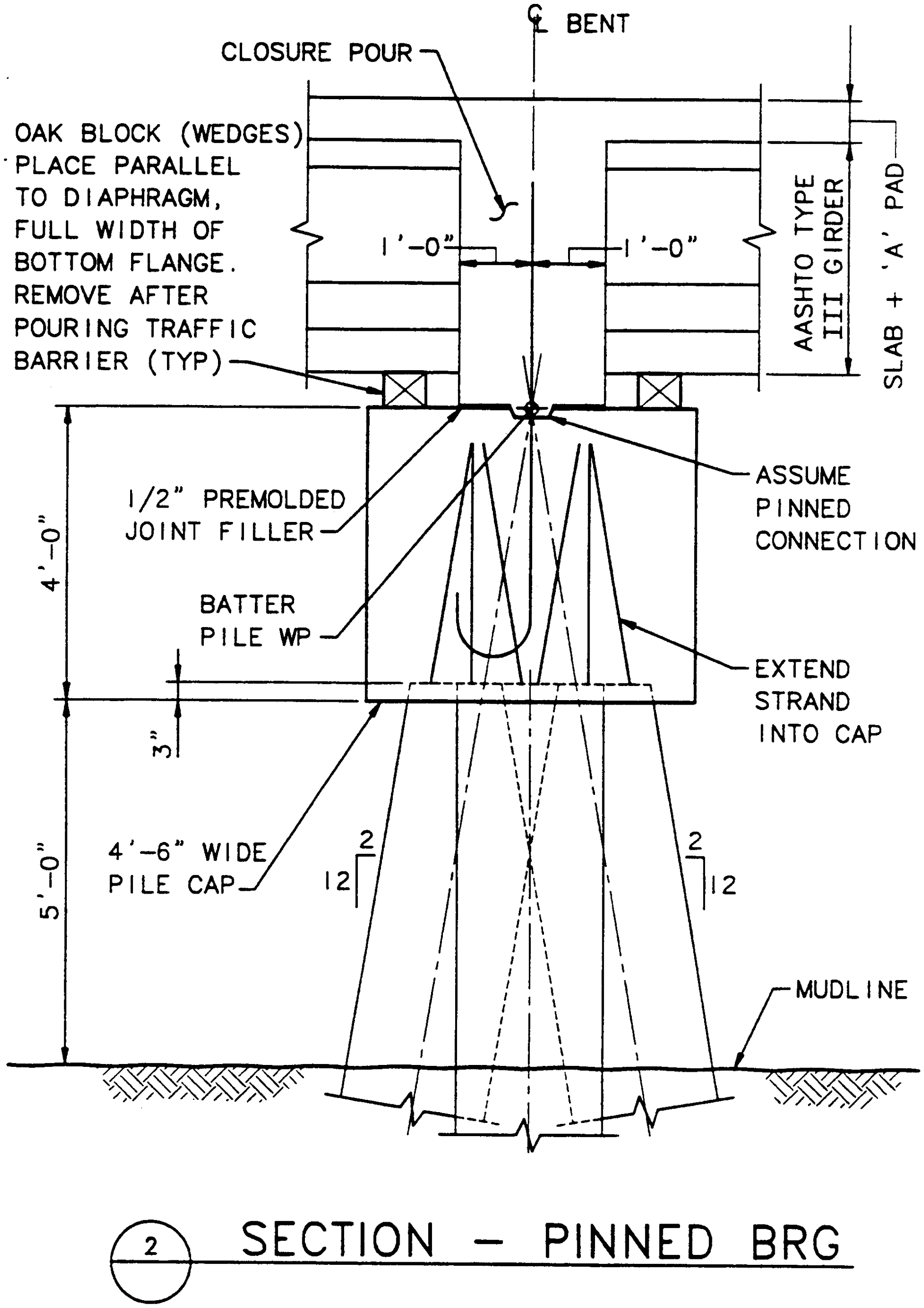


Figure 2d — Bridge No. 7 - Section at Bent with Pinned Bearings

BRIDGE DATA
(continued)

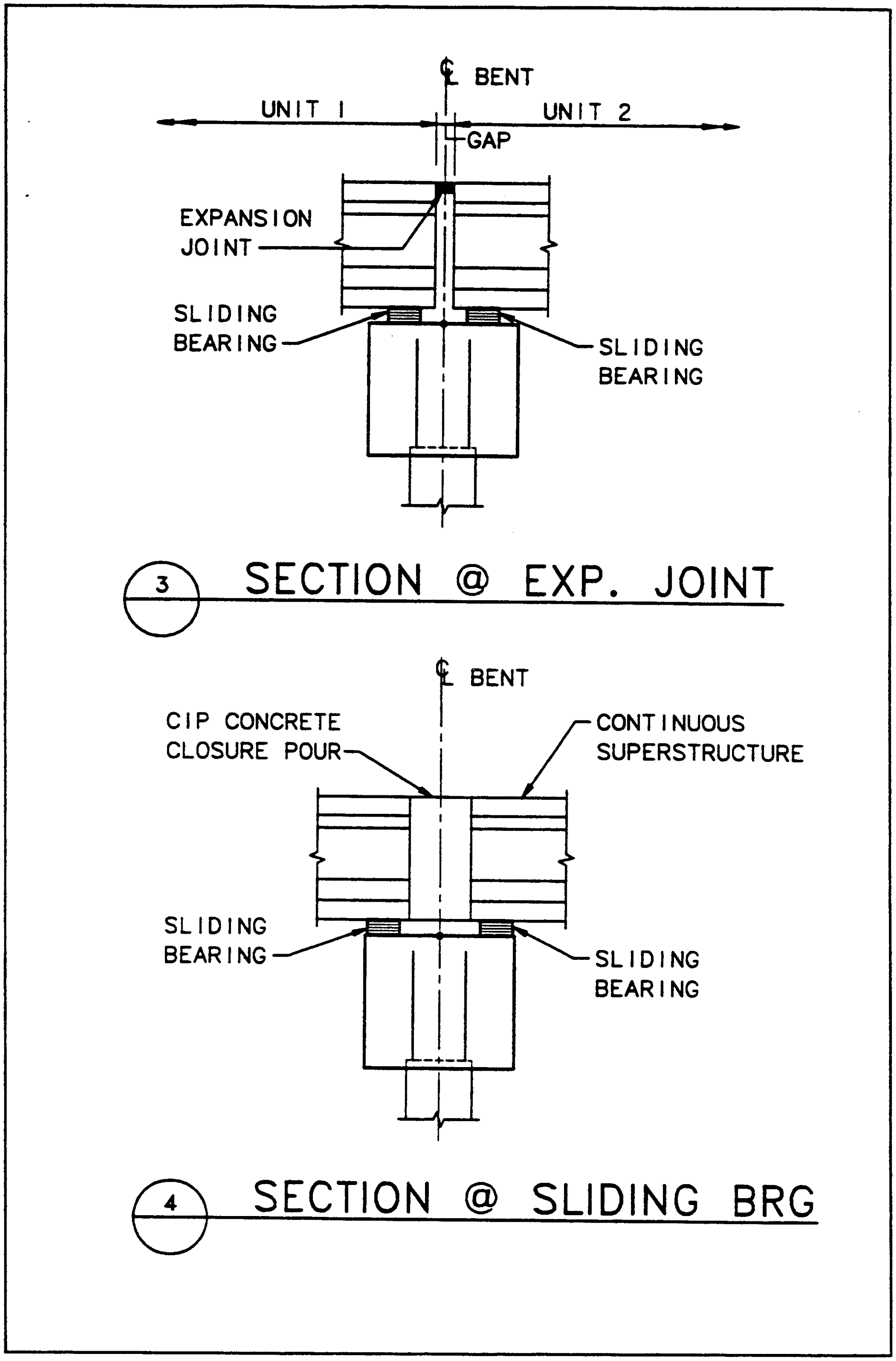


Figure 2e — Bridge No. 7 - Sections at Bents with Sliding Bearings

BRIDGE DATA
(continued)

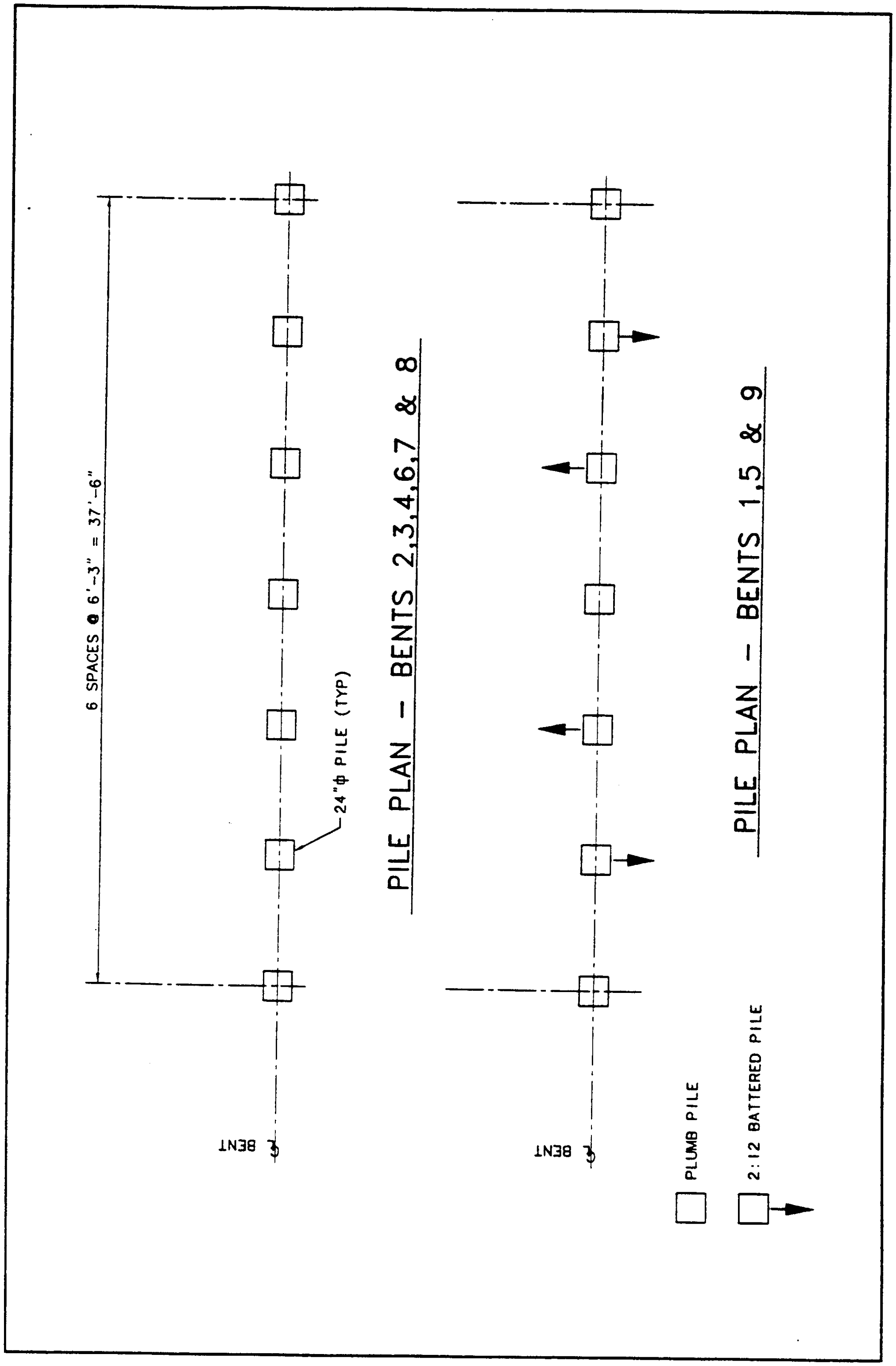


Figure 2f — Bridge No. 7 - Pile Plan at Bents

DESIGN STEP 1 PRELIMINARY DESIGN

Design Step 1.2 Basic Requirements

Design Step 1.2.1 Applicability of Specification

[Division I-A, Article 3.1]

The bridge consists of 10 spans, each approximately 72 feet in length, and the bridge is made of reinforced concrete. Thus, the Specification applies.

Design Step 1.2.2 Acceleration Coefficient

[Division I-A, Article 3.2]

The bridge is sited in an area where the Acceleration Coefficient A is 0.10g.

Design Step 1.2.3 Importance Classification

[Division I-A, Article 3.3]

The Importance Classification (IC) of this bridge is taken to be II, because it is assumed not to be essential for use following an earthquake.

Design Step 1.2.4 Seismic Performance Category

[Division I-A, Article 3.4]

The Seismic Performance Category (SPC) is B, as taken from Table 1 of the Specification.

Design Step 1.2.5 Site Effects

[Division I-A, Article 3.5]

The site conditions affect the design through a coefficient based on the soil profile. In this case, SOIL PROFILE TYPE II corresponds to the deep cohesionless soils that are under the site.

The Site Coefficient S for this type of soil is 1.2, per Table 2 of the Specification.

Design Step 1.2.6 Response Modification Factors

[Division I-A, Article 3.7]

Because this bridge is classified as SPC B, appropriate Response Modification Factors (R Factors) must be selected for use later in establishing appropriate design force levels.

Design Step 1.2.6 (continued)

In this case, Table 3 of the Specification gives the following R Factors.

For Reinforced Concrete Pile Bents

R = 3	For vertical piles only
R = 2	For one or more batter piles

For Steel Pile Bents

R = 5	For vertical piles only
R = 3	For one or more batter piles

AASHTO Division I-A, Article 6.2.2, under the heading "Exceptions", states that the "R" factor is not to be reduced for design forces for foundations. In this example, no "R" factor was used to reduce the seismic axial forces in the batter piles. See Design Step 3.1.9 for a related discussion.

Design Step 1.3

Basic Unit Dead Loads of Bridge

The dead load of the superstructure consists of seven precast concrete AASHTO beams, a CIP concrete deck, four transverse intermediate diaphragms and a traffic barrier on each side of the bridge. See Figure 2 (a to f) for the details.

Calculate the unit weight along the length of the bridge using a typical span length of 72 feet.

The following assumptions are used in the weight calculations.

$\gamma_c := 150 \cdot pcf$	Unit weight of the concrete
$L_{span} := 72.0 \cdot ft$	Span length of bridge from centerline of bent to centerline of bent
$b_{super} := 43.83 \cdot ft$	Overall width of bridge superstructure
$t_{slab} := 8 \cdot in$	Average thickness of top slab, including girder pad
$A_g := 560 \cdot in^2$	Area of each girder

Design Step 1.3 (continued)

The weight of each AASHTO Type 3 girder is

$$w_g := \left(A_g \cdot \frac{ft^2}{144 \cdot in^2}\right) \cdot \gamma_c \qquad w_g = 0.583 \cdot \frac{kip}{ft}$$

Therefore, the weight of all seven girders in each span of the bridge is

$$L_{span} = 72.0 \cdot ft$$

$$W_g := 7 \cdot w_g \cdot L_{span} \qquad W_g = 294.0 \cdot kip$$

For the weight of the deck slab, assume an average thickness of 8 inches, including girder pads.

$$L_{span} = 72.0 \cdot ft$$

$$b_{super} = 43.8 \cdot ft$$

$$t_{slab} = 8.0 \cdot in$$

$$W_s := L_{span} \cdot b_{super} \cdot t_{slab} \cdot \gamma_c \qquad W_s = 315.6 \cdot kip$$

For the weight of two traffic barriers, assume 0.40 kip per foot each.

$$W_b := 2 \cdot \left(0.389 \cdot \frac{kip}{ft}\right) \cdot (72.0 \cdot ft) \qquad W_b = 56.0 \cdot kip$$

For the weight of four diaphragms, assume each to be 12 inches wide and full depth.

$$W_d := 4 \cdot (34 \cdot ft) \cdot (3.75 \cdot ft) \cdot (1.0 \cdot ft) \cdot \gamma_c \qquad W_d = 76.5 \cdot kip$$

Design Step 1.3 (continued)

Therefore, the total weight of one typical span of the bridge superstructure is

$W_{span} := W_g + W_s + W_b + W_d$

$W_g = 294.0 \cdot kip$ Weight of AASHTO girders

$W_s = 315.6 \cdot kip$ Weight of deck slab

$W_b = 56.0 \cdot kip$ Weight of traffic barriers

$W_d = 76.5 \cdot kip$ Weight of all diaphragms

Therefore, the total weight of one span is $W_{span} = 744.1 \cdot kip$

The unit weight along the length of the superstructure is

$L_{span} = 72.0 \cdot ft$

$w_{super} := \dfrac{W_{span}}{L_{span}}$ $w_{super} = 10.3 \cdot \dfrac{kip}{ft}$

The reaction at each end of a typical 72-foot span is

$P_{brg} := \dfrac{W_{span}}{2}$ $P_{brg} = 371\ kip$

The weight of one bent cap is

$\gamma_c = 0.15 \cdot kcf$

$W_{cap} := 4.0 \cdot ft \cdot 4.5 \cdot ft \cdot 42 \cdot ft \cdot \gamma_c$ $W_{cap} = 113.4 \cdot kip$

Design Step 1.3 (continued)

The weight of one 2-foot-square concrete pile is

$w_{pile} := 2 \cdot ft \cdot 2 \cdot ft \cdot \gamma_c$ $\qquad w_{pile} = 0.60 \cdot klf$

The weight of a 10-foot length of pile (5-foot clear length to mudline plus first 5 feet of pile in the soil), which is an estimate of the pile seismic mass.

$W_{pile} := w_{pile} \cdot 10 \cdot ft$ $\qquad W_{pile} = 6.0 \cdot kip$

A summary of the weights of the bridge components is

$W_{span} = 742.1$ kip — Weight of one span of the superstructure

$W_{cap} = 113.4 \cdot kip$ — Weight of one bent cap beam

$W_{pile} = 6.0 \cdot kip$ — Weight of one 10-foot length of concrete pile

Calculate the total dead load on each of the bent piles. Assume that each of the seven piles carries the same load.

The tributary dead load at each pile bent system is

$P_{bent} := W_{span} + W_{cap} + 7 \cdot W_{pile}$ $\qquad P_{bent} = 897.5$ kip

$P_{DL} := \dfrac{P_{bent}}{7}$ $\qquad P_{DL} = 128 \cdot kip$

It should be noted that for battered piles, the actual axial dead load in the pile should be divided by the cosine of the angle of the batter pile. This refinement is small and has not been included in this design example.

Design Step 1.4 — Analysis Procedure for Preliminary Design

No computer analyses are presented in this design example. The equations for stiffness of both plumb and batter piles are developed in Design Steps 2 and 6, using basic analysis theory. Once the stiffness of each member is developed, calculations of the seismic forces are performed using a simplified version of Procedure 1 found in AASHTO Division I-A, Article 4.3.

In the longitudinal direction, the superstructure is assumed to move as a rigid body, and the deflections are assumed to be a constant. Each of the three continuous span bridges is assumed to act independently of each other; therefore, each can be analyzed separately.

In the transverse direction, the middle structure is assumed to translate as a rigid body under a seismic load. The longer the structure, the closer this assumption is to the actual behavior, provided that the stiffness of each substructure member is the same from bent to bent.

Design Step 1.5 — Ultimate Soil Capacities of Piles

These soil capacities were provided by the geotechnical engineer. See Appendix A.

Design Step 1.5.1 — Concrete Piles

The ultimate soil capacities are based on a 60-foot-long, 24-inch-square concrete pile embedded 55 feet into the soil.

Tension $T = (-)213$ kips
Compression $P = (+)767$ kips

Design Step 1.5.2 — Steel Piles

The ultimate soil capacities are based on a 60-foot-long, 24-inch-diameter, closed end steel pipe pile embedded 55 feet into the soil.

Tension $T = (-)135$ kips
Compression $P = (+)528$ kips

DESIGN STEP 2 HORIZONTAL STIFFNESS OF CONCRETE PILES

In this design step, the horizontal stiffness of each individual pile is calculated. To illustrate the relative differences of each component, the horizontal stiffness due to the flexural component is calculated separately from the horizontal stiffness due to the axial component. The total stiffness of an individual pile is then estimated by using the principle of superposition.

For seismic forces in the longitudinal direction, determining where to locate the work point for batter piles is critical. If the work point is located too low relative to the location of the horizontal force, there will be an additional twisting moment on the cap under horizontal seismic loads. This will cause additional bending moments in the batter piles.

If the connection of the superstructure to the pile cap is fixed against rotation, the connection will transfer a moment into the pile cap and pile system equal to the horizontal force multiplied by the distance from the c.g.c. of the load to the top of the cap. In this scenario, the work point should be moved up to match the c.g.c. of the horizontal force in the superstructure.

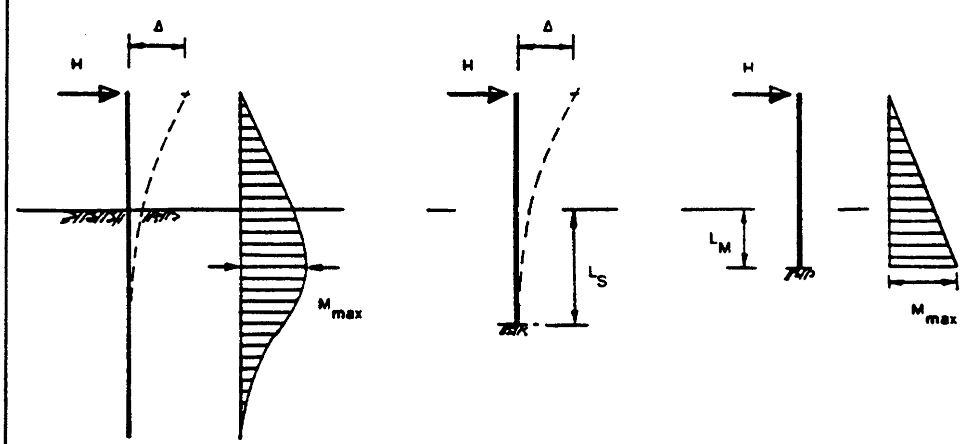
In this design example, the connection of the superstructure to the pile cap is considered pinned. Refer to Figure 2d. Only horizontal shear is assumed to be transferred through this connection. With the work point located at the centroid of this shear, the additional twist in the pile cap is zero, and the horizontal force is resisted by axial loads in the batter piles. Equilibrium in the system is satisfied by a couple, formed by axial loads in adjacent bents.

See Appendix B for the development of the equation for horizontal stiffness in the batter piles.

Design Step 2.1 Horizontal Stiffness Due to Flexure

Determine the pile depth-to-fixity in the soil by approximating the relative horizontal stiffness using the equivalent Cantilever Beam Method. The equation is found in the Seismic Design and Retrofit Manual for Highway Bridges (FHWA, 1987). See Figure 3.

Design Step 2.1
(continued)



	L_S	L_M
Cohesive Soil Constant K_h	$1.4\sqrt[4]{\frac{EI}{K_h}}$	$0.44\sqrt[4]{\frac{EI}{K_h}}$
Cohesionless Soil Constant n_h	$1.8\sqrt[5]{\frac{EI}{n_h}}$	$0.78\sqrt[5]{\frac{EI}{n_h}}$

Ref. FHWA-1P-87-6, Seismic Design and Retrofit Manual for Highway Bridges, May 1987

Figure 3 — Equivalent Pile Lengths for Flexure

Basic data for the concrete pile is given below.

$f_c := 5000 \cdot psi$ Concrete strength of prestress pile

$E_p := 4030 \cdot ksi$ Modulus of elasticity of pile

$t := 2 \cdot ft$ Dimension of square pile

The cross-sectional area of the pile is

$A_p := t^2$ $A_p = 4.0 \cdot ft^2$

Design Step 2.1 (continued)

The moment of inertia of the pile is

$$I_p := \frac{t^4}{12} \qquad I_p = 1.33 \cdot ft^4$$

The constant for horizontal subgrade reaction of the pile, as provided by the geotechnical engineer, is

$$n_h := 8 \cdot pci$$

Note that the value 8 pci is a low value, indicating low soil strength (loose deposits) under seismic loading. See Appendix A for further discussion.

The equation for the flexural length of the pile in soil, L_s, which approximates the horizontal flexural stiffness of the pile, is taken from Figure 3.

$$L_s := 1.8 \cdot \left(\frac{E_p \cdot I_p}{n_h}\right)^{\frac{1}{5}} \qquad L_s = 16.0 \cdot ft$$

The dimension "Lf_s" is the flexural length along the pile, from the pile top to the assumed point of fixity in the soil. It includes the 5-foot pile length from the mudline to the bottom of the pile cap. See Figure 4.

$$Lf_s := L_s + 5 \cdot ft \qquad Lf_s = 21.0 \cdot ft$$

The length of pile used for the flexural stiffness depends on the direction under consideration. The length used is different for the longitudinal and transverse directions because of the assumed behavior of the pile cap, as discussed in the next design step.

Design Step 2.1 (continued)

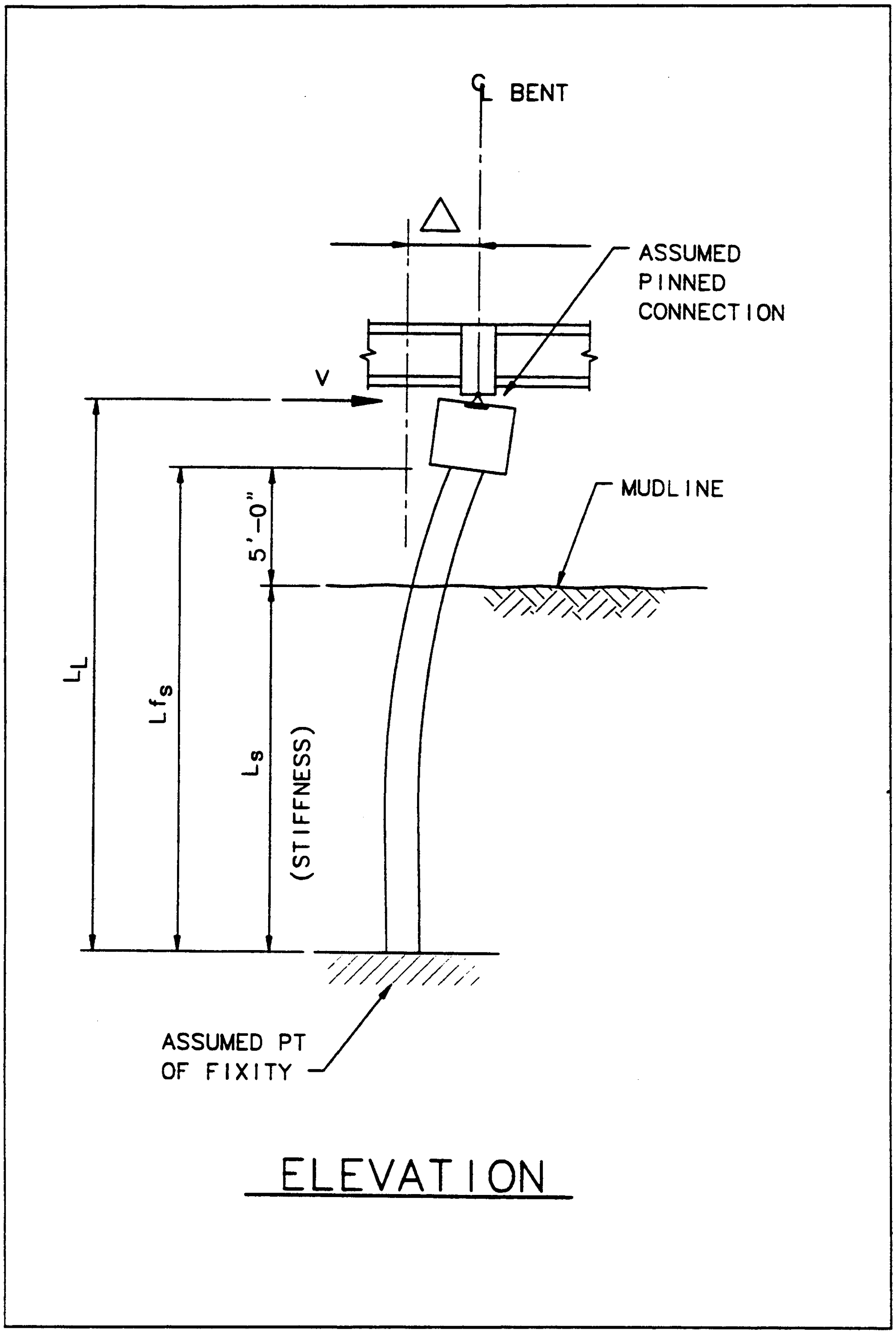


Figure 4 — Plumb Pile Deflected Shape (Longitudinal Direction)

Design Step 2.1.1

Flexural Stiffness Component (Plumb Pile)

a) Longitudinal Direction

The connection of the pile to the pile cap is always fixed against rotation. But in the longitudinal direction, the connection of the superstructure to the pile cap is assumed to be pinned. See Figure 2d. This connection allows the top of the pile cap to rotate about the transverse axis. Hence, the tops of the piles can also rotate, and tend to behave as if they were pinned at the top of the pile cap. The equivalent length of pile, then, includes the depth of the pile cap to this point of rotation.

First, calculate the equivalent pile length, L_L, for horizontal stiffness in the longitudinal direction. The 4-foot depth of the pile cap must be added to the previously calculated Lf_S. See Figure 4.

$Lf_S = 21.0 \cdot ft$ — Equivalent flexural length due to stiffness

$L_L := Lf_S + (4 \cdot ft)$ — Equivalent flexural length for longitudinal stiffness

$L_L = 25.0 \cdot ft$

For a plumb pile with a pinned pile top, the equation for the horizontal stiffness due to flexure is the same as that for a free cantilever (note the subscripts "O" for zero batter and "p" for pinned pile top).

$E_p = 4030 \cdot ksi$ — Modulus of elasticity of pile

$I_p = 1.33 \cdot ft^4$ — Moment of inertia of pile

$L_L = 25.0 \cdot ft$ — Equivalent pile length for longitudinal direction

$$k_{Op} := \frac{3 \cdot E_p \cdot I_p}{\left(L_L\right)^3} \qquad k_{Op} = 12.3 \cdot \frac{kip}{in}$$

Note: If the detail shown in Figure 2d were such that it restrained the top of the pile cap from rotating, the stiffness of the system would be

Design Step 2.1.1 (continued)

greater. Not only would the equivalent length of the pile be shorter (taken to the bottom of the cap), but the equation for stiffness would be 12EI instead of 3EI.

b) Transverse Direction

In the transverse direction parallel with the pile cap, the cap is restrained against rotation about the longitudinal axis. The pile cap only translates, it does not rotate in this direction. The top of the pile is fixed at the bottom of the concrete pile cap by embedding the perimeter reinforcement of the pile up into the pile cap; therefore, the equivalent length of pile calculated for stiffness is taken from the bottom of the cap, and is the same length as Lf_s, which was previously calculated. See Figure 5.

$Lf_s = 21.0 \cdot ft$ Equivalent length due to stiffness

$L_T := Lf_s$ Equivalent length for transverse stiffness

$L_T = 21.0 \cdot ft$

For a plumb pile with a restrained (or fixed) pile top, the equation for the horizontal stiffness due to flexure is the same as for a fixed-fixed beam. Note the subscripts "O" for zero batter and "r" for restrained pile top.

$E_p = 4030 \cdot ksi$ Modulus of elasticity of pile

$I_p = 1.33 \cdot ft^4$ Moment of inertia of pile

$L_T = 21.0 \cdot ft$ Equivalent length for transverse stiffness

$$k_{Or} := \frac{12 \cdot E_p \cdot I_p}{(L_T)^3} \qquad k_{Or} = 83.3 \cdot \frac{kip}{in}$$

Design Step 2.1.1 (continued)

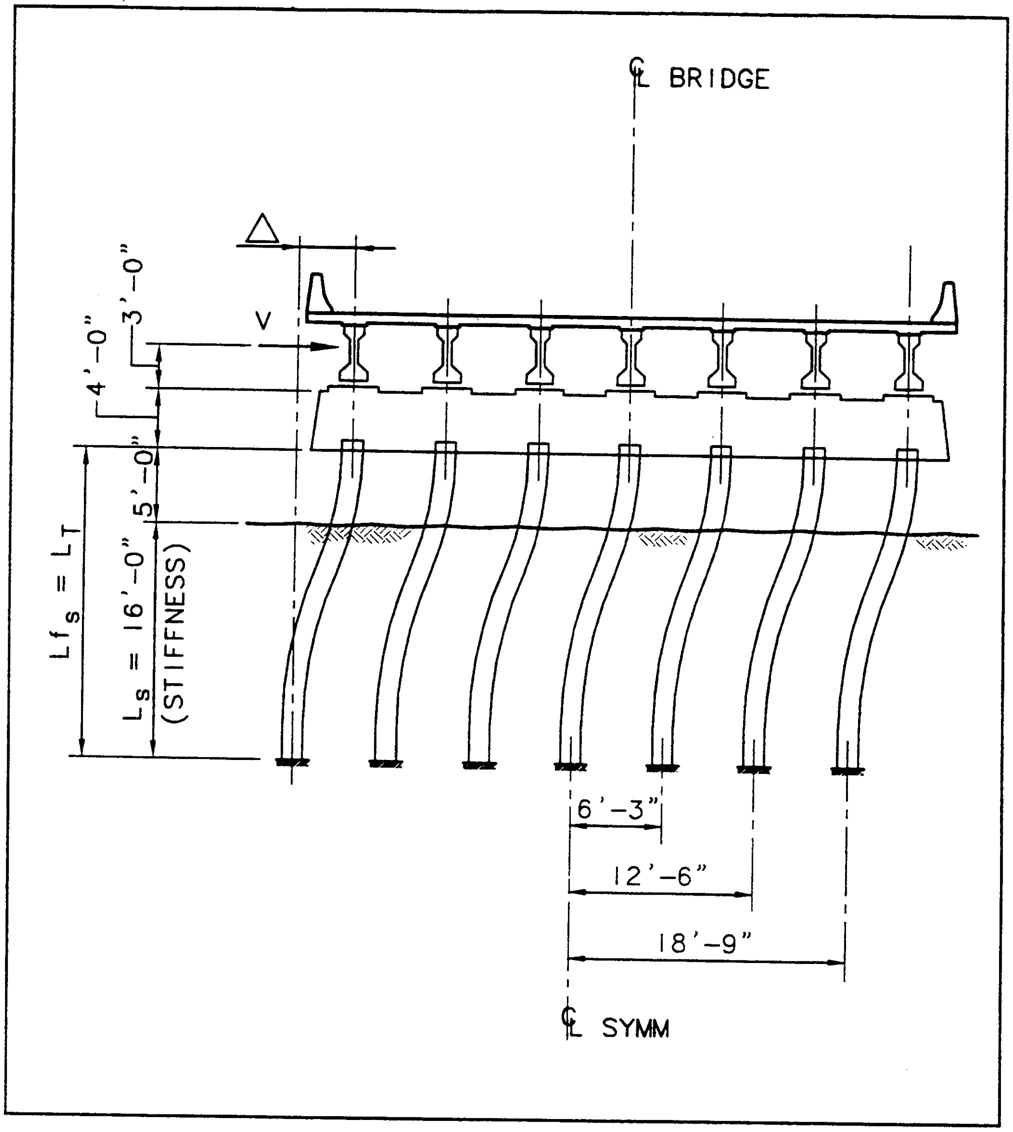


Figure 5 — Plumb Pile Deflected Shape (Transverse Direction)

Design Step 2.1.2

Flexural Stiffness Component (Batter Pile)

When a pile is battered 2:12 in the longitudinal direction, the horizontal component of the flexural stiffness is reduced as the slope of the batter increases. (Note: The horizontal stiffness due to axial loads in the pile will be considered separately in Design Step 2.2.) The first step is to calculate the slope or angle of the batter.

Design Step 2.1.2 (continued)

For a 2 on 12 batter pile, the angle of the batter is

$run_2 := 2$ Horizontal run relative to vertical rise

$rise := 12$ Vertical rise of 12 relative to variable horizontal run

The angle of the batter, measured from a vertical line, is

$$\alpha_2 := \text{atan}\left(\frac{run_2}{rise}\right) \qquad \alpha_2 = 9.46 \cdot deg$$

From analysis theory, the horizontal stiffness of a battered pile, due to flexural effects, is proportional to the flexural stiffness of a pile multiplied by $\cos^2(\alpha)$. See Appendix B.

$$k_{batter} := k_{flexure} \cdot (\cos(\alpha))^2$$

As was discussed in Design Step 2.1.1, in the longitudinal direction, the connection of the superstructure to the pile cap does not restrain the top of the cap from rotating. The pile behaves as if pinned at the top of the cap. The horizontal stiffness, due to the "flexural" component of a battered pile with a pinned pile top, is the same as for a free cantilever, with an adjustment for the angle of the batter, α. Note: The length of the batter pile is defined along the pile, not along the vertical length. See Figure 6.

$$kf_p := \frac{3 \cdot E_p \cdot I_p}{Lf_s^{\,3}} \cdot (\cos(\alpha))^2$$

Design Step 2.1.2 (continued)

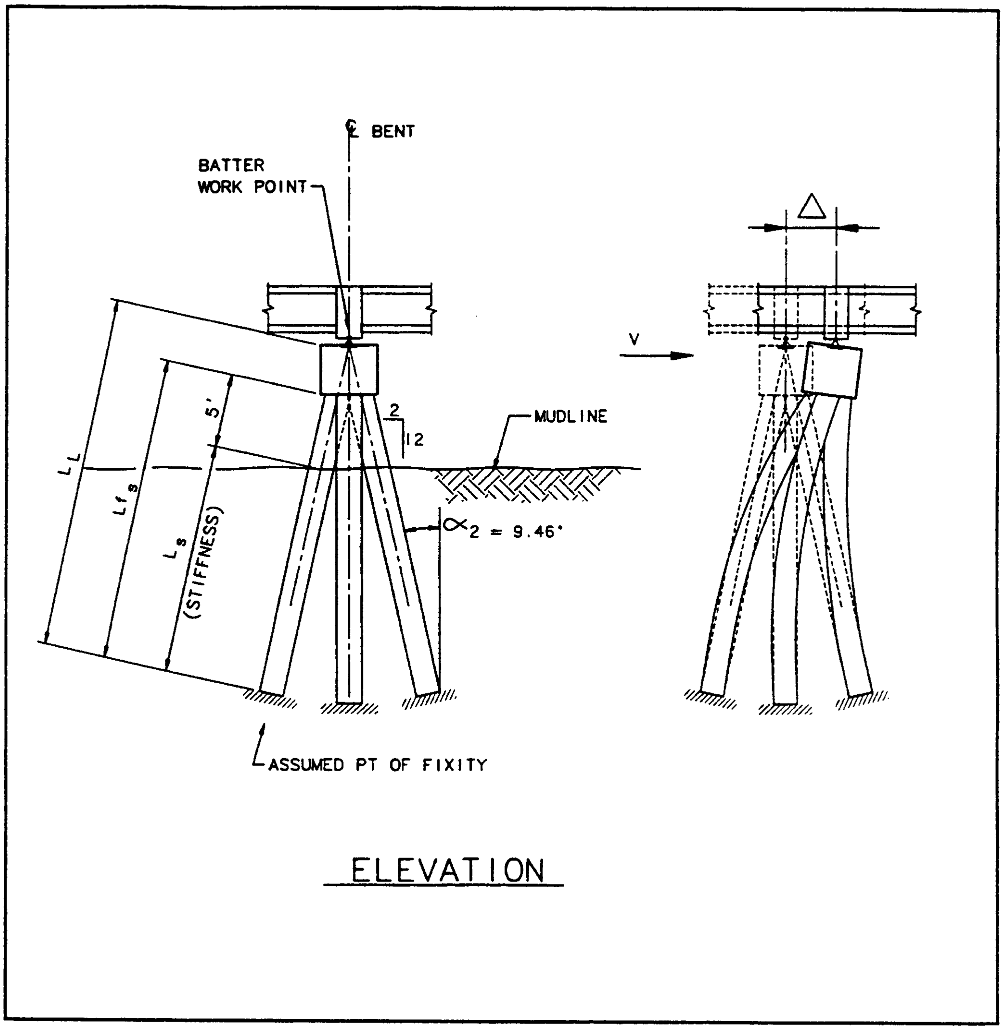


Figure 6 — Batter Pile Length Definitions

Or, written in terms of the previously calculated term k_{Op}, the horizontal stiffness due to the flexural component of batter pile with a pinned top is

$$kf_p := k_{Op} \cdot (\cos(\alpha))^2$$

$k_{Op} = 12.3 \cdot \frac{kip}{in}$ Horizontal stiffness of plumb pile with pinned top

$\alpha_2 = 9.46 \cdot deg$ Angle of batter pile

Design Step 2.1.2 (continued)

For a 2:12 batter (Note: The 2 in the subscript is for the batter.).

$$kf_{2p} := k_{Op} \cdot (\cos(\alpha_2))^2$$

$$kf_{2p} = 12.0 \cdot \frac{kip}{in}$$

Versus 12.3 for a plumb pile

Note: For most pile bent configurations, the effect of the batter on the flexural stiffness can be neglected. In this design example, the actual calculated stiffness is used for the sake of consistency.

Design Step 2.2

Axial Stiffness Component

Determine the horizontal stiffness of one batter pile, due to the axial "truss action" of the pile. It is assumed that there are "pairs" of batter piles, with each pile having another pile with an opposite batter to cancel the net vertical deflection. See Figure 2d.

The axial stiffness calculated in this design example accounts only for the axial stiffness of the pile itself. It does not take into account that the soil supporting the pile also acts as an elastic spring as the pile is loaded. For this example, with a maximum load of 600 kips compression, the soil will deflect approximately 1/4 inch due to the elastic nature of the soil surrounding the pile.

$$k_{soil} := \frac{600 \cdot kip}{.25 \cdot in} \qquad k_{soil} = 2400 \cdot \frac{kip}{in}$$

This soil spring would act in "series" with the structural spring of the pile, k_{pile}.

$$\left(\frac{1}{k_{total}}\right) = \left(\frac{1}{k_{pile}}\right) + \left(\frac{1}{k_{soil}}\right)$$

For simplification, this additional soil spring makes only a small contribution to the stiffness and, therefore, has not been included in the calculation of the axial spring for this design example. The designer should consult with the geotechnical engineer to determine if the effect of the soil spring is significant.

Design Step 2.2 (continued)

The length of pile used for the axial stiffness is longer than the length used for the flexural stiffness. If the piles were end bearing piles, with no friction capacity, the entire length of the pile would be used, resulting in a softer spring. As the friction capacity of the soil increases, the effective length of the pile for calculating stiffness decreases.

Calculate the equivalent pile length of the pile for axial loads. Refer to Figure 7.

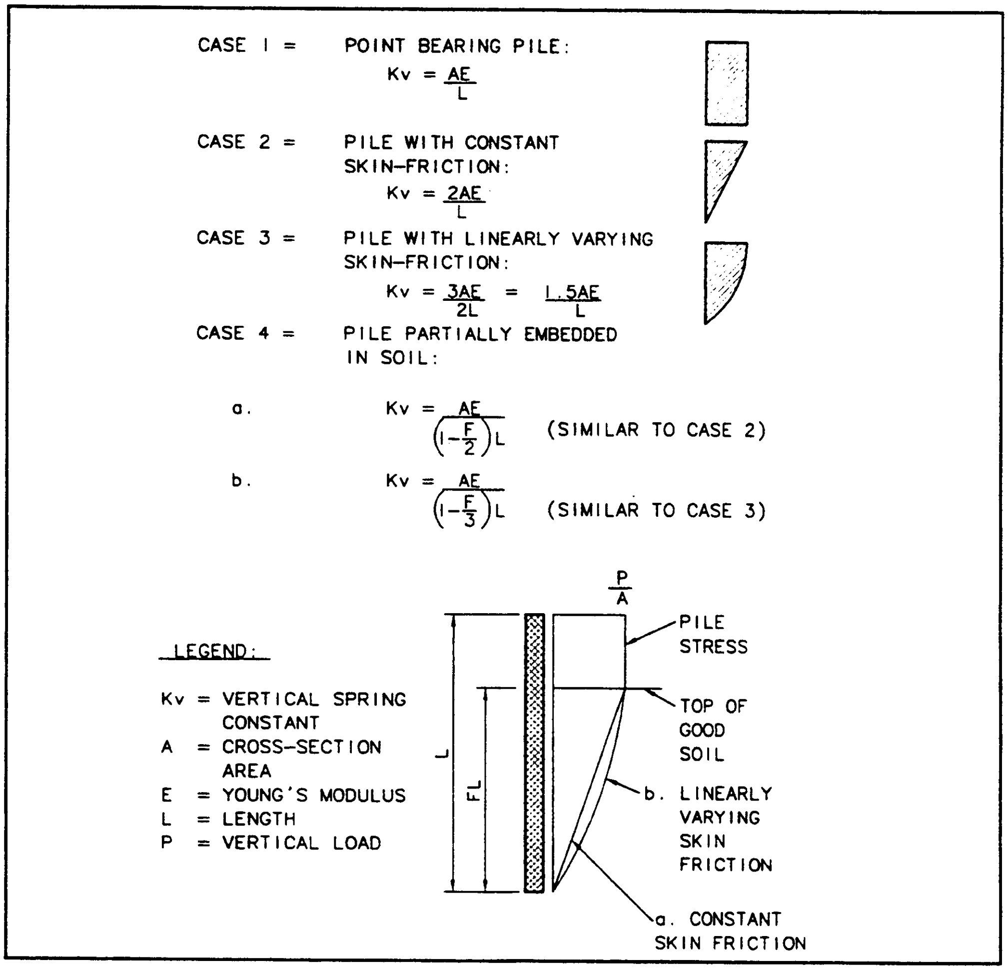


Figure 7 — Equivalent Pile Axial Stiffness

Design Step 2.2 (continued)

$L := 60 \cdot ft$ Actual pile length

$a := 5 \cdot ft$ Clear length from bottom of cap to mudline

The fraction of the total pile length that is embedded in the soil is F.

$F := \frac{L - a}{L}$ $F = 0.92$

A direct way of calculating the equivalent pile length, L_a, for axial stiffness is

$L_a := \left(1 - \frac{F}{3}\right) \cdot L$ $L_a = 41.67 \cdot ft$

Design Step 2.2.1

Axial Stiffness Component (Plumb Pile)

For simplification, the plumb piles are assumed not to develop any axial forces when displaced horizontally; therefore, $K_{ao} = 0$.

Design Step 2.2.2

Axial Stiffness Component (Batter Pile)

Before the horizontal stiffness of a batter pile is calculated, the pure axial stiffness of a pile must be developed. From basic analysis theory

$A_p = 4.0 \cdot ft^2$ Cross-sectional area of pile

$E_p = 4030 \cdot ksi$ Modulus of elasticity of a pile

$L_a = 41.67 \cdot ft$ Equivalent pile length for axial stiffness

$k_{axial} := \frac{A_p \cdot E_p}{L_a}$ $k_{axial} = 4643 \cdot \frac{kip}{in}$

Design Step 2.2.2 (continued)

Axial Stiffness Component (Batter Pile)

The horizontal stiffness of a battered pile, due to axial effects only, is the pure axial stiffness of the pile multiplied by $\sin^2(\alpha)$. See Appendix B.

$$k_{batter} := \frac{A_p \cdot E_p}{L_a} \cdot (\sin(\alpha))^2$$

Written another way

$$k_{batter} := k_{axial} \cdot (\sin(\alpha))^2$$

Calculate the horizontal stiffness due to axial truss action for a 2 on 12 batter pile.

$k_{axial} = 4643 \cdot \frac{kip}{in}$ Axial stiffness of a pile

$\alpha_2 = 9.46 \cdot deg$ Angle of batter pile

The horizontal stiffness due to the axial stiffness component is

$ka_2 := k_{axial} \cdot (\sin(\alpha_2))^2$ $ka_2 = 125.5 \cdot \frac{kip}{in}$

Design Step 2.3

Combined Stiffness of the Concrete Pile

The total horizontal stiffness of a pile, due to the combined effects of both flexural and axial stiffness, is the sum of the individual components. See Appendix B.

Calculate the combined stiffness for a plumb pile. Because $K_{ao} = 0$, then the combined stiffness of a plumb pile = K_{op} = 12.3 kip/in (from Design Step 2.1.1).

Calculate the combined stiffness for a longitudinal 2 on 12 batter pile with pinned top.

Design Step 2.3 (continued)

$$k_{2p} := \frac{3 \cdot E_p \cdot I_p}{(L_L)^3} \cdot (\cos(\alpha_2))^2 + \frac{A_p \cdot E_p}{L_a} \cdot (\sin(\alpha_2))^2$$

Another way to express the same equation, in terms of previously defined horizontal stiffness components, is

$$k_{2p} := kf_{2p} + ka_2$$

$kf_{2p} = 12.0 \cdot \frac{kip}{in}$ — Flexural stiffness component of a 2:12 batter pile with a pinned top

$ka_2 = 125.5 \cdot \frac{kip}{in}$ — Axial stiffness component of a 2:12 batter pile

Therefore, the total horizontal stiffness is

$k_{2p} := kf_{2p} + ka_2$ $\qquad$ $k_{2p} = 137.5 \cdot \frac{kip}{in}$

Design Step 2.4

Summary of Concrete Pile Stiffness

A summary of horizontal pile stiffness used in this design example is given in Table 1.

Table 1
Summary of Concrete Pile Stiffness Components

Support	Batter	Angle	Horiz Component Due to Flexural Stiff k_f (kip/in)	Horiz Component Due to Axial Stiff k_a (kip/in)	**Total Horiz Stiffness (kip/in)**
Pinned Top	0	0.00	12.3	0.0	**12.3**
	2	9.46	12.0	125.5	**137.5**
Fixed Top	0	0.00	83.2	0.0	**83.3**

DESIGN STEP 3

BRIDGE OPTION NO. 1
One Bent with 2:12 Concrete Batter Piles

Figure 2 (a through f) shows the layout for a conventional bridge, with only one bent of each bridge unit incorporating batter piles. As was mentioned in the Introduction, this design example will concentrate on the "Unit 2" bridge structure, a four-span continuous unit in the middle of the 10-span structure.

Because it is uniform and is separated by expansion joints at each end, bridge Unit 2 will be isolated and analyzed as a separate structure in this design example. It is assumed that there is an adequate expansion joint gap that allows it to move independently of the other units. Figure 8 shows the conceptual layout for Option No. 1.

Bent 5 is the one bent of bridge Unit 2 that contains batter piles and will resist horizontal force in the longitudinal direction.

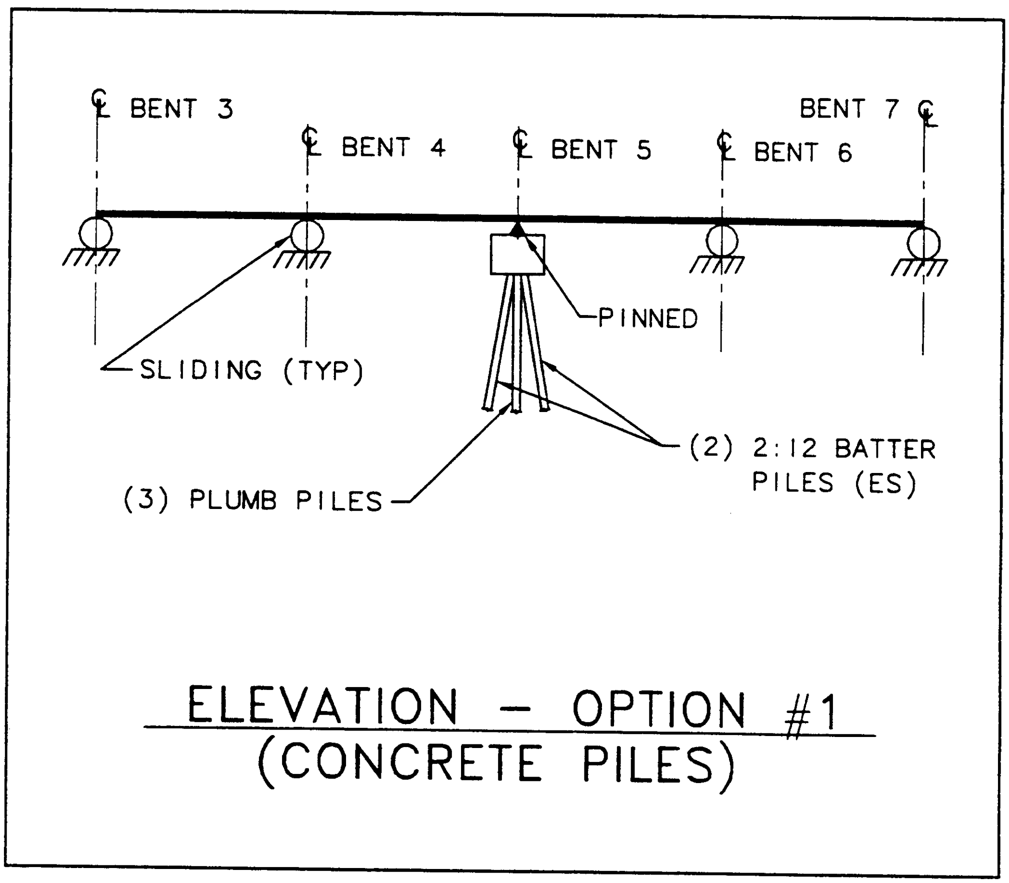


Figure 8 — Conceptual Layout of Piles

Design Step 3.1 — Longitudinal Seismic Force

The following items will be calculated for the bridge Unit 2 structure. Note that only forces in the longitudinal direction are calculated in this design step. Forces for the transverse direction are calculated in Design Step 4.2.

1. Seismic mass
2. Total horizontal stiffness
3. Period of the structure
4. Total seismic shear force
5. Elastic seismic deflection
6. Shear force in the batter pile
7. Axial and flexural components of the horizontal shear force
8. Corresponding axial force in the pile
9. Pile capacity as controlled by the soil

Design Step 3.1.1 — Seismic Mass W_{unit}*g

In Design Step 1.3, the basic unit dead loads were calculated for different components of the bridge. The seismic mass of bridge Unit 2 depends on which bents are pinned to the superstructure. When "sliding" bearings are used at a bent, the superstructure is assumed to move independently of the bent substructure in the longitudinal direction. Therefore, the bent does not add longitudinal stiffness to the system, nor does its mass add to the overall mass of the system.

When "pinned" bearings are used at a bent, the substructure restrains the superstructure movement. Not only does the stiffness of the bent add to the longitudinal stiffness of the system, but also a portion of the substructure mass must be added to the total mass of the system. The entire pile cap and 10-foot pile length will be included in this mass calculation.

In the configuration for Option No. 1, the participating mass of the system in the longitudinal direction consists of the following.

1. Four spans of the superstructure
2.. One cap beam at Bent 5
3. The top 10 feet of seven piles in the Bent 5 cap

See Figure 9.

Design Step
3.1.1
(continued)

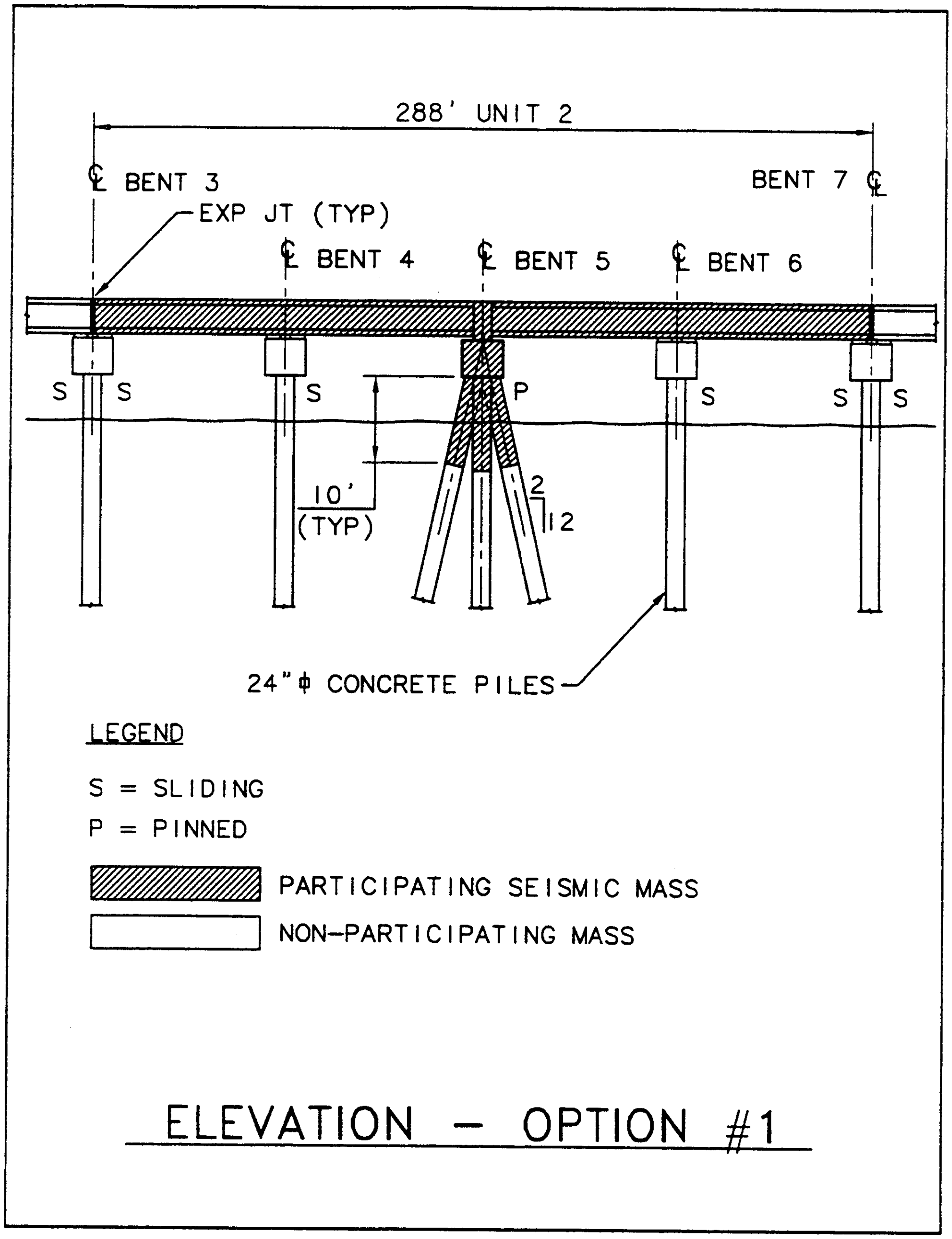


Figure 9 — Seismic Mass Used for Longitudinal Direction, Option No. 1

Design Step 3.1.1 (continued)

Therefore, the total dead load weight of the system, W_{unit}, is

$W_{span} = 742.1$ kip — Weight of one span of the superstructure

$W_{cap} = 113.4 \cdot kip$ — Weight of one bent cap beam

$W_{pile} = 6.0 \cdot kip$ — Weight of one 10-foot length of pile

$$W_{unit} := (4 \cdot W_{span}) + W_{cap} + (7 \cdot W_{pile}) \qquad W_{unit} = 3124\ kip$$

Design Step 3.1.2

Total Horizontal Stiffness k_{unit}

Calculate the stiffness in the longitudinal direction. Only Bent 5 is pinned to the superstructure; therefore, it is the only bent that resists the longitudinal seismic forces. Below is a summary of the pile types that are oriented in the longitudinal direction.

One bent of four 2 on 12 batter piles plus three plumb piles, all with pinned tops.

$k_{2p} = 137.5 \cdot \frac{kip}{in}$ — Horizontal stiffness of a 2 on 12 batter pile, pinned at the top

$k_{0p} = 12.3 \cdot \frac{kip}{in}$ — Horizontal stiffness of a plumb pile, pinned at the top

The total stiffness of bridge Unit No. 2 is

$$k_{unit} := 4 \cdot k_{2p} + 3 \cdot k_{0p} \qquad k_{unit} = 586.9 \cdot \frac{kip}{in}$$

Design Step 3.1.2 (continued)

See Table 2 for a summary of the stiffness components.

Table 2
Option No. 1, Longitudinal Stiffness

Bearing System Used: Bent 5 pinned, sliding bearings all other bents
Pile Configuration: (4) 2:12 batter piles + (3) equivalent plumb piles at Bent 5 only

Concrete Pile Option No. 1	Unit No. 2 w/ Concrete Piles, Longitudinal Stiffness (kip/in)					
Bent Number Bearing Restraint at Bent	Bent 3 Sliding	Bent 4 Sliding	Bent 5 Pinned	Bent 6 Sliding	Bent 7 Sliding	Total
Number Plumb Piles, N_p	0	0	3	0	0	3
Horizontal Stiffness per Pile, k (kip/in)	0	0	12.3	0	0	
Stiffness of Plumb Piles = N_p*k (kip/in)	0	0	37	0	0	37
Number of 2:12 Batter Piles, N_b	0	0	4	0	0	4
Horizontal Stiffness per Pile, k	0	0	137.5	0	0	
Stiffness of Batter Piles = N_b*k (kip/in)	0	0	550	0	0	550
Longitudinal Horizontal Stiffness of Unit No. 2 Bridge (kip/in) =						587

Design Step 3.1.3

Period of the Structure T

$W_{unit} := 3124 \cdot kip$ Total weight of the unit

$g := 32.2 \cdot \frac{ft}{sec^2}$ Acceleration of gravity

The period of the structure in the longitudinal direction, per Equation 4-3 of Division I-A is

$$T := 2 \cdot \pi \cdot \sqrt{\frac{W_{unit}}{k_{unit} \cdot g}} \qquad T = 0.74 \cdot sec$$

Design Step 3.1.4

Total Seismic Shear Force Veq_L

$A := 0.10$ The seismic acceleration coefficient

$S := 1.2$ Soil site coefficient

The elastic seismic response coefficient is calculated from AASHTO Division I-A, Equation 3-1. Note that the units of seconds to the (2/3) power maintains consistent units in Mathcad®.

$$C_s := \frac{1.2 \cdot sec^{\frac{2}{3}} \cdot A \cdot S}{T^{\frac{2}{3}}} \qquad C_s = 0.176$$

Per AASHTO Division I-A, Article 3.6.1, the maximum value of C_s need not exceed

$$C_{smax} := 2.5 \cdot A \qquad C_{smax} = 0.25$$

Therefore, use the actual value $C_s = 0.176$

The total equivalent static earthquake force is then

$$Veq_L := C_s \cdot W_{unit} \qquad Veq_L = 550 \text{ kip}$$

Design Step 3.1.5

Elastic Seismic Deflection ρeq

$k_{unit} = 587 \cdot \frac{kip}{in}$ Horizontal stiffness of the unit

The elastic deflection due to the seismic shear force is then the force divided by the stiffness. See Figure 10.

$$\Delta_{eq} := \frac{Veq_L}{k_{unit}} \qquad \Delta_{eq} = 0.94 \cdot in$$

Design Step
3.1.5
(continued)

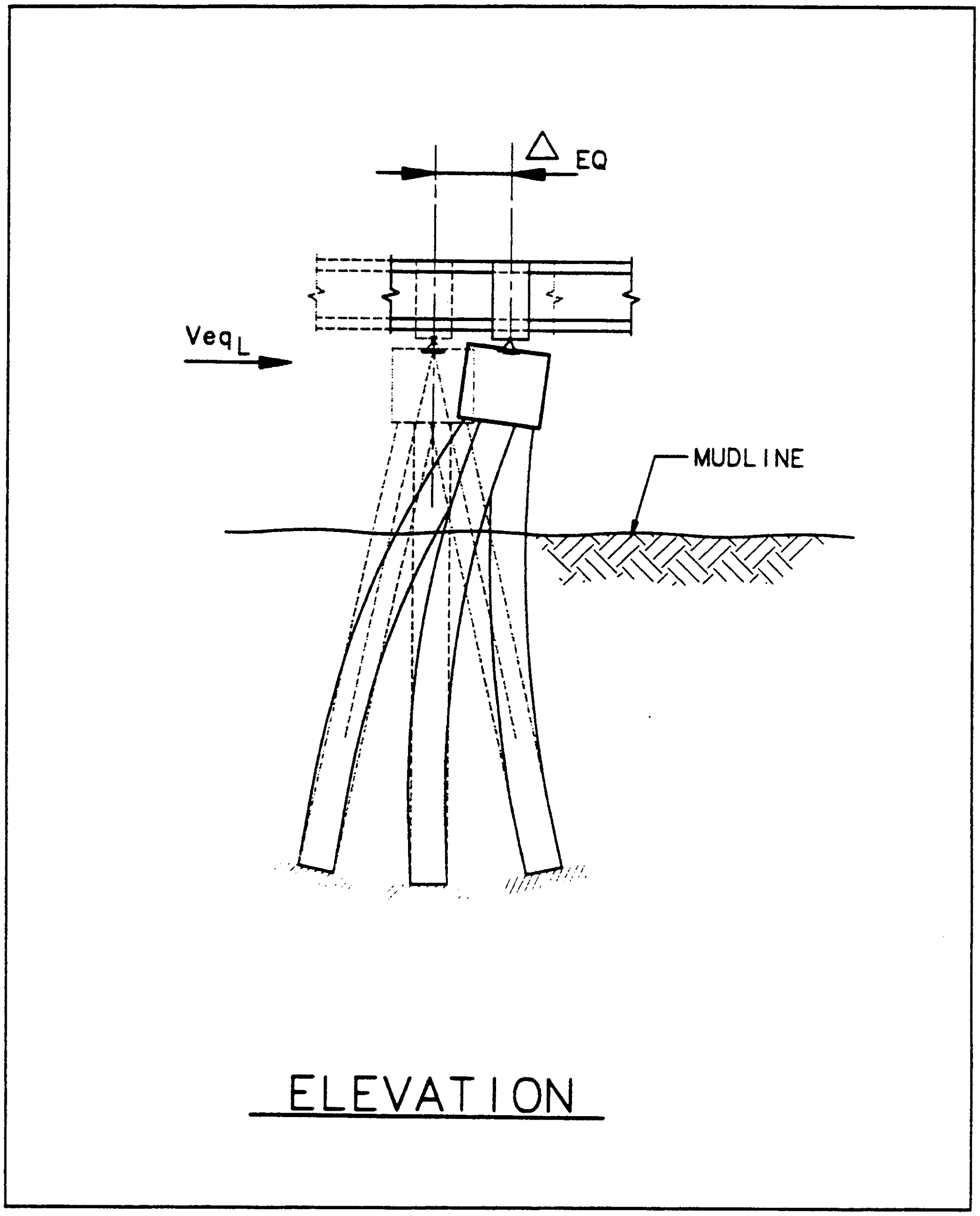


Figure 10 — Deflected Shape of Batter Pile Bent

Design Step 3.1.6

Shear Force in the Batter Pile Veq

The total seismic shear is distributed proportionately to the seven piles in Bent 5, based on their relative stiffnesses. The individual shear forces can be calculated by multiplying the stiffness of the individual piles by the total deflection.

The resulting horizontal force resisted by each 2:12 batter pile is

$k_{2p} = 137.5 \cdot \frac{kip}{in}$ Horizontal stiffness of a 2:12 batter pile with pinned top

$\Delta_{eq} = 0.94 \cdot in$ Seismic deflection of the bridge

$V_{eq} := k_{2p} \cdot \Delta_{eq}$ $V_{eq} = 129.3 \cdot kip$

The individual shear force in each plumb pile is 11.6^{k}, which is small relative to the shear force in the batter piles; therefore, it will not be addressed further in this example.

Design Step 3.1.7

Axial and Flexural Components of Horizontal Shear Forces V_a and V_m

A portion of the horizontal force is resisted by an axial load in the batter pile while the remainder is resisted by flexure in the pile. The shear carried by each component is expressed as a simple ratio of its stiffness to the total stiffness.

The total horizontal stiffness of one 2:12 batter pile with a pinned top is

$$k_{2p} = 137.5 \cdot \frac{kip}{in}$$

The horizontal component of the total stiffness due to axial stiffness in the pile is

$$ka_2 = 125.5 \cdot \frac{kip}{in}$$

Design Step 3.1.7 (continued)

The horizontal shear associated with the axial deformation is a ratio of the axial to total horizontal stiffness of the batter pile.

$$V_a := \frac{ka_2}{k_{2p}} \cdot V_{eq} \qquad V_a = 118.0 \cdot kip$$

The horizontal component of the total stiffness due to flexural stiffness in the pile is

$$kf_{2p} = 12.0 \cdot \frac{kip}{in}$$

The horizontal shear associated with the flexural deformation is expressed as a ratio of the flexural to total horizontal stiffness of the batter pile.

$$V_m := \frac{kf_{2p}}{k_{2p}} \cdot V_{eq} \qquad V_m = 11.3 \cdot kip$$

Check that the total horizontal seismic force equals Veq.

$$V := V_a + V_m \qquad V = 129.3 \cdot kip \quad \text{okay}$$

Design Step 3.1.8

Axial Forces

The resulting axial forces in the 2:12 batter pile are based on the shear force Va and simple trigonometry.

$V_a = 118.0 \cdot kip$ — Horizontal shear force

$\alpha_2 = 9.46 \cdot deg$ — Angle of batter

The elastic axial force in the pile is either tension or compression, depending on the direction of the EQ load.

$$P_a := \frac{V_a}{\sin(\alpha_2)} \qquad P_a = 718 \cdot kip \quad \text{Very large}$$

Design Step 3.1.9

Pile Capacity — as Controlled by the Soil

From Design Step 1.5, the capacity of the pile (as controlled by the soil) is (-)213 kips in tension, and (+)767 kips in compression. Determine the magnitude of the final axial loads. The axial loads in the batter piles should not be reduced by the Response Modification Factor "R" because there is little energy dissipated in "truss" type action of the batter piles unless the soil fails. The full elastic axial forces are, therefore, used in this design example.

For Group VII loading, per AASHTO Division I-A, Article 6.2.2, the load factors are all 1.0. The maximum compression in the 2:12 batter piles, using the full elastic force, is

$P_{DL} = 128 \cdot kip$ DL reaction in pile (unfactored)

$P_a = 718 \cdot kip$ Full elastic seismic axial load

$P_{compr} := (1.0 \cdot P_{DL}) + (1.0 \cdot P_a)$ Maximum compressive axial load

$P_{compr} = 846 \cdot kip$ > 767 kip, therefore N.G.

The maximum tension in the 2:12 batter piles, under Group VII loading using the full elastic force is

$P_{ten} := P_{DL} - P_a$ Maximum tensile axial load

$P_{ten} = -590$ kip > (-)213 kip, therefore N.G.

Both the axial tension and compression exceed the soil capacity for the Bent 5 piles.

Design Step 3.2

Implications

The tension load exceeds the soil capacity by a large margin (-590 kip load verses -213 kip capacity). The compression load in the pile also exceeds the soil capacity (846 kip verses 767 kip). During a seismic event, the pile may tend to walk its way out of the soil (following he principle of tension failure before compression failure). This behavior would dissipate energy, but might result in damage to piles and an uneven structure profile.

Therefore, look at other concrete pile layout options at a preliminary level.

- Option No. 2: Seven plumb piles at each bent, Bents 4, 5, and 6, pinned to the superstructure (Design Step 4).

- Option No. 3: Four batter piles and three plumb piles at each bent, Bents 4, 5, and 6, pinned to the superstructure (Design Step 5).

DESIGN STEP 4

BRIDGE OPTION NO. 2
Bents 4, 5, and 6 with All Concrete Plumb Piles

Figures 11 shows the conceptual layout for Option No. 2. The three middle bents of the four-span bridge Unit No. 2 are pinned to the superstructure. All the piles are plumb piles.

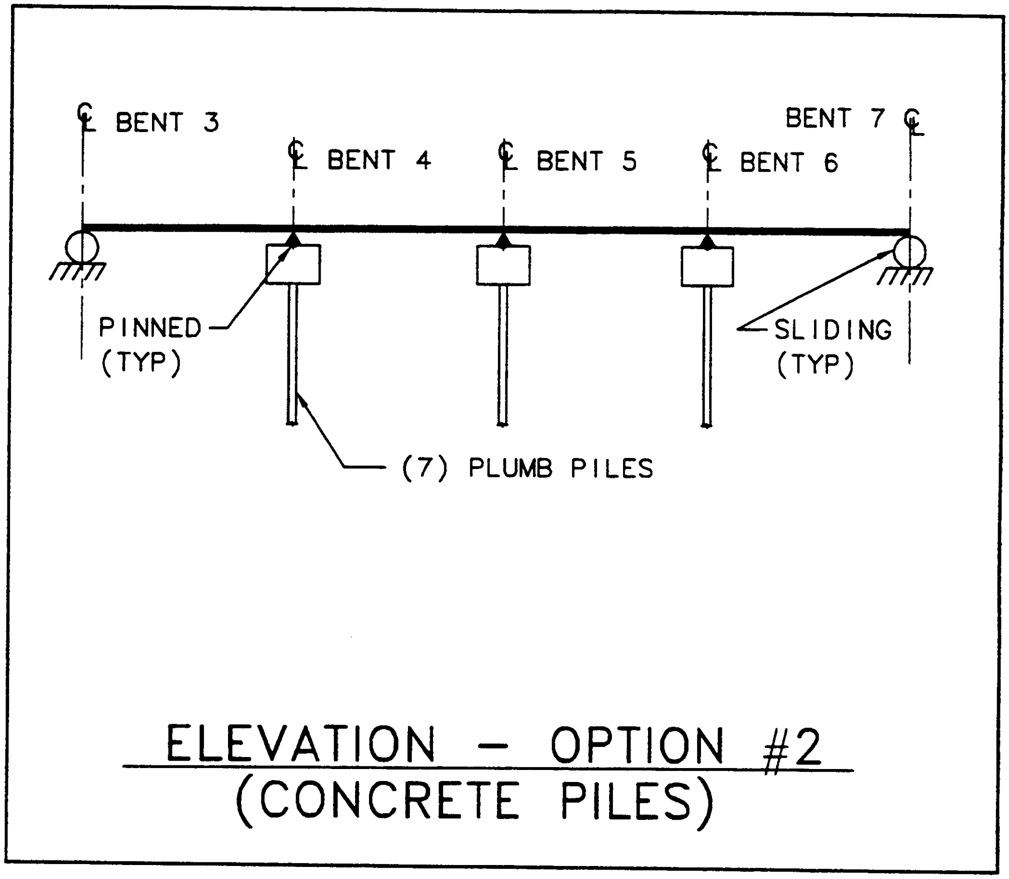


Figure 11 — Conceptual Layout of Piles, Option No. 2

Design Step 4.1

Longitudinal Seismic Force on Pile Bents 4, 5, and 6

The following items will be calculated for the bridge Unit 2 structure. Note that only forces in the longitudinal direction are calculated in this design step. Forces for the transverse direction are calculated in Design Step 4.2.

1. Seismic mass
2. Total horizontal stiffness

3. Period of the structure
4. Total seismic shear force
5. Elastic seismic deflection
6. Shear force in the pile
7. Axial and flexural components of the horizontal shear force
8. Corresponding axial force and flexural moment in the pile

Design Step 4.1.1

Seismic Mass W_{unit} * g

Refer to Design Step 3.1.1 for a more in-depth discussion of seismic mass. For Option No. 2 , the middle three bents have pinned bearings, and all contribute to the seismic mass of bridge Unit 2 structure.

In this design configuration, the participating mass of the system in the longitudinal direction consists of the following.

1. Four spans of the superstructure
2. The three cap beams of Bents 4, 5, and 6
3. The top 10 feet of 21 piles in Bents 4, 5, and 6

See Figure 12.

$W_{span} = 742.1$ kip — Weight of one span of the superstructure

$W_{cap} = 113.4 \cdot \text{kip}$ — Weight of one bent cap beam

$W_{pile} = 6.0 \cdot \text{kip}$ — Weight of one 10-foot length of pile

Therefore, the total dead load weight of the system, W_{unit}, is

$$W_{unit} := \left(4 \cdot W_{span}\right) + \left(3 \cdot W_{cap}\right) + \left(21 \cdot W_{pile}\right)$$

$W_{unit} = 3435$ kip

Design Step 4.1.1 (continued)

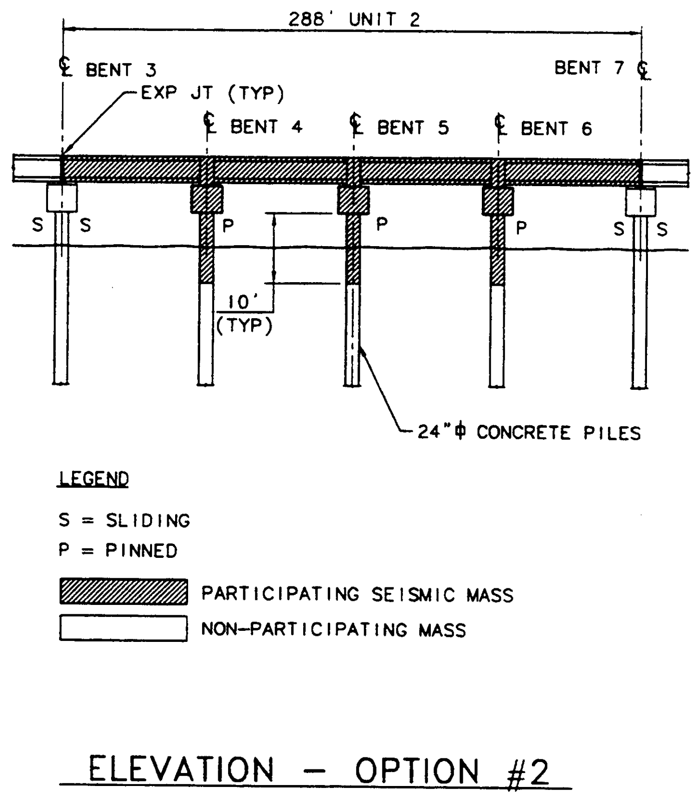


Figure 12 — Seismic Mass Used for Longitudinal Direction, Option No. 2

Design Step 4.1.2

Total Horizontal Stiffness k_{unit}

Calculate the stiffness in the longitudinal direction. Bents 4, 5, and 6 are pinned to the superstructure; therefore, they all contribute to the longitudinal seismic stiffness. In addition, the weight of the pile caps and piles of these bents contribute to the seismic mass of the bridge structure. Refer to Table 1 in Design Step 2.3 for individual stiffness.

Design Step 4.1.2 (continued)

Three bents of seven plumb piles each = 21 total plumb piles, all with pinned tops.

$k_{Op} = 12.3 \cdot \frac{kip}{in}$ — Horizontal stiffness of a plumb pile, pinned at the top

The total stiffness of the Bridge Unit No. 2 is

$k_{unit} := 21 \cdot k_{Op}$ $\qquad k_{unit} = 258.3 \cdot \frac{kip}{in}$

See Table 3.

Table 3
Option No. 2, Longitudinal Stiffness

Bearing System Used: Sliding bearings at Bents 3 and 7, pinned at Bents 4 to 6
Pile Configuration: All plumb piles participate at Bents 4, 5, and 6

Concrete Pile Option No. 2	Unit No. 2 w/ Concrete Piles, Longitudinal Stiffness (kip/in)					
Bent Number	Bent 3	Bent 4	Bent 5	Bent 6	Bent 7	Total
Bearing Restraint at Bent	Sliding	Sliding	Pinned	Sliding	Sliding	
Number Plumb Piles, N_p	0	7	7	7	0	21
Horizontal Stiffness per Pile, k (kip/in)	0	12.3	12.3	12.3	0	
Stiffness of Plumb Piles = N_p*k (kip/in)	0	86.1	86.1	86.1	0	258.3
Number of 2:12 Batter Piles, N_b	0	0	0	0	0	0
Horizontal Stiffness per Pile, k	0	0	0.0	0	0	
Stiffness of Batter Piles = N_b*k (kip/in)	0	0	0.0	0	0	0.0
Longitudinal Horizontal Stiffness of Unit No. 2 Bridge (kip/in) =						258

Design Step 4.1.3

Period of the Structure T

$W_{unit} := 3435 \cdot kip$ Total weight of the unit

$k_{unit} = 258 \cdot \frac{kip}{in}$ Horizontal stiffness of bridge Unit No. 2

$g := 32.2 \cdot \frac{ft}{sec^2}$ Acceleration of gravity

The period of the structure in the longitudinal direction, per Equation 4-3 of Division I-A is

$$T := 2 \cdot \pi \cdot \sqrt{\frac{W_{unit}}{k_{unit} \cdot g}} \qquad T = 1.17 \cdot sec$$

Design Step 4.1.4

Total Seismic Shear Force V_{eqL}

$A := 0.10$ Seismic acceleration coefficient

$S := 1.2$ Soil site coefficient

The elastic seismic response coefficient is calculated from AASHTO Division I-A, Equation 3-1. Note that the units of seconds to the (2/3) power maintains consistent units in Mathcad®.

$$C_s := \frac{1.2 \cdot sec^{\frac{2}{3}} \cdot A \cdot S}{T^{\frac{2}{3}}} \qquad C_s = 0.130$$

Per AASHTO Division I-A, Article 3.6.1, the maximum value of C_s need not exceed

$$C_{smax} := 2.5 \cdot A \qquad C_{smax} = 0.25$$

Design Step 4.1.4 (continued)

Therefore, use the actual value $C_s = 0.130$

The total equivalent static earthquake force is then

$$Veq_L := C_s \cdot W_{unit} \qquad Veq_L = 447\ \text{kip}$$

Design Step 4.1.5

Elastic Seismic Deflection ρeq

$$k_{unit} = 258 \cdot \frac{kip}{in}$$ Horizontal stiffness of the unit

The elastic deflection due to the seismic shear force is, then, the force divided by the stiffness

$$\Delta_{eq} := \frac{Veq_L}{k_{unit}} \qquad \Delta_{eq} = 1.73 \cdot in$$

Note that this deflection is much larger than the deflection associated with Option No. 1, which has batter piles (1.73 verses 0.88 inch). This deflection must be accommodated at the expansion joints at Bents 3 and 7. The absolute value of the seismic deflection of the two adjoining bridge units must be added together. The gap between the ends of the superstructure units should be equal to or greater than this value.

Design Step 4.1.6

Shear Force in Each Plumb Pile Veq

This seismic load is distributed proportionately to each of the three bents resisting the seismic force, based on their relative stiffnesses. Because the stiffnesses of all bents are equal, each will take an equal proportion of the seismic shear. Because all piles are identical, each will take an equal proportion of the bent seismic shear.

The flexural shear force resisted by each of the 21 plumb piles in the longitudinal direction is

$$V_{eq} := \frac{Veq_L}{21} \qquad V_{eq} = 21.3 \cdot kip$$

Design Step 4.1.6 (continued)

This result should be the same as that yielded by multiplying the deflection times the stiffness of an individual plumb pile.

$k_{Op} = 12.3 \cdot \frac{kip}{in}$ Stiffness of a plumb pile

$\Delta_{eq} = 1.73 \cdot in$ Seismic deflection of the bridge

$V_{eq} := k_{Op} \cdot \Delta_{eq}$ $V_{eq} = 21.3 \cdot kip$ The same, okay!

Design Step 4.1.7

Axial and Flexural Components of Horizontal Shear Force V_a and V_L

For Option No. 2, with all plumb piles, it is assumed that there are no axial forces due to seismic loads in the longitudinal direction.

$V_a := 0 \cdot kip$

$V_L := V_{eq}$ $V_L = 21.3 \cdot kip$

Design Step 4.1.8

Axial Force and Flexural Moment in Each Pile P_a and M_L

There are no axial forces in the plumb piles for Option No. 2.

$P_a := 0 \cdot kip$

The bending moment in each of these piles is based on the length in the soil for flexure, L_m, shown in Figure 13.

$E_p = 4030 \cdot ksi$

$I_p = 1.33 \cdot ft^4$

$n_h = 8.0 \cdot pci$

$$L_m := 0.78 \cdot \left(\frac{E_p \cdot I_p}{n_h} \right)^{\frac{1}{5}}$$ $L_m = 6.94\ ft$

Design Step
4.1.8
(continued)

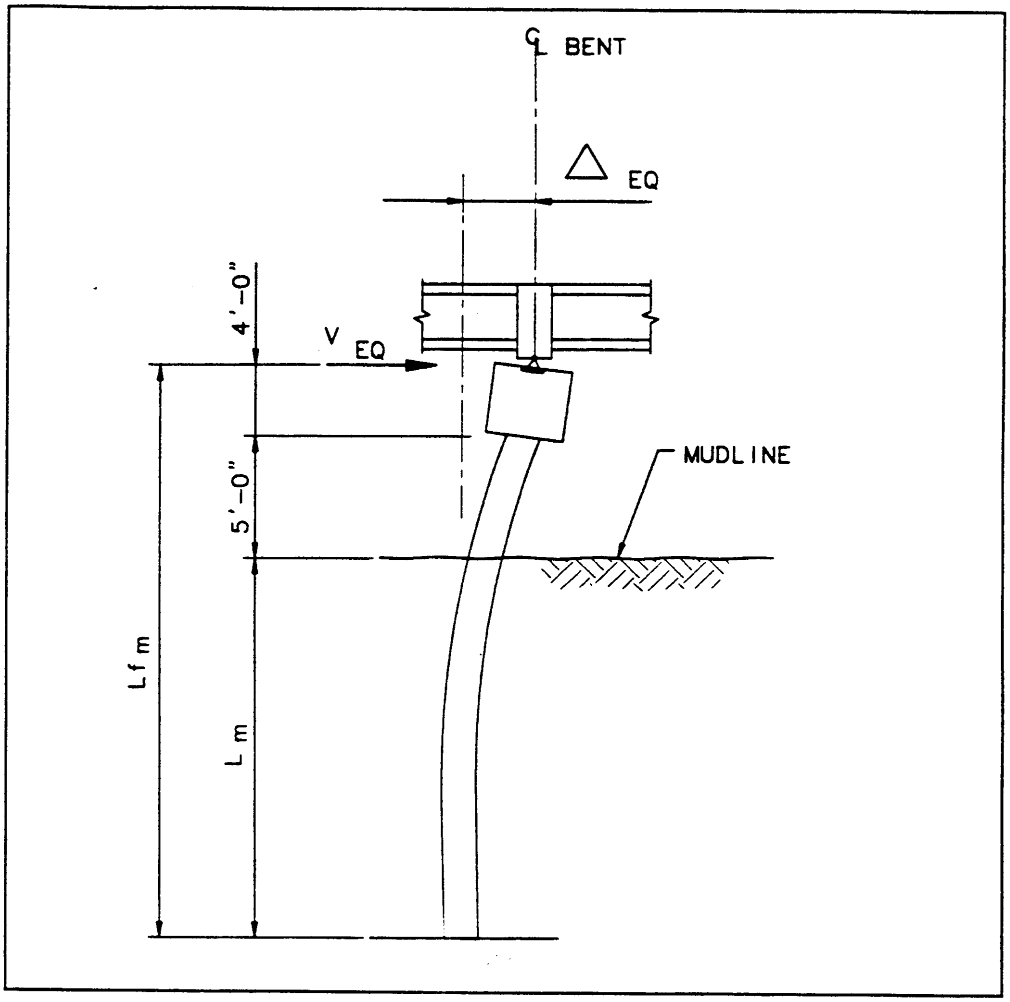


Figure 13 — Deflected Shape of All Plumb Pile Bent

Then the dimension Lf_m is conservatively taken as the flexural length along the pile, from the top of the pile cap to the assumed point of maximum bending moment.

$Lf_m := L_m + 5 \cdot ft + 4 \cdot ft$ $\qquad$ $Lf_m = 15.94\ ft$

The moment in the longitudinal direction for each pile (assuming no fixity at the top of the pile) is

$M_L := V_L \cdot Lf_m$ $\qquad$ $M_L = 340 \cdot kip \cdot ft$

Design Step 4.2

Transverse Seismic Force on Pile Bent 5

Forces in the transverse direction are calculated in this design step. Forces in the longitudinal direction were calculated in Design Step 4.1. In this design step, the following items will be calculated for pile Bent 5. Because the pile bents are identical and the superstructure is assumed to act as a rigid body (superstructure order of magnitude stiffer than piles), tributary lengths will be used to calculate values for pile Bent 5. (Note that because the span lengths are almost identical, all bents will have nearly identical values.)

1. Seismic mass
2. Total horizontal stiffness
3. Period of the structure
4. Total seismic shear force
5. Elastic seismic deflection
6. Shear force in each pile
7. Corresponding axial force and flexural moment in the outboard piles

Design Step 4.2.1

Seismic Mass P_{bent} *g

Calculate the tributary weight of each bent. Refer to Design Step 3.1.1 for a more in-depth discussion of seismic mass.

In this design configuration, the participating mass of each bent in the transverse direction consists of the following.

1. One tributary span of the superstructure
2. One cap beam
3. The top 10 feet of seven piles

The tributary weight at each pile bent, P_{bent}, is

$W_{span} = 742.1\ \text{kip}$

$W_{cap} = 113.4 \cdot \text{kip}$

$W_{pile} = 6.0 \cdot \text{kip}$

$P_{bent} := W_{span} + W_{cap} + 7 \cdot W_{pile} \qquad P_{bent} = 898\ \text{kip}$

Design Step 4.2.2

Total Horizontal Stiffness k_{bent}

Calculate the stiffness of each bent in the transverse direction. Each bent resists its own tributary seismic shear.

Each bent has seven plumb piles.

$$k_{Or} = 83.3 \cdot \frac{kip}{in}$$

Horizontal stiffness of a plumb pile, fixed at the top

The stiffness of each bent is

$$k_{bent} := 7 \cdot k_{Or} \qquad k_{bent} = 583 \cdot \frac{kip}{in}$$

Design Step 4.2.3

Period of the Structure T

The tributary weight of each bent is

$$P_{bent} = 899 \cdot kip$$

$$g := 32.2 \cdot \frac{ft}{sec^2}$$

Acceleration of gravity

The period of the structure in the transverse direction, per Equation 4-3 of Division I-A, is

$$T := 2 \cdot \pi \cdot \sqrt{\frac{P_{bent}}{k_{bent} \cdot g}} \qquad T = 0.40 \cdot sec$$

Design Step 4.2.4

Seismic Shear Force Veq_T

$A := 0.10$ — Seismic acceleration coefficient

$S := 1.2$ — Soil site coefficient

Design Step 4.2.4 (continued)

The elastic seismic response coefficient is calculated from AASHTO Division I-A, Equation 3-1. Note that the units of seconds to the (2/3) power maintains consistent units in Mathcad®.

$$C_s := \frac{1.2 \cdot sec^{\frac{2}{3}} \cdot A \cdot S}{T^{\frac{2}{3}}} \qquad C_s = 0.267$$

Per AASHTO Division I-A, Article 3.6.1, the maximum value of C_s need not exceed

$$C_{smax} := 2.5 \cdot A \qquad C_{smax} = 0.25$$

Therefore, use the maximum value $\qquad C_{smax} = 0.25$

The total equivalent static earthquake force is then

$$Veq_T := C_{smax} \cdot P_{bent} \qquad Veq_T = 225 \cdot kip$$

Design Step 4.2.5

Elastic Seismic Deflection ρeq

$k_{bent} = 583 \cdot \frac{kip}{in}$ Horizontal stiffness of each bent

The elastic deflection due to the seismic shear force is, then, the force divided by the stiffness.

$$\Delta_{eq} := \frac{Veq_T}{k_{bent}} \qquad \Delta_{eq} = 0.39 \cdot in$$

See Figure 14.

Design Step
4.2.5
(continued)

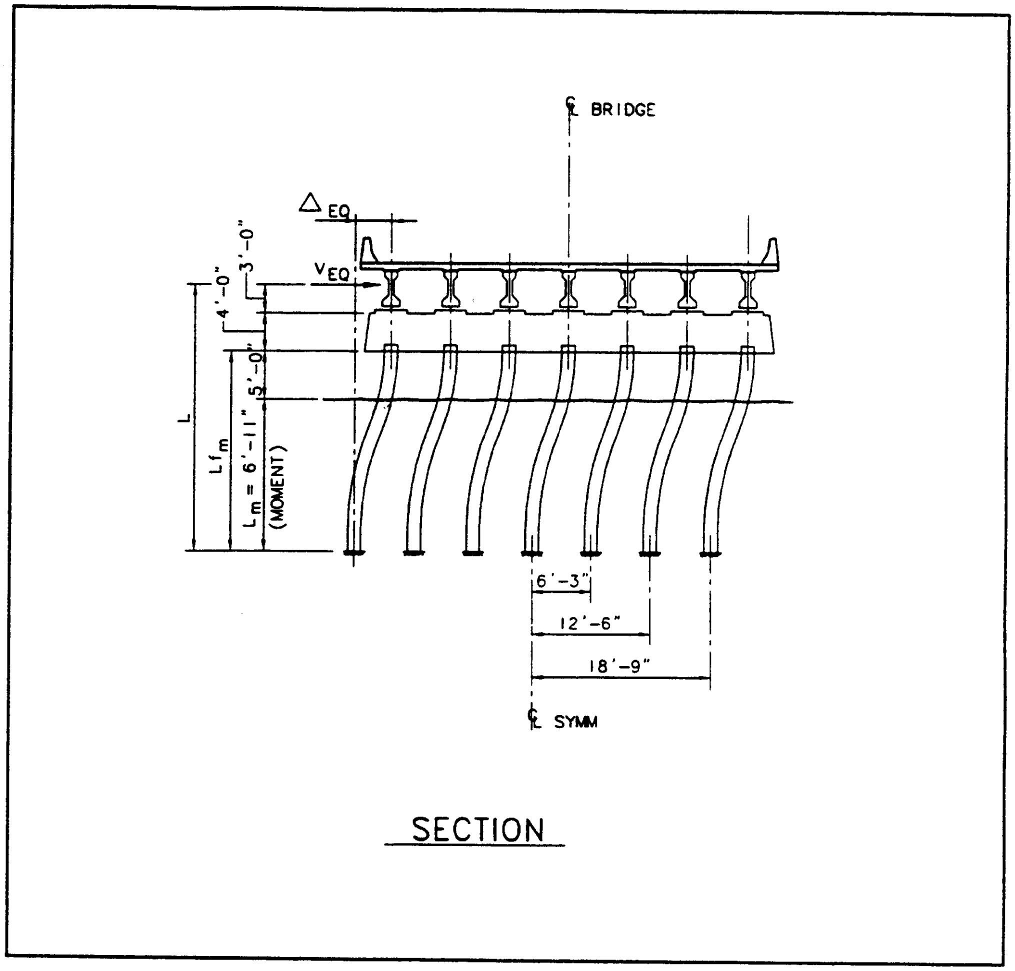


Figure 14 — Deflected Shape of Transverse Bent

Design Step
4.2.6

Shear Force in Each Plumb Pile V_{eq}

This seismic load is distributed evenly among the seven plumb piles in each bent.

The flexural shear force resisted by each of the seven plumb piles in the transverse direction is

$$V_{eq} := \frac{Veq_T}{7} \qquad V_{eq} = 32.1 \cdot kip$$

Design Step 4.2.6 (continued)

This result should be the same as that yielded multiplying the deflection times the stiffness of an individual plumb pile.

$k_{Or} = 83.3 \cdot \frac{kip}{in}$ Stiffness of plumb pile, fixed at the top

$\Delta_{eq} = 0.39 \cdot in$ Seismic deflection of the bridge

$V_{eq} := k_{Or} \cdot \Delta_{eq}$ $V_{eq} = 32.1 \cdot kip$ The same, okay!

$V_T := V_{eq}$ $V_T = 32.1 \cdot kip$

Design Step 4.2.7

Corresponding Axial Force and Flexural Moment in the Outboard Piles

a) Check the Outboard Piles for Axial Tension Due to Frame Action

$Veq_T = 225 \cdot kip$ Shear on the bent

The distance from the assumed point of fixity of the pile to the center of the mass of the superstructure is L, from Design Step 4.1.8.

$L_m = 6.94\ ft$ Length due to flexure

$L := L_m + (5 \cdot ft) + (4 \cdot ft) + (3 \cdot ft)$ $L = 18.94\ ft$

The overturning moment in the bent is

$M_{ot} := Veq_T \cdot L$ $M_{ot} = 4260 \cdot kip \cdot ft$

The section modulus of the seven piles resisting overturning in the transverse direction is

$$Adsq := 2 \cdot \left(18.75^2 + 12.5^2 + 6.25^2\right) \cdot ft^2$$

$$Adsq = 1094 \cdot ft^2$$

Design Step 4.2.7 (continued)

The exterior piles in a bent are spaced a distance of z.

$z := 6 \cdot 6.25 \cdot ft$ $\quad\quad z = 37.5 \cdot ft$

The outside piles will have a seismic axial load (tension) of

$$Pa := \frac{M_{ot} \cdot \frac{z}{2}}{Adsq} \quad\quad Pa = 73.0 \cdot kip \quad\quad \text{okay}$$

$P_{DL} = 128.5 \cdot kip$ $\quad\quad$ Dead load of pile

$P_{ton} = P_{DL} - P_a$ $\quad\quad$ > 0; therefore, zero tension

The axial load due to the transverse seismic shear does not overcome the axial dead load in the piles. Because it was assumed there is no axial load in the plumb piles due to longitudinal seismic forces, the combined axial load effect is insignificant by inspection.

b) The Bending Moment in Each of These Piles Is Based on the Length in the Soil for Flexure, Lm

The equivalent length of the pile in the soil, L_m, to approximate the flexural length of the pile, from Design Step 4.1.8, is

$L_m = 6.94\ ft$

The dimension "Lf_m" is the flexural length along the pile, from the bottom of the pile cap to the assumed point of fixity in the soil, for bending moment.

$Lf_m := L_m + 5 \cdot ft$ $\quad\quad Lf_m = 11.94\ ft$

The moment in the transverse direction is approximately

$$M_T := V_{eq} \cdot \frac{Lf_m}{2} \quad\quad M_T = 192 \cdot kip \cdot ft$$

Design Step 4.3

Combination of Orthogonal Forces
[AASHTO Division I-A, Article 3.9]

Because there is no cross coupling of forces in the transverse and longitudinal directions (no transverse moments caused by longitudinal forces and vice versa), all the piles are assumed to carry the same values of transverse shear and moment.

Design Step 4.3.1

Pile Shear

A summary of the shear forces on a Bent 5 pile is given below.

$V_L = 21.3 \cdot kip$ Longitudinal shear on pile

$V_T = 32.1 \cdot kip$ Transverse shear on pile

a) Load Case 1 (100 Percent Global Longitudinal plus 30 Percent Global Transverse)

Resulting longitudinal shear force

$V_{1L} := (1.0 \cdot V_L)$ $V_{1L} = 21.3 \cdot kip$

Resulting transverse shear force

$V_{1T} := (0.3 \cdot V_T)$ $V_{1T} = 9.6 \cdot kip$

b) Load Case 2 (30 Percent Global Longitudinal plus 100 Percent Global Transverse)

Resulting longitudinal shear force

$V_{2L} := (0.3 \cdot V_L)$ $V_{2L} = 6.4 \cdot kip$

Resulting transverse shear force

$V_{2T} := (1.0 \cdot V_T)$ $V_{2T} = 32.1 \cdot kip$

Design Step 4.3.2

Pile Moment

A summary of the moment forces on Bent 5 pile is given below.

$M_L = 340 \cdot ft \cdot kip$ Longitudinal moment on pile

$M_T = 192 \cdot ft \cdot kip$ Transverse moment on pile

a) Load Case 1 (100 Percent Global Longitudinal plus 30 Percent Global Transverse)

Resulting longitudinal moment

$M_{1L} := \left(1.0 \cdot M_L\right)$ $M_{1L} = 340.0 \cdot ft \cdot kip$

Resulting transverse moment

$M_{1T} := \left(0.3 \cdot M_T\right)$ $M_{1T} = 57.6 \cdot ft \cdot kip$

b) Load Case 2 (30 Percent Global Longitudinal plus 100 Percent Global Transverse)

Resulting longitudinal moment

$M_{2L} := \left(0.3 \cdot M_L\right)$ $M_{2L} = 102.0 \cdot ft \cdot kip$

Resulting transverse moment

$M_{2T} := \left(1.0 \cdot M_T\right)$ $M_{2T} = 192 \cdot ft \cdot kip$

Design Step 4.4

Modified Design Forces in Bent 5 Plumb Pile

[AASHTO Division I-A, Article 6.2.2]

Design Step 4.4.1

Summary of Elastic Forces

a) Elastic Shear Force in Pile

Summary for Load Case 1

$V_{1T} = 9.6 \cdot kip$ $\qquad V_{1L} = 21.3 \cdot kip$

For preliminary design it is conservative to calculate the resultant shear in the pile as shown below, and check it against the shear capacity in the major axes. The resultant shear is

$V_{u1} := \sqrt{V_{1T}^2 + V_{1L}^2}$ $\qquad V_{u1} = 23.4 \cdot kip$

Summary for Load Case 2

$V_{2T} = 32.1 \cdot kip$ $\qquad V_{2L} = 6.4 \cdot kip$

Again, find the resultant shear force.

$V_{u2} := \sqrt{V_{2T}^2 + V_{2L}^2}$ $\qquad V_{u2} = 32.8 \cdot kip$ <--- Controls

b) Elastic Biaxial Moments in Pile

Summary for Load Case 1

$M_{1T} = 57.6 \cdot ft \cdot kip$ $\qquad M_{1L} = 340 \cdot ft \cdot kip$

For preliminary design it is conservative to calculate the resultant moment in the pile as shown below, and check it against the moment capacity about a major axis. If the moments about the two major axes were similar in magnitude, then it would be necessary to create a

Design Step 4.4.1 (continued)

biaxial moment diagram. Since the longitudinal moment dominates in this example, the method below is conservative. The resultant moment is

$$M_{u1} := \sqrt{M_{1T}^2 + M_{1L}^2}$$

$M_{u1} = 345 \cdot ft \cdot kip$ <--- Controls

Summary for Load Case 2

$M_{2T} = 192 \cdot ft \cdot kip$ $M_{2L} = 102 \cdot ft \cdot kip$

Again, find the resultant moment force.

$M_{u2} := \sqrt{M_{2T}^2 + M_{2L}^2}$ $M_{u2} = 217 \cdot ft \cdot kip$

Design Step 4.4.2

Modified Design Forces
[AASHTO Division I-A, Article 6.2.1]

According to Table 3 of AASHTO Division 1-A, Article 3.7, the "R" factor for reinforced concrete pile bents, with all vertical piles, is R=3. Per the exception noted in this referenced section, the full "R" value is used for pile bents. Assume dead load shear and moment forces equal to zero in the piles.

a) Modified Shear Forces in Pile, V_u

$V_{u2} = 32.8 \cdot kip$ Controlling resultant shear force in pile

$R := 3$ Response Modification Reduction Factor

$V_u := \frac{V_{u2}}{R}$ $V_u = 10.9 \cdot kip$

It should be noted that discussion is presently taking place regarding whether the shear force should be divided by R for shear in SPC B. See Gieger et al.

Design Step 4.4.2 (continued)

b) Modified Moment Forces in Pile, M_u

The controlling resultant moment is from Load Case 1.

$M_{u1} = 345 \cdot kip \cdot ft$ Controlling resultant moment in pile

$R := 3$ Response Modification Reduction Factor

$M_u := \frac{M_{u1}}{R}$ $M_u = 115 \cdot ft \cdot kip$

Design Step 4.5

Design the Bent 5 Piles

Design Step 4.5.1

Shear Design

The shear design of the pile is not shown in this design example. On inspection, it can be seen that the seismic factored shear loads do not control the design of tie reinforcement. The design requirements of AASHTO Division I-A, Article 6.6.2 should be used to calculate the minimum transverse reinforcement requirements.

Design Step 4.5.2

Flexure Design

At its top, the pile acts as a regular reinforced section because the unstressed strand is extended into the pile cap. Where the moment is a maximum at the assumed point of fixity below mudline, the pile is a prestressed section. In Design Step 4.4.2.b, the factored resultant moment was calculated, and bending about one major axis will be considered. See Figure 15 for the pile cross section.

Design Step
4.5.2
(continued)

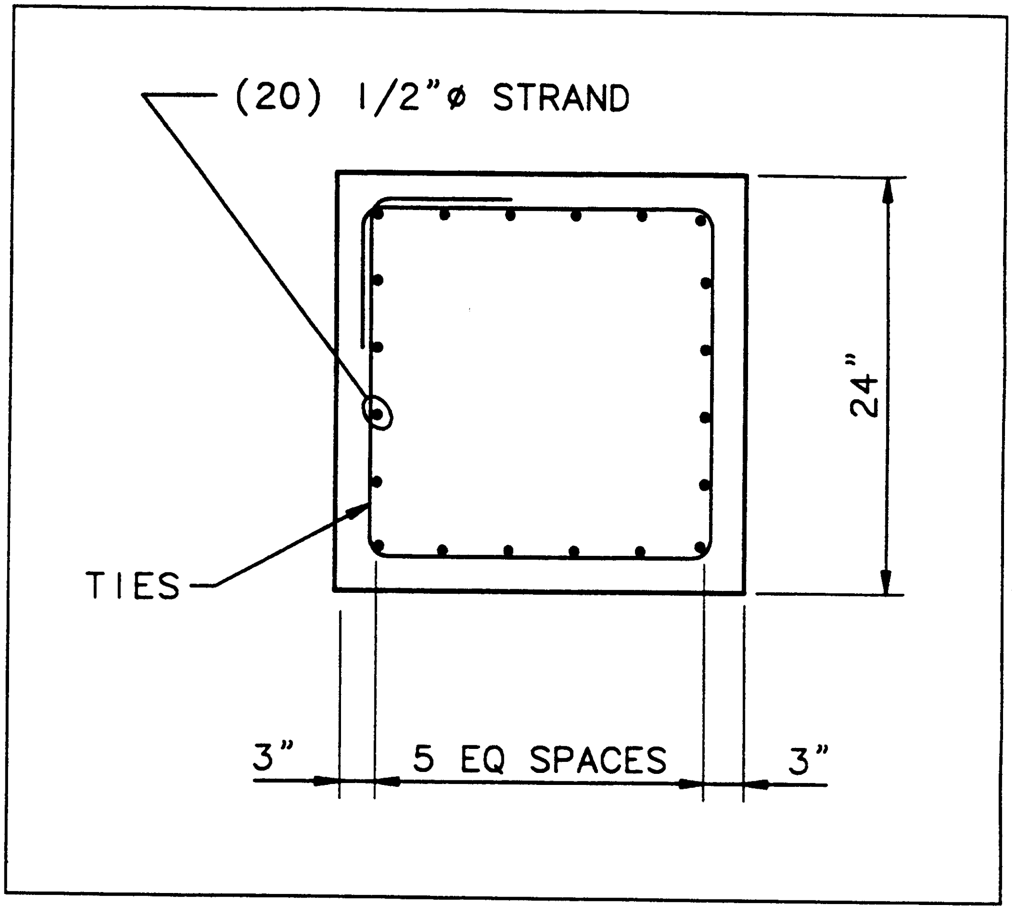


Figure 15 — Cross Section of Concrete Pile

Figure 16 shows the capacity curve for a 24-inch-square prestressed concrete pile with 20 1/2-inch strands. The curves do not account for pile slenderness, because most of the pile is surrounded by competent soil. For longer unsupported length piles, pile slenderness must be considered.

The curve considers only bending about one major axis. As was discussed above, because the moment is predominant about one axis, it is not necessary to plot a biaxial bending curve. Notice that for small axial loads, the moment capacity of the pile is similar for both fully prestressed and zero prestressed conditions.

Design Step
4.5.2
(continued)

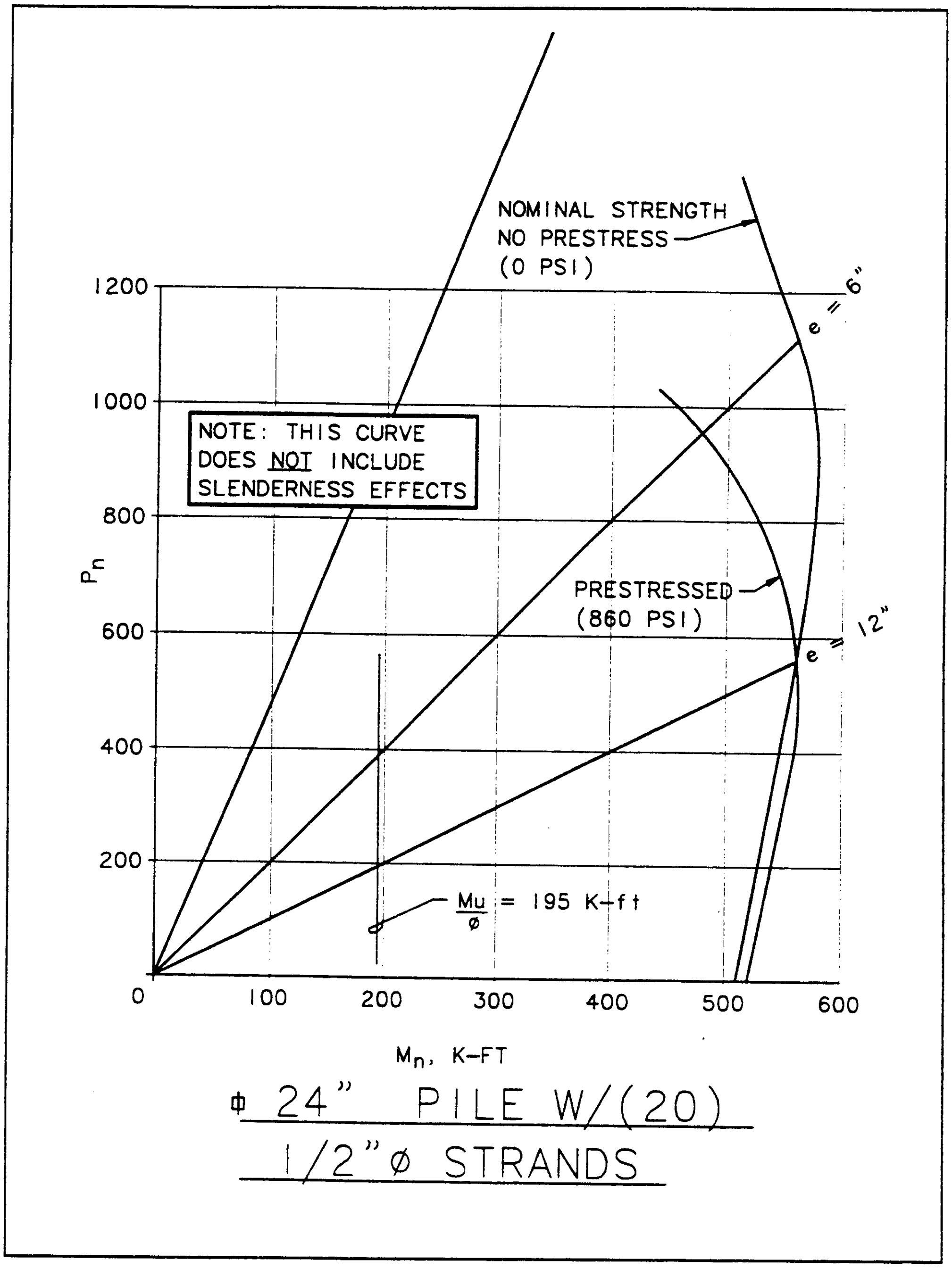


Figure 16 — Interaction Diagram of Concrete Pile

Design Step 4.5.2 (continued)

The maximum factored moment in the longitudinal direction is

$$M_u = 115 \cdot kip \cdot ft$$

The interaction plot is for nominal capacity, therefore M_u must be divided by the phi factor. For SPC B, the minimum phi factor is

$$\phi_{min} := 0.7$$

Therefore,

$$\frac{M_u}{\phi_{min}} = 164 \cdot kip \cdot ft$$

On inspection, it can be seen that the moment capacity of the pile (approximately 500 kip*ft) is much larger than the applied moment of 164 kip*feet. Therefore, the 24-inch-square pile with 20 1/2-inch-diameter strands can be used. Because of the short clear height above the soil, it was assumed that the moment magnification is small, and that the pile is supported by the soil. The dead load times the elastic deflection could have been added, but its effect is insignificant in this case.

DESIGN STEP 5

BRIDGE OPTION NO. 3

Figure 17 shows the conceptual layout for Option No. 3. All the middle bents of the four-span bridge Unit 2 are pinned to the superstructure. Four of the seven piles in the bents are 2:12 batter piles. The other three are plumb piles.

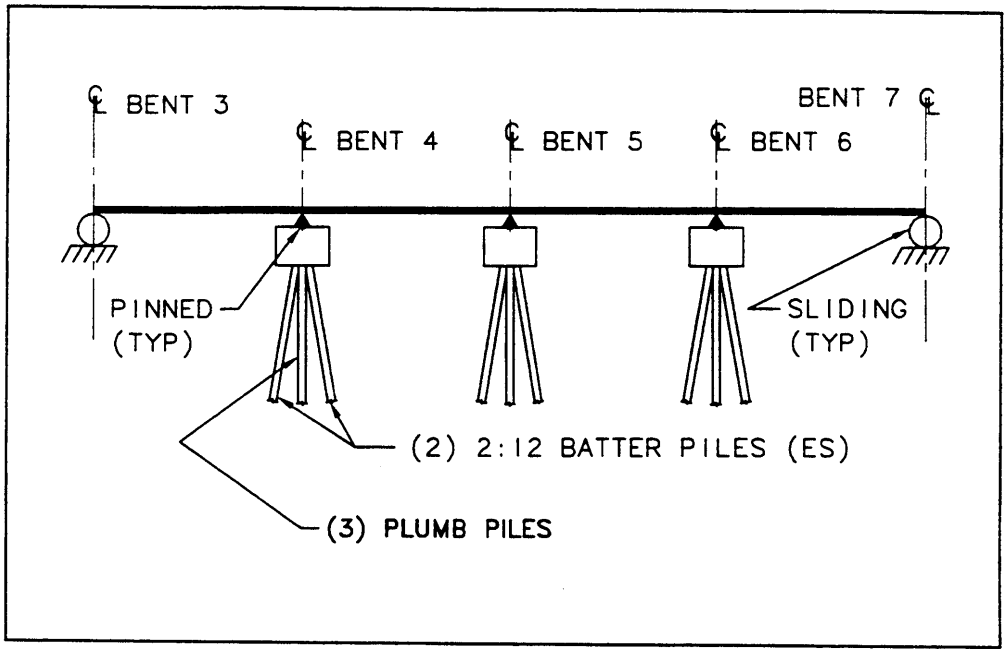


Figure 17 — Conceptual Layout of Piles, Option No. 3

Design Step 5.1

Longitudinal Seismic Force on Pile Bent 5

The following items will be calculated for the bridge Unit 2 structure. Note that only forces in the longitudinal direction are calculated in this design step. Forces for the transverse direction are calculated in Design Step 4.2.

1. Seismic mass
2. Total horizontal stiffness
3. Period of the structure
4. Total seismic shear force
5. Elastic seismic deflection
6. Shear force in the batter pile
7. Axial and flexural components of the horizontal shear force
8. Corresponding axial force in the pile
9. Pile capacity as controlled by the soil

Design Step 5.1.1

Seismic Mass $W_{unit} * g$

The seismic mass for Option No. 3 is the same as for Option No. 2, see Design Step 4.1.1. See Figure 18.

$W_{unit} := 3435 \cdot kip$

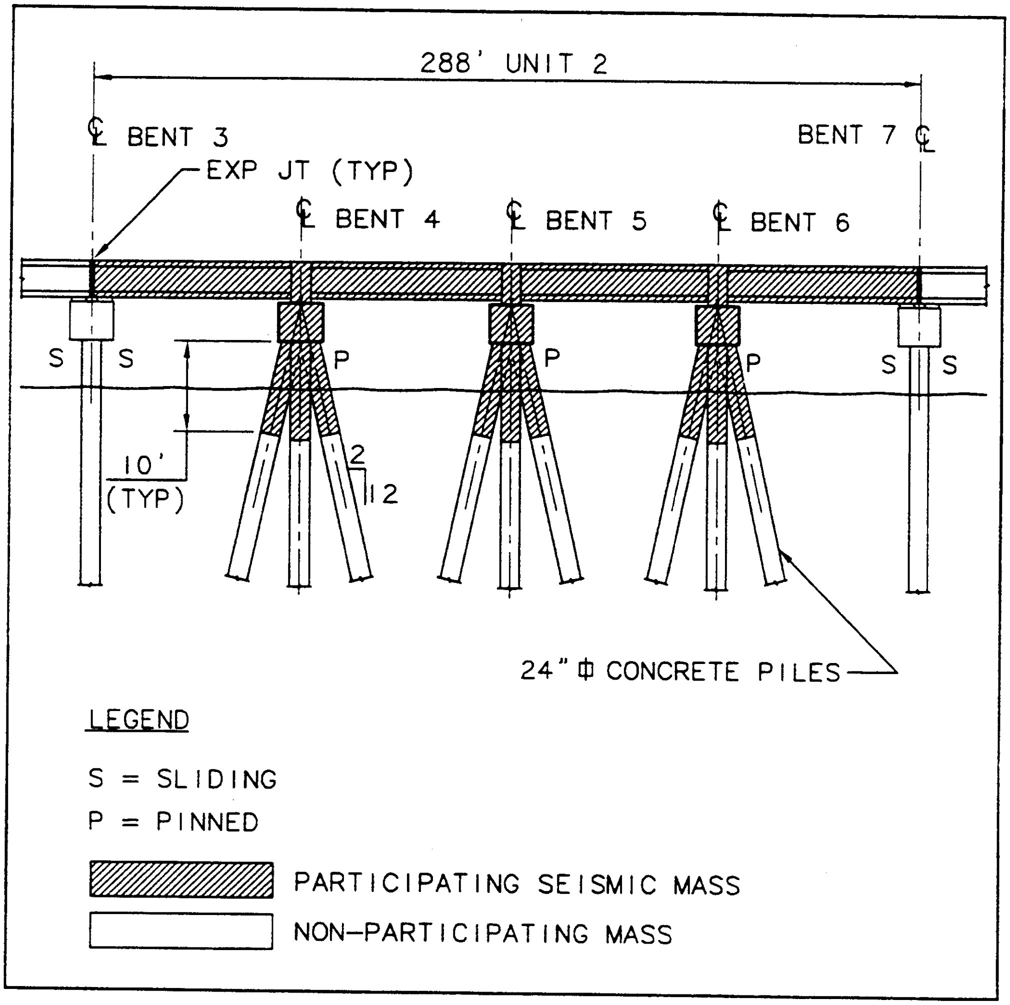


Figure 18 — Seismic Mass Used for Longitudinal Direction, Option No. 3

Design Step 5.1.2

Total Horizontal Stiffness k_{unit}

Calculate the stiffness of bridge Unit 2 in the longitudinal direction. Bents 4, 5, and 6 are pinned to the superstructure; therefore, they all contribute to

Design Step 5.1.2 (continued)

the longitudinal seismic stiffness. Refer to Table 1 in Design Step 2.3 for individual stiffnesses. Each of the three bents consist of four piles with 2 on 12 batter plus three plumb piles.

$$k_{2p} = 137.5 \cdot \frac{kip}{in}$$

Horizontal stiffness of a 2 on 12 batter pile, pinned at the top

$$k_{Op} = 12.3 \cdot \frac{kip}{in}$$

Horizontal stiffness of a plumb pile, pinned at the top

Total stiffness of bridge Unit No. 2 is

$$k_{unit} := 3 \cdot \left(4 \cdot k_{2p} + 3 \cdot k_{Op}\right) \qquad k_{unit} = 1761 \cdot \frac{kip}{in}$$

See Table 4.

Table 4
Option No. 3, Longitudinal Stiffness

Bearing System Used: Sliding bearings at Bents 3 and 7, pinned at Bents 4 to 6
Pile Configuration: (4) 2:12 batter piles + (3) equivalent plumb piles at Bents 4, 5, and 6

Concrete Pile Option No. 3	Unit No. 2 w/Concrete Piles, Longitudinal Stiffness (kip/in)					
Bent Number	Bent 3	Bent 4	Bent 5	Bent 6	Bent 7	Total
Bearing Restraint at Bent	Sliding	Sliding	Pinned	Sliding	Sliding	
Number Plumb Piles, N_p	0	3	3	3	0	9
Horizontal Stiffness per Pile, k (kip/in)	0	12.3	12.3	12.3	0	
Stiffness of Plumb Piles = N_p*k (kip/in)	0	36.9	36.9	36.9	0	110.7
Number of 2:12 Batter Piles, N_b	0	4	4	4	0	12
Horizontal Stiffness per Pile, k	0	137.5	137.5	137.5	0	
Stiffness of Batter Piles = N_b*k (kip/in)	0	550.0	550.5	550.5	0	16.5
Longitudinal Horizontal Stiffness of Unit No. 2 Bridge (kip/in) =						1761

Design Step 5.1.3

Period of Structure T

$W_{unit} := 3435 \cdot kip$ — Total weight of the unit

$g := 32.2 \cdot \frac{ft}{sec^2}$ — Acceleration of gravity

The period of the structure in the longitudinal direction, per Equation 4-3 of Division I-A, is

$$T := 2 \cdot \pi \cdot \sqrt{\frac{W_{unit}}{k_{unit} \cdot g}} \qquad T = 0.45 \cdot sec$$

Design Step 5.1.4

Total Seismic Shear Force V_{eqL}

$A := 0.10$ — Seismic acceleration coefficient

$S := 1.2$ — Soil site coefficient

The elastic seismic response coefficient is calculated from AASHTO Division I-A, Equation 3-1. Note that the units of seconds to the (2/3) power maintains consistent units in Mathcad®.

$$C_s := \frac{1.2 \cdot sec^{\frac{2}{3}} \cdot A \cdot S}{T^{\frac{2}{3}}} \qquad C_s = 0.246$$

Per AASHTO Division I-A, Article 3.6.1, the maximum value of C_s need not exceed

$$C_{smax} := 2.5 \cdot A \qquad C_{smax} = 0.25$$

Therefore, use the actual value $\qquad C_{smax} = 0.246$

The total equivalent static earthquake force is then

$Veq_L := C_{smax} \cdot W_{unit}$ $\qquad Veq_L = 845 \text{ kip}$

Design Step 5.1.5

Elastic Seismic Deflection ρeq

$k_{unit} = 1761 \cdot \frac{kip}{in}$ $\qquad$ Horizontal stiffness of the unit

The elastic deflection due to the seismic shear force is, then, the force divided by the stiffness.

$\Delta_{eq} := \frac{Veq_L}{k_{unit}}$ $\qquad \Delta_{eq} = 0.48 \text{ in}$

See Figure 10 in Design Step 3.1.5.

Design Step 5.1.6

Shear Force in the Batter Pile Veq

The total seismic shear is distributed proportionately to each pile based on its relative stiffness. The force in each pile can be calculated by multiplying the stiffness of the individual pile by the total deflection. The resulting horizontal force resisted by each 2:12 batter pile with a pinned top is

$k_{2p} = 137.5 \cdot \frac{kip}{in}$ $\qquad$ Stiffness of a 2:12 batter pile

$\Delta_{eq} = 0.48 \text{ in}$ $\qquad$ Seismic deflection of the bridge

$V_{eq} := k_{2p} \cdot \Delta_{eq}$ $\qquad V_{eq} = 66.0 \text{ kip}$

Design Step 5.1.7

Axial and Flexural Components of Horizontal Shear Force V_a and V_m

A portion of the horizontal force is resisted by an axial load in the batter pile while the remainder is resisted by flexure in the pile. Before the axial force and flexural moment can be calculated, the relative stiffness of the axial and flexural contribution to the horizontal stiffness must be considered. The shear carried by each component is expressed as a simple ratio of its stiffness to the total stiffness.

Design Step 5.1.7 (continued)

The total stiffness of one 2 on 12 batter pile with pinned top is

$$k_{2p} = 137.5 \cdot \frac{kip}{in}$$

The horizontal component of total stiffness due to axial stiffness in the pile is

$$ka_2 = 125.5 \cdot \frac{kip}{in}$$

The horizontal shear associated with the axial deformation is a ratio of the axial to total horizontal stiffness of the batter pile.

$$V_a := \frac{ka_2}{k_{2p}} \cdot V_{eq} \qquad V_a = 60.2\ kip$$

The horizontal component of total stiffness due to flexural stiffness in the pile is

$$kf_{2p} = 12.0 \cdot \frac{kip}{in}$$

The horizontal shear associated with the flexural deformation is a ratio of the flexural to total horizontal stiffness of the batter pile. This force is used to calculate the flexural shear and moment in the pile.

$$V_m := \frac{kf_{2p}}{k_{2p}} \cdot V_{eq} \qquad V_m = 5.8\ kip$$

Check that the total horizontal seismic force equals Veq.

$$V := V_a + V_m \qquad V = 66.0\ kip \qquad \text{okay}$$

Design Step 5.1.8

Axial Forces in the Pile

a) The Resulting Axial Forces in the 2 on 12 Batter Pile Are Based on the Shear Force V_a and Simple Trigonometry

$V_a = 60.2$ kip — Horizontal shear force

$\alpha_2 = 9.46 \cdot deg$ — Angle of batter

The elastic axial force in the pile is either tension or compression, depending on the direction of the EQ load.

$$P_a := \frac{V_a}{\sin(\alpha_2)} \qquad P_a = 366 \text{ kip}$$

Design Step 5.1.9

Pile Capacity (as controlled by the soil)

From Design Step 1.5, the capacity of the soil is (-)213 kips in tension, and (+)767 kips in compression. Determine the magnitude of the final axial loads.

The Response Modification Factor "R" should not apply to the axial component of the batter piles in a pile bent. There is little energy dissipated in "truss-type" action of batter piles. The full elastic axial forces are therefore used.

For Group VII loading, per AASHTO Division I-A, Article 6.2.2, the load factors are all 1.0. The maximum compression in the 2:12 batter piles, under Group VII loading using the full elastic force, is

$P_{DL} = 128 \cdot kip$ — Dead load reaction in a pile (unfactored)

$P_a = 366$ kip — Full elastic seismic axial load

$P_{compr} := (1.0 \cdot P_{DL}) + (1.0 \cdot P_a)$ — Maximum compressive axial load

$P_{compr} = 494$ kip — < 767 kip, therefore, ok

Design Step 5.1.9 (continued)

The maximum tension in the 2:12 batter piles, under Group VII loading using the full elastic force, is

$P_{ten} := P_{DL} - P_a$ Maximum tensile axial load

$P_{ten} = -238$ kip > (-)213 kip, close but N.G.

The maximum compression load in the pile is less than the soil compression capacity, but the tension capacity of the soil exceeds the capacity by a small amount.

Therefore, go on to Design Step 5.3 for the implications of the pile layout for Option No. 3, and for other options to investigate.

Design Step 5.3

Implications

The full elastic forces were used in the analysis in Design Step 5.1.8(a). The capacity of the soil exceeded or was close to these loads. The pile length could be increased to provide the necessary capacity.

Therefore, this pile layout can be designed to resist the seismic forces of the bridge. Unlike Option No. 1, where only one bent had batter piles, this option contains more batter piles to share the horizontal shear forces. The additional length of pile required to resist the tension loads is also reasonable.

DESIGN STEP 6 HORIZONTAL STIFFNESS OF STEEL PILES

In Design Step 2, the horizontal stiffness components for a 24-inch-square concrete pile were calculated.

In this design step, the horizontal stiffness components of a steel pipe pile are calculated. The horizontal stiffness due to the flexural component is calculated separately from the horizontal stiffness due to the axial component. All the steel piles are assumed to be 24-inch O.D. pipes with a 1/2-inch wall thickness.

See Appendix B for the development of the equation for horizontal stiffness in the batter piles.

Design Step 6.1 Horizontal Stiffness Due to Flexural Component

Determine the pile depth-to-fixity in the soil by approximating the relative horizontal stiffness. The basic data for the steel pile is given below.

$E_p := 29000 \cdot ksi$ Modulus of elasticity of pile

Cross-sectional area of a 24-inch O.D. pipe pile with 1/2-inch wall thickness

$A_p := 36.9 \cdot in^2$

Moment of inertia of pile

$I_p := 2550 \cdot in^4$

The constant of horizontal subgrade reaction of the pile, provided by the geotechnical engineer, is the same for the steel pile as it was for the concrete pile, and is

$n_h := 8 \cdot pci$

The equation for the flexural length of the pile in soil, L_s, to approximate the horizontal flexural stiffness of the pile, is taken from Figure 3 in Design Step 2.1.

Design Step 6.1 (continued)

$$L_s := 1.8 \cdot \left(\frac{E_p \cdot I_p}{n_h} \right)^{\frac{1}{5}} \qquad L_s = 14.77 \cdot ft$$

The dimension Lf_s is the flexural length along the pipe pile, from the pile top to the assumed point of fixity in the soil. It includes the 5-foot pile length from the mudline to the bottom of the pile cap. See Figure 4 in Design Step 2.1.

$$Lf_s := L_s + 5 \cdot ft \qquad Lf_s = 19.77 \cdot ft$$

Design Step 6.1.1

Flexural Stiffness Component (Plumb Pile)

a) Longitudinal Direction

As with the concrete pile, the connection of the steel pile-to-pile cap is considered fixed against rotation, but in the longitudinal direction, the connection of the superstructure to the pile cap is assumed to be pinned.

First, calculate the equivalent pile length, L_L, for horizontal stiffness in the longitudinal direction. The 4-foot depth of the pile cap must be added to the previously calculated Lf_s.

$Lf_s = 19.77 \cdot ft$ — Equivalent flexural length due to stiffness

$L_L := Lf_s + (4 \cdot ft)$ — Equivalent flexural length for longitudinal stiffness

$L_L = 23.77 \cdot ft$

For a plumb pile with a pinned pile top, the equation for the horizontal stiffness due to flexure is the same as that for a free cantilever. (Note the subscripts 0 for zero batter and p for pinned pile top.)

$E_p = 29000 \cdot ksi$ — Modulus of elasticity of pile

$I_p = 2550 \cdot in^4$ — Moment of inertia of pile

Design Step 6.1.1 (continued)

$L_L = 23.77\ ft$ Equivalent pile length for longitudinal direction

$$k_{Op} := \frac{3 \cdot E_p \cdot I_p}{\left(L_L\right)^3} \qquad k_{Op} = 9.6 \cdot \frac{kip}{in}$$

b) Transverse Direction

In the transverse direction parallel with the pile cap, the cap is restrained against rotation about the longitudinal axis. The pile cap only translates, it does not rotate in this direction. The top of the steel pipe pile is, therefore, assumed fixed at the bottom of the concrete pile cap.

Note: Achieving fixity of a steel pipe pile connected to a concrete pile cap is more difficult than with concrete piles. One method is to weld reinforcement bars to the inside face of the pile and embed them into the pile cap.

Therefore, the equivalent length of pile calculated for stiffness is taken from the bottom of the cap, and is the same length as Lf_s, previously calculated. See Figure 5 in Design Step 2.1.1.

$Lf_s = 19.77 \cdot ft$ Equivalent length due to stiffness

$L_T := Lf_s$ Equivalent length for transverse stiffness

$L_T = 19.77 \cdot ft$

For a plumb pile with a restrained (or fixed) pile top, the equation for the horizontal stiffness due to flexure is the same as for a fixed-fixed beam. (Note the subscripts O for zero batter and r for restrained pile top.)

$E_p = 29000 \cdot ksi$ Modulus of elasticity of pile

$I_p = 2550 \cdot in^4$ Moment of inertia of pile

$L_T = 19.77 \cdot ft$ Equivalent length for transverse stiffness

Design Step 6.1.1 (continued)

$$k_{Or} := \frac{12 \cdot E_p \cdot I_p}{\left(L_T\right)^3} \qquad k_{Or} = 66.5 \cdot \frac{kip}{in}$$

Design Step 6.1.2

Flexural Stiffness Component (Batter Pile)

When a pile is battered 2:12 in the longitudinal direction (parallel to the length of the bridge), the horizontal component of the flexural stiffness is reduced as the slope of the batter increases. See Appendix B.

As was discussed in Design Step 2.1.2, the horizontal stiffness, due to the flexural component of a battered pile with a pinned pile top, is the same as for a free cantilever, with an adjustment for the angle of the batter, α. (Note: The length of the batter pile is defined along the pile, not along the vertical length. See Figure 6 in Design Step 2.1.2.)

$$kf_p := \frac{3 \cdot E_p \cdot I_p}{Lf_s^{\,3}} \cdot (\cos(\alpha))^2$$

Or, written in terms of the previously calculated term k_{Op}, the horizontal stiffness due to the flexural component of batter pile with a pinned top is

$$kf_p := k_{Op} \cdot (\cos(\alpha))^2$$

$k_{Op} = 9.6 \cdot \frac{kip}{in}$ Horizontal stiffness of plumb pile with pinned top

$\alpha_2 = 9.46 \cdot deg$ Angle of batter pile

For a 2:12 batter (Note: The "2" in the subscript is for the batter.)

$$kf_{2p} := k_{Op} \cdot (\cos(\alpha_2))^2$$

$kf_{2p} = 9.3 \cdot \frac{kip}{in}$ Versus 9.6 for a plumb pile

Again, for most pile bent configurations, the effect of the batter on the flexural stiffness can be neglected. In this design example, the actual calculated stiffness is used for consistency.

Design Step 6.2

Axial Stiffness Component

Determine the horizontal stiffness of one batter pile, due to the axial "truss action" of the pile. It is assumed that there are "pairs" of batter piles, with each pile having another pile with an opposite batter to cancel the net vertical deflection.

Because the work point of the steel batter pile is the same as for the concrete piles, the equivalent pile length of pile for axial loads is the same as in Design Step 2.2.

$L_a = 41.67 \cdot ft$ Equivalent pile length for axial stiffness

Design Step 6.2.1

Axial Stiffness Component (Plumb Pile)

For simplification, the plumb piles are assumed not to develop any axial forces when displaced horizontally.

Design Step 6.2.2

Axial Stiffness Component (Batter Pile)

First the pure axial stiffness of a pile must be developed.

$A_p = 36.9 \cdot in^2$ Cross-sectional area of pile

$E_p = 29000 \cdot ksi$ Modulus of elasticity of a pile

$L_a = 41.67 \cdot ft$ Equivalent pile length for axial stiffness

$k_{axial} := \dfrac{A_p \cdot E_p}{L_a}$ $k_{axial} = 2140 \cdot \dfrac{kip}{in}$

Design Step 6.2.2 (continued)

The horizontal stiffness component of a battered pile, due to axial effects only, is the pure axial stiffness of the pile multiplied by $\sin^2(\alpha)$. See Appendix B.

$$k_{batter} := \frac{A_p \cdot E_p}{L_a} \cdot (\sin(\alpha))^2$$

Written another way

$$k_{batter} := k_{axial} \cdot (\sin(\alpha))^2$$

Calculate the horizontal stiffness due to axial truss action for a 2 on 12 batter pile.

$k_{axial} = 2140 \cdot \frac{kip}{in}$ Axial stiffness of a pile

$\alpha_2 = 9.46 \cdot deg$ Angle of batter pile

The horizontal stiffness due to the axial stiffness component

$ka_2 := k_{axial} \cdot (\sin(\alpha_2))^2$ $ka_2 = 57.8 \cdot \frac{kip}{in}$

Design Step 6.3

Combined Stiffness of the Steel Pile

The total horizontal stiffness of a pile, due to the combined effects of both flexural and axial stiffness, is the sum of the individual components. See Appendix B.

Calculate the combined stiffness for a longitudinal 2 on 12 batter pile with pinned top.

$$k_{2p} := \frac{3 \cdot E_p \cdot I_p}{(L_L)^3} \cdot (\cos(\alpha_2))^2 + \frac{A_p \cdot E_p}{L_a} \cdot (\sin(\alpha_2))^2$$

Design Step 6.3 (continued)

Another way to express the same equation, in terms of previously defined horizontal stiffness components, is

$$k_{2p} := kf_{2p} + ka_2$$

$kf_{2p} = 9.3 \cdot \frac{kip}{in}$ Flexural stiffness component, of a 2:12 batter pile with a pinned top

$ka_2 = 57.8 \cdot \frac{kip}{in}$ Axial stiffness component of a 2:12 batter pile

Therefore, the total horizontal stiffness is

$k_{2p} := kf_{2p} + ka_2$ $\qquad k_{2p} = 67.1 \cdot \frac{kip}{in}$

Design Step 6.4

Summary of Steel Pile Stiffness

A summary of horizontal steel pipe pile stiffness components used in this design example is given in Table 5.

Table 5
Summary of Steel Pipe Pile Stiffness Components

Support	Batter	Angle	Horiz Component Due to Flexural Stiff k_f (kip/in)	Horiz Component Due to Axial Stiff k_a (kip/in)	**Total Horiz Stiffness (kip/in)**
Pinned Top	0	0.00	9.6	0.0	**9.6**
	2	9.46	9.3	57.8	**67.1**
Fixed Top	0	0.00	66.5	0.0	**66.5**

DESIGN STEP 7

BRIDGE OPTION NO. 4
One Bent with 2:12 Steel Batter Piles

The pile layout of Option No. 4 is the same as for Option No. 1 in Design Step 3, except that steel pile piles are used instead of concrete piles. In Bent 5, which is restrained to the Unit 2 superstructure, the layout consists of a combination of batter and plumb piles. The other bents under the Unit 2 bridge have sliding bearings. Figure 19 shows the conceptual layout for Option No. 4.

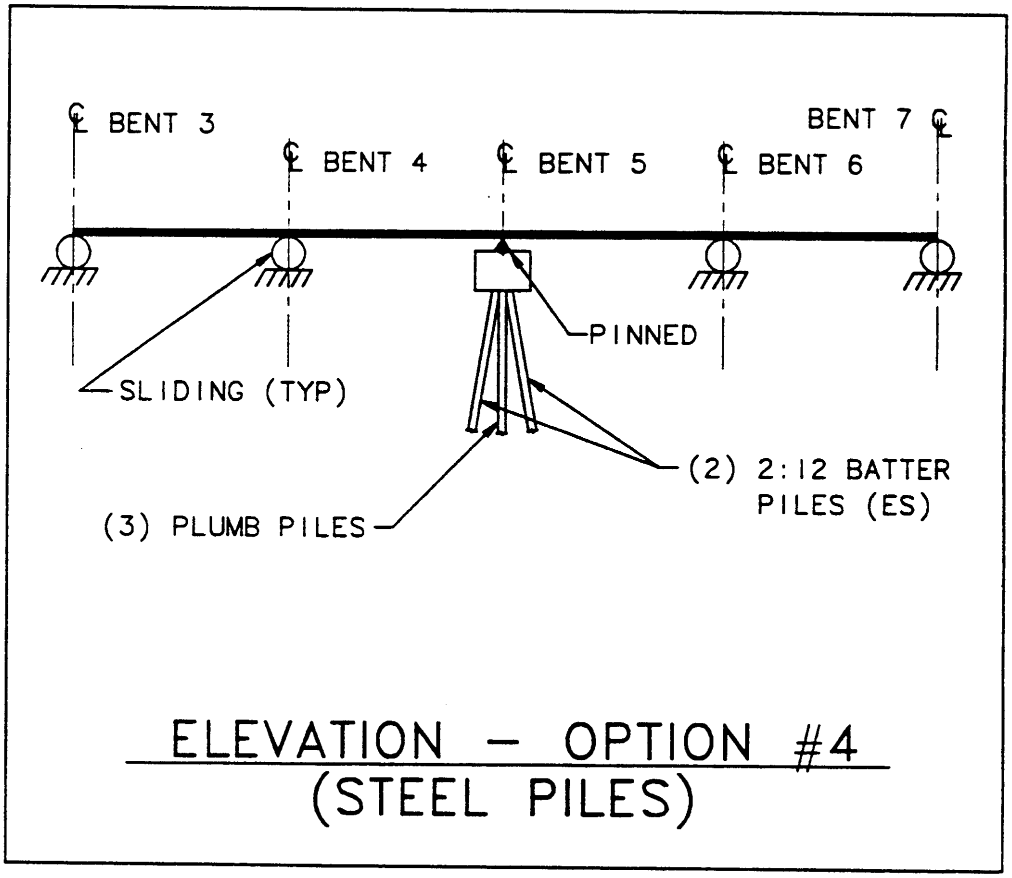


Figure 19 — Conceptual Layout of Piles, Option No. 4

Design Step 7.1 Longitudinal Seismic Force on Bent 5 Piles

The following items will be calculated for the bridge Unit 2 structure in the longitudinal direction. Note that only forces in the longitudinal direction are calculated in this design step. Forces for the transverse direction are calculated in Design Step 8.2.

1. Seismic mass
2. Total horizontal stiffness
3. Period of the structure
4. Total seismic shear force
5. Elastic seismic deflection
6. Shear force in the batter pile
7. Axial and flexural components of the horizontal shear force
8. Corresponding axial force in the pile
9. Pile capacity as controlled by the soil

Design Step 7.1.1 Seismic Mass $W_{unit}*g$

In the configuration for Option No. 4, the participating mass of the system in the longitudinal direction consists of the following.

1. Four spans of the superstructure
2. One cap beam at Bent 5
3. The top 10 feet of seven piles in the Bent 5 cap

See Figure 20.

Therefore, the total dead load weight of the system, W_{unit}, is

$W_{span} = 742.1\ \text{kip}$ Weight of one span of the superstructure

$W_{cap} = 113.4 \cdot \text{kip}$ Weight of one bent cap beam

Note: The weight of a steel pipe pile is less than that of the concrete piles.

$W_{pile} := 1 \cdot \text{kip}$ Weight of one 10-foot length of steel pile pile

$$W_{unit} := (4 \cdot W_{span}) + W_{cap} + (7 \cdot W_{pile}) \qquad W_{unit} = 3089\ \text{kip}$$

Design Step 7.1.1 (continued)

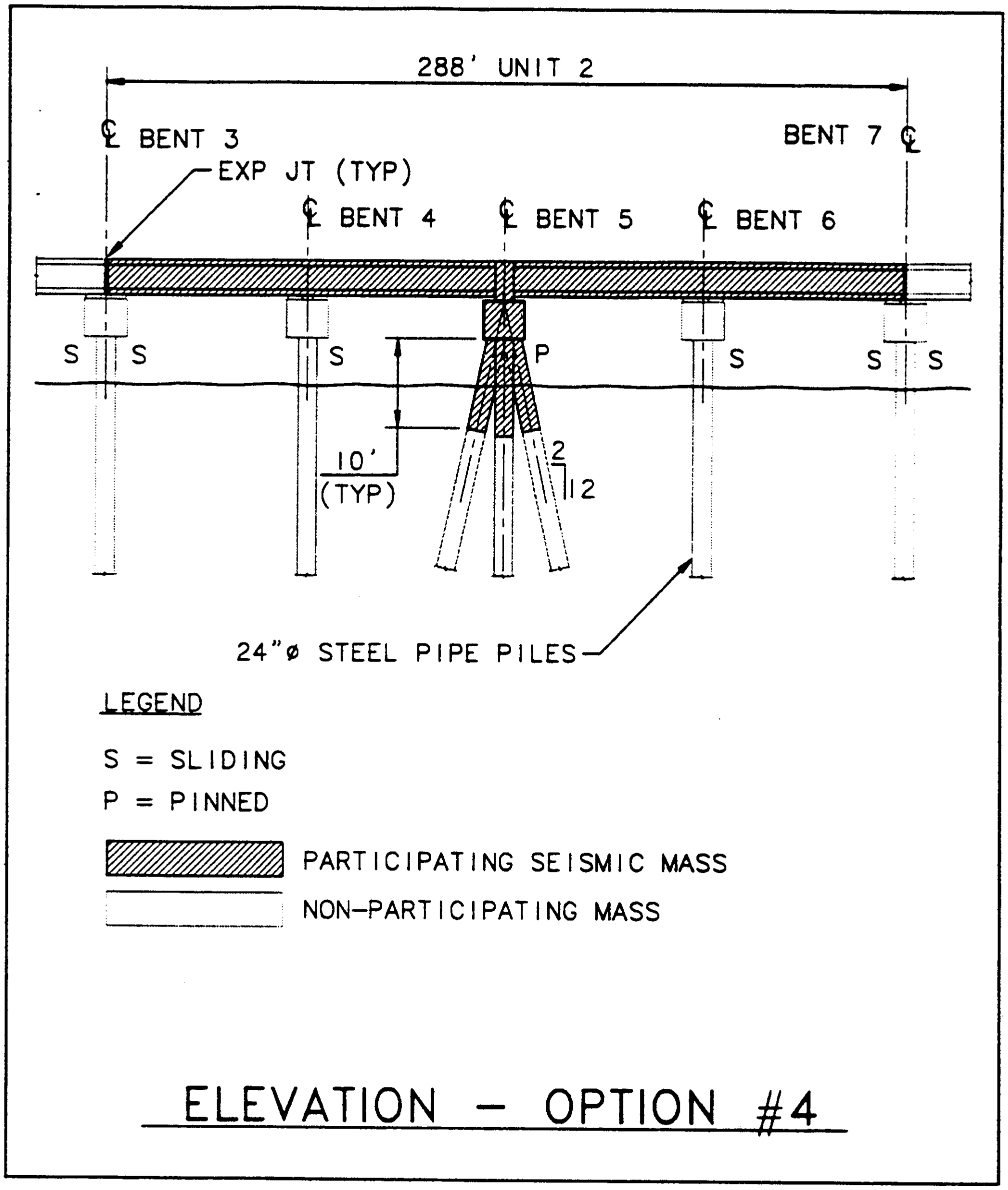


Figure 20 — Seismic Mass Used for Longitudinal Direction, Option No. 4

Design Step 7.1.2

Total Horizontal Stiffness k_{unit}

Calculate the total stiffness in the longitudinal direction. Only Bent 5 is pinned to the superstructure; therefore, it is the only bent that resists the longitudinal seismic forces. Below is a summary of the pile types that are oriented in the longitudinal direction.

Design Step 7.1.2 (continued)

One bent of four 2:12 batter piles plus three plumb piles, all with pinned tops.

$k_{2p} = 67.1 \cdot \frac{kip}{in}$ Horizontal stiffness of a 2:12 batter pile, pinned at the top

$k_{Op} = 9.6 \cdot \frac{kip}{in}$ Horizontal stiffness of a plumb pile, pinned at the top

Total stiffness of Bridge Unit No. 2 is

$k_{unit} := 4 \cdot k_{2p} + 3 \cdot k_{Op}$ $k_{unit} = 297 \cdot \frac{kip}{in}$

See Table 6 for a summary of the stiffness components.

Table 6
Option No. 4, Longitudinal Stiffness

Bearing System Used: Bent 5 pinned, sliding bearings all other bents
Pile Configuration: (4) 2:12 batter piles + (3) equivalent plumb piles at Bent 5 only

Steel Pile Option No. 4	Unit No. 2 w/Steel Piles, Longitudinal Stiffness (kip/in)					
Bent Number	Bent 3	Bent 4	Bent 5	Bent 6	Bent 7	Total
Bearing Restraint at Bent	Sliding	Sliding	Pinned	Sliding	Sliding	
Number Plumb Piles, N_p	0	0	3	0	0	3
Horizontal Stiffness per Pile, k (kip/in)	0	0	9.6	0	0	
Stiffness of Plumb Piles = N_p*k (kip/in)	0	0	28.8	0	0	28.8
Number of 2:12 Batter Piles, N_b	0	0	4	0	0	4
Horizontal Stiffness per Pile, k	0	0	67.1	0	0	
Stiffness of Batter Piles = N_b*k (kip/in)	0	0	268.4	0	0	268.4
Longitudinal Horizontal Stiffness of Unit No. 2 Bridge (kip/in) =						297

Design Step 7.1.3

Period of the Structure T

$W_{unit} = 3089\ kip$ Total weight of the unit

$g := 32.2 \cdot \frac{ft}{sec^2}$ Acceleration of gravity

The period of the structure in the longitudinal direction, per Equation 4-3 of Division I-A, is

$$T := 2 \cdot \pi \cdot \sqrt{\frac{W_{unit}}{k_{unit} \cdot g}} \qquad T = 1.03\ sec$$

Design Step 7.1.4

Total Seismic Shear Force Veq_L

$A := 0.10$ Seismic acceleration coefficient

$S := 1.2$ Soil site coefficient

The elastic seismic response coefficient is calculated from AASHTO Division I-A, Equation 3-1. Note that the units of seconds to the (2/3) power maintains consistent units in Mathcad®.

$$C_s := \frac{1.2 \cdot sec^{\frac{2}{3}} \cdot A \cdot S}{T^{\frac{2}{3}}} \qquad C_s = 0.141$$

Per AASHTO Division I-A, Article 3.6.1, the maximum value of C_s need not exceed

$$C_{smax} := 2.5 \cdot A \qquad C_{smax} = 0.25$$

Therefore, use the actual value $C_s = 0.141$

The total equivalent static earthquake force is then

$Veq_L := C_s \cdot W_{unit}$ $\qquad Veq_L = 436 \text{ kip}$

Design Step 7.1.5

Elastic Seismic Deflection ρeq

$k_{unit} = 297 \cdot \frac{kip}{in}$ $\qquad$ Horizontal stiffness of the unit

The elastic deflection due to the seismic shear force is, then, the force divided by the stiffness. See Figure 10 in Design Step 3.1.5.

$\Delta_{eq} := \frac{Veq_L}{k_{unit}}$ $\qquad \Delta_{eq} = 1.47 \text{ in}$

Design Step 7.1.6

Shear Force in the Batter Pile Veq

The total seismic shear is distributed proportionately to the relative stiffness of the seven piles in Bent 5, which resist the entire seismic load of bridge Unit No. 2. This force can be calculated by multiplying the stiffness of the individual piles by the total deflection. The resulting horizontal force resisted by each 2:12 batter pile with a pinned top is

$k_{2p} = 67.1 \cdot \frac{kip}{in}$ $\qquad$ Horizontal stiffness of a 2:12 batter pile with pinned top

$\Delta_{eq} = 1.47 \text{ in}$ $\qquad$ Seismic deflection of the bridge

$V_{eq} := k_{2p} \cdot \Delta_{eq}$ $\qquad V_{eq} = 98.6 \text{ kip}$

Design Step 7.1.7

Axial and Flexural Components of Horizontal Shear Force V_a and V_m

A portion of the horizontal force is resisted by an axial load in the batter pile, while the remainder is resisted by flexure in the pile. The shear carried by each component is a simple ratio of its stiffness to the total stiffness.

Design Step 7.1.7 (continued)

The total horizontal stiffness of one 2:12 batter pile with pinned top is

$$k_{2p} = 67.1 \cdot \frac{kip}{in}$$

The horizontal component of the total stiffness due to axial stiffness in the pile is

$$ka_2 = 57.8 \cdot \frac{kip}{in}$$

The horizontal shear associated with the axial deformation is a ratio of the axial to total horizontal stiffness of the batter pile.

$$V_a := \frac{ka_2}{k_{2p}} \cdot V_{eq} \qquad V_a = 84.9\ kip$$

The horizontal component of the total stiffness due to flexural stiffness in the pile is

$$kf_{2p} = 9.3 \cdot \frac{kip}{in}$$

The horizontal shear associated with the flexural deformation is a ratio of the flexural to total horizontal stiffness of the batter pile.

$$V_m := \frac{kf_{2p}}{k_{2p}} \cdot V_{eq} \qquad V_m = 13.7\ kip$$

Check that the total horizontal seismic force equals Veq.

$$V := V_a + V_m \qquad V = 98.6\ kip \qquad \text{okay}$$

Design Step 7.1.8

Axial Forces

The resulting axial forces in the 2:12 batter pile are based on the shear force V_a and simple trigonometry.

$V_a = 84.9$ kip — Horizontal shear force

$\alpha_2 = 9.46 \cdot deg$ — Angle of batter

The elastic axial force in the pile is either tension or compression, depending on the direction of the EQ load.

$$P_a := \frac{V_a}{\sin(\alpha_2)}$$

$P_a = 516$ kip — Very large

Design Step 7.1.9

Pile Capacity, as Controlled by the Soil

From Design Step 1.5, the capacity of the pile (as controlled by the soil) is (-)135 kips in tension, and (+)528 kips in compression. On inspection, it can be seen that the required tension capacity of the soil will be much too large with this pile layout. Determine the order of magnitude of the final axial loads. The axial loads in the batter piles should not be reduced by the Response Modification Factor R. This criterion should not be applied to the axial component of batter piles in a pile bent. There is little energy dissipated in truss-type action of batter piles except if the soil fails. The full elastic axial forces are therefore used in this design example.

For Group VII loading, per AASHTO Division I-A, Article 6.2.2, the load factors are all 1.0. The maximum compression in the 2:12 batter piles, using the full elastic force, is

$P_{DL} = 128 \cdot kip$ — DL reaction in pile (unfactored)

$P_a = 516$ kip — Full elastic seismic axial load

$P_{compr} := (1.0 \cdot P_{DL}) + (1.0 \cdot P_a)$ — Maximum compressive axial load

$P_{compr} = 644$ kip — > 528 kip, therefore NG

Design Step 7.1.9 (continued)

The maximum tension in the 2:12 batter piles, under Group VII loading using the full elastic force is

$P_{ten} := P_{DL} - P_a$ Maximum tensile axial load

$P_{ten} = -388$ kip > (-)135 kip, therefore NG

Both the axial tension and compression exceed the soil capacity for the Bent 5 piles.

Design Step 7.2

Implications

The tension load exceeds the soil capacity by a large margin (-388 kip load verses -135 kip capacity). The compression load in the pile also exceeds the soil capacity (644 kip verses 528 kip).

Therefore, look at other steel pile layout options at a preliminary level.

- In Design Step 8, seven plumb piles are used in each bent and Bents 4, 5, and 6 are pinned to the superstructure.
- In Design Step 9, four 2:12 batter piles and three plumb piles are used in Bents 4, 5, and 6, and all three bents are pinned to the superstructure.

DESIGN STEP 8

BRIDGE OPTION NO. 5
Bents 4, 5, and 6 with All Steel Plumb Piles

Figure 21 shows the conceptual layout for Option No. 5. The three middle bents of the four-span bridge Unit No. 2 are pinned to the superstructure. All the piles in all the bents are plumb piles. The pile layout of Option No. 5 is the same as for Option No. 2 in Design Step 4, except that steel pile piles are used instead of concrete piles.

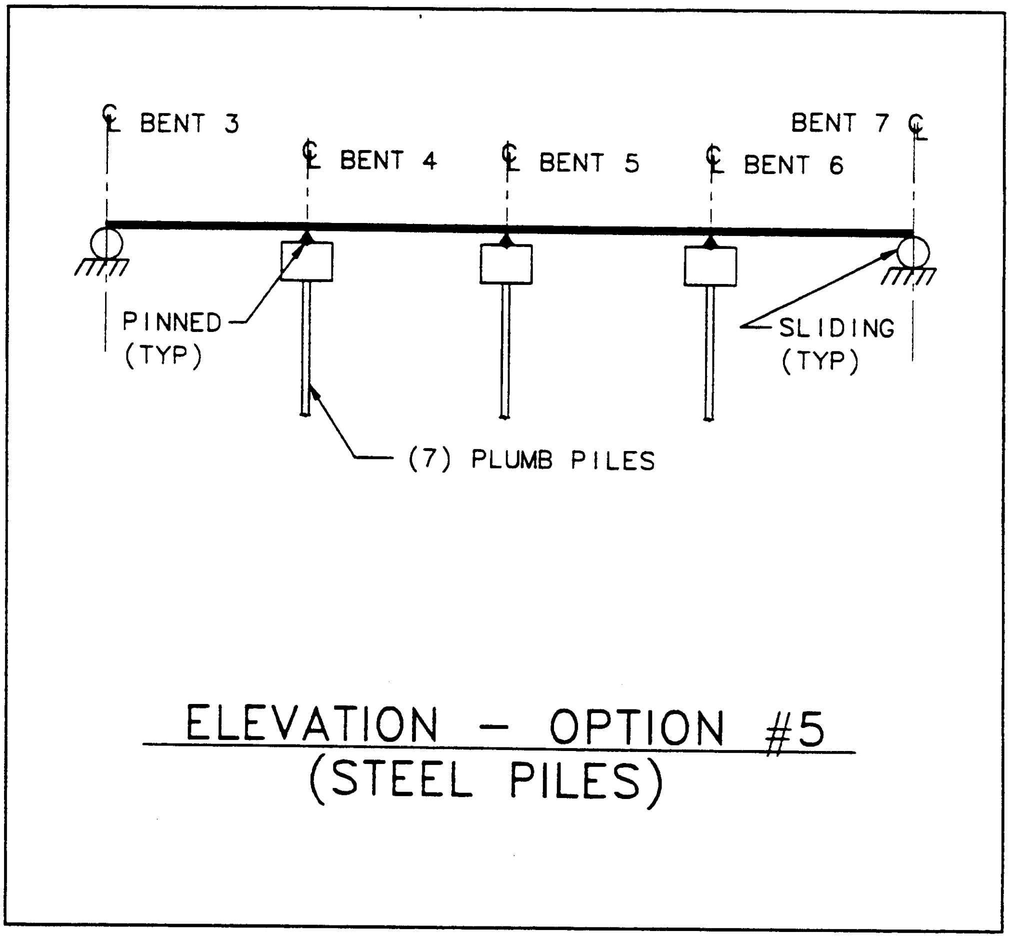


Figure 21 — Conceptual Layout of Piles, Option No. 5

Design Step 8.1 Longitudinal Seismic Force on Pile Bents 4, 5, and 6

The following items will be calculated for the four-span bridge Unit 2 structure. Note that only forces in the longitudinal direction are calculated in this design step. Forces for the transverse direction are calculated in Design Step 8.2.

1. Seismic mass
2. Total horizontal stiffness
3. Period of the structure
4. Total seismic shear force
5. Elastic seismic deflection
6. Shear force in the pile
7. Axial and flexural components of the horizontal shear force
8. Corresponding axial force and flexural moment in the pile

Design Step 8.1.1 Seismic Mass W_{unit}*g

In this design configuration, the participating mass of the system in the longitudinal direction consists of the following.

1. Four spans of the superstructure
2. The three cap beams of Bents 4, 5, and 6
3. The top 10 feet of 21 piles in Bents 4, 5, and 6

See Figure 22.

$W_{span} = 742.1$ kip — Weight of one span of the superstructure

$W_{cap} = 113.4 \cdot kip$ — Weight of one bent cap beam

$W_{pile} = 1.0 \cdot kip$ — Weight of one 10-foot length of pile

Therefore, the total dead load weight of the system, W_{unit}, is

$$W_{unit} := (4 \cdot W_{span}) + (3 \cdot W_{cap}) + (21 \cdot W_{pile})$$

$$W_{unit} = 3330 \text{ kip}$$

Design Step 8.1.1 (continued)

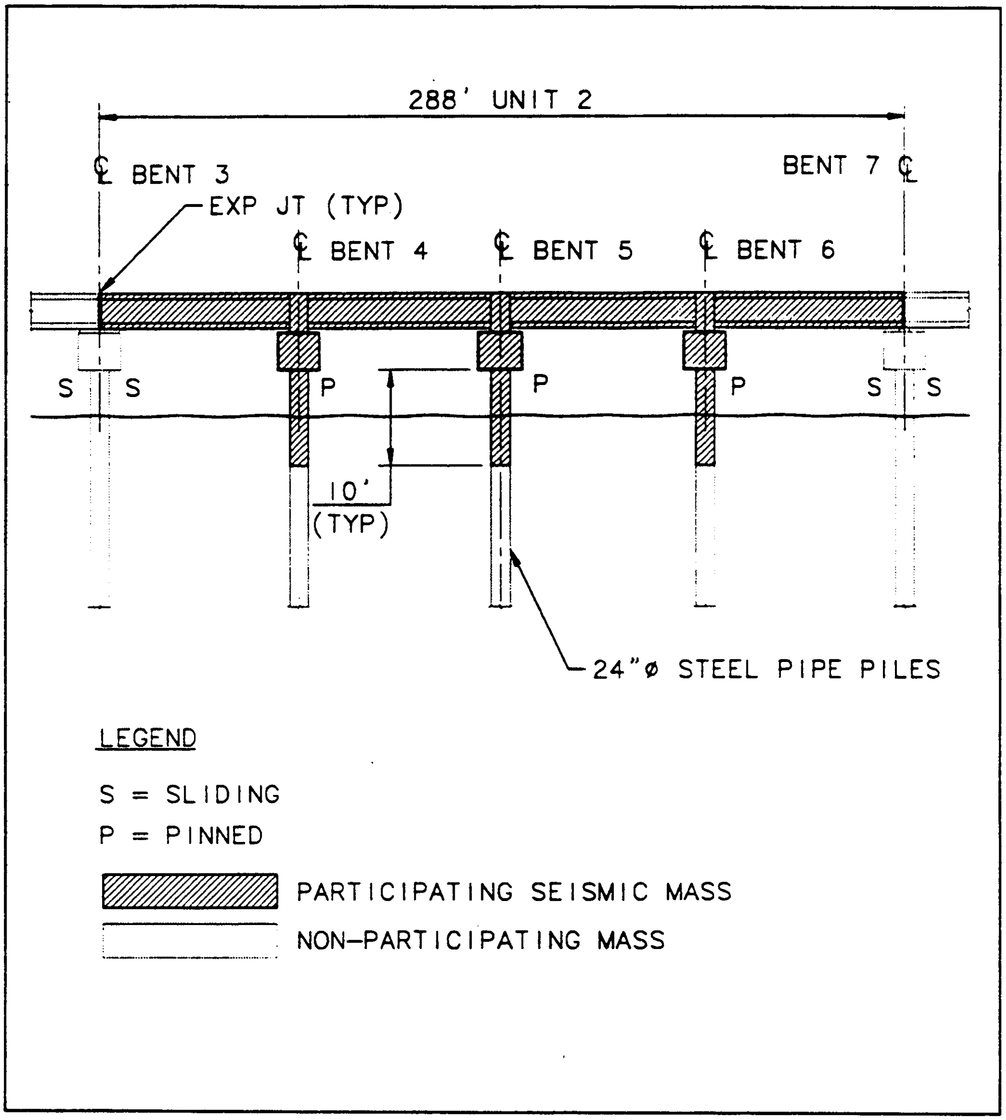


Figure 22 — Seismic Mass Used for Longitudinal Direction, Option No. 5

Design Step 8.1.2

Total Horizontal Stiffness k_{unit}

Calculate the total stiffness in the longitudinal direction. Bents 4, 5, and 6 are pinned to the superstructure; therefore, they all contribute to the longitudinal seismic stiffness. In addition, the weight of the pile caps and piles of these bents contribute to the seismic mass of the bridge structure. Refer to Table 5 in Design Step 6.3 for individual stiffness.

Design Step 8.1.2 (continued)

Total Horizontal Stiffness k_{unit}

Three bents of seven plumb piles = 21 total plumb piles, all with pinned tops.

$k_{Op} = 9.6 \cdot \frac{kip}{in}$ Horizontal stiffness of a plumb pile, pinned at the top

Total stiffness of the bridge Unit No. 2

$k_{unit} := 21 \cdot k_{Op}$ $k_{unit} = 202 \cdot \frac{kip}{in}$

See Table 7.

Table 7
Option No. 5, Longitudinal Stiffness

Bearing System Used: Sliding bearings at Bents 3 and 7, pinned at Bents 4 to 6
Pile Configuration: All plumb piles participate at Bents 4, 5, and 6

Steel Pile Option No. 5	Unit No. 2 w/ Steel Piles, Longitudinal Stiffness (kip/in)					
Bent Number Bearing Restraint at Bent	Bent 3 Sliding	Bent 4 Sliding	Bent 5 Pinned	Bent 6 Sliding	Bent 7 Sliding	Total
Number Plumb Piles, N_p	0	7	7	7	0	21
Horizontal Stiffness per Pile, k (kip/in)	0	9.6	9.6	9.6	0	
Stiffness of Plumb Piles = N_p*k (kip/in)	0	67.2	67.2	67.2	0	201.6
Number of 2:12 Batter Piles, N_b	0	0	0	0	0	0
Horizontal Stiffness per Pile, k	0	0	0.0	0	0	
Stiffness of Batter Piles = N_b*k (kip/in)	0	0	0.0	0	0	0.0
Longitudinal Horizontal Stiffness of Unit No. 2 Bridge (kip/in) =						202

Design Step 8.1.3

Period of the Structure T

$W_{unit} = 3330$ kip — Total weight of the unit

$k_{unit} = 202 \cdot \frac{kip}{in}$ — Horizontal stiffness of bridge Unit No. 2

$g := 32.2 \cdot \frac{ft}{sec^2}$ — Acceleration of gravity

The period of the structure in the longitudinal direction, per Equation 4-3 of Division I-A, is

$$T := 2 \cdot \pi \cdot \sqrt{\frac{W_{unit}}{k_{unit} \cdot g}} \qquad T = 1.30 \text{ sec}$$

Design Step 8.1.4

Total Seismic Shear Force Veq_L

$A := 0.10$ — Seismic acceleration coefficient

$S := 1.2$ — Soil site coefficient

The elastic seismic response coefficient is calculated from AASHTO Division I-A, Equation 3-1. Note that the units of seconds to the (2/3) power maintains consistent units in Mathcad®.

$$C_s := \frac{1.2 \cdot sec^{\frac{2}{3}} \cdot A \cdot S}{T^{\frac{2}{3}}} \qquad C_s = 0.121$$

Per AASHTO Division 1-A, Article 3.6.1, the maximum value of C_s need not exceed

$$C_{smax} := 2.5 \cdot A \qquad C_{smax} = 0.25$$

Therefore, use the actual value $\quad C_s = 0.121$

The total equivalent static earthquake force is then

$Veq_L := C_s \cdot W_{unit}$ $\qquad Veq_L = 403\ \text{kip}$

Design Step 8.1.5

Elastic Seismic Deflection ρeq

$k_{unit} = 202 \cdot \frac{kip}{in}$ $\qquad$ Horizontal stiffness of the unit

The elastic deflection due to the seismic shear force is then the force divided by the stiffness

$\Delta_{eq} := \frac{Veq_L}{k_{unit}}$ $\qquad \Delta_{eq} = 2.00\ \text{in}$

Note that this deflection is much larger than the deflection associated with Option No. 4, which has batter piles (2.05 verses 1.48 inches). This deflection must be accommodated at the expansion joints at Bents 3 and 7. The absolute value of the seismic deflection of the two adjoining bridge units must be added together. The gap between the ends of the superstructure units should be equal to or greater than this dimension.

Design Step 8.1.6

Shear Force in Each Plumb Pile Veq

This seismic load is distributed proportionately to each of the three bents resisting the seismic force based on their relative stiffnesses. Because the stiffnesses of all bents are equal, each will take an equal proportion of the seismic shear. Because all piles are identical, each will take an equal proportion of the bent seismic shear.

The flexural shear force resisted by each of the 21 plumb piles in the longitudinal direction is

$V_{eq} := \frac{Veq_L}{21}$ $\qquad V_{eq} = 19.2\ \text{kip}$

Design Step 8.1.6 (continued)

Shear Force in Each Plumb Pile Veq

This result should be the same as that yielded by multiplying the deflection times the stiffness of an individual plumb pile.

$k_{Op} = 9.6 \cdot \frac{kip}{in}$ Stiffness of a plumb pile

$\Delta_{eq} = 202\ in$ Seismic deflection of the bridge

$V_{eq} := k_{Op} \cdot \Delta_{eq}$ $V_{eq} = 19.2\ kip$ The same, okay

Design Step 8.1.7

Axial and Flexural Components of Horizontal Shear Force V_a and V_m

For Option No. 5, with all plumb piles, it is assumed that there are no axial forces due to seismic loads in the longitudinal direction.

$V_a := 0 \cdot kip$

$V_L := V_{eq}$ $V_L = 19.2\ kip$

Design Step 8.1.8

Axial Forces and Flexural Moment in Each Pile P_a and M_L

There are no axial forces in the plumb piles for Option No. 5.

$P_a := 0 \cdot kip$

The bending moment in each of these piles is based on the length in the soil for flexure, L_m, shown in Figure 13 in Design Step 4.1.8.

$E_p = 29000 \cdot ksi$

$I_p = 2550 \cdot in^4$

$n_h = 8.0 \cdot pci$

Design Step 8.1.8 (continued)

$n_h = 8.0 \cdot pci$

$$L_m := 0.78 \cdot \left(\frac{E_p \cdot I_p}{n_h}\right)^{\frac{1}{5}} \qquad L_m = 6.40 \cdot ft$$

Then the dimension Lf_m is conservatively taken as the flexural length along the pile, from the top of the pile cap to the assumed point of maximum bending moment.

$$Lf_m := L_m + 5 \cdot ft + 4 \cdot ft \qquad Lf_m = 15.4 \cdot ft$$

The moment in the longitudinal direction (assuming no fixity at the top of the pile) is

$$M_L := V_L \cdot Lf_m \qquad M_L = 296\ kip\ ft$$

Design Step 8.2

Transverse Seismic Force on Pile Bent 5

Only forces in the transverse direction are calculated in this design step. Forces for the longitudinal direction were calculated in Design Step 8.1.

In this design step, the following items will be calculated for the typical bent in the transverse direction, assuming that each bent takes a tributary load. Tributary load is used, because, by inspection, the transverse stiffness of the superstructure is so large by comparison to the transverse stiffness of the pile that the structure will move as a rigid body.

1. Seismic mass
2. Total horizontal stiffness
3. Period of the bent
4. Total seismic shear force
5. Elastic seismic deflection
6. Shear force in each pile
7. Corresponding axial force and flexural moment in the outboard piles

Design Step 8.2.1

Seismic Mass $P_{bent}*g$

In this design configuration, the participating mass of the bent in the transverse direction consists of the following.

1. One tributary span of the superstructure
2. One cap beam
3. The top 10 feet of seven piles in each bent

The tributary dead load at each pile bent system, P_{bent}, is

$W_{span} = 742.1\ kip$

$W_{cap} = 113.4 \cdot kip$

$W_{pile} = 1.0 \cdot kip$

$P_{bent} := W_{span} + W_{cap} + 7 \cdot W_{pile}$ $\qquad P_{bent} = 863\ kip$

Design Step 8.2.2

Total Horizontal Stiffness k_{bent}

Calculate the stiffness of a typical bent in the transverse direction. Each typical bent resists its own tributary seismic shear.

Each typical bent has seven plumb piles

$k_{Or} = 66.5 \cdot \frac{kip}{in}$ Horizontal stiffness of a plumb pile, fixed at the top

Stiffness of a typical bent

$k_{bent} := 7 \cdot k_{Or}$ $k_{bent} = 466 \cdot \frac{kip}{in}$

Design Step 8.2.3

Period of the Structure T

The tributary weight of the bent is

$P_{bent} = 863\ kip$

$g := 32.2 \cdot \frac{ft}{sec^2}$ Acceleration of gravity

The period of the structure in the transverse direction, per Equation 4-3 of Division I-A

$T := 2 \cdot \pi \cdot \sqrt{\frac{P_{bent}}{k_{bent} \cdot g}}$ $T = 0.44 \cdot sec$

Design Step 8.2.4

Seismic Shear Force Veq_T

$A := 0.10$ Seismic acceleration coefficient

$S := 1.2$ Soil site coefficient

Design Step 8.2.4 (continued)

The elastic seismic response coefficient is calculated from AASHTO Division I-A, Equation 3-1. Note that the units of seconds to the (2/3) power maintains consistent units in Mathcad®.

$$C_s := \frac{1.2 \cdot sec^{\frac{2}{3}} \cdot A \cdot S}{T^{\frac{2}{3}}} \qquad C_s = 0.251$$

Per AASHTO Division I-A, Article 3.6.1, the maximum value of C_s need not exceed

$$C_{smax} := 2.5 \cdot A \qquad C_{smax} = 0.25$$

Therefore, use the maximum value $\quad C_{smax} = 0.25$

The total equivalent static earthquake force is then

$$Veq_T := C_{smax} \cdot P_{bent} \qquad Veq_T = 216 \cdot kip$$

Design Step 8.2.5

Elastic Seismic Deflection ρeq

$$k_{bent} = 466 \cdot \frac{kip}{in}$$ Horizontal stiffness of the bent

The elastic deflection due to the seismic shear force is then the force divided by the stiffness.

$$\Delta_{eq} := \frac{Veq_T}{k_{bent}} \qquad \Delta_{eq} = 0.46 \cdot in$$

See Figure 14 in Design Step 4.2.5.

Design Step 8.2.6

Shear Force in Each Plumb Pile Veq

This seismic load is distributed evenly between the seven plumb piles in each bent.

Design Step 8.2.6 (continued)

Shear Force in Each Plumb Pile Veq

The flexural shear force resisted by each of the seven plumb piles in the transverse direction is

$$V_{eq} := \frac{Veq_T}{7} \qquad V_{eq} = 30.9 \cdot kip$$

This result should be the same as that yielded by multiplying the deflection times the stiffness of an individual plumb pile.

$k_{Or} = 66.5 \cdot \frac{kip}{in}$ Stiffness of plumb pile, fixed at the top

$\Delta_{eq} = 0.46 \cdot in$ Seismic deflection of the bridge

$V_{eq} := k_{Or} \cdot \Delta_{eq}$ $V_{eq} = 30.9 \cdot kip$ The same, okay

$V_T := V_{eq}$ $V_T = 30.9 \cdot kip$

Design Step 8.2.7

Corresponding Axial Forces and Flexural Moment in the Outboard Piles

a) Check the Outboard Piles for Axial Tension Due to Frame Action

$Veq_T = 216 \cdot kip$ Shear on the bent

The distance from the assumed point of fixity of the pile to the center of the mass of the superstructure is from Design Step 8.1.8.

$L_m = 6.40 \cdot ft$ Length due to flexure

$L := L_m + (5 \cdot ft) + (4 \cdot ft) + (3 \cdot ft)$ $L = 18.4 \cdot ft$

The overturning moment in the bent is

$M_{ot} := Veq_T \cdot L$ $M_{ot} = 3974\ kip\ ft$

Design Step 8.2.7 (continued)

Corresponding Axial Forces and Flexural Moment in the Outboard Piles

The section modulus of the seven piles resisting overturning in the transverse direction is

$$Adsq := 2 \cdot \left(18.75^2 + 12.5^2 + 6.25^2\right) \cdot ft^2$$

$$Adsq = 1094 \cdot ft^2$$

The outside piles are spaced a distance of

$z := 6 \cdot 6.25 \cdot ft$ $\qquad\qquad$ $z = 37.5 \cdot ft$

The outside piles will have a seismic axial load (tension) of

$Pa := \dfrac{M_{ot} \cdot \frac{z}{2}}{Adsq}$ $\qquad\qquad$ $Pa = 68.1\ kip$ $\qquad$ Okay

$P_{DL} = 128.5 \cdot kip$ $\qquad\qquad$ Dead load of pile

$P_{ten} = P_{DL} - Pa$ $\qquad\qquad$ > 0; therefore, zero tension

The axial load due to the transverse seismic shear is small, and does not overcome the axial dead load in the piles. Because it was assumed there is no axial load in the plumb piles due to longitudinal seismic forces, the combined axial load effect is okay by inspection.

b) The Bending Moment in Each of These Piles Is Based on the Length in the Soil for Flexure, L_m

The equivalent length of pile in the soil, L_m, to approximate the flexural length of the pile, from Design Step 8.1.8, is

$$L_m = 6.4 \cdot ft$$

Design Step 8.2.7 (continued)

The dimension Lf_m is the flexural length along the pile, from the bottom of the pile cap to the assumed point of fixity in the soil, for bending moment.

$Lf_m := L_m + 5 \cdot ft$ $\qquad$ $Lf_m = 11.4 \cdot ft$

The moment in the transverse direction is approximately

$M_T := V_{eq} \cdot \frac{Lf_m}{2}$ $\qquad$ $M_T = 176 \cdot kip \cdot ft$

Design Step 8.3

Combination of Orthogonal Forces
[AASHTO Division I-A, Article 3.9]

Because there is no cross coupling of forces in the transverse and longitudinal directions (no transverse moments caused by longitudinal forces and vice versa), all the piles are assumed to carry the same shear and moment.

Design Step 8.3.1

Pile Shear

A summary of the shear forces on a Bent 5 pile is given below.

$V_L = 19.2\ kip$ Longitudinal shear on pile

$V_T = 30.9 \cdot kip$ Transverse shear on pile

a) Load Case 1 (100 Percent Global Longitudinal plus 30 Percent Global Transverse)

Resulting longitudinal shear force

$V_{1L} := (1.0 \cdot V_L)$ $V_{1L} = 19.2\ kip$

Resulting transverse shear force

$V_{1T} := (0.3 \cdot V_T)$ $V_{1T} = 9.3 \cdot kip$

b) Load Case 2 (30 Percent Global Longitudinal plus 100 Percent Global Transverse)

Resulting longitudinal shear force

$V_{2L} := (0.3 \cdot V_L)$ $V_{2L} = 5.8\ kip$

Resulting transverse shear force

$V_{2T} := (1.0 \cdot V_T)$ $V_{2T} = 30.9 \cdot kip$

Design Step 8.3.2

Pile Moment

A summary of the moment forces on Bent 5 pile is given below.

$M_L = 296\ \text{ft kip}$ — Longitudinal moment on pile

$M_T = 176 \cdot \text{ft} \cdot \text{kip}$ — Transverse moment on pile

a) Load Case 1 (100 Percent Global Longitudinal plus 30 Percent Global Transverse)

Resulting longitudinal moment

$M_{1L} := (1.0 \cdot M_L)$ $\qquad M_{1L} = 296\ \text{ft kip}$

Resulting transverse moment

$M_{1T} := (0.3 \cdot M_T)$ $\qquad M_{1T} = 52.8 \cdot \text{ft} \cdot \text{kip}$

b) Load Case 2 (30 Percent Global Longitudinal plus 100 Percent Global Transverse)

Resulting longitudinal moment

$M_{2L} := (0.3 \cdot M_L)$ $\qquad M_{2L} = 88.8\ \text{ft kip}$

Resulting transverse moment

$M_{2T} := (1.0 \cdot M_T)$ $\qquad M_{2T} = 176.0 \cdot \text{ft} \cdot \text{kip}$

Design Step 8.4

Modified Design Forces in Bent 5 Plumb Pile

[AASHTO Division I-A, Article 6.2.2]

Design Step 8.4.1

Summary of Elastic Forces

a) Elastic Shear Force in Pile

Summary for Load Case 1

$V_{1T} = 9.3 \cdot kip$ $\qquad$ $V_{1L} = 19.2\ kip$

For preliminary design, it is conservative to calculate the resultant shear in the pile as shown below, and check it against the shear capacity in the major axes. The resultant shear is

$V_{u1} := \sqrt{V_{1T}^2 + V_{1L}^2}$ $\qquad$ $V_{u1} = 21.3\ kip$

Summary for Load Case 2

$V_{2T} = 30.9 \cdot kip$ $\qquad$ $V_{2L} = 5.9 \cdot kip$

Again, find the resultant shear force.

$V_{u2} := \sqrt{V_{2T}^2 + V_{2L}^2}$

$V_{u2} = 31.4 \cdot kip$ $\qquad$ <--- Controls

b) Elastic Biaxial Moments in Pile

Summary for Load Case 1

$M_{1T} = 52.8 \cdot ft \cdot kip$ $\qquad$ $M_{1L} = 296\ ft\ kip$

Design Step 8.4.1 (continued)

Refer to Design Step 4.4.1(b) for commentary on calculating the resultant moment.

$$M_{u1} := \sqrt{M_{1T}^2 + M_{1L}^2}$$

$M_{u1} = 301\ \text{ft kip}$ <--- Controls

Summary for Load Case 2

$M_{2T} = 176 \cdot \text{ft} \cdot \text{kip}$ $M_{2L} = 88.8\ \text{ft kip}$

Again, find the resultant moment force.

$$M_{u2} := \sqrt{M_{2T}^2 + M_{2L}^2}$$ $M_{u2} = 197\ \text{ft kip}$

Design Step 8.4.2

Modified Design Forces [AASHTO Division I-A, Article 6.2.1]

According to Table 3 of AASHTO Division I-A, Article 3.7, the R Factor for steel pile bents, with all vertical piles, is R = 5. Per the exception noted in this referenced section, the full R value is used for pile bents. Assume that the dead load shear and moment forces equal zero in the piles.

a) Modified Shear Forces in Pile V_u

$V_{u2} = 31.4 \cdot \text{kip}$ Controlling resultant shear force in pile

$R := 5$ Response Modification Reduction Factor

$$V_u := \frac{V_{u2}}{R}$$ $V_u = 6.3 \cdot \text{kip}$

It should be noted that discussion is presently taking place regarding whether the shear force should be divided by R for shear in SPC B. See Gieger et al.

Design Step 8.4.2 (continued)

b) Modified Moment Forces in Pile M_u

The controlling resultant moment is from Load Case 1.

$M_{u1} = 301$ kip ft — Controlling resultant moment in pile

$R = 5$ — Response Modification Reduction Factor

$$M_u := \frac{M_{u1}}{R} \qquad M_u = 60 \text{ ft kip}$$

Design Step 8.5

Design the Bent 5 Piles

Design Step 8.5.1

Shear Design

The shear design of the pile is not shown in this design example.

Design Step 8.5.2

Design the Pile for Flexure

The maximum factored moment in the longitudinal direction is

$M_u = 60$ kip ft

The section modulus of the 24-inch pipe pile is

$S := 212.5 \cdot in^3$

The bending stress in the pile is

$$f_b := \frac{M_u}{S} \qquad f_b = 3.4 \text{ ksi}$$

The bending stress is so low, that by inspection, the combined stresses are okay. Because of the short clear height above the soil, it was assumed that the moment magnification is small, and that the pile is supported by the soil. The dead load times the elastic deflection was not included in this design example.

DESIGN STEP 9

BRIDGE OPTION NO. 6
Bents 4, 5, and 6 with 2:12 Steel Batter Piles

Figure 23 shows the conceptual layout for Option No. 6. All the middle bents of the four-span Bridge Unit 2 are pinned to the superstructure. Four of the seven piles in the bents are 2:12 batter piles. The other three are plumb piles. The pile layout of Option No. 6 is the same as for Option No. 3 in Design Step 5, except that steel pile piles are used instead of concrete piles.

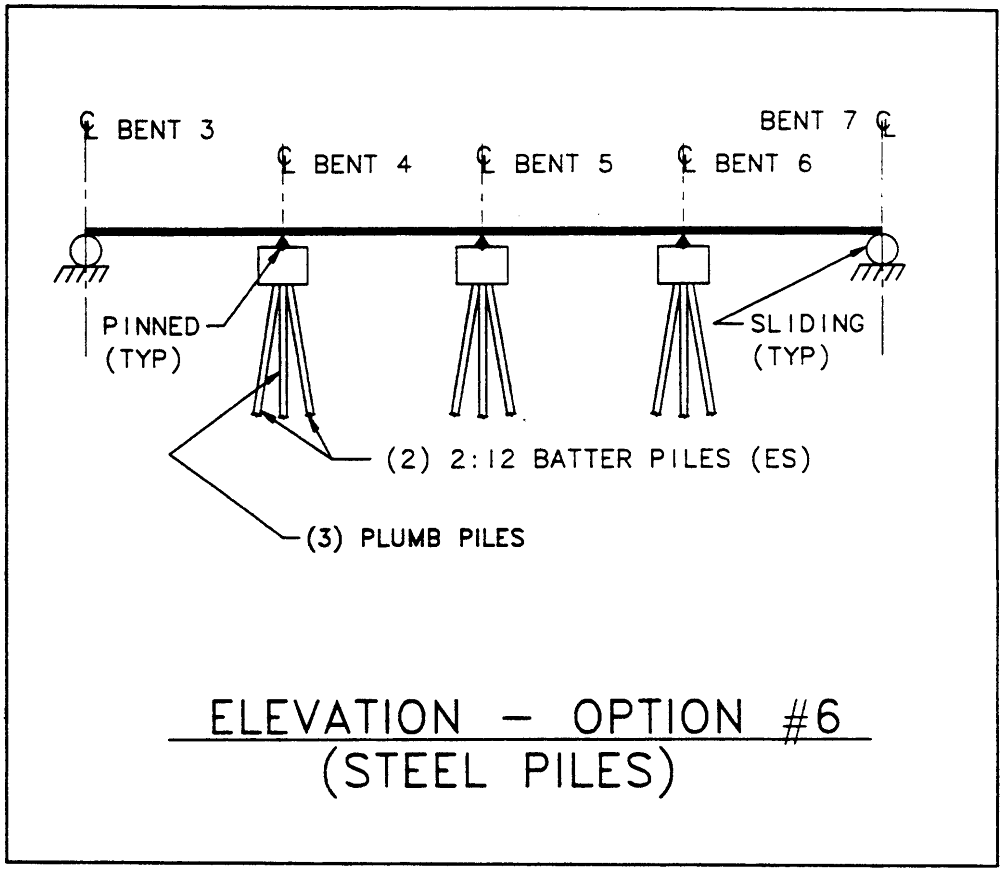


Figure 23 — Conceptual Layout of Piles, Option No. 6

Design Step 9.1

Longitudinal Seismic Force on Pile Bent 5

The following items will be calculated for the four-span bridge Unit 2 structure. Note that only forces in the longitudinal direction are calculated in this design step. Forces for the transverse direction are calculated in Design Step 8.2.

Design Step 9.1 (continued)

1. Seismic mass
2. Total horizontal stiffness
3. Period of the structure
4. Total seismic shear force
5. Elastic seismic deflection
6. Shear force in the batter pile
7. Axial and flexural components of the horizontal shear force
8. Corresponding axial force in the pile
9. Pile capacity as controlled by the soil

Design Step 9.1.1

Seismic Mass $W_{unit}*g$

The seismic mass for Option No. 6 is the same as for Option No. 5. See Design Step 8.1.1 and Figure 24.

$W_{unit} := 3330 \cdot kip$

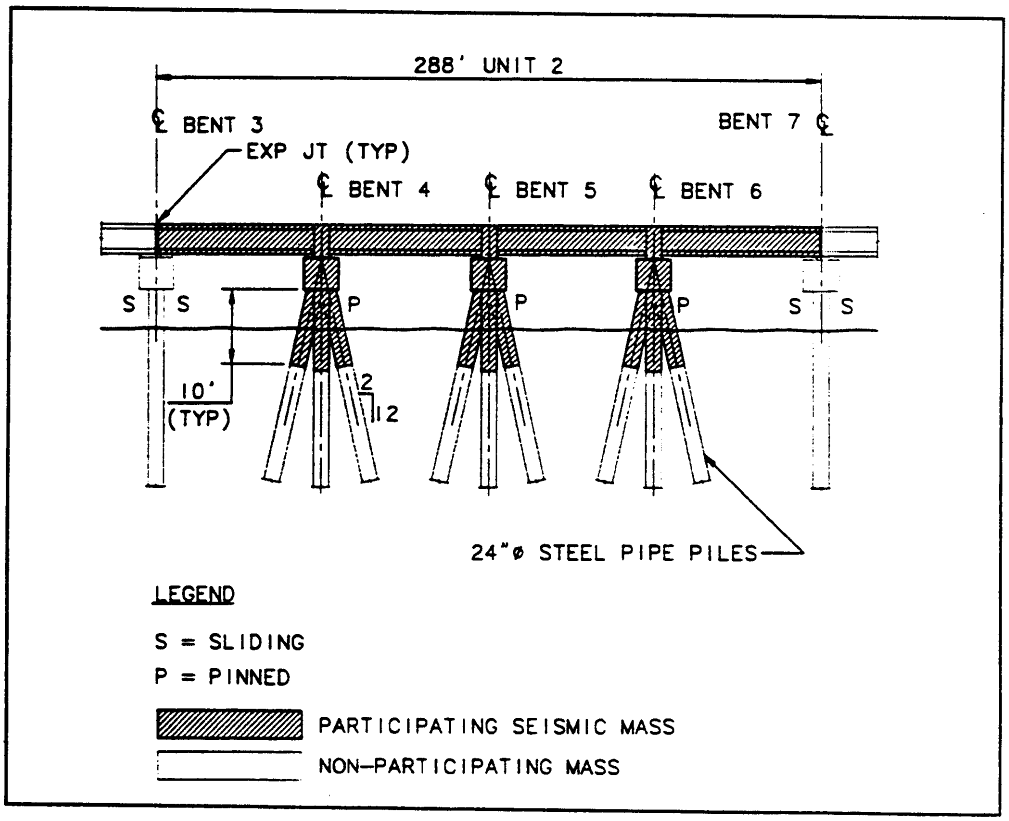


Figure 24 — Seismic Mass Used for Longitudinal Direction, Option No. 6

Design Step 9.1.2

Total Horizontal Stiffness k_{unit}

Calculate the total stiffness of bridge Unit 2 in the longitudinal direction. Bents 4, 5, and 6 are pinned to the superstructure; therefore, they all contribute to the longitudinal seismic stiffness. Refer to Table 5 in Design Step 6.3 for individual stiffnesses. Each of the three bents consists of four piles with 2:12 batter plus three plumb piles.

$k_{2p} = 67.1 \cdot \frac{kip}{in}$ Horizontal stiffness of a 2 on 12 batter pile, pinned at the top

$k_{0p} = 9.6 \cdot \frac{kip}{in}$ Horizontal stiffness of a plumb pile, pinned at the top

Total stiffness of bridge Unit No. 2 is

$k_{unit} := 3 \cdot \left(4 \cdot k_{2p} + 3 \cdot k_{0p}\right)$ $k_{unit} = 892 \cdot \frac{kip}{in}$

See Table 8.

Design Step 9.1.2 (continued)

Table 8
Option No. 6, Longitudinal Stiffness

Bearing System Used: Sliding bearings at Bents 3 and 7, pinned at Bents 4 to 6
Pile Configuration: (4) 2:12 batter piles + (3) equivalent plumb piles at Bents 4, 5, and 6

Steel Pile Option No. 6	Unit No. 2 w/Steel Piles, Longitudinal Stiffness (kip/in)					
Bent Number	Bent 3	Bent 4	Bent 5	Bent 6	Bent 7	Total
Bearing Restraint at Bent	Sliding	Sliding	Pinned	Sliding	Sliding	
Number Plumb Piles, N_p	0	3	3	3	0	9
Horizontal Stiffness per Pile, k (kip/in)	0	9.6	9.6	9.6	0	
Stiffness of Plumb Piles = N_p*k (kip/in)	0	28.8	28.8	28.8	0	86.4
Number of 2:12 Batter Piles, N_b	0	4	4	4	0	12
Horizontal Stiffness per Pile, k	0	67.1	67.1	67.1	0	
Stiffness of Batter Piles = N_b*k (kip/in)	0	268.4	268.4	268.4	0	805.2
Longitudinal Horizontal Stiffness of Unit No. 2 Bridge (kip/in) =						892

Design Step 9.1.3

Period of the Structure T

$W_{unit} := 3330 \cdot kip$ — Total weight of the unit from Design Step 8.1.1

$g := 32.2 \cdot \frac{ft}{sec^2}$ — Acceleration of gravity

The period of the structure in the longitudinal direction, per Equation 4-3 of Division I-A, is

$$T := 2 \cdot \pi \cdot \sqrt{\frac{W_{unit}}{k_{unit} \cdot g}} \qquad T = 0.62\ sec$$

Design Step 9.1.4

Total Seismic Shear Force Veq_L

$A := 0.10$ Seismic acceleration coefficient

$S := 1.2$ Soil site coefficient

The elastic seismic response coefficient is calculated from AASHTO Division I-A, Equation 3-1. Note that the units of seconds to the (2/3) power maintains consistent units in Mathcad®.

$$C_s := \frac{1.2 \cdot sec^{\frac{2}{3}} \cdot A \cdot S}{T^{\frac{2}{3}}} \qquad C_s = 0.198$$

Per AASHTO Division I-A, Article 3.6.1, the maximum value of C_s need not exceed

$$C_{smax} := 2.5 \cdot A \qquad C_{smax} = 0.25$$

Therefore, use the actual value $C_s = 0.198$

The total equivalent static earthquake force is then

$$Veq_L := C_{smax} \cdot W_{unit} \qquad Veq_L = 659\ kip$$

Design Step 9.1.5

Elastic Seismic Deflection ρeq

$k_{unit} = 892 \cdot \frac{kip}{in}$ Horizontal stiffness of the unit

The elastic deflection due to the seismic shear force is, then, the force divided by the stiffnesses. See Figure 10 in Design Step 3.1.5.

$$\Delta_{eq} := \frac{Veq_L}{k_{unit}} \qquad \Delta_{eq} = 0.74\ in$$

Design Step 9.1.6

Shear Force in the Batter Pile Veq

The total seismic shear is distributed proportionately to each pile based on its relative stiffness. The force in each pile can be calculated by multiplying the stiffness of the individual pile by the total deflection. The resulting horizontal force resisted by each 2:12 batter pile with a pinned top is

$k_{2p} = 67.1 \cdot \frac{kip}{in}$ Stiffness of a 2:12 batter pile

$\Delta_{eq} = 0.74\ in$ Seismic deflection of the bridge

$V_{eq} := k_{2p} \cdot \Delta_{eq}$ $V_{eq} = 49.7\ kip$

Design Step 9.1.7

Axial and Flexural Components of Horizontal Shear Force V_a and V_m

A portion of the horizontal force is resisted by an axial load in the batter piles while the remainder is resisted by flexure in the pile. Before the axial force and flexural moment can be calculated, the relative stiffness of the axial and flexural contribution to the horizontal stiffness must be considered. The shear carried by each component is expressed as a simple ratio of its stiffness to the total stiffness

Total stiffness of one 2:12 batter pile with pinned top

$k_{2p} = 67.1 \cdot \frac{kip}{in}$

The horizontal component of total stiffness due to axial stiffness in the pile is

$ka_2 = 57.8 \cdot \frac{kip}{in}$

Design Step 9.1.7 (continued)

The horizontal shear associated with the axial deformation is expressed as a ratio of the axial to total horizontal stiffness of the batter pile.

$$V_a := \frac{ka_2}{k_{2p}} \cdot V_{eq} \qquad V_a = 42.8 \text{ kip}$$

The horizontal component of total stiffness due to flexural stiffness in the pile is

$$kf_{2p} = 9.3 \cdot \frac{\text{kip}}{\text{in}}$$

The horizontal shear associated with the flexural deformation is expressed as a ratio of the flexural to total horizontal stiffness of the batter pile. This force is used to calculate the flexural shear and moment in the pile.

$$V_m := \frac{kf_{2p}}{k_{2p}} \cdot V_{eq} \qquad V_m = 6.9 \text{ kip}$$

Check that the total horizontal seismic force equals Veq.

$$V := V_a + V_m \qquad V = 49.7 \text{ kip} \qquad \text{okay}$$

Design Step 9.1.8

Axial Forces and Flexural Moment in the Piles

The resulting axial forces in the 2 on 12 batter pile are based on the shear force V_a and simple trigonometry.

$V_a = 49.7$ kip — Horizontal shear force

$\alpha_2 = 9.46 \cdot deg$ — Angle of batter

The elastic axial force in the pile is either tension or compression, depending on the direction of the EQ load.

$$P_a := \frac{V_a}{\sin(\alpha_2)} \qquad P_a = 302 \text{ kip}$$

Design Step 9.1.9

Pile Capacity, As Controlled by the Soil

From Design Step 1.5, the capacity of the soil is (-)135 kips in tension, and (+)528 kips in compression. Determine the magnitude of the final axial loads.

The Response Modification Factor R should not apply to the axial component of the batter piles in a pile bent. There is little energy dissipated in truss-type action of batter piles. The full elastic axial forces are, therefore, used.

For Group VII loading, per AASHTO Division I-A, Article 6.2.2, the load factors are all 1.0. The maximum compression in the 2:12 batter piles, under Group VII loading using the full elastic force, is

$P_{DL} = 128 \cdot kip$ Dead load reaction in a pile (unfactored)

$P_a = 302\ kip$ Full elastic seismic axial load

$P_{compr} := (1.0 \cdot P_{DL}) + (1.0 \cdot P_a)$ Maximum compressive axial load

$P_{compr} = 430\ kip$ < 528 kip, therefore, ok

The maximum tension in the 2:12 batter piles, under Group VII loading using the full elastic force is

$P_{ten} := P_{DL} - P_a$ Maximum tensile axial load

$P_{ten} = -174\ kip$ > (-)135 kip, close but NG

The maximum compression load in the pile is less than the soil compression capacity, but the tension capacity of the soil exceeds the capacity by a small amount.

Therefore, go on to Design Step 9.3 for the implications of the pile layout for Option No. 6.

Design Step 9.2

Implications

The full elastic forces were used in the analysis in Design Step 9.1.8(a). The capacity of the soil exceeded or was close to the loads. The pile length could be increased slightly to provide the necessary capacity.

Therefore, this pile layout can be made to resist the seismic forces of the bridge. Unlike Option No. 4 where only one bent had batter piles, this option contains more batter piles to share the horizontal shear force. The additional length of pile required to resist the tension loads is reasonable.

DESIGN STEP 10

SUMMARY OF RESULTS

Six pile options were looked at in the preliminary design: three pile layouts with concrete piles and three with steel piles. Figure 25 shows a summary of the conceptual pile layout options. Table 9 is a summary of the number of restrained piles in each bent.

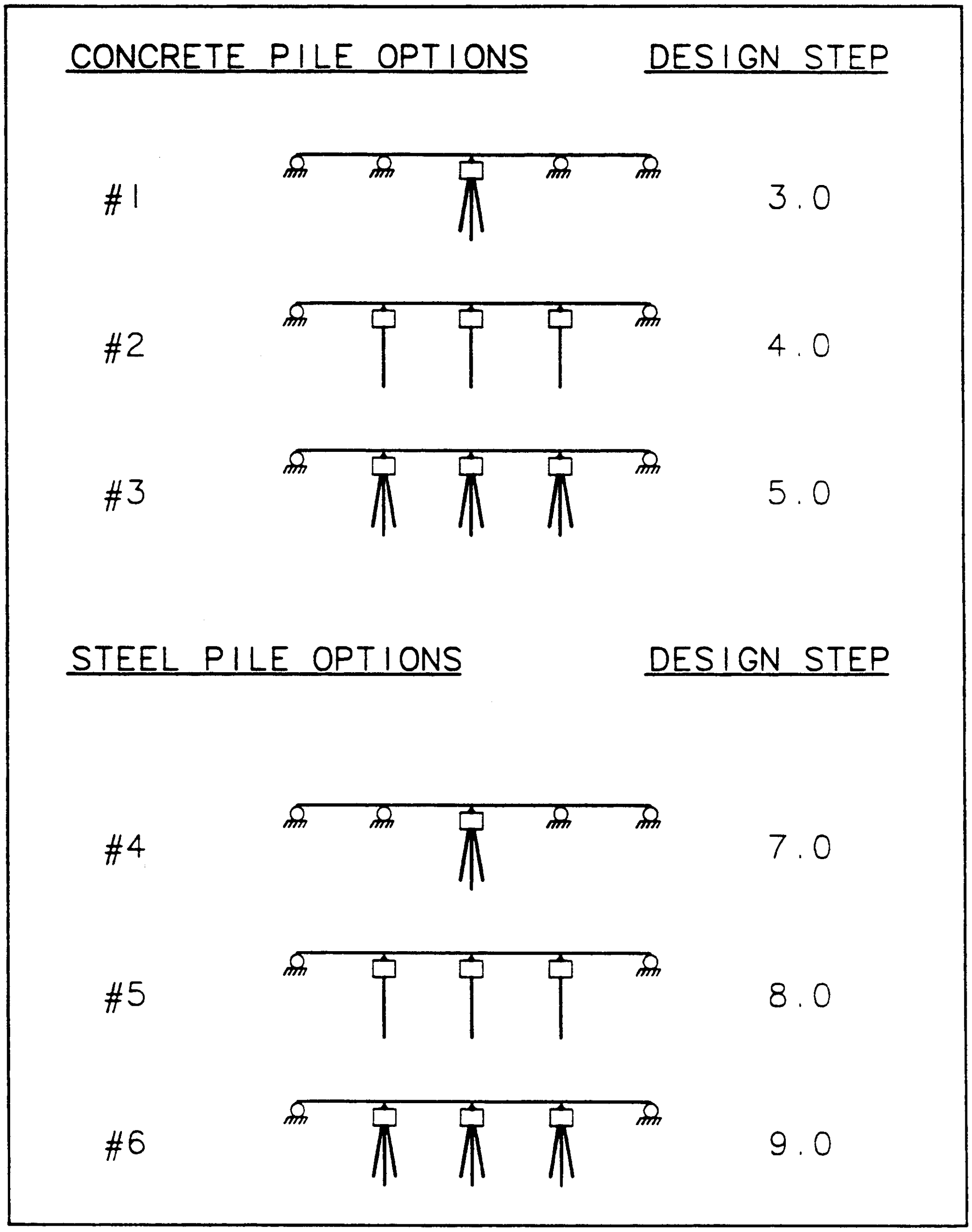


Figure 25 — Summary of Pile Layout Options

DESIGN STEP 10
(continued)

Table 9
Summary of Restrained Piles

Pile Layout Option No.	Pile Material	No. of Bents Restrained Longit.	No. of Longitudinally Restrained Piles	
			Plumb	2:12 Batter
1	Concrete	1	3	4
2		3	21	0
3		3	9	12
4	Steel	1	3	4
5		3	21	0
6		3	9	12

Below are two tables that summarize the analysis results. Table 10 summarizes the results for bridge Option Nos. 1, 2, and 3 using 24-inch-square concrete piles. Table 11 summarizes the results for bridge Option Nos. 4, 5, and 6 using 24-inch-diameter steel pipe piles.

An important observation is that the decision to use batter piles must be made carefully in a seismic region. This is not to say they should not be used. There are nonseismic issues not addressed in this example that may make it desirable to use batter piles. However, if batter piles are used, the designer must be aware of the seismic issues. This effect becomes more important as the seismic acceleration coefficient increases.

In this design example, even with a very low seismic coefficient of A = 0.10, all four options with batter piles (Option Nos. 1, 3, 4, and 6) had pile loads that exceeded their capacity. All had tension loads in the pile that were too large. The all-batter pile options (Nos. 3 and 6) had compression loads that are acceptable, and tension loads that are manageable with longer piles.

DESIGN STEP 10
(continued)

Table 10
Summary of Option Nos. 1, 2, and 3
with Concrete Piles

Concrete Pile Options	Units	Longitudinal Direction			Transverse
		Option No. 1	Option No. 2	Option No. 3	Direction
Seismic Mass, W	kip	3124	3435	3435	898
Total Stiffness, k	kip/in	587	258	1761	583
Period, T	sec	0.74	1.17	0.45	0.40
Seismic Response Coef, C_s	(Max. 0.25)	0.176	0.13	0.246	0.25
Total Seismic Shear, V_{eq}	kip	550	447	845	225
Elastic Deflection, Δ	in	0.94	1.73	0.48	0.39
Seismic Shear per Pile, V_{eq}	kip	129.3	21.3	66.0	32.1
Seismic Axial Load in Pile, P_a	kip	718	NA	366	73
Max. Pile Tension, P_{ten}	kip	-590	NA	-238	NA
Max. Pile Compression, P_{compr}	kip	846	NA	494	NA
Max. Pile Moment, M	kip-ft	NA	340	NA	192

DESIGN STEP 10
(continued)

Table 11
Summary of Option Nos. 4, 5, and 6 with Steel Piles

Steel Pile Options	Units	Longitudinal Direction			Transverse Direction
		Option No. 4	Option No. 5	Option No. 6	
Seismic Mass, W	kip	3089	3330	3330	863
Total Stiffness, k	kip/in	297	202	892	466
Period, T	sec	1.03	1.30	0.62	0.44
Seismic Response Coef, C_s	(Max. 0.25)	0.141	0.121	0.198	0.25
Total Seismic Shear, Veq	kip	436	403	659	216
Elastic Deflection, Δ	in	1.47	2.00	0.74	0.46
Seismic Shear per Pile, Veq	kip	98.6	19.2	49.7	30.9
Seismic Axial Load in Pile, P_a	kip	516	NA	302	68
Max. Pile Tension, P_{ten}	kip	-388	NA	-174	NA
Max. Pile Compression, P_{compr}	kip	644	NA	430	NA
Max. Pile Moment, M	kip-ft	NA	296	NA	176

DESIGN STEP 10
(continued)

In Option Nos. 1 and 4, using only one bent with batter piles to resist all the longitudinal seismic shear, the pile tensions were very large in comparison to the capacities. Any time that batter piles are present, the resulting stiffness of the system will be large, which in turn results in a large seismic shear. If batter piles are used, there should be enough of them to share the load.

Option Nos. 2 and 5, with all plumb piles in the bents, had low axial loads and acceptable moments in the piles. Because of the flexibility in the system, the stiffness was low enough not to attract large seismic shears. However, if the clear height of the piles were much longer, it would become more difficult to use all plumb piles without going to a larger pile diameter. The disadvantage of using all plumb piles is that the deflections are much larger than with the stiffer batter pile systems.

In this example, where the clear distance between the top of the pile and the mudline is small, the flexural stiffness of the pile can be significant relative to the magnitude of the horizontal component caused by the axial stiffness. With tall clear height piles, the flexural component becomes insignificant compared to the stiff axial load path.

Appendix A
Geotechnical Data

APPENDIX A GEOTECHNICAL DATA

SUBSURFACE CONDITIONS

Subsurface conditions were derived from two borings drilled along the bridge alignment. As shown on Figure A1, the subsurface conditions consist of 200 feet of fine-grained, cohesionless alluvial deposits consisting of alternating layers of medium dense, silty fine sand, and fine sandy silt. The water table is essentially at the ground surface.

SOIL PROPERTIES

Soil properties for the subsurface materials are shown on Figure A1. These properties were estimated from empirical correlations to the standard penetration test resistance values (N-values) in the borings. Laboratory tests may provide more detailed design values.

SOIL PROFILE TYPE

Type II — Deep cohesionless where the soil depth exceeds 200 feet and the soil types overlying the rock are stable deposits of sand and gravel.

SITE ACCELERATION

0.10g — Taken from AASHTO seismicity map.

FOUNDATION DESIGN

Abutments

Axial capacity based on U.S. Army Corps of Engineers (1991).

Alternative 1: 24-inch-square prestressed concrete pile foundations, 60-foot design length, embedded 55 feet in the soil.

Tension

Critical Depth = 15 x diameter = 30 feet (assumed for medium dense sand)

$$Q_{T\,ult} = f_s pL = K_t \sigma_{v\,avg} (\tan \delta)\, pL$$

where

$Q_{T\,ult}$ ultimate tension capacity of single pile (kips)

K_t coefficient of lateral earth pressure (assumed as 0.65 for tension)

$\sigma_{v\,avg}$ average effective vertical stress over the length of the pile; effective stress increases linearly to the critical depth of 30 feet and is constant below this depth (60´-5´-30´ = 25´) (ksf)

FOUNDATION DESIGN (continued)

δ	angle of friction between soil and steel pile (= 0.9ϕ)
p	surface area of pile (8 feet)
L	length of embedment of pile below ground surface (feet)

$$Q_{T\,ult} = 0.65\,(30' \times 0.0576\text{kcf} \times 1/2)\,[\tan(0.9 \times 34)]\,(8')(30') + 0.65\,(30' \times 0.0576\text{kcf})[\tan(0.9 \times 34)](8')(25') = 79.7 + 132.8 = 213 \text{ kips}$$

Compression

$$Q_{C\,ult} = A_t q + K_c \sigma_{v\,avg}\,(\tan\delta)\,pL$$

where

$Q_{C\,ult}$	ultimate compression capacity of single pile (kips)
K_c	coefficient of lateral earth pressure (assumed as 1.5 for compression)
q_t	tip resistance (ksf) $\sigma_v N_q$ = where $N_q = 40$ for $\phi = 34°$
A_t	area of tip of pile = 4ft^2

$$Q_{C\,ult} = (4\text{ ft}^2)(30' \times 0.0576\text{kcf})(40) + (1.5)(30' \times 0.0576\text{kcf} \times 1/2)[\tan(0.9 \times 34)](8')(30') + (1.5)(30' \times 0.0576\text{kcf})[\tan(0.9 \times 34)](8')(25') = 276.5 + 183.9 + 306.6 = 767 \text{ kips}$$

Alternative 2: 24-inch-diameter closed-end pipe pile foundations, 60-foot design length, embedded 55 feet in the soil.

Tension

Critical Depth = 15 x diameter = 30 feet (assumed for medium design sand)

$$Q_{T\,ult} = f_s pL = K_t \sigma_{v\,avg}\,(\tan\delta)\,pL$$

FOUNDATION DESIGN (continued)

where

$Q_{T\,ult}$ ultimate tension capacity of single pile (kips)

K_t coefficient of lateral earth pressure (assumed as 0.65 for tension)

$\sigma_{v\,avg}$ average effective vertical stress over the length of the pile; effective stress increases linearly to the critical depth of 30 feet and is constant below this depth (60′-5′-30′ = 25′) (ksf)

δ average of the angle of friction between soil and steel pile (= 0.9ϕ)

p surface area of pile (6.28 feet)

L length of embedment of pile below ground surface (feet)

$Q_{T\,ult}$ = 0.65 (30′ x 0.0576kcf x 1/2)[tan (0.75 x 34)](6.28′)(30′)
+ 0.65 (30′ x 0.0576kcf)[tan (0.75 x 34)](6.28′)(25′)
= 50.5 + 84.1 = 135 kips

Compression

$$Q_{C\,ult} = A_t q + K_c \sigma_{v\,avg} (\tan \delta)\, pL$$

Where:

$Q_{C\,ult}$ ultimate compression capacity of single pile (kips)

K_c coefficient of lateral earth pressure (assumed as 1.5 for compression)

q tip resistance (ksf)
$\sigma_v N_q$ = where N_q = 40 for ϕ = 34°

A_t area of tip of pile = 3.14 ft^2

$Q_{C\,ult}$ = (3.14 ft^2)(30′ x 0.0576kcf)(40)
+ (1.5)(30′ x 0.0576kcf x 1/2)[tan (0.75 x 34)](6.28′)(30′)
+ (1.5)(30′ x 0.0576kcf)[tan (0.75 x 34)](6.28′)(25′)
= 217.0 + 116.5 + 194.1 = 528 kips

FOUNDATION DESIGN (continued)

Lateral Pile Resistance

The constant of horizontal subgrade reaction, n_h, depends on the relative density of the soil, the location of the groundwater table, the nature of the loading (static versus cyclic), the spacing between adjacent piles, and the slope of the ground in front of the piles. For instance, the value of n_h of submerged sand is about one-half the value of dry sand. Similarly, then value of n_h for cyclic loading conditions is about one-half the value for static loading conditions for certain methods of analysis. In addition, some methods reduce the value of n_h if the pile spacing in the direction of loading is less than about 6- to 8-pile diameters. Finally, some methods take into account a reduction in the value of n_h for sloping ground conditions in front of the pile. For this example, the value of n_h has been reduced by a factor of two to account for cyclic loading conditions.

There are several methods available to compute pile stiffness — for example, those used in simplified procedures (NAVFAC Design Manual 7.02, 1986), those indicated in this example, and those found in computer programs such as COM624 or LPILE^{plus} (Reese and Wang, 1993). All are widely used by various Departments of Transportation and design consultants. Depending on the method of pile analysis, different deflections, shears, and moments in the pile may be obtained even when the same value of n_h is used. Therefore, communication between the geotechnical engineer and the structural engineer is essential in determining the value(s) of n_h to be used in the structural analysis.

OTHER CONCERNS

Liquefaction potential of the medium dense, silty sand was analyzed using the procedures of Seed, et. al. (1984) and Seed and Harder (1990). The results indicated an adequate factor of safety against liquefaction.

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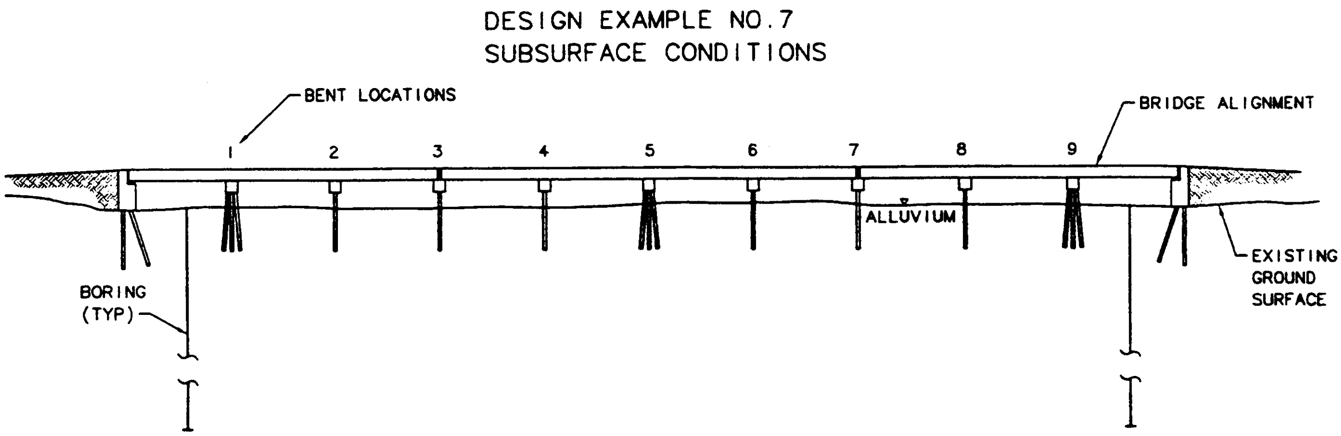

SOIL PROPERTIES

Stratum	Depth (ft)	Soil Description	N (bpf)	γ (pcf)	ϕ (deg)	c (pcf)	n_h (pci)
Alluvium	0 to > 100	Medium dense, silty sand	20	122.4	34	0	15
Nonfill	Above grade	Medium dense sand and gravel	20	122.4	34	0	23

Where:

N standard penetration resistance (blows per foot)
γ total unit weight (pounds per cubic foot)
ϕ internal angle of friction (degrees)
c cohesion (pounds per square foot)
n_h constant of horizontal subgrade reaction (pounds per cubic foot)

Figure A1 — Subsurface Conditions

Appendix B
Derivation of Lateral Stiffness Relations for Battered Piles

APPENDIX B DERIVATION OF LATERAL STIFFNESS RELATIONS FOR BATTERED PILES

Lateral Stiffness Derivation

This appendix contains the derivation of the lateral stiffness of a battered pile with the following boundary conditions: The base of the pile is fixed against both translation and rotation. The tip of the pile is free to translate horizontally and to rotate, but is fixed against vertical translation. This condition replicates the symmetric arrangement of two battered piles connected - via a pin - at their tips.

For a lateral deflection of 1 inch applied at the tip of the pile, as shown in Figure B1, the deflection components are

$\Delta := 1$ Unit horizontal deflection

$\Delta_f := \cos(\theta)\cdot\Delta$ Flexural deflection of the pile

$\Delta_a := \sin(\theta)\cdot\Delta$ Axial extension of the pile

Recall that the equation for flexural stiffness of a cantilever, written in terms of the load, is

$$P_f := \frac{3\cdot E\cdot I}{L^3}\cdot\Delta_f$$

Substitution of the flexural component, Δ_f, of the applied 1-inch deflection yields

$$P_f := \left(\frac{3\cdot E\cdot I}{L^3}\right)\cdot\cos(\theta)\cdot\Delta$$

Recall that the equation for axial stiffness, written in terms of the load can be expressed as

$$P_a := \frac{A\cdot E}{L}\cdot\Delta_a$$

Lateral Stiffness Derivation (continued)

Substitution of the axial component, Δ_a, of the applied 1 inch deflection yields

$$P_a := \left(\frac{A \cdot E}{L}\right) \cdot \sin(\theta) \cdot \Delta$$

The two forces P_f and P_a are the respective bending and axial forces necessary to produce the 1-inch horizontal deflection, while simultaneously preventing any vertical deflection. These two forces are thus directed as shown in Figure B2. Typically, the horizontal and vertical forces are the ones of interest. Thus P_f and P_a can be expressed in this form as shown below.

The horizontal components of the flexural and axial forces, respectively, are

$$H_f := P_f \cdot \cos(\theta)$$

$$H_a := P_a \cdot \sin(\theta)$$

The total horizontal force is then

$$H := H_f + H_a$$

In terms of the flexural and axial forces, H is

$$H := P_f \cdot \cos(\theta) + P_a \cdot \sin(\theta)$$

In terms of the properties and geometry of the system, H can then be reduced to

$$H := \frac{3 \cdot E \cdot I}{L^3} \cdot \left(\cos(\theta)^2\right) \cdot \Delta + \frac{A \cdot E}{L} \cdot \left(\sin(\theta)^2\right) \cdot \Delta$$

If like terms multiplying the displacement are collected, then the lateral stiffness can be calculated as shown below.

$$H := \left(\frac{3 \cdot E \cdot I}{L^3} \cdot \cos(\theta)^2 + \frac{A \cdot E}{L} \cdot \sin(\theta)^2\right) \cdot \Delta$$

Lateral Stiffness Derivation (continued)

The lateral stiffness of the battered pile subject to lateral loading is then

$$k_h := \left(\frac{3 \cdot E \cdot I}{L^3} \cdot \cos(\theta)^2 + \frac{A \cdot E}{L} \cdot \sin(\theta)^2\right)$$

Also, the flexural and axial contributions to the lateral stiffness are given by the two terms in the expression. These can be related as k_f and k_a, respectively, as shown below.

$$k_f := \frac{3 \cdot E \cdot I}{L^3} \cdot \cos(\theta)^2$$

$$k_a := \frac{A \cdot E}{L} \cdot \sin(\theta)^2$$

The vertical force necessary to hold the vertical deflection to zero has not been derived, although it can be determined through the same process as that used for the horizontal stiffness. Typically, battered piles are arranged so that the vertical restraining force is developed by an opposing pile (i.e., a pile sloping in the direction opposite the one in question).

Lateral Stiffness Derivation (continued)

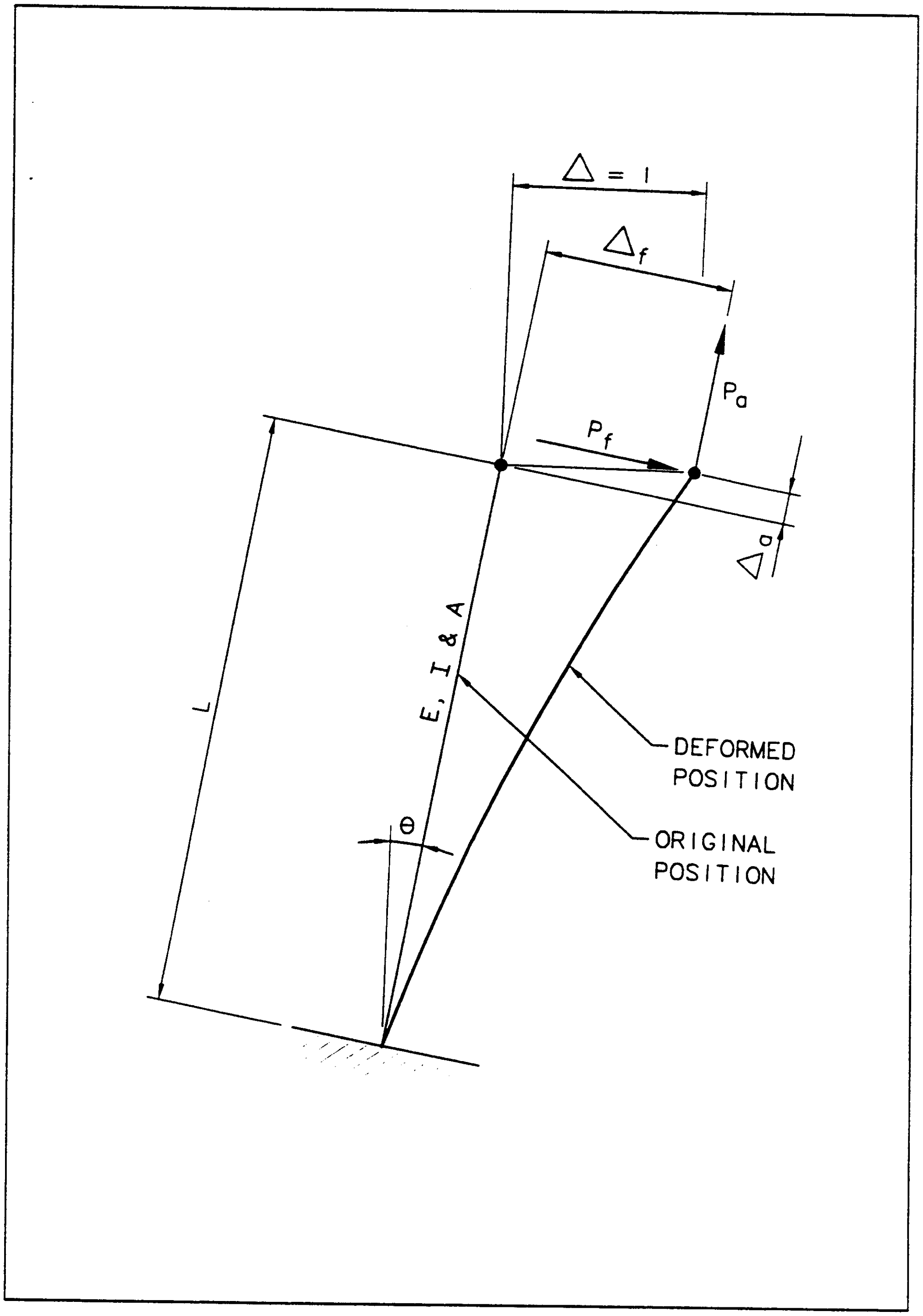

Figure B1 — Horizontal Deflected Shape of Batter Pile

Lateral Stiffness Derivation
(continued)

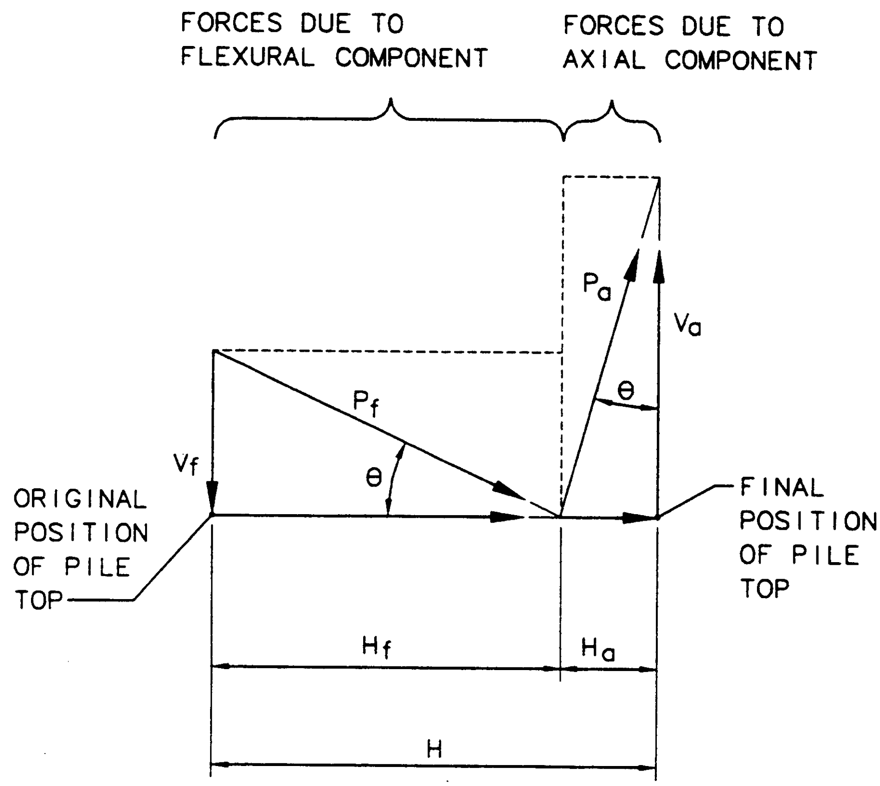

Figure B2 — Forces at Pile Tip Due to Horizontal Deflection