Design Requirements for Column Braces

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Summary—The elastic behaviour of steel column-brace assemblies is analysed. Both column and brace have initial crookedness, and there is an initial lack of fit between column and brace. Realistic magnitudes of crookedness and lack of fit are proposed, and criteria are developed for assessing the strength of the assembly under the braced column load permitted by the SAA Steel Structures Code. Values of the minimum brace areas which satisfy these criteria are determined for assemblies with given column and brace slendernesses and ratios of brace length to column height. It is found that special care may be required when connecting the braces of stocky columns to ensure that overstressing does not occur. Comparisons with the rules of the SAA Steel Structures Code indicate that its brace strength requirements are conservative and its brace stiffness requirements inadequate. A new basis for the design of column braces is proposed.

SYMBOLS

A_B	area of the brace section
A_M	area of the column section
B	width of the section in the bending plane
b	transverse deflection of the brace
b_i	transverse crookedness of the brace
c_1, c_2	coefficients in column-displacement brace-force relationship (14)
E	Young's modulus of elasticity
F_Y	yield stress
H	force in the brace with the column loaded
H_C	ultimate compressive load of the brace
H_E	Euler buckling load of the brace
H_o	force in the brace on assembly
H_T	ultimate tensile load of the brace
I	minor axis moment of inertia
L_B	nominal length of the brace
L_M	overall height of the column
l	effective length of the column
M	m/L_M
M_i	m_i/L_M
m	deflection of the loaded column
m_i	column crookedness
m_o	deflection of the column on assembly
P	axial load on the column
P_k	critical load of the column with effective length l
P_E	Euler load of the column for buckling in the plane of the brace $= \pi^2(EI)_M/L_M^2$
P_u	ultimate design load of the column
P_Y	squash load $= (EA)_M \varepsilon_Y$
r	minimum radius of gyration of the brace
u	displacement of the end point of the brace with the column loaded
u_o	displacement of the end point of the brace at assembly
u_i	error in fit of the brace to a straight column
x	distance along the length of the brace
Z	z/L
z	distance along the height of the column
ε_a	uniform axial strain in the column
ε_c	curvature strain in the extreme fibre of the column
ε_o	strain in the column at assembly
ε_Y	yield strain of the component material
η	geometrical imperfection parameter for a column
λ	$\dfrac{\pi^2}{48}\left(\dfrac{EA}{L}\right)_B \dfrac{L_M}{P_E}$ dimensionless stiffness of the straight brace
λ_c	critical dimensionless stiffness (see (50)).

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1.—INTRODUCTION

The design ultimate load carrying capacity P_u (Refs. 5, 7, 9 and 12) of a slender steel column can be significantly increased by bracing it at its mid height. This is demonstrated by the comparison shown in Fig. 1 between the ultimate load carrying capacity of an unbraced pin-ended column and that of a rigidly braced column, which indicates that the strength of a slender column can approach four times the strength of the corresponding unbraced column. These ultimate strengths are related to the elastic buckling strengths P_k of perfectly straight columns, and these are also shown in Fig. 1. Provided the stiffness of the brace exceeds a certain minimum value (Refs. 2, 10, 14 and 18), the braced straight column buckles with a node at the brace at a load which is equal to the buckling load of an equivalent pin-ended column of half the height.

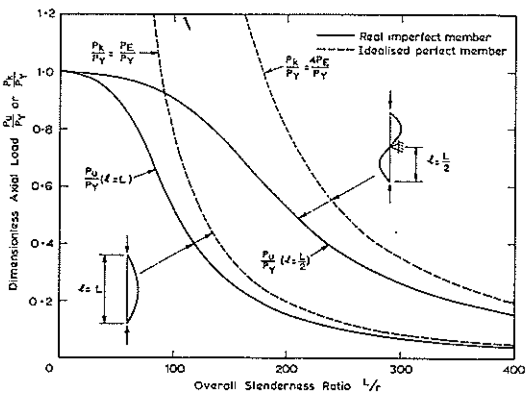

Fig. 1—Strengths of real and perfect columns.

A real column is not straight and the fit of the brace is uncertain, as shown in Fig. 2(*a*). If the initial crookedness of the column is one of symmetrical single curvature, then the column will deflect symmetrically under load, and a force will act between the brace and column, as indicated in Figs. 2(*b*) and 2(*c*). The column deflections and brace force depend on the stiffness of the brace and so the design stiffness should be sufficient to ensure that the design ultimate strength of the column can be reached. The brace should also have sufficient strength to transmit the maximum brace force.

A variety of column models with intermediate restraints have been studied (Refs. 1, 2, 6, 10, 11, 14, 15 and 17). A simplified model of a column-brace assembly comprising a column with sinusoidal crookedness and a perfectly fitting straight brace at mid height was studied by Green *et al* (Ref. 6) and Zuk (Ref. 19). Both column and brace were assumed to remain elastic and the magnitude of the force in the brace was related to the axial load in the column for different values of the brace stiffness. In a later paper, Winter (Ref. 18) developed simplified expressions for the minimum brace stiffness and the corresponding brace force for a column loaded by its elastic buckling load, by assuming that a hinge formed at the brace point. The effect of the decreased axial stiffness of a crooked compression brace was investigated by Swannell (Ref. 13), who presented graphs relating the elastic buckling load of a straight braced column to the section properties of a crooked brace.

In this paper, the behaviour of the idealised pin-ended and pin-connected column-brace assembly (the strengthening effects of moment continuity in the connections are ignored) shown in Fig. 2 is investigated. Both the column and brace have initial crookedness, and there is also an initial lack of fit between them. It is assumed that there are no twists (Ref. 8) and no deflections of the column out of the plane of the assembly. The column and brace deflections of the assembly are determined by elastic analyses and the maximum stresses are evaluated. This information is then used to formulate stiffness and strength requirements for the brace which will allow the column to be designed as an equivalent pin-ended column of half the height.

2.—ELASTIC ANALYSIS OF A COLUMN-BRACE ASSEMBLY

The nominal brace length L_B shown in Fig. 2(*a*) is based on the assumption that both the column and brace will be perfectly straight and that there will be perfect fit between them. However because of fabrication and other errors, the column has a central crookedness of m_{im}, while the chord length of the brace is $L_B - u_i$. Thus there is an initial lack of fit $(u_i - m_{im})$ between the column and brace.

When the column and brace are assembled there is a force H_o in the brace and the assembled column's profile is $(m_i + m_o)$ measured from the load axis, where

$$m_o = \frac{\pi^2}{48}\frac{H_o}{P_E} L_M\left[3\left(\frac{z}{L}\right)_M - 4\left(\frac{z}{L}\right)^3_M\right] \tag{1}$$

in which

$$P_E = \pi^2(EI)_M/L_M^2 \tag{2}$$

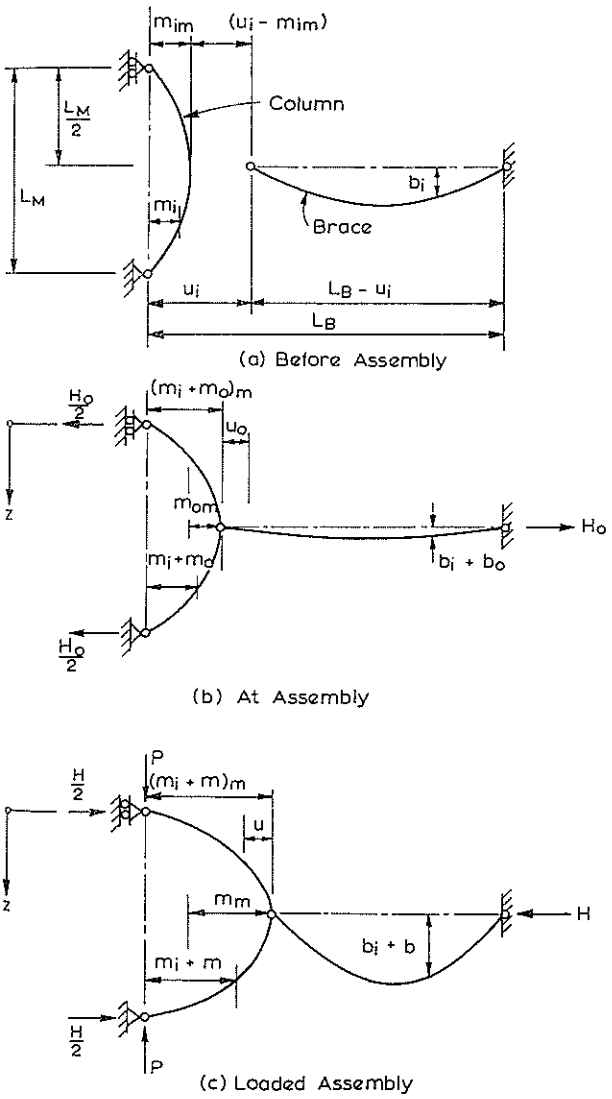

Fig. 2—Column-brace assembly.

while the end of the brace is displaced u_o from its initial position (see Fig. 2(*b*)). An axial load P is applied to the column after assembly and the brace force changes to H. The column's profile is then $(m_i + m)$ where m is the deflection due to P and H, while the end of the brace is displaced u from its position before assembly (see Fig. 2(*c*)). Once the column and brace are assembled, the distance of the brace point from its nominal position for perfect geometry is

$$m_{im} + m_m = u_i + u \tag{3}$$

which can be written in dimensionless terms as

$$\frac{L_M}{L_B}\left(\frac{m_{im}}{L_M} + \frac{m_m}{L_M}\right) = \left(\frac{u_i}{L_B} + \frac{u}{L_B}\right) \tag{4}$$

If the brace has an initial crookedness

$$b_i = b_{im} \sin \pi x/L_B \tag{5}$$

(it is assumed that for the purpose of analysing the behaviour of the brace, its length can be taken as L_B instead of L_B-u_i), then, when a compression force is transmitted by the brace, its shape changes as shown in Fig. 2, from b_i to

$$b + b_i = \frac{b_i}{(1 - H/H_E)} \tag{6}$$

where

$$H_E = \frac{\pi^2 (EI)_B}{L_B^2} = \frac{\pi^2 (EA)_B}{(L/r)_B^2} \tag{7}$$

is the Euler buckling load of the brace. In this case the brace point deflection u is related to H by

$$\frac{u}{L_B} = \frac{(2 - H/H_E)}{(1 - H/H_E)^2}\frac{H}{H_E}\frac{\pi^2}{4}\left(\frac{b_{im}}{L_B}\right)^2 + \frac{\pi^2}{(L/r)_B^2}\frac{H}{H_E} \tag{8}$$

the first term in which represents the foreshortening due to the change in shape (Refs. 2 and 13)

$$\frac{1}{2}\int_0^{L_B}\left(\frac{d}{dx}(b + b_i)\right)^2 dx - \frac{1}{2}\int_0^{L_B}\left(\frac{d}{dx}b_i\right)^2 dx$$

while the second term represents the deflection due to uniform axial straining $L_B H/(EA)_B$.

The relation between the deflections of the column and the brace force H can be determined from the differential equation for bending of one half of the pin-ended column (see Fig. 2(*c*)) which is

$$-(EI)_M \frac{d^2m}{dz^2} = P(m_i + m) - \frac{Hz}{2} \tag{9}$$

for which the boundary conditions are

$$(m)_o = \left(\frac{dm}{dz}\right)_{L/2} = 0 \tag{10}$$

Expressed non-dimensionally, (9) and (10) become

$$M'' + \pi^2\frac{P}{P_E}M = -\pi^2\frac{P}{P_E}M_i + \frac{\pi^2}{2}\frac{HZ}{P_E} \tag{11}$$

and

$$(M)_o = (M')_{\frac{1}{2}} = 0 \tag{12}$$

in which

$$M = m/L_M$$
$$M_i = m_i/L_M = \left(\frac{m_{im}}{L_M}\right)f(Z)$$
$$Z = z/L_M \tag{13}$$

where $f(Z)$ is some function of Z, and each prime denotes one differentiation with respect to Z.

This set of equations can be solved numerically by the method of finite integrals (Ref. 3). An integrating operator is used to express the values of M at a series of equally spaced points along the column's half height as linear combinations of the values of M'' at these points, the constants of integration being determined from the boundary conditions of (12). These values can then be substituted into (11), and a series of simultaneous equations in the unknown values of M'' can be obtained. If these are triangulated using a Gauss-Jordan reduction scheme, then the values of M'' can be expressed as linear combinations of H/P_E and $M_{i\frac{1}{2}} = m_{im}/L_M$. The values of M'' can then be substituted in the finite integral form of $M_{\frac{1}{2}}$ to give a relation between the column deflection and the brace force of the form

$$M_{\frac{1}{2}} = c_1 H/P_E + c_2 M_{i\frac{1}{2}} \tag{14}$$

The value of the dimensionless brace force H/H_E can be determined by substituting the column and brace characteristics ((14) and (8)) into the compatibility condition (4), to give

$$\frac{L_M}{L_B}\left(M_{t_{\frac{1}{2}}} + c_1\frac{H_E}{P_E}\frac{H}{H_E} + c_2 M_{t_{\frac{1}{2}}}\right) =$$

$$\frac{u_t}{L_B} + \frac{(2 - H/H_E)}{(1 - H/H_E)^2}\frac{H}{H_E}\frac{\pi^2}{4}\left(\frac{b_{im}}{L_B}\right)^2 + \frac{\pi^2}{(L/r)_B{}^2}\frac{H}{H_E} \qquad (15)$$

The solution of (15) for H/H_E can be used to find

$$\frac{H}{P_E} = \frac{H_E}{P_E}\frac{H}{H_E} \qquad (16)$$

and this can be substituted into the simultaneous equations representing (11), which can then be solved for the dimensionless curvatures M''. The maximum dimensionless curvatures M''_{max} can be found by interpolating between these values of M'' by assuming the same parabolic variations (Ref. 3) as those used to derive the finite integral operator.

The maximum bending strain $\varepsilon_{c_{max}}$ is

$$\varepsilon_{c_{max}} = \left(\frac{B}{2r}\right)_M \frac{1}{(L/r)_M}\left|M''_{max}\right| \qquad (17)$$

where B is the column width. The maximum longitudinal strain ε_{max} is

$$\varepsilon_{max} = \varepsilon_a + \varepsilon_{c_{max}} \qquad (18)$$

where ε_a is the uniform axial strain

$$\varepsilon_a = \frac{P}{(EA)_M} = \frac{\pi^2}{(l/r)_M{}^2}\frac{P}{P_k} \qquad (19)$$

where

$$P_k = \pi^2(EI)_M/l^2 \qquad (20)$$

is the elastic critical load for a straight column whose effective length is l. When the column is just assembled ($P = \varepsilon_a = 0$), the curvature varies linearly along the column (see (1)) and (17) can be expanded to give the maximum strain as

$$\varepsilon_0 = \left(\frac{B}{2r}\right)_M \frac{1}{(L/r)_M}\frac{12 m_{om}}{L_M} = \frac{\pi^2}{4}\left(\frac{B}{2r}\right)_M \frac{1}{(L/r)_M}\frac{H_o}{P_E} \qquad (21)$$

3.—LOADS AND IMPERFECTIONS OF A COLUMN-BRACE ASSEMBLY

The design of a restrained compression member according to Rule 6.1 of the SAA Steel Structures Code (Refs. 7, 12 and 16) is based on the ultimate load capacity P_u given by

$$\frac{P_u}{P_k} = \frac{\left(\frac{\varepsilon_Y}{\varepsilon_k} + \eta + 1\right)}{2} - \sqrt{\left(\frac{\frac{\varepsilon_Y}{\varepsilon_k} + \eta + 1}{2}\right)^2 - \frac{\varepsilon_Y}{\varepsilon_k}} \qquad (22)$$

where

$$\varepsilon_k = \frac{\pi^2}{(l/r)^2{}_M} = \frac{P_k}{(EA)_M} \qquad (23)$$

where the effective length l is the distance between points of effective lateral restraint, ε_Y is the yield strain F_Y/E, and the geometrical imperfection parameter η is given by

$$\eta = 0.00003\left(\frac{l}{r}\right)^2 \qquad (24)$$

Thus if the brace in the assembly shown in Fig. 2 is effective, then the column should be able to support a load P_u which is given by (22), (23) and (24) with (see Fig. 1)

$$l = L_{M/2} \qquad (25)$$

The ultimate load capacity P_u given by (22) is equal to the load which causes first yield in a hypothetical stress-relieved column which has an initial crookedness

$$m_i = m_{im}\sin\ \pi z/l \qquad (26)$$

in which

$$\frac{m_{im}}{l} = \frac{1}{\left(\frac{B}{2r}\right)_M}\ \frac{1}{\left(\frac{l}{r}\right)_M}\ \eta \qquad (27)$$

This is the result of a series of simplifications, since real columns have residual stresses, and may have different initial crookedness shapes and magnitudes, while the failure loads of real columns are higher than their first yield loads (Refs. 5, 7 and 16).

The braced column shown in Fig. 2 is assumed to have an initial crookedness

$$m_i = m_{im}(3(z/L) - 4(z/L)^3) \qquad (28)$$

where m_{im} has a maximum value of

$$m_{im} = 0.0013\ L_M \qquad (29)$$

The shape given by (28) is the same as the force fit deflection shape of (1), and has been chosen for simplicity so that the profile of the assembled unloaded column is of the same shape as its initial crookedness. The magnitude of the maximum initial crookedness given by (29), which is 30% higher than the fabrication tolerance specified in Rule 11.2.2 of the SAA Steel Structures Code, was chosen somewhat arbitrarily. It does, however, make some allowance for the eccentricity of the load in the plane of assembly, and for the effect on the deflections of the residual stresses in the column which are ignored in the elastic analysis of the column brace assembly. These are similar to the allowances made by Robertson (Ref. 7) for the original version of the imperfection parameter η (see (22)) used in (29).

The brace crookedness is assumed to be the sine shape given in (5) with its maximum magnitude equal to

$$b_{im} = 0.0026\ L_B \qquad (30)$$

This value is 30% higher than the fabrication tolerance allowed in Rule 11.2.2 of the SAA Steel Structures Code and this makes some allowance for the self-weight of the brace and for the effect of residual stresses on the axial stiffness of the brace.

The error u_t in the chord length of the brace is assumed to have a maximum value of

$$u_t = \pm\ 0.0005\ L_B \qquad (31)$$

This corresponds to $u_t = 4.5$ mm when $L_B = 9$ m, which is comparable with the values of 3 mm for $L_B < 9$ m and 5 mm for $L_B > 9$ m given as fabrication tolerances for the shortness in length in Rule 11.2.3 of the SAA Steel Structures Code. While the error was chosen to be proportional to the brace length in (31) for simplicity in presenting the results, the amount by which it exceeds the 5 mm fabrication tolerance for long braces makes some allowance for setting out and erection errors. On the other hand, the small amount by which the assumed error of (31) is exceeded by the combined effects of fabrication and erection errors in short braces can usually be taken up by the connection clearances.

The initial lack of fit ($u_t - m_{im}$) between the column and the brace (see Fig. 2(*a*)) depends on both the maximum initial crookedness m_{im} of the column and the chord length error u_t of the brace, and a number of different imperfection geometries can occur, as shown in Fig. 3, depending on the relative magnitudes of m_{im} and u_t. The assembling of the column and brace may increase or decrease the lack of straightness of the column and may place the brace in compression or tension, while the subsequent loading of the column may increase or decrease the bending in the column, or may increase or decrease the brace force. It is not obvious which is the most dangerous combination of m_{im} and u_t and so all combinations of

$$\frac{m_{im}}{0.0013\ L_M} = -1.0, -0.5, 0, 0.5, 1.0$$

and

$$\frac{u_t}{0.0005\ L_B} = -1.0, 0, 1.0 \qquad (32)$$

are investigated. Some of these cases would however lead to excessive errors in the alignment of the unloaded column. In these cases, then, it is assumed that the value of u_t is reduced until

$$(m_t + m_o)_m = \pm\ 0.0013\ L_M \qquad (33)$$

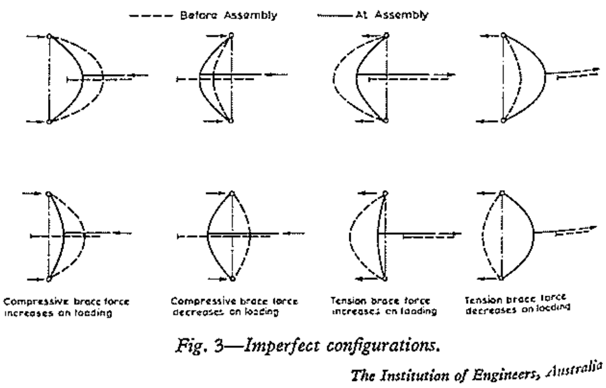

Fig. 3—Imperfect configurations.

this reduction corresponding to the correction that would be made during erection. The special cases where

$$m_{im} = u_i = \pm\, 0.0013\, L_M \qquad (34)$$

so that there is perfect fit (rather than the cases given by (32) or (33)) are also investigated.

4.—DESIGN CRITERIA FOR COLUMN AND BRACE

The column-brace assembly may be considered safe if its components satisfy those established rules of the SAA Steel Structures Code (Ref. 12) which apply and if they meet the criteria developed in this section. Two situations are considered, that when the column and brace are first connected, and that when the design ultimate column load P_u is applied. The criteria developed in this section are derived from the elastic behaviour analysed in Section 2 of the assembly discussed in Section 3.

At assembly, the column acts as a beam loaded by the brace force H_o, and in this case Rules 5.2 and 5.3 of the SAA Steel Structures Code permit a maximum stress under the working load of between 0.60 F_Y and 0.66 F_Y. If the lower of these two is taken, then the maximum strain ε_o must satisfy

$$\varepsilon_o \leqslant 0.60\, \varepsilon_Y \qquad (35)$$

which is the design criterion for the column when it is first connected to the brace.

The criterion used to assess the safety of the braced column when loaded with its design working load 0.60 P_u is the relative magnitude of the maximum elastic strain induced by the ultimate load P_u and the corresponding brace force. It is argued that although the Perry-Robertson relation (Ref. 7) for P_u assumes a sine shape crookedness of magnitude given by (24) and (27) to determine the maximum strain, some other crookedness shape and magnitude (Ref. 4) could equally well be chosen, and some other strain level than the constant yield strain ε_Y could be used to determine the design ultimate load P_u.

When the crookedness of the half-height model column has the sine shape given by (26) and shown in Fig. 4(*a*), then the maximum strain induced by a load P is

$$\varepsilon_{\max} = \varepsilon_a + \frac{m_{im}}{l}\left(\frac{B}{2r}\right)_M \left(\frac{l}{r}\right)_M \frac{\varepsilon_k}{\varepsilon_k/\varepsilon_a - 1} \qquad (36)$$

If the magnitude of the crookedness is given by (24) and (27) and P is equal to the design ultimate load P_u (case a), then

$$\varepsilon_{\max(a)} = \varepsilon_Y \qquad (37)$$

as shown in Fig. 5. This of course is the usual elastic criterion used to assess the safety of the column. However if the crookedness magnitude m_{im} of the half-height column with the sine shape of (26) is given by (case b)

$$m_{im} = 0.0013\, l = 0.00065\, L_M \qquad (38)$$

which is 30% greater than the fabrication tolerance specified in Rule 11.2.2 of the SAA Steel Structures Code, then the maximum strain $\varepsilon_{\max(b)}$ induced by the ultimate load P_u is given by the appropriate curve shown in Fig. 5 when $B/2r = 2.0$ (which is representative of most universal sections) and the material has a yield strain $\varepsilon_Y = 0.00125$ (which is typical of the most common structural steel). Thus the maximum strains given by this curve can provide a basis for assessing the design ultimate strength of any half-height column which is assumed to have the crookedness of (38). This basis is equivalent to above (37), and gives the same ultimate load P_u.

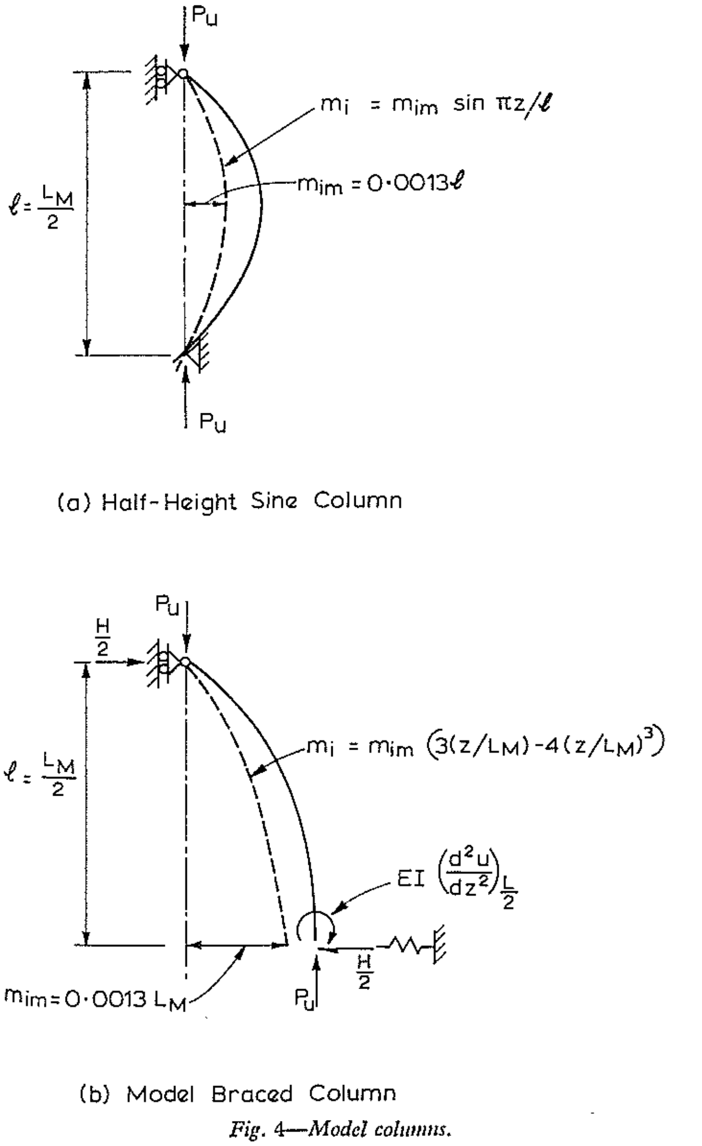

Fig. 4—Model columns.

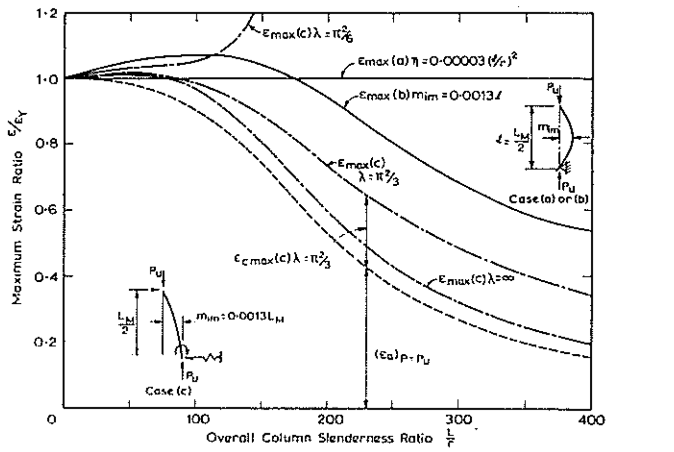

Fig. 5—Strain levels in model columns caused by the design ultimate load P_u.

The crookedness of a real braced column will differ in shape as well as magnitude from the sine crookedness given by (26) and (38) for the half-height model shown in Fig. 4(*a*). A more appropriate shape for the braced model is that of (28) shown in Fig. 4(*b*). The deflections of this model depend on the brace characteristic and on the initial lack of fit between the column and the brace, and can be determined as indicated in Section 2. The maximum strains $\varepsilon_{\max(c)}$ in a model column which has an initial crookedness of the magnitude given by (29) and a perfectly fitting straight brace with dimensionless stiffness

$$\lambda = \frac{\pi^2}{48}\left(\frac{EA}{L}\right)_B \frac{L_M}{P_E} = \frac{\pi^2}{6}, \frac{\pi^2}{3}, \text{ or } \infty \qquad (39)$$

are shown in Fig. 5 (case c). It can be seen that when the dimensionless stiffness λ is equal to or greater than $\pi^2/3$ the maximum strains vary in a similar way to the strains of the sine shaped model (case b). Thus it seems reasonable to assume that a real braced column with a perfectly fitting brace will be safe if the maximum elastic strain induced by the design ultimate load P_u does not exceed the strain for case b shown in Fig. 5.

Real braced columns will have imperfectly fitting braces, and the total strain under load will be influenced by the force fitting of the brace. Nevertheless, it is proposed that the same maximum levels of total strain (case b) should be adopted to assess the safety of these columns. The general criterion is then that the strain in the braced column must satisfy the inequality

$$\varepsilon_{\max} - \varepsilon_{\max(b)} < 0 \qquad (40)$$

where ε_{max} is given by (17), (18) and (19), and $\varepsilon_{max(b)}$ by (36) and (38). This criterion can be expressed as

$$\frac{\varepsilon_{max} - \varepsilon_{max(b)}}{\varepsilon_{c_{max}}} < 0 \tag{41}$$

where $\varepsilon_{c_{max}}$ is the maximum bending strain given by (17). When the expressions for the strains are substituted and rearranged, this inequality becomes

$$\frac{|M''_{max}|}{2\pi^2\left(\dfrac{0.0013}{\varepsilon_k/\varepsilon_a - 1}\right)} < 1 \tag{42}$$

which is independent of the value of $B/2r$.

The criterion for the design of the brace depends on whether the force in it is compressive or tensile. When the brace force is compressive, the brace acts as a pin-ended strut, and if the self weight is ignored (which may not be insignificant in a long horizontal brace), then its ultimate compressive strength H is given by Rule 6.1.1 of the SAA Steel Structures Code, which can be written in terms of the brace parameters as

$$\frac{H_C}{H_E} = \frac{\left(\dfrac{\varepsilon_Y}{\varepsilon_E} + \eta + 1\right)}{2} - \sqrt{\left(\frac{\dfrac{\varepsilon_Y}{\varepsilon_E} + \eta + 1}{2}\right)^2 - \frac{\varepsilon_Y}{\varepsilon_E}} \tag{43}$$

$$\text{where}\quad \varepsilon_E = \frac{\pi^2}{(L/r)_B{}^2} = \frac{H_E}{(EA)_B} \tag{44}$$

and $\eta = 0.00003\ (L/r)_B^2$. When the brace is in tension, its ultimate tensile strength H_T is determined from Rule 7.1, which permits a maximum stress of 0.60 F_Y under the working load. Thus the ultimate strength is

$$\frac{H_T}{H_E} = -\frac{\varepsilon_Y}{\varepsilon_E} = -(L/r)_B{}^2\frac{\varepsilon_Y}{\pi^2} \tag{45}$$

The force in the brace when first connected must not exceed the design working load capacity and so

$$0.60\frac{H_T}{H_E} \leqslant \frac{H_o}{H_E} \leqslant 0.60\frac{H_C}{H_E} \tag{46}$$

Furthermore when the design ultimate load P_u is applied to the column the brace force H must not exceed the ultimate strength of the brace and so

$$\frac{H_T}{H_E} \leqslant \frac{H}{H_E} \leqslant \frac{H_C}{H_E} \tag{47}$$

5.—DESIGN BRACE AREAS

A column will be adequately braced if the brace has sufficient stiffness to ensure that the column strength criteria established in Section 4 are satisfied, and if the brace satisfies the brace strength criteria. The column geometry may be specified by its unbraced slenderness $(L/r)_M$, while the design variables for the brace may be chosen as the brace slenderness $(L/r)_B$ and the dimensionless brace area A_B/A_M, so that the term H_E/P_E in the brace force equation (16) can be expressed as

$$\frac{H_E}{P_E} = \frac{(L/r)_M{}^2}{(L/r)_B{}^2}\left(\frac{A_B}{A_M}\right) \tag{48}$$

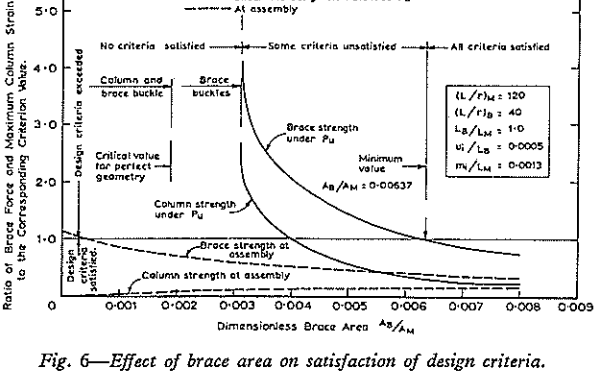

Fig. 6—Effect of brace area on satisfaction of design criteria.

The performance of an assembly with given values of these variables will also depend on the ratio L_B/L_M of the brace length to column height, and on the particular geometrical imperfections of the column and brace. The design dimensionless brace area for a given assembly is taken as the lowest value of A_B/A_M for which all the design criteria established in Section 4 are satisfied for all the geometrical imperfection sets specified in Section 3.

A computer program has been written which determines for each imperfection set the minimum value of A_B/A_M which satisfies all of the design criteria. An initial value is taken as

$$\frac{A_B}{A_M} = 48\frac{L_B}{L_M}\frac{\lambda_C}{(L/r)_M{}^2} \tag{49}$$

in which

$$\lambda_C = \frac{\dfrac{1}{3}\left(\dfrac{\pi}{2}\sqrt{P_u/P_E}\right)^2}{\left(1 - \dfrac{\tan\dfrac{\pi}{2}\sqrt{P_u/P_E}}{\dfrac{\pi}{2}\sqrt{P_u/P_E}}\right)} \tag{50}$$

is the critical dimensionless stiffness (Refs. 2, 10 and 14) of a straight brace which exactly fits a straight column bearing the load P_u. This initial value is progressively increased until all the design criteria are satisfied. The computation process is illustrated in Fig. 6 by the extent to which the different design criteria are satisfied as the value of A_B/A_M increases for the particular assembly specified by $(L/r)_M = 120$, $(L/r)_B = 40$, $L_B/L_M = 1$ which has the imperfection combination $m_i/L_M = 0.0013$, $u_i/L_B = 0.0005$. In this case, the minimum value of A_B/A_M is 0.00637, and this is determined by the criterion for the safety of the brace at the design ultimate load.

This computation process is repeated for all the imperfection sets, and the design dimensionless brace area is taken as the greatest value of the minimum values of A_B/A_M. A comparison of the minimum values of A_B/A_M required for different imperfection sets is shown in Fig. 7 for assemblies with $(L/r)_B = 160$, $L_B/L_M = 1$. This figure demonstrates the general conclusion reached that the perfect fit set of (34) is the most critical, except for some columns with $(L/r)_M$ less than 150, when some of the force-fit sets become more important.

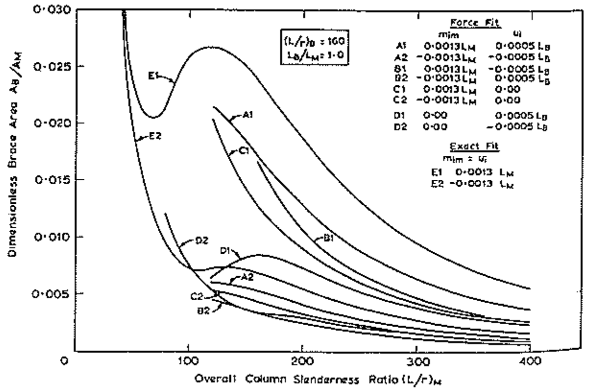

Fig. 7—Minimum dimensionless brace areas for different imperfection sets.

Further investigations of these stocky columns revealed that in some cases a satisfactory brace could not be designed. This situation arises for some force-fit sets for which small brace areas lead to column overstress under the design ultimate load, while large braces overstress the column on assembly. Thus the forced connection of a brace of a stocky column may decrease the column's reserve of strength. The increased strength of an effectively braced stocky column is only marginally greater than that of the unbraced member (see Fig. 1), and so if a brace can not be satisfactorily fitted without excessive force, the assembly would be better designed and built as an unbraced column. It is concluded that provided care is taken to avoid any excessive force fit of a brace to a stocky column, then the design of all column braces may be based on the minimum values of A_B/A_M obtained for the perfect fit imperfection set of (34). The variations of these minimum values with $(L/r)_M$ are shown in Fig. 8 for all the sets of $(L/r)_B = 40, 120, 160, 280$ with $L_B/L_M = 0.5, 1.0$ and 2.0.

The dimensionless design brace areas shown in Fig. 8 are determined by the brace strength criterion when the column is slender, and by the column strength criterion when the column is stocky. Thus the function of the brace is either to transmit the brace force or to stiffen the column so that it can transmit its load. These functions correspond to the brace strength and stiffness requirements specified in design codes.

The SAA Steel Structures Code (Ref. 12) requires the brace to be capable of safely transmitting a force equal to 0.025 times the maximum working load on the column (Rule 3.3.4.2), and so the design ultimate compressive strength of the brace must be

$$H_C = 0.025\, P_u \tag{51}$$

The dimensionless area of a brace with this strength can be expressed as

$$\frac{A_B}{A_M} = 0.025\,\frac{(L/r)_B{}^2}{(L/r)_M{}^2}\,\frac{P_u/P_E}{H_C/H_E} \tag{52}$$

where the terms P_u/P_E and H_C/H_E are the functions of the respective member slendernesses given in (22) and (43). Values of A_B/A_M which just satisfy (52) are plotted in Fig. 9. If these are compared with the values shown in Fig. 8 which satisfy the criteria developed in Section 4 of this paper, then it can be seen that the Code brace strength requirements are greater than necessary.

The SAA Steel Structures Code also requires that the brace should have a minimum force-extension stiffness of

$$\frac{H}{u} = 10\,\frac{P}{L_M} = 6\,\frac{P_u}{L_M} \tag{53}$$

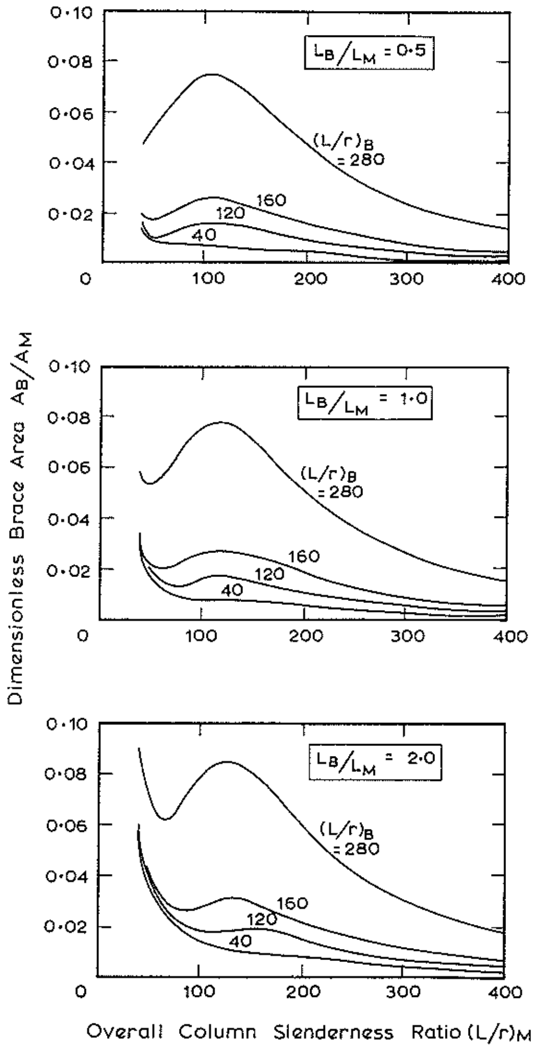

Fig. 8—Design dimensionless brace areas.

If the brace is straight so that

$$\frac{H}{u} = \left(\frac{EA}{L}\right)_B \tag{54}$$

then the dimensionless area of the brace which satisfies (53) and (54) is given by

$$\frac{A_B}{A_M} = 6\,\frac{\pi^2}{(L/r)_M{}^2}\,\frac{L_B}{L_M}\,\frac{P_u}{P_E} \tag{55}$$

These values are also plotted in Fig. 9, and if they are compared with the values which satisfy the criteria of Section 4, then it can be seen that the Code brace stiffness requirements are inadequate. Fortunately, the Code brace strength requirements always govern the design of the brace, and the inadequacy of the stiffness requirements is unimportant.

6.—CONCLUSIONS

An elastic analysis has been made of the behaviour of a steel column-brace assembly comprised of an initially crooked pin-ended column braced at its half height by an imperfectly fitting crooked brace. A method has been developed of obtaining numerical solutions for the column deflections and strains and the brace force under applied load.

The SAA Steel Structures Code rules for member imperfections and tolerances were used as starting points to establish realistic and analytically suitable magnitudes for the geometrical imperfections and lack of fit. The load on the column was taken to be the full design ultimate load of a pin-ended column of half the overall height. Appropriate criteria for assessing the adequacy of the column and brace at assembly and under the design ultimate load were obtained from the established code rules for simple beams and pin-ended columns and tension members.

Minimum values of the dimensionless design brace area A_B/A_M for which all the design criteria are satisfied have been obtained for a range of braced-column assemblies with various imperfection combinations. It was found that some stocky columns with imperfectly fitting braces can not be satisfactorily designed unless special care is taken to reduce the lack of fit before assembly. Graphs of values of the brace area ratios A_B/A_M have been prepared which can be used to proportion the braces for stocky columns with closely fitting braces and for slender columns. Comparisons of these values with those obtained from the rules of the SAA Steel Structures Code indicate that the brace strength rule is conservative and the brace stiffness rule is inadequate.

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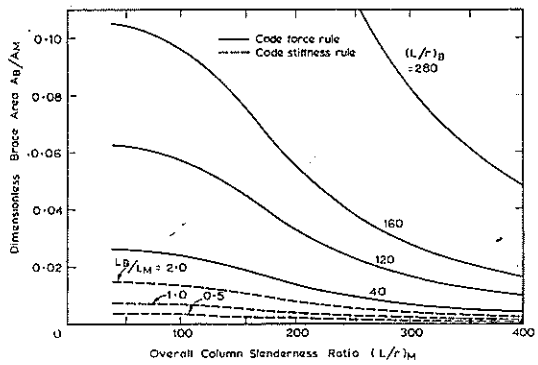

Fig. 9—Dimensionless brace areas proportioned by Code AS 1250.

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