

In the NBC 2005 and NBC 2010 provisions, the positions of the centres of resistance at each floor do not need to be determined. Because there is no multiplier applied to e_x , the combination of lateral and torsional effects in each direction of loading can be obtained directly by two applications of the lateral loads, one set located $+0.10 D_{nx}$ from the centres of mass and the other located $-0.10 D_{nx}$ from the centres of mass. Conveniently, this is exactly the same set of load applications required for the determination of the torsional sensitivity parameter, B (see Sentence 4.1.8.11.(9)).

Clause (b): This Clause requires that the Dynamic Analysis Procedure be used for cases where $B > 1.7$ and $I_E F_a S_a(0.2) \geq 0.35$ as given in Article 4.1.8.12. The reasons why dynamic analysis is required are discussed earlier in the Commentary (see also Humar, Yavari and Saatcioglu^[16]). However, as indicated in Clause 4.1.8.7.(1)(a), the ESFP may be used in regions of low seismic hazard regardless of whether torsional sensitivity or any other permissible irregularity exists; the approximations inherent in the ESFP are unlikely to have serious consequences for such relatively small ground motions.

Dynamic Analysis Procedures (4.1.8.12.)

174. As indicated in Article 4.1.8.7., dynamic analysis is mandatory for the determination of earthquake design actions except for situations in which the simplified ESFP is adequate, as detailed in that same Article. Dynamic analysis must be conducted in accordance with the procedures of Article 4.1.8.12. using two different models, one in which all lateral displacements other than those in the direction of the earthquake forces are restrained and the floor and roof rotations about a vertical axis are restrained, and the other in which the floors and roof(s) are unrestrained. The first analysis is used to determine the scale factor that must be applied to the results of the second analysis. The following steps are involved in a linear dynamic analysis procedure:

- Step 1:** Construct a structural model of the building taking into account the requirements in Sentence 4.1.8.3.(8).
- Step 2:** Using the model from Step 1 with all displacements other than those in the direction of earthquake forces restrained, carry out a linear dynamic analysis to determine the fundamental lateral period, T_a and the base shear, V_e .
- Step 3:** If Sentence 4.1.8.12.(6) applies, determine the factor described therein using the value of T_a from Step 2 in the following formula:

$$\frac{2 S(0.2)}{3 S(T_a)} \leq 1.0$$

Multiply the value of V_e from Step 2 by this factor to determine the design elastic base shear, V_{ed} .
Step 4: Determine the equivalent static design base shear, V , as per Article 4.1.8.11. In determining V , the fundamental period may be taken as the smaller of the value obtained in Step 2 and the applicable upper limit specified in Subclauses 4.1.8.11.(3)(d)(i) to (iv). For the purpose of calculating the scale factor to be applied to deflections in Step 5, V may be taken as the smaller of the value obtained in Step 2 and the applicable upper limit specified in Subclause 4.1.8.11.(3)(d)(v).

If required for the calculation of the effects of accidental torsion as per Sentence 4.1.8.12.(4), determine the floor level forces, F_x , by distributing V across the height of the structure. If the torsional sensitivity factor, B, was not determined earlier, calculate its value using these values of F_x .

- Step 5:** Using the value of V_{ed} from Step 3, obtain the base shear, V_d , in accordance with Sentences 4.1.8.12.(7) and (8). Note that V_d can be no less than $0.8 V$ for regular structures and irregular structures that can be designed using the Equivalent Static Force Procedure, and no less than V for irregular structures requiring dynamic analysis, where V is the base shear calculated in Step 4. Determine the scale factor, V_d/V_e , using the value of V_e obtained in Step 2. This scale factor will be applied to the elastic storey shears, storey forces, member forces, and deflections obtained from a dynamic analysis of the model in which the floors are unrestrained.
- Step 6:** If accidental torsion is to be accounted for using the procedure specified in Clause 4.1.8.12.(4)(a), carry out a three-dimensional elastic linear dynamic analysis on the model constructed in Step 1 to obtain the elastic storey shears, storey forces, member forces and deflections; otherwise go to Step 8.
- Step 7:** Calculate the effects of accidental torsion in accordance with Clause 4.1.8.12.(4)(a) and add these effects to those determined in Step 6 to obtain the elastic storey shears, storey forces,

member forces and deflections which now include the effect of accidental torsion. The floor level forces required for the calculation of accidental torsion effects may be taken as the forces determined in Step 4 multiplied by $R_d R_o / I_E$. Alternatively, they may be obtained from the storey shears calculated in Step 6, the force at level x being taken as the difference between the maximum dynamic shear in the storey below level x and that in the storey above level x .

- Step 8:** If accidental torsion is to be accounted for using the procedure specified in Clause 4.1.8.12.(4)(b), carry out two separate elastic linear dynamic analyses, one on the model constructed in Step 1 with the mass centres shifted by $-0.05 D_{nx}$, and another on that same model with the mass centres shifted by $+0.05 D_{nx}$ (the mass centres are to be shifted in the same direction for all storeys). The larger of the values obtained from the two analyses provide the elastic storey shears, storey forces, member forces and deflections including the effect of accidental torsion.
- Step 9:** Scale the storey shears, storey forces, member forces and deflections obtained in Step 7 or Step 8 using the scale factors obtained in Step 5. Note that these deflections are elastic and need to be multiplied by $R_d R_o$.

Sentence 4.1.8.12.(1)

175. This Sentence indicates that it is permissible to do either a Linear or a Nonlinear Dynamic Analysis.
Clause (a): A Linear Dynamic Analysis, using either the Modal Response Spectrum Method or the Numerical Integration Linear Time History Method, is the normal approach because the analysis procedures are straightforward and can be found in texts on structural dynamics (Chopra,^[73] Humar^[74]). Also, standard software used for structural analysis often includes Linear Dynamic Analysis as one of the options; the Modal Response Spectrum Method is more commonly included in such software. The structural model used in Linear Dynamic Analysis must comply with the requirements of Sentence 4.1.8.3.(8) to ensure that it represents the actual structure in a realistic manner. The other Sentences in Article 4.1.8.3. prescribe how the dynamic excitation is to be determined and how the results are to be used in design, including how accidental torsion is to be taken into account. Saatcioglu and Humar^[12] discuss various of these requirements including considerations such as the number of modes required to accurately represent the dynamic response of the structure.

The Modal Response Spectrum Method is based on the fact that the response of a linear elastic system is made up of the superposition of the responses of individual natural modes of vibration, each mode responding at its natural frequency with its own pattern of deformation, i.e. its mode shape. The most common form of this method involves the combination of the maximum response parameters in each mode to determine the maximum values of the response parameters for the structure as a whole. Only a small number of modes (e.g. 3 to 5) are required to provide a good approximation of the total response; Chopra^[73] discusses the factors involved in selecting the number of modes, which are affected both by the desired accuracy and the response quantity of interest. NEHRP 2000 (Building Seismic Safety Council^[26]) provides a simple rule for the determination of the number of modes required: the normal requirement is that the combined participating mass of all the modes included in the analysis should total at least 90% of the total mass. The primary sources of uncertainty in this method are: the validity of the structural model, the validity of the modal combination rule, and the value of damping in each mode.

The Numerical Integration Linear Time History Method involves the determination of the response of a structural model to a specific earthquake ground motion accelerogram through the numerical integration of the equations of motion. The primary advantage of this method, compared with the Modal Response Spectrum Method, is that the various response parameters are obtained as time-histories, providing information on the time-wise fluctuation of the state of deformation of the structure. There are several disadvantages, most notably: i) such analyses produce voluminous amounts of data to be interpreted, and ii) the results depend greatly on the characteristics of the individual ground shaking accelerograms so that analyses need to be done using a number of different time-histories (see the Commentary section on Sentence 4.1.8.12.(3)). Due to these disadvantages and the resulting increased costs of analysis, this method is rarely used for the design of ordinary building structures.

Clause (b): A Nonlinear Dynamic Analysis is an acceptable alternative provided that a special study is performed. Since such analyses are still primarily done in a research environment, it is essential that the special study be conducted by individuals who are competent and experienced in making the necessary judgments and decisions. In addition, the resulting