5. ANALYSIS OF RAFT AND STRIP FOUNDATIONS

5.1 Introduction

The analysis and design of raft and strip foundations usually involves the following assessments:

- bearing capacity under the design loadings
- settlements and differential settlements
- bending moments and shears for the structural design of the foundation.

Attention will be focussed here on the latter two aspects. Ideally, analyses should take account of the stiffness of the raft or strip, together with the stiffness of the structure being supported. Such structure-foundation-soil interaction analyses, while becoming more common with major structures, are still the exception rather than the rule, and most analyses ignore the effects of superstructure stiffness.

5.2 Subgrade Reaction versus Elastic Continuum Soil Models

Table 8 summarizes and categorizes a number of methods commonly used for the analysis and design of raft and strip foundations. All but the simple rigid footing approximation give settlements and differential settlements, as well as moments and shear forces. The majority of these methods consider the stiffness of the raft or strip, and differ primarily in the manner in which the supporting soil is modelled. There are two usual methods of modelling of the soil:

1. by use of the subgrade reaction method, in which the soil is modelled as a series of independent springs (often called the "Winkler spring model" after one of the originators of the concept)
2. by use of elastic continuum theory, in which the soil is modelled as an elastic continuum.

The first approach has long been favoured by many structural and foundation engineers because of its theoretical convenience, and because, prior to the computer age, analytical solutions were available for strip foundations resting on a Winkler soil model. However, despite its theoretical convenience, the Winkler soil model has a number of important limitations which are not always appreciated. These include the following:

1. A Winkler soil model only deflects if a pressure is applied to it. Thus unloaded areas in a Winkler soil model do not deflect, and hence there is no stress transmission or interaction within the soil
2. A Winkler soil responds to loading only in the direction of that loading. Thus, vertical loading will produce only vertical displacements, and no horizontal displacements, and vice-versa
3. A Winkler soil is usually characterised by the modulus of subgrade reaction, which has units of force/length^3. The modulus of subgrade reaction is NOT a fundamental soil parameter, but is dependent on the dimensions of the foundation.

A Winkler soil model cannot incorporate properly the effects of soil layering since it does not allow stress transmission. The assessment of the modulus of subgrade reaction for a layered soil profile therefore involves considerable uncertainty which is sometimes resolved by resorting to elastic theory to obtain an equivalent value.

Table 5. Variables Used in Methods of Estimating Settlements of Footings on Sand
(Tan and Duncan, 1991)

Method (reference)	Variables Used											
	N	N_{cor}	q_c	B	D_W	D_f	γ_t	L	T	Soil Type	Str. Hist	Time
Alpan (1964)		Y		Y	Y	Y	Y				Y	
Burland and Burbridge (1985)	Y			Y	Y	Y	Y	Y	Y	Y	Y	Y
D'Appolonia & D'Appolonia (1970)	Y			Y	Y	Y			Y			
Duncan & Buchignani (1976)	Y			Y						Y		Y
Meyerhof (1956)	Y			Y								
NAVFAC (1982)	Y			Y	Y							
Parry (1971)	Y			Y	Y				Y			
Peck & Bazaraa (1969)		Y		Y	Y	Y	Y			Y		
Peck, Hanson, Thornburn (1974)		Y		Y	Y	Y	Y					
Schmertmann (1978)			Y	Y	Y	Y	Y					Y
Schultz & Sherif (1973)	Y			Y		Y			Y			
Terzaghi and Peck (1967)	Y			Y	Y					Y		

N	=	SPT Blow Count	B	=	footing width
q_c	=	Cone Penetration Test tip resistance	γ_t	=	total unit weight of sand
D_f	=	depth of footing below ground surface	Soil Type	=	silty or clean sand
T	=	thickness of sand layer below footing	D_W	=	depth of water table
Time	=	duration of loading	L	=	footing length
N_{cor}	=	SPT Blow Count corrected for overburden pressure	Stress Hist.	=	max. previous load

Y = Yes

Table 6. Computation Times for Methods Based on SPT Blow Count (Tan and Duncan, 1991)

Method	Computation Time (minutes)
Alpan (1964)	29
Burland & Burbridge (1985)	14
D'Appolonia & D'Appolonia (1970)	8
Duncan & Buchignani (1976)	9
Meyerhof (1956)	6
NAVFAC (1982)	8
Parry (1971)	9
Peck & Bazaraa (1969)	25
Peck, Hanson, Thornburn (1974)	25
Schultze & Sherif (1973)	6
Terzaghi & Peck (1967)	11

Table 7. Summary of Calculated & Measured Settlement of 3m Square Footing

Method	Settlement for P = 4MN	Notes
Terzaghi & Peck (1957)	39	Av. N = 20
Schmertmann (1978)	28	
Burland & Burbridge (1985)	21	Average value (range 10-58 mm)
Elastic Theory, using E_s = 3N MPa	18	Decourt (1989)
Elastic Theory, using PMT data	24	Reload modulus values
Strain-dependent modulus	32	Poulos (1996), Class A prediction
Finite Element Analysis	75	Chang (1994), Class A prediction, using constitutive soil model
Measured	14	After 30 minutes.

Table 8. Method of Analysis of Raft and Strip Foundations

Method	Category	Remarks	Typical References
Rigid footing assumption	1	Does not give settlements	Bowles (1984)
Strip on Winkler Soil	2A	Closed form solutions	Bowles (1984)
Strip on Elastic Soil	2A	Approximate equations for deep layer	Vesic (1961)
Design Charts for Strip on Elastic Soil	2A	Concentrated loadings, deep layer	Brown (1975)
Design Charts for Raft on Elastic Soil	2A	Uniform loadings only, finite layer	Fraser & Wardle (1976); Brown (1969)
Strip on Elastic Soil or Winkler Soil	3A	Computer program GASP	Poulos (1991)
Raft on Winkler Soil	3A	Computer program based on finite elements	Bowles (1984)
Raft on Elastic Soil	3A	Finite elements for raft	Wood (1977)
Raft on Nonlinear Soil	3B	Approx. allowance for local soil yield and raft lift-off; program GARP	Poulos (1994a)

The first two limitations are at variance with our knowledge of real soil behaviour, while the third has led to some significant difficulties, with inadequate designs arising from the use of subgrade reaction moduli which have not been corrected for the footing dimensions.

It is of interest to examine the relationship between solutions for a loaded strip foundation on Winkler and elastic continuum soil models. Brown (1977) has presented comparisons between the computed bending moments for a strip footing subjected to increasing numbers of concentrated loads. The relative stiffness of the strip, K, is defined as follows:

$$K = 16\, EI\, (1-\nu_s^2) / \pi\, E_s L^4 \quad (8)$$

where EI = bending stiffness of strip
E_s = Young's modulus of soil
ν_s = Poisson's ratio of soil
L = length of strip.

The Young's modulus and modulus of subgrade reaction values have been chosen such that the settlements of a rigid strip with a single central load are equal.

Figure 9 shows the comparison for a single central load and reveals quite reasonable agreement for a variety of relative stiffness values K of the strip. Figures 10 and 11 show similar comparisons for 3 and 5 loads equally spaced along the strip. The differences between the solutions becomes greater as the number of loads increases, and the general "dishing" effect which the elastic model reveals is not exhibited by the Winkler model, because the latter cannot consider interaction and stress transmission through the soil. In the extreme case of a uniform loading along the entire strip, the Winkler soil model predicts ZERO bending moment at all points in the strip, whereas the elastic model gives significant moments. In general, it may be concluded that the subgrade reaction approach may provide reasonable estimates of bending moment (and shear force) for strips subjected to isolated concentrated loads, but it becomes increasingly unsatisfactory as the loading becomes more distributed in nature.

5.3 The Analysis of a Raft as a Series of Strip Footings

It is common design practice to analyse a raft foundation by dividing it up into a series of strip footings and analysing each strip as an independent foundation subjected to the loadings applied on that strip. A simple example of this procedure is illustrated in Figure 12. While convenient, this procedure has a number of limitations, including:

- the strip method cannot give torsional moments in the raft
- there will generally be an incompatibility between the computed settlements at the junction of the intersecting strips.

Assuming the case shown in Figure 12, and an elastic continuum soil model, Table 9 compares the key performance characteristics computed from the strip analysis and that computed from a proper analysis of the raft as a plate. The strip solutions have been obtained from the computer program GASP (Poulos, 1991) while the raft solutions are from the program GARP (Poulos, 1994).

Two solutions from the strip analysis are shown, one in which the strip sections are assumed to be isolated independent strips, and the other in which the effects of loads on the raft area outside the strip is taken into account (the 'interacting strip' solution). Assuming that the GARP analysis is the 'benchmark' solution, the following observations are made:

a) the analysis using isolated independent strips underestimates both the settlement and bending moments
b) the interacting strip solution gives a good estimation of the maximum settlement, but under-estimates the minimum settlement
c) the interacting strip solution tends to under-estimate the maximum bending moments.

Overall, the performance of the strip analysis is disappointing and of some concern since it tends to err on the unconservative side as far as bending moments and structural design are concerned, although conversely it tends to be conservative when estimating the differential settlement between the columns in the case considered. In general, it would appear that strip analyses used to be viewed with caution, and it may be appropriate for some further research to be carried out in order to develop better procedures of adaptation of the strip method to raft analysis.

Table 9. Comparison of Computed Performance of Raft

Quantity	Calculated Value		
	Raft Analysis elastic cont'm soil	Strips with Extl. Areas	Isolated Strips
Settlement at EC mm centre col.	88.8	88.4	68.2
Settlement at A mm outer col.	75.2	55.0	33.6
M_{XX} at AC MNm/m	2.90	1.83	1.57
M_{yy} at AC MNm/m	2.40	1.08	1.12
M_{xx} at EA MNm/m	0.22	0.18	0.16
M_{yy} at A MNm/m	0.32	0.21	0.19

5.4The Effects of Structure-Foundation-Soil Interaction

It has been recognised for many years that the stiffness of a structure will affect the distribution of settlements along a strip or raft foundation, and that in turn, the distribution of structural loads and moments will be affected by the foundation flexibility. Methods of incorporating the foundation-soil interaction into a settlement analysis have been described by several authors, including, Lee and Brown (1972), Lee (1975) and Poulos (1975). In general, it has been found that the stiffness of the structure generally leads to a reduction in the differential settlements, compared to the usual methods which take the structural loads as being constant and statically determinant. An excellent example of the improvement in differential settlement prediction which may result from incorporating the structural stiffness is presented by Lopes and Gusmao (1991). For a 15 storey apartment building in Brazil, supported by a system of strip footings, the settlement distribution is predicted more closely if the stiffness of the structure is included in the settlement analysis (see Figure 13).

Lee (1975) has studied the effects of raft flexibility on the column loads in two-dimensional and three-dimensional structural frames, and has found that increasing raft flexibility leads to a more uniform distribution of structural loads than is the case for a rigid foundation (the usual case assumed by structural analysts). Lee also found that the use of the Winkler soil model predicted the reverse trend, and attributed this incorrect trend to the different settlement profiles which emerge from the subgrade reaction theory. Lee made the following observation: "With the advent of large high speed computers, the justification for the Winkler model is removed, and it is clear that it is now only of historical importance…this is no real reason for its continued use". In the intervening 25 years, computer power has increased by orders of magnitude, yet there is still an unfortunate but widespread persistence with the Winkler concept because of its convenience and simplicity. The price of this simplicity is high, given the potential for unreliable and unrealistic results and the enduring problem of assessing an appropriate modulus of subgrade reaction. The time has come for the Winkler concept to be consigned to history, and not to be perpetuated in modern-day structural and geotechnical analyses.