STRUCTURES ENGINEERING SOFTWARE MANUAL

PROGRAM APA115

THE ANALYSIS OF STIFFENER PANELS IN SHEAR

(Vers. 1.0)

Volume 3 - Theory and Validation

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SUMMARY

This report describes the theory and method used in the analysis of panels with B-, C-, H-, I-, J-, L, T- or Z-section stiffeners as implemented in the program APA115.

APA115 provides a complete instability analysis of stiffened panels, flat or curved, under shear with additional flange and stiffener direct loads. The initial panel buckling analysis is followed by a post-buckled analysis under incomplete diagonal tension, providing permanent deformation stress and failure stress for the panel. Comprehensive edge member, stiffener and attachment analyses are also included leading to true reserve factors for the complete panel assembly.

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APA115 Volume 3: THEORY AND VALIDATION

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1.0 INTRODUCTION

The preferred analysis performed by program APA115 and summarised here is based on E.S.D.U. data sheets. These have largely been derived from two N.A.C.A. reports (Ref.2 and Ref.3) which combine both theoretical and empirical results. This report outlines the basic equations and principles. For a description of the individual methods in detail and a comparison of the different methods the reader is referred to the appendices.

Shear webs may carry load in two separate modes. Prior to buckling, all the load may be carried as shear in a 'shear-resistant' web. In a completely buckled panel, load is carried in tension in a 'diagonal-tension' web. The latter case is an idealised one and is only approached asymptotically. In practice, buckled panels fall into an intermediate category of incomplete diagonal tension where the load is carried in a combination of pure shear and pure diagonal tension.

A stiffener/panel assembly can fail in a variety of modes under shear. As the applied load is increased, the first mode of instability found is normally the shear buckling of the panel. At this stage, the panel takes on the shape of waves at about 40°-45° (for a flat panel) which carry further applied load by diagonal tension. Some load is then transferred to the stiffeners and edge members by these waves, sometimes passing across the stiffeners into adjacent panels. Panel buckling itself does not necessarily constitute a failure but it may be undesirable for other reasons.

Panels are sometimes stiffened with intermediate light stiffeners. In this case it is possible that the panels buckle across the stiffeners. This may be acceptable in some situations, but in any event the waves must eventually reach boundary members adequate to contain them. If the intermediate stiffeners are very strong, they can provide a restraint greater than simple-support. APA115 checks the intermediate stiffener properties for the degree of this restraint and whether it is enough to hold the panel buckles.

Figures are presented in this report which are based on the quoted references. In most cases, the equations behind these figures have been established and the figures themselves act as a verification of the analysis procedure. Other figures are extracted directly from the embedded data in the program. These serve as a verification of the program data. The interpolation routines used are all linear. This ensures reliability and, with sufficient data points, accuracy is also maintained.

2.0 REFERENCES

1 D.M.A. Leggett. THE STRESSES IN A FLAT PANEL UNDER SHEAR WHEN THE BUCKLING LOAD HAS BEEN EXCEEDED
A.R.C. Technical Report No. 2430, September 1940

2 P. Kuhn, J.P. Peterson and L.Ross Levin. A SUMMARY OF DIAGONAL TENSION
PART 1 - METHODS OF ANALYSIS
N.A.C.A Report No. TN 2661, May 1952

3 P. Kuhn, J.P. Peterson and L.Ross Levin. A SUMMARY OF DIAGONAL TENSION
PART 2 - EXPERIMENTAL EVIDENCE
N.A.C.A Report No. TN 2662, May 1952

4 PERMANENT DEFORMATION AND FAILURE OF FLAT PLATES IN SHEAR
E.S.D.U. 02.03.13 (superseded), 1958

5 FLAT PANELS IN SHEAR. PERMANENT DEFORMATION AND FAILURE OF THE PLATE
E.S.D.U. 02.03.03 (superseded), September 1958

6 R.N. Hadcock and R.M. Hare. THE ANALYSIS OF PANELS UNDER SHEAR LOADS
Hawker Siddeley Aviation report no. S.O.R. 51, Part 1 Iss.2, Jan 1968

7 BUCKLING OF FLAT PLATES IN SHEAR
E.S.D.U. 71005, Amends A & B September 1976

8 FLAT PANELS IN SHEAR, BUCKLING OF LONG PANELS WITH TRANSVERSE STIFFENERS
E.S.D.U. 02.03.02, Amends A & B, April 1983

9 BUCKLING STRESS FOR CURVED PLATES IN SHEAR (AXIAL LENGTH EXCEEDING CIRCUMFERENTIAL LENGTH, EDGES SIMPLY-SUPPORTED)
E.S.D.U. 02.03.18, Amend A, April 1977

10 BUCKLING STRESS FOR CURVED PLATES IN SHEAR (CIRCUMFERENTIAL LENGTH EXCEEDING AXIAL LENGTH, EDGES SIMPLY-SUPPORTED)
E.S.D.U. 02.03.19, Amend B, April 1979

11 FLAT PANELS IN SHEAR. POST-BUCKLING ANALYSIS
E.S.D.U. 77014, Amend A, April 1983

12 CURVED PANELS IN SHEAR. POST-BUCKLING ANALYSIS
E.S.D.U. 77018, Amend A, August 1978

13 THE GENERALISATION OF CONTINUOUS STRESS-STRAIN CURVES FOR METALLIC MATERIALS

E.S.D.U. 76016, Amend A, May 1985

14 M. Stein and R.W. Fralich. CRITICAL SHEAR STRESS OF INFINITELY LONG, SIMPLY-SUPPORTED PLATE WITH TRANSVERSE STIFFENERS.
N.A.C.A Report No. TN 1851, April 1949

15 I.T. Cook and K.C. Rockey. SHEAR BUCKLING OF CLAMPED AND SIMPLY-SUPPORTED INFINITELY LONG PLATES REINFORCED BY TRANSVERSE STIFFENERS.
Aeronautical Quarterly, Vol.13, Part 1, pages 41-70, February 1962

16 I.T. Cook and K.C. Rockey. SHEAR BUCKLING OF CLAMPED AND SIMPLY-SUPPORTED INFINITELY LONG PLATES REINFORCED BY CLOSED-SECTION TRANSVERSE STIFFENERS.
Aeronautical Quarterly, Vol.13, Part 3, pages 212-222, August 1962

17 B.A.C. STRUCTURAL DESIGN DATA VOL. 3
Section 35. February 1966

18 PROGRAM APA114: THE ANALYSIS OF STIFFENER PANELS IN COMPRESSION
Doc.No. , Issue 1, July 1995

19 PLASTICITY CORRECTION FACTORS FOR PLATE BUCKLING
E.S.D.U. 83044, Issue 1, December 1983

20 BOEING DESIGN MANUAL
VOLUME 1, Section 8 October 1977

3.0 NOTATION

It was intended to ensure that the notation used would correspond broadly to that in the references. However, there are some inconsistences in notation between flat and curved panel notation in the E.S.D.U. data sheets. The flat panel notation corresponds more closely to N.A.C.A. and this has been adopted. The notation for shear stress has been changed from q to τ.

The general term 'stiffener' refers to any member bounding a panel. The general term 'edge member' refers to a stiffener which has a panel on one side only. The term ‘flange’ used by E.S.D.U. normally refers to an edge member of a flat web beam, such as a spar cap on a wing spar. The term ‘upright’ refers to a stiffener joining the two flanges. The term ‘frame’ normally refers to a circular frame and the term ‘stringer’ refers to the straight continuous stiffening member in a curved web system, such as a fuselage, in which the panels continue round the section.

The uprights in the flat panel correspond to the frames in the curved panel. The flanges correspond to the frames, although without the panel continuity across them. See Figs.2,3.

The same notation has generally been adopted in the program with the addition of further suffices to identify applicability. For example, panel properties are suffixed $-_p$, and stiffener properties are suffixed $-_s$. To avoid conflict with other suffices, edge member properties are suffixed $-_f$ (for frame).

NOTATION	DESCRIPTION

Material Properties, input

E	Young's modulus
f_n	reference stress
m	material characteristic
f_{all}	allowable stress, usually ultimate stress (f_{tu})

Panel Geometry, input:

t	thickness
d	depth: pitch of stiffeners S_a (usually stiffener)
h	height: pitch of stiffeners S_b (usually edge member)
b_p	width of stiffener pad
t_p	thickness of stiffener pad
R	panel radius

Stiffener Geometry, input:

h_s	height between free and attached flanges
b_f	free flange width (total for I)
b_a	attached flange width (total for I, J, T)
B_f	free flange bulb area (total for I)
B_a	attached flange bulb area (total for I, J, T)
t_w	web thickness
t_f	free flange thickness
t_a	attached flange thickness

Attachment Geometry, input:

s	pitch
p	offset from stiffener web
d_s	diameter
P_{ult}	tensile strength
P_{ults}	shear strength
P_{sfall}	stiffener to edge member joint allowable load

Applied Loads, input:

Q	applied shear load
PF	proof factor
P_s	externally applied compression load in stiffener
P_f	externally applied compression load in edge member

Material Properties, derived:

E_T	tangent modulus
G	shear modulus
G_S	secant shear modulus
G_T	tangent shear modulus
t_2	0.2% proof stress
ν	Poisson's ratio (fixed at 0.3 for this analysis)

Panel Geometry, derived:

a	free panel depth (between stiffeners S_b)
a_e	equivalent panel size for simply-supported panel
b	free panel width (between stiffeners S_a)
b_e	equivalent panel size for simply-supported panel
h_e	effective distance between stiffener centroids
l	strut length
l_e	effective strut length

Section Constants, derived:

A_f	area of edge member (flange) or frame
A_{fe}	area of edge member or frame, plus effective panel
A_s	area of stiffener or stringer
A_{se}	effective area of stiffener allowing for offset from panel [$A_s/(1 + A_s \cdot y_s^2/I_s)$]
(A_{se}	area of stiffener plus effective panel)
I_f	inertia of edge member about own neutral axis
I_{fe}	inertia of edge member about own neutral axis including panel contribution
I_{ps}	polar 2nd moment of area of stiffener plus effective panel through shear centre
I_x	inertia of stiffener about mid-plane of panel, actual
I'_x	inertia of stiffener about mid-plane of panel, required to prevent buckling
I_{xs}	inertia of stiffener about own neutral axis parallel to plane of panel
I_{xse}	inertia of stiffener plus effective panel about neutral axis parallel to panel
I_{ys}	inertia of stiffener about own neutral axis normal to plane of panel
I_{yse}	inertia of stiffener plus effective panel about neutral axis normal to panel
J_s	torsion constant of stiffener
J_{se}	torsion constant of stiffener plus effective panel under attached flange
Γ_s	torsion-bending constant (warping) of stiffener plus effective panel
k	radius of gyration, general
t_s	thickness of stiffener, mean
x_f	fibre distance for edge member bending stress
x_{fre}	fibre distance for edge member bending stress
x_s	centroid of stiffener from stiffener web centreline
x_{se}	centroid of stiffener plus effective panel from stiffener web centreline
y_s	centroid of stiffener from mid-panel
y_{se}	centroid of stiffener plus effective panel from mid-panel

Stresses & Strains

ε	strain, general
ε_f	strain in edge member (flange) or frame
ε_s	strain in stiffener or stringer
f	direct stress, general
f_{all}	allowable direct stress
f_{fi}	allowable flexural instability stress
f_{ir}	allowable inter-attachment buckling stress
f_{li}	allowable local instability stress
f_{sc}	allowable forced crippling stress
f_{ti}	allowable torsional instability stress
f_{tu}	allowable ultimate tensile stress
σ	direct stress, general
σ_{all}	allowable stress, general
σ_f	direct stress in edge member (flange) or frame
σ_{fr}	direct stress in edge member at attachment line
σ_{pr}	applied direct stress in panel at edge member or stringer
σ_s	direct stress in stiffener or stringer
σ_{sh}	direct stress in stiffener or frame heel
σ_x	direct stress along buckle
σ_y	direct stress across buckle
σ_1,σ_2	principal stresses
θ	angle of maximum principal stress
τ	shear stress, general
τ_{all}	allowable shear stress
τ_{app}	applied shear stress
τ_b	shear buckling stress
τ_{xy}	shear stress across buckle

General

C	stiffener peak stress factor
C2	stress concentration factor
c	constant in secant formula ($=k^2/y$)
e	eccentricity
F	load in edge member (flange) due to diagonal tension
G_{DT}	effective shear modulus under pure diagonal tension
G_{IDT}	effective shear modulus under incomplete diagonal tension
M	moment in edge member (flange) due to diagonal tension
K	buckling coefficient, general
K_1	single flat panel buckling coefficient
K_2	stiffened flat panel buckling coefficient
K_3	single curved panel buckling coefficient
K_p	stiffened flat panel simply-supported buckling coefficient

K_r	stiffened curved panel buckling coefficient, tentative
k_d	diagonal tension factor
k_G	factor for effective shear modulus to allow for edge member flexibility
k_r	inter-attachment buckling coefficient
P_{fs}	stiffener to edge member attachment load
R_{all}	allowable running load for edge member attachment
S_a	notation for stiffener A
S_b	notation for stiffener B
W_s	total inward radial load on stringer
w_s	uniform inward radial load/unit length on centre portion of stringer
w_f	uniform inward radial load/unit length on frame
α	angle of diagonal tension, measured to horizontal stiffener (edge member or stringer)
β	factor for panel width acting with stiffener
ξ	factor on b_a, b_p or p for determining plain panel width
μ	elastic restraint coefficient
μ_c	critical elastic restraint coefficient (buckling across stiffeners)
ω_d	edge member flexibility factor

4.0 PANEL GEOMETRY AND SECTION CONSTANTS

Some rationalisation of panel and stiffener geometry regarding effective stiffener spacing, panel widths, plain panel widths, stiffener section constants and effective panel restraints needs to be made for each stiffener and panel configuration. Assumptions need to be made when automating such an analysis. This section lists these assumptions and the reasoning behind them.

4.1 STIFFENER TYPES

The program deals with seven stiffener types: I-, J-, Z-, C-, L-, T- or B- (integral blade) section stiffeners. The notation for these stiffeners is given in Fig.1. Program APA114 (see Ref.18) performs a compression panel analysis covering four of these stiffener types. This shear panel analysis also allows for double stiffeners: dI-, dJ-, dZ-, dC-, dL-, dT- or dB-. The reinforcement pad on the panel remains on one side whether single or double stiffeners.

For the derivation of the section constants for single stiffeners, a full description can be found in Ref.18, Volume 3, Appendix B. An additional constant, I_{yse}, is also required. This depends on the number of attached flanges. For double attached flanges (I-, J-, T-sections):

$$x_p = \frac{A_{se} \cdot x_{se} - A_s \cdot x_s}{A_{se} - A_s}$$

$$I_{yse} = I_{ys} + A_s \cdot (x_s - x_{se})^2 + A_{se} \cdot (x_p - x_{se})^2 + \min(b_a, b_p)^3 \cdot \frac{t_p}{12} + \max(0, b_a - b_p)^3 \cdot \frac{t}{12} \quad (1)$$

where x_p is a temporary variable. For single attached flanges (C-, L-, Z-sections), the $A_y \cdot x$ terms representing the effect of the panel between attachments are removed

$$I_{yse} = I_{ys} + \min(b_a, b_p)^3 \cdot \frac{t_p}{12} + \max(0, b_a - b_p)^3 \cdot \frac{t}{12} \quad (2)$$

For the section constants of double stiffeners, without panel contribution, the section constants are obtained from the single section as follows:

$$\begin{aligned} A_s &\rightarrow 2 \cdot A_s \\ x_s &\rightarrow x_s \\ y_s &\rightarrow 0 \\ I_{xs} &\rightarrow 2 \cdot \left(I_{xs} + A_s \cdot {y_s}^2\right) \\ I_{ys} &\rightarrow 2 \cdot I_{ys} \\ J_s &\rightarrow 2 \cdot J_s \end{aligned} \quad (3)$$

Note that the torsion constant is conservative because it doesn't allow for coupling of the two single stiffener sections. For the section constants of double stiffeners, including panel contribution, the section constants are obtained from the single section as follows:

$$
\begin{aligned}
A_{se} &\rightarrow A_{se} + A_s \\
x'_{se} &\rightarrow \frac{A_s \cdot x_s + A_{se} \cdot x_{se}}{A_s + A_{se}} \\
y_{se} &\rightarrow 0 \\
I_{xse} &\rightarrow I_{xs} + I_{xse} + A_s \cdot {y_s}^2 + A_{se} \cdot {y_{se}}^2 \\
I_{yse} &\rightarrow I_{ys} + I_{yse} + A_s \cdot (x'_{se} - x_s)^2 + A_{se} \cdot (x'_{se} - x_{se})^2 \\
J_{se} &\rightarrow J_s + \min(b_a, b_p) \cdot \frac{{t_p}^3}{3} + \max(0, b_a - b_p) \cdot \frac{t^3}{3}
\end{aligned}
\qquad (4)
$$

The use of these section constants will become clear later in the analysis.

4.2 STIFFENER CONFIGURATION

Some typical examples of overall stiffener and panel layout are given in Fig.2 and Fig.3. There are three main aspects to the local configuration of the stiffener: stiffener reversal, stiffener offset, stiffener as edge member. These are selectable options in the APA115 input file. Also considered are the number of attached flanges and whether the stiffener is single or double.

The configuration in Fig.2 is typical of a wing spar. It has light single vertical stiffeners and heavy double horizontal edge members. The vertical stiffeners are orientated symmetrically, i.e. they are all facing the same way whereas the edge members are reversed such that the attached flanges are orientated inwards towards each other.

The configuration in Fig.3 is typical of a fuselage shell. It has light single horizontal stringers and heavy single vertical frames. Both pairs of stiffeners are orientated symmetrically and the panel is continuous across both of them.

The orientation influences the effective stiffener pitches. The stiffener pitches are defined with respect to the stiffener web centreline. The effective pitch is:

$$\text{for symmetric stiffeners: } h_e = h$$

$$\text{for reversed stiffeners: } h_e = h - 2 \cdot x_s \qquad (5)$$

The configuration also affects the plain panel width, as shown in Fig.4. This is covered in detail in Section 5.1.1, where the implication of plain panel width on panel restraints is also discussed.

5.0 INITIAL PANEL BUCKLING

APA115 calculates the initial panel buckling stress with reference to several curves across a range of data sheets, in dependence on the particular configuration of the panel and the degree of complexity used for the idealisation of the edge conditions.

The panel for analysis needs to be clearly defined. A single panel with no adjacent panels capable of transferring shear at its edges is clearly defined. Its behaviour as a shear panel is limited by its edge members. If these should buckle or fail, the structure is usually inadequate. A panel which is bounded by light stiffeners and part of larger panel could be viewed as an individual panel if its edges are adequately restrained. This usually requires that the stiffeners provide simple-support under the applied load. If this support is not provided, either buckling occurs across the stiffeners or the stiffeners themselves buckle. The boundaries defining the panel are then no longer valid for the purposes of the analysis. An analysis of the larger panel as a whole, considering these stiffeners distributed within it, may be more suitable.

In some structures, engineering judgement may be appropriate to decide that a panel is adequately supported or isolated from the adjacent structure by its edge members. In this case, the panel is considered as a single panel and an initial assumption of simple-support may be sufficient. However, in some cases this may be unduly pessimistic and the assumption of clamped edges may be more appropriate. Thus, although the panel geometry is well-defined, the restraints could be better idealised leading to a lighter structure.

Some analyses use equivalent panel sizes (see S.O.R. 51, Ref.6). This introduces the concept of an imaginary panel which has the same buckling stress as the physical panel under analysis, but has a buckling coefficient equal to a simply-supported panel. The dimensions of this imaginary panel are the equivalent panel sizes. A panel with an equivalent size greater than the physical size indicates that buckling has occurred across its edges. A panel with an equivalent size less than the physical size indicates restraints greater than simple-support. Although APA115 may quote equivalent panel sizes, these are for reference only. They are derived from the actual buckling coefficient and their values are not used in the analysis.

Inevitably, assumptions need to be made in the analysis. Some of these are concerned with initial panel buckling. The support from the stiffeners and the effective plain panel sizes are related to the type, orientation and dimensions of the stiffeners. These assumptions are incorporated in APA115 and are listed in this section.

In the flat panel analysis, more information is available to determine the effects of buckling across stiffeners. The curved panel analysis lacks this information but some tentative suggestions have been made which incorporate features from the flat panel analysis.

The output of APA115 can be brief or extended. The extended output provides more detail about assumptions and calculations performed by the analysis. Several buckling coefficients based on different edge restraints are determined. From these, a coefficient applicable for the structure is selected directly or otherwise derived.

5.1 FLAT PANELS

The elastic buckling stress is expressed in general form as follows:

$$\tau = K \ \cdot E \cdot \left(\frac{\tau}{b}\right)^2 \tag{6}$$

where K is the buckling coefficient obtained from theoretical or experimental data. The panel dimension b is the shorter of the two sides, or plain panel widths. The value of K depends on assumptions about panel geometry and edge restraint. In its simplest form, it may be related purely to aspect ratio. In more detailed analyses, the flexural and torsional properties of stiffeners are taken into consideration.

5.1.1 Plain Panel Width

The plain panel width is based on stiffener configuration: single or double, offset to web, orientation, number of attached flanges. Fig.4 shows the plain panel width for all possible configurations, apart from the reinforcement pad geometry. Note that the height, h, is always between the stiffener web centrelines. There is a minimum and a maximum value for the plain panel width for most of these configurations. These limits are based on nominal assumptions regarding the relative thicknesses of the plain panel and the attached flange of the stiffener.

The assumptions regarding double stiffeners is that they can reduce the effective plain panel to the edge of the attached flanges or heel of the stiffener if the attached flanges are thick enough in comparison with the panel thickness. The assumption made is that the attached flange thickness must be greater than or equal to the plain panel thickness for this to begin to have an effect. If the stiffener attached flange thickness approaches zero, it is assumed that the plain panel size extends fully to the stiffener pitch. The stiffener attached flange is usually a gauge or two higher than the panel, thus falling in between these two extremes. In such a case, a linear interpolation is performed giving the following factor which is applied to the difference between the minimum and maximum plain panel sizes.

$$\xi = 1 - \max\left(\min\left(1 - \frac{t_a}{t}, 1\right) 0\right) \tag{7}$$

Single stiffeners are considered to be capable of reducing the effective plain panel size to the attachment as the panel may buckle on the underside between attachment and the free edge of the attached flange. Again, a limit on the attached flange thickness is set. In this case, full reduction is assumed to be provided if the attached flange thickness is twice as much as the plain panel thickness. This is equivalent to the total of the attached flange thicknesses of the double stiffener layout. In the same way, a linear interpolation is performed where the attached flange thickness falls in between these limiting values. In this case, the following factor is used:

$$\xi = 1 - \max\left(\min\left(1 - \frac{t_a}{2 \cdot t}, 1\right) 0\right) \tag{8}$$

There may also be an effect on plain panel size due to the reinforcement pad underneath the stiffener. In practice this will act together with the attached flange. However, the analysis considers it as a separate condition and uses the maximum panel size reduction from either attached flange of reinforcement pad individually. The maximum panel size reduction is assumed to be provided when the pad thickness is greater than or equal to twice the plain panel thickness. The factor to be applied to the reinforcement pad width, b_p, is obtained as follows.

$$\xi = 1 - \max\left(\min\left(2 - \frac{t_p}{t}, 1 \right), 0 \right) \tag{9}$$

These adjustments to the plain panel width are intended to make an allowance for effective panel thickness at the edge of the panel. They are not intended to represent different support idealisations. They are based on engineering judgement with a view to alleviating pessimism in the analysis.

5.1.2 Single Flat Panels

A simple analysis is found in E.S.D.U. 71005 (Ref.7). This provides the shear buckling stress for panels with a combination of either simply-supported or clamped edges, but no intermediate edge restraint between these extremes. The buckling coefficient is only dependent on the panel aspect ratio. As the minimum restraint is a simple-support, it is implicitly assumed that buckling across the stiffeners does not occur. If this analysis were to be used in isolation, some justification of the adequacy of stiffener restraint would be necessary.

The curves of buckling coefficient are represented in Fig.5, obtained directly from data embedded in the program. In the extended output, APA115 quotes all four buckling coefficients.

5.1.3 Multiple Flat Panels

The results of a more detailed analysis are presented in E.S.D.U. 02.03.02 (Ref.8). This considers long panels with transverse stiffeners in which the stiffeners are considered to act in conjunction with the panel to restrain its overall buckling. The buckling modes may be such that the complete panel buckles or one individual bay buckles or some intermediate mode. The curves in Ref.8 were obtained from Refs.14-16. Although Refs.14-16 go into great depth, the precise derivation of the curves has not been identified at this stage. The curves are represented in Figs.6a-6j, obtained from embedded data in the program.

The limiting curves for μ_c on Fig.6a and Fig.6b represent the limit at which buckling occurs across the transverse stiffeners (normally the stiffeners S_a, of length a and pitch b). At this boundary, the stiffeners are providing minimal restraint - i.e. simple-support. The figure for simply-supported edge members (Fig.6b) is the figure used by APA115 to determine the inertia required to prevent buckling across the stiffeners.

A value of μ = 0 on Fig.6a and Fig.6b represents a complete loss of restraint on the transverse stiffeners. This effectively implies that these stiffeners are no longer present in the capacity of boundary members. The buckling stress must be based on an extended panel, using the shortest dimension (which may then become a instead of b) on Fig.6a and Fig.6b.

The buckling coefficients so far considered have been referred to transverse stiffeners with no torsional stiffness. Ref.8 provides a range of curves which make allowance for this. These are reproduced in Figs.6c-6j.

APA115 performs the analysis twice, considering in turn stiffeners S_a and S_b as the transverse stiffeners. Thus criteria can be established for the stiffness of each stiffener. This may be conservative when the stiffener is an edge member and is thus only a guideline. One pair of stiffeners will not meet the aspect ratio limit of a/b > 1. In which case a/b = 1 is assumed.

In the extended output, program APA115 provides values of buckling coefficient from all the figures for comparison - a total of eight from this reference. The entry values to the figures are also quoted, the stiffener section constants being calculated as explained in Section 4. Note the definition of the stiffener inertia used by Ref.8. The stiffener inertia about the panel mid-plane is suggested, but it is stated that a combination of half the panel width plus stiffener about the combined neutral axis provides similar results. Program APA115 uses the stiffener inertia alone about the panel mid-plane.

5.1.4 Equivalent Panel Size

An equivalent panel is defined as a simply-supported panel which has the same buckling stress as the physical panel under analysis. The dimensions of this panel are the equivalent panel sizes:

$$a_e = a \cdot \sqrt{\frac{K_1}{K_p(b)}}, \quad b_e = b \cdot \sqrt{\frac{K_1}{K_p(a)}} \qquad (10)$$

where K_1 is the single panel, simply-supported buckling coefficient from Fig.4 using the smallest of a/b or b/a and K_p is the multiple panel, simply-supported buckling coefficient from Fig.6b. $K_p(a)$ is the value found using S_a as the transverse stiffener and $K_p(b)$ is the value found using S_b.

APA115 quotes the equivalent flat panel dimensions for reference in the extended output. These may be directly compared with the output which comes from the S.O.R. 51 analysis (Ref.6). Whereas Ref.6 uses the increased effective panel width as part of its method of analysis, it is not an inherent part of the calculations in this analysis but an extension of it.

5.1.5 Selection of Buckling Coefficient

A total number of twelve possible buckling coefficients are calculated by APA115. It then remains to choose the most appropriate. The broad generalisation is made that double stiffeners provide clamping and single stiffeners provide simple support. Thus, if both pairs of stiffeners are double, the buckling coefficient chosen is the minimum of the two fully-clamped coefficients from Ref.8 obtained when considering each stiffener as the transverse stiffener. If only stiffeners S_a are double, the minimum of the coefficient from longitudinal edges clamped curves for S_a and the coefficient from longitudinal edges simply-supported curves for S_b is used. Likewise, the buckling coefficients for other stiffener combinations are found.

Although the assumptions regarding restraint are of a very general nature, the user may decide to over-ride that chosen by the program by selecting a value from the range of buckling coefficients presented, or even one from some other source.

5.1.6 Plasticity Correction

The plasticity correction for shear buckling is η_1, obtained from E.S.D.U. 83044 (Ref.19) using C = 2. Although η_8 is quoted as the coefficient for shear, this value of η_1 gives an identical set of curves to E.S.D.U. 71005 (Ref.7). The value of η_1 is nearly identical to the secant modulus ratio. That is, η_1 times the elastic stress is approximately equal to the stress on the stress-strain curve at a strain equal to the elastic strain. The slight difference is due to allowance for variation in the plastic value of Poisson's ratio.

The plasticity correction curves generated by MathCAD using Ref.19 are given in Fig.7.

5.1.7 Stiffener Inertia Requirement

The values of μ and μ_c obtained from Ref.8 Fig.1 provide a means of determining if the stiffeners have enough inertia to prevent buckling across them. The inertia here is that of the stiffener alone about the mid-plane of the panel (see 5.1.3). The minimum stiffener inertia required is given by:

$$I'_x = \frac{\mu_c}{\mu} \cdot I_x \qquad (11)$$

This is equivalent to having an increased effective panel size greater than the actual panel size (see 5.1.4). The above expression directly leads to the determination of the minimum stiffener inertia whereas that of Ref.6 require the intermediate calculation of increased effective panel width.

APA115 quotes the ratios of actual inertias to required inertias. These are presented as reserve factors but are in parentheses as they are not, strictly speaking, true reserve factors.

5.2 CURVED PANELS

E.S.D.U. 02.03.18 and E.S.D.U. 02.03.19 (Ref.9 and Ref.10) provide the buckling stress for curved panels in shear. These references correspond to the flat panel data sheet in Ref.7 for the simply-supported case only. No reference has been found which considers the possibility of buckling across stiffeners for curved panels in the same manner as Ref.8.

Curved panels are less prone to buckle in shear than flat panels. This may be appreciated by considering that such a panel will tend to flatten itself before being capable of developing a tension field requiring a greater energy input.

The references have investigated shear buckling of flat panels in greater depth than curved panels. The greater allowable stress of a curved panel based on the references quoted will not allow for buckling across the stiffeners. APA114 makes a tentative allowance for this effect in curved panels based on the effect it would have if the panel were flat. The ratio of the buckling coefficients for the flat panel, with and without consideration of stiffeners flexibility, will be used to factor the buckling coefficient for the curved panel. Thus the net buckling coefficient, K_r, is:

$$K_r = \frac{K_2}{K_1} \cdot K_3 \qquad (12)$$

where K_1 is obtained for a simple panel from Ref.7, K_2 is obtained for a long panel with transverse stiffeners from Ref.8 and K_3 is for a simple curved panel from Ref.9 or Ref.10.

Although no empirical verification of this procedure is available, it is expected to provide conservative results. Justification for this is as follows. Should buckling occur across one stiffener, for example, the effective panel size increases to two bays. This requires significantly greater energy to flatten it than the flat panel as the central stiffener has to be brought in line with the two outer stiffeners, against the curvature. As more stiffeners are subjected to panel buckling, an increasing resistance to further buckling is presented. This resistance is not present in flat panels so an allowance based on flat panels is thus conservative. However, the stiffeners in a curved panel are subject to a lateral load requiring a more detailed analysis than the flat panel.

Several other aspects of the curved panel analysis are based on flat panels. Following on from the allowance for buckling across stiffeners, the required stiffener inertias are calculated. The correction for plasticity follows the same procedure as for flat panels.

The curved panel buckling coefficients are reproduced from Ref.9 and Ref.10 in Fig.8 and Fig.9 obtained from data embedded in the program.

6.0 POST-BUCKLED ANALYSIS

As with initial buckling, the post-buckled behaviour of shear panels has been investigated in greater depth for flat panels than for curved panels. APA115 follows the method in two E.S.D.U. data sheets. These are 77014 for flat panels (Ref.11) and 77018 for curved panels (Ref.12). They are based almost entirely on two N.A.C.A. reports (Ref.2 and Ref.3). An initial buckling stress is required for the post-buckled analysis. This has been detailed in Section 5.

Due to the lack of information on curved panels, the analysis has been extended to provide a similar level of detail in the strength summary as for the flat panels. This means several assumptions have had to be made.

Post-buckled shear analysis is based on the degree of buckling, measured by the diagonal tension factor, k_d ($0 \leq k_d \leq 1$). Two systems are considered to be acting concurrently to react the applied shear load. A fraction, $(1 - k_d)$, is reacted in pure shear and a fraction, k_d, is reacted in pure diagonal tension.

The diagonal tension acts at an angle α to the edge member or stringer. This is around 40° for a flat web system but can vary a lot more for curved web systems around 20° to 30°. The determination of this angle is fundamental to the analysis of post-buckled behaviour. Once it has been established, many of the design parameters can be derived.

There is a difference in notation between flat and curved panels in E.S.D.U. Ref.11 and Ref.12. The more substantial stiffeners acting as edge members in a flat panel, such as the wing spar in Fig.2, are the spar caps at the wing skin and the panel is continuous in a spanwise sense for the purposes of Ref.8. The more substantial stiffeners in a curved panel, such as the fuselage skin in Fig.3, are the frames and the panel is continuous in a circumferential sense for the purposes of Ref.8. The angle of diagonal tension is referred to the edge members in the flat panel and to the stringer in the curved panel. When considering flexural instability, the vertical stiffeners in the flat panel are critical and the horizontal stringers in the curved panel are critical.

6.1 FLAT PANELS

Flat panels are covered in Ref.11. In general, this reference is consistent with the N.A.C.A. reference (Ref.2), but some minor discrepancies have been noted which were reported to E.S.D.U. in 1990 and are still waiting to be resolved. As program APA115 has been based on routines emulating the Ref.11 analysis by generating the figures analytically where possible, this section will concentrate on presenting the theoretical relationships leading to the equations and figures representing those in Ref.11. The greater depth of Ref.2 has been used to this end.

Two figures are used in program APA115 for which no theoretical relationship is available. These are shown in Fig.10 and Fig.11. Fig.10 provides concentration factors and comes from Ref.2. It is based on empirical data. Fig.11 comes from Ref.11 and is basically identical to that in S.O.R.51 (Ref.6). This figure is based in part on a long analysis by Legget (Ref.1).

6.1.1 Diagonal Tension Factor

The diagonal tension factor, k_d, of a flat web is based on the buckling ratio (τ/τ_b) and given by the empirical relationship from Ref.2 equation (27):

$$k_d = \tanh\left[0.5 \cdot \log_{10}\left(\frac{\tau}{\tau_b}\right)\right] \tag{13}$$

If the applied stress is less than the buckling stress, k_d is set to zero for later calculations.

6.1.2 Angle of Diagonal Tension

The angle of diagonal tension, α, is calculated from the consideration of the strains acting on the flange, stiffener and web (ε_f, ε_s, ε respectively). These are obtained from the Ref.2 equations 30a - 30d:

$$\frac{E \cdot \varepsilon_f}{\tau} = -\frac{k_d \cdot \cot\alpha}{2 \cdot \frac{A_f}{h \cdot t} + 0.5 \cdot (1 - k_d)} \tag{14}$$

$$\frac{E \cdot \varepsilon_s}{\tau} = -\frac{k_d \cdot \tan\alpha}{\frac{A_{se}}{d \cdot t} + 0.5 \cdot (1 - k_d)} \tag{15}$$

$$\frac{E \cdot \varepsilon}{\tau} = \frac{2 \cdot k_d}{\sin 2\alpha} + (1 - k_d) \cdot (1 + \nu) \cdot \sin 2\alpha \tag{16}$$

$$\tan^2 \alpha = \frac{\varepsilon - \varepsilon_f}{\varepsilon - \varepsilon_s} \tag{17}$$

These equations are not directly solvable. APA115 solves them by iteration. A starting value of 45° is used. If the applied stress is less than the buckling stress, α is set to 45 ° for later calculations.

6.1.3 Edge Member Strength

Two calculations are performed in the case of an edge member. Under the action of induced end-load and bending in conjunction with the applied end-load, critical fibres are checked. The inter-rivet buckling of the attachment to the panel is then considered.

6.1.3.1 Maximum Applied Edge Member Stress

The maximum applied stress at the most critical locations on the edge member are considered in comparison to the maximum allowable edge member stress. The average compressive load in a flange due to the web shear is given by a derivation of Ref.2 equation 30b using loads instead of stresses:

$$\frac{F}{Q} = -\frac{\frac{A_f}{h \cdot t} \cdot k_d \cdot \cot\alpha}{2 \cdot \frac{A_f}{h \cdot t} + 0.5 \cdot (1 - k_d)} \tag{18}$$

There are some differences between the figures presented in Ref.11 Figs.10-12 and the equations in Ref.2. This equation has been recalculated and the results are given in Fig.12 in the same format as Ref.11 but the figure for $A_f/ht = 0$ has been replaced by a figure at $A_f/ht = 0.1$. The differences between the figures and equations are assessed in more detail in Appendix B.

There is also a bending moment acting on the edge member due to the panel shear as the shear cannot be carried across the edge member as it can across the stiffener. For a continuous edge member, this should be considered to act above the stiffener attachment causing tension in the fibres away from the panel and with the same magnitude but opposite sense in the central portion of the edge member. A curve for this is given in Ref.11 Fig.13. This is represented as follows and reproduced in Fig.13:

$$\frac{M}{\tau \cdot t \cdot b^2} = 0.054\sqrt{k_d} \tag{19}$$

Ref.2 Section 4.16 suggests the moment is proportional to k_d and reaches a maximum possible value of $0.083k_d$ but also considers this to be conservative. Equation (19) will provide a greater estimate of bending moment than this and is the calculation adopted by APA115.

There may be additional loads acting on the flange due to influences from external structure (e.g. bending moments). These should be included in the edge member analysis.

The net direct stress at a chosen fibre in the edge member due to diagonal tension comes from a combination of end-load and moment, each being a function of shear stress τ. This is combined with the applied direct stress factored to remain in proportion to the applied shear stress. In order to consider the worst possible case, the sense of end-load and moment are kept the same.

$$\sigma_f = \frac{F(\tau)}{A_{fe}} + \frac{M(\tau) \cdot x_f}{I_{fe}} + \frac{|P_f|}{A_{fe}} \cdot \frac{\tau}{\tau_{app}} \tag{20}$$

This expression is iterated for τ until σ_f reaches the maximum allowable value.

6.1.3.2 Edge Member Inter-Attachment Buckling

Ref.11 does not consider inter-attachment buckling at the edge member attachment to the panel. Considering the sometimes high applied loads in edge members, it is thought that this is just as important as consideration of the inter-attachment buckling in the stiffener. The allowable inter-attachment buckling stress is given by:

$$\frac{f_{ir}}{E_t} = \frac{k_r}{12} \cdot \left(\frac{\pi \cdot t}{s}\right)^2 \tag{21}$$

The coefficient depends on the type of attachment: for flat head rivets, $k_r = 4.0$; for mushroom or snaphead head rivets, $k_r = 3.0$; for countersunk or dimpled rivets, $k_r = 1.5$. The above expression is for the panel and uses panel thickness and material properties. The edge member is also checked using the attached flange thickness and edge member properties. The lower of the two is used. This is iterated against the net end load in the edge member which assumed to act at the attachment line for the purposes of this calculation.

6.1.4 Panel Strength

The stresses in the panel are considered in two locations - in the centre of the panel away from the influence of the stiffeners or edge members and near the edge member where there is influence from the edge member loading.

6.1.4.1 Panel Stress at Centre of Panel

Ref.11 Fig.1 provides a series of curves for allowable stresses for the panel away from the stiffeners, not allowing for edge member flexibility. These are reproduced in Fig.11. It gives the panel stress for permanent deformation based on the proof stress and the panel stress for failure based on the ultimate stress. Permanent deformation causes yielding at the crests of the waves and leaves a slight waviness in the plate after unloading. Failure is caused by a high average stress through the thickness of the panel and results in rupture approximately normal to the waves. There is also an allowable stress for general permanent deformation which uses the curves for failure together with the proof stress. The minimum value of A_s/bt and A_f/ht should be used for these curves. Also, to make allowance for different stiffener and edge member materials, the values of A_s/bt and A_f/ht should be factored by the ratio of their respective Young's modulus to the panel's Young's modulus.

These curves were originally presented in a different format in earlier E.S.D.U. data sheets. They are derived partly from theoretical work by Leggett (Ref.1), but there is no direct analytical solution to the curves. Limited values of A_s/bt and A_f/ht are available at present but for values below 0.3, a linear interpolation down to the $\tau = \tau_b$ line is thought adequate. This will tend to be conservative.

6.1.4.2 Panel Stress near Edge Member

Ref.11 Fig.2 provides a curve for allowable stresses at the plate to flange joint. This considers the interaction of the web shear stress with the direct stress in the flange. This curve is given by:

$$\begin{gathered}\frac{\sigma_{pr}}{\sigma_{all}} > 0.5: \quad \left(\frac{\sigma_{pr}}{\sigma_{all}}\right)^2 + 3\cdot\left(\frac{\tau_{all}}{\sigma_{all}}\right)^2 = 1 \\ \frac{\sigma_{pr}}{\sigma_{all}} <= 0.5: \quad \left(\frac{\tau_{all}}{\sigma_{all}}\right)^2 = 0.5\end{gathered} \qquad (22)$$

σ_{all} should be taken as the tensile strength of the plate material, f_{allp}.

The method of using this curve, employed in the example in Ref.11, takes no account of the fact that σ_{pr} is a function of τ_{all}. σ_{pr} is a combination of stresses from the additional applied stress and that due to diagonal tension. It is not correct to use a value of σ_{pr} determined from the applied stress to obtain τ_{all} from the curve directly because the value of τ_{all} so obtained would imply a different value of σ_{all}. An iterative method is used by program APA115. The figure representing the above equations is given in Fig.14.

The direct stress used in conjunction with the shear stress is that at the flange rivet line. This comes from combination of end-load and moment, each as a function of shear stress τ, with the applied direct stress factored to remain in proportion to the applied shear stress. In order to consider the worst possible case, the sense of end-load and moment are kept the same.

$$\sigma_{fr} = \frac{F(\tau)}{A_{fe}} + \frac{M(\tau) \cdot x_{fre}}{I_{fe}} + \frac{|P_f|}{A_{fe}} \cdot \frac{\tau}{\tau_{app}} \qquad (23)$$

This expression is iterated for τ, together with the interaction equations above, until σ_{fr} reaches the maximum allowable value.

6.1.5 Stiffener Strength

There are three types of applied stiffener stresses considered in the analysis. There is an average stiffener stress along its length, which is suitable for comparison with overall stiffener stability, and a peak stiffener stress which may be associated with local modes of failure and instability. Additionally, a local stiffener heel stress may be found which is suitable for comparison with attached flange criteria.

There may be additional loads acting on the stiffener due to influences from external structure (e.g. Brazier loads on wing ribs). These must be included in the stiffener analysis together with the loads induced by shear.

There are several possible criteria to consider when analysing the stiffener. The overall modes are flexural and torsional instability. Combined flexure and torsion is not considered in this analysis. The local modes considered are attached flange instability and inter-rivet buckling. Forced crippling is also considered - often the main criterion for thin, fabricated panels.

All the stiffener criteria are solved by iterative methods. For most of the criteria, it is possible to determine the allowable load in the stiffener. However, to find the shear load at which the stiffener load is induced makes the iteration necessary.

6.1.5.1 Average Stiffener Stress

The average compressive stress in a single or double stiffener due to the web shear is shown in Fig.15. It is obtained from Ref.2 equation 30a:

$$\frac{\sigma_s}{\tau} = -\frac{k_d \cdot \tan\alpha}{2 \cdot \frac{A_{se}}{d \cdot t} + 0.5 \cdot (1 - k_d)} \qquad (24)$$

This equation is represented in Ref.11 Figs.3-5, subject to the small differences mentioned in Section 6.1. Equation (24) results in curves which are between 8.6% and 12.8% lower. The value of A_{se} is normally taken as A_s, the actual stiffener area, for double stiffeners.

6.1.5.2 Stiffener Heel Stress

The stress at the heel of a single stiffener, σ_{sh}, is obtained by using a value of A_{se}, an equivalent stiffener area, in equation (24) determined as follows:

$$A_{se} = \frac{A_s}{1 + \frac{A_s \cdot y_s^{2}}{I_{xs}}} \qquad (25)$$

6.1.5.3 Peak Stiffener Stress

The stress in the stiffener reaches a peak in the central portion. This is known as the 'gusset effect' where the local panel begins to carry a greater proportion of the load. This effect is difficult to quantify precisely. The peak factor used comes from Ref.11 Fig.6. There is a different factor in Ref.2, but both expressions are rather tentative.

$$C = 1 + 0.57 \cdot (1 - k_d) \cdot \left(1 - \frac{b}{h}\right) \qquad (26)$$

This expression is shown graphically in Fig.16.

6.1.5.4 Stiffener Flexural Instability

Flexural instability is considered using the secant formula method. Due to effective panel width and strut length variation, the analysis does not provide an allowable stiffener load independent of the applied shear load. Therefore the iteration internally generates allowable stiffener loads in dependence on the shear.

The effective panel width to be used in conjunction with the basic stiffener properties is taken from Ref.11 Fig.7. This agrees with Ref.2 and is simply expressed as follows:

$$\beta = 0.5 \cdot (1 - k_d) \qquad (27)$$

β is a factor applied to the stiffener pitch b. This expression is shown graphically in Fig.17.

The effective strut length reduces with applied shear stress and as the stiffeners become long with respect to their pitch. This is due, in part, to the restraining effect of diagonal tension waves. These behave like corrugations providing lateral bending stiffness to the panel and consequently a restraint on the stiffener. The effective strut length is found from Ref.11 Fig.8. It agrees with Ref.2 for double stiffeners. It is also used for single stiffeners with eccentric loading whereas Ref.2 uses 0.5·h with no eccentricity. The strut length factor, shown graphically in Fig.18, is:

$$\frac{b}{h} < 1.5: \quad \frac{l_e}{l} = \frac{1}{\sqrt{1 + k_d^{2} \cdot \left(3 - 2 \cdot \frac{b}{h}\right)}}, \qquad \frac{b}{h} \geq 1.5: \quad \frac{l_e}{l} = 1 \qquad (28)$$

The secant formula strut analysis solves the following equation for f_{fi}:

$$f_{all} = f_{fi} \cdot \left[1 + \frac{e}{c} \cdot \sec\left(\frac{l_e}{2 \cdot k} \cdot \sqrt{\frac{f}{E_T}}\right)\right] \qquad (29)$$

The shear stress is iterated until the net average applied stiffener stress reaches this value. The analysis considers both stiffener and panel materials and fibre offsets.

6.1.5.5 Stiffener Torsional Instability

The torsional instability analysis is applicable to open sections only. The allowable torsional instability stress of the stiffener is not dependent on the applied shear load. The torsional instability stress is given by:

$$\frac{f_{ti}}{E_T} = \frac{1}{I_{ps}} \cdot \left[\frac{J_s}{2 \cdot (1+\nu)} + \Gamma_s \cdot \left(\frac{\pi}{h} \right)^2 \right] \tag{30}$$

This is corrected for plasticity using the usual Ramberg-Osgood tangent modulus method. The shear stress is iterated until the net average applied stiffener stress reaches this value.

6.1.5.6 Stiffener Local Instability

The local instability analysis considers the attached flange only, being the highest-stressed location on the stiffener. The allowable local instability stress is given by:

$$\text{C, L, Z sections}: \quad \frac{f_{li}}{E_T} = 0.58 \cdot \left(\frac{t_a}{\frac{1}{2} \cdot b_a} \right)^2, \qquad \text{I, J, T sections}: \quad \frac{f_{li}}{E_T} = 0.58 \cdot \left(\frac{t_a}{b_a} \right)^2 \tag{31}$$

This is corrected for plasticity using the usual Ramberg-Osgood tangent modulus method. The shear stress is iterated until the net peak applied stiffener heel stress reaches this value.

6.1.5.7 Stiffener Forced Crippling

The forced crippling allowable is dependent on the applied shear stress. Moreover, as this mode depends on the shear buckles deforming the attached flange, the forced crippling allowable is compared with the stiffener stress induced by the buckled panel. That is, any existing applied load in the stiffener is not included. The effect of combining applied stiffener load and induced stiffener load is thought to be adequately covered by the local instability analysis of the attached flange.

The forced crippling allowable is given in Ref.11 Fig.9. The expression for these curves comes from Ref.2 where it is considered applicable for single stiffeners. Double stiffeners are considered to be 80% less than this value:

$$\frac{f_{sc}}{t_{2s}} = 0.5 \cdot k_d^{2/3} \cdot \left(\frac{t_a}{t} \right)^{1/3} \tag{32}$$

The shear stress is iterated until the peak applied stiffener stress due to shear buckling reaches this value. Fig.19 plots this expression for various t_s/t.

6.1.5.8 Stiffener Inter-Attachment Buckling

This follows the same procedure as the local instability analysis, being centred around the attached flange. The allowable inter-attachment buckling stress is given by:

$$\frac{f_{ir}}{E_t} = \frac{k_r}{12} \cdot \left(\frac{\pi \cdot t}{s}\right)^2 \tag{33}$$

The coefficient depends on the type of attachment: for flat head rivets, k_r = 4.0; for mushroom or snaphead head rivets, k_r = 3.0; for countersunk or dimpled rivets, k_r = 1.5. The above expression is for the panel and uses panel thickness and material properties. The stiffener is also checked using the attached flange thickness and stiffener properties. The lower of the two is used.

This is corrected for plasticity using the usual Ramberg-Osgood tangent modulus method. The shear stress is iterated until the net peak applied stiffener heel stress reaches this value.

6.1.6 Attachment Strength

6.1.6.1 Edge Member Attachment

Ref.11 Fig. 14 presents a specific criterion for the edge member attachment. This appears to have been derived from Ref.2 Equation 19a which is as follows:

$$\frac{R_{all}}{\tau \cdot t} = (1 + k_d \cdot C_2) \cdot (1 + 0.414 \cdot k_d) \tag{34}$$

R is the running load along the attachment. C_2 is the concentration factor from Fig.10. This is obtained from the following parameter based on edge member stiffness:

$$\omega_d = 0.7 \cdot b \cdot \sqrt[4]{\frac{t}{2 \cdot h \cdot I_{fe}}} \tag{35}$$

Again, the expression for R requires iteration as R is a function of τ. R may be obtained from Fig.20, which is a representation of Ref.11 Fig.14. There are a few discrepancies between the equation from Ref.2 and the figure from Ref.11. The analytical expression has been followed.

6.1.6.2 Stiffener/Edge Member Load

The load in the stiffener due to shear buckling results in a load at the attachment to the edge member. The example in Ref.11 gives rise to the following expression which is an interpolation between a reduction of 25% in stiffener end-load at τ/τ_b=2 and a reduction of 10% at τ/τ_b=20. The load at the attachment is thus:

$$P_{fs} = \frac{\sigma_s \cdot A_s}{120} \cdot \left(88 + \frac{\tau}{\tau_b}\right) \tag{36}$$

The iteration to find the allowable shear at which the attachment fails uses the average stiffener stress in the above expression *without* any added applied load. The applied load in the stiffener is not considered to be input through this joint.

6.1.6.3 Stiffener Attachment Tension Criteria

The stiffener attachment needs to be capable of carrying a degree of tension from the buckled panel. Ref.11 uses the same criteria as Ref.2. From test results, it was found that failure is unlikely if the tensile load capacity per unit run of the attachment exceeded $0.22 \cdot t \cdot f_{tu}$ of the stiffener for single stiffeners and $0.15 \cdot t \cdot f_{tu}$ of the panel for double stiffeners.

APA115 calculates the margin beyond these criteria as a 'reserve factor' although it is not strictly speaking a reserve factor in the normal sense.

6.1.7 Panel Stiffness

The panel stiffness reduces once shear buckling is initiated. Ref.11 provides figures for determining the secant and tangent shear moduli of the buckled panel. APA115 presents these stiffnesses in the extended output. The following expressions come from Ref.2 pages 56-57:

$$\frac{E}{G_{DT}} = \frac{4}{\sin^4 \alpha} + \frac{\tan^2 \alpha}{\frac{A_s}{b \cdot t} + 0.5 \cdot (1 - k_d)} + \frac{\cot^2 \alpha}{2 \cdot \frac{A_f}{h \cdot t} + 0.5 \cdot (1 - k_d)}$$

$$G_{DT} = \frac{2 \cdot (1 + \nu)}{\frac{E}{G_{DT}}} \tag{37}$$

$$\frac{G_{IDT}}{G} = \frac{1}{1 + k_d \cdot \left(\frac{1}{G_{DT}} - 1 \right)}$$

These are given in Fig.21. The tangent moduli may be obtained numerically by calculating the change in secant modulus over a small range of buckling ratio:

$$\left(\frac{G_T}{G} \right)_i = \frac{(G_s/G)_{i+1} \cdot (q/q_b)_{i+1} - (G_s/G)_i \cdot (q/q_b)_i}{(q/q_b)_{i+1} - (q/q_b)_i} \tag{38}$$

Fig.22 reproduces Ref.11 Figs.18-20 using this expression. There is, however, some slight deviation. APA115 uses these expressions, but they do not form part of the strength analysis.

A allowance is made due to flange flexibility following Ref.12. This is based on a parameter relating panel stiffness to flange stiffness. When this parameter is less than 0.5 the reduction is no more than 10% and when it is less than 0.2 the reduction is no more than 5%. This provides the following reduction factor to be applied to G_S/G and G_T/G:

$$k_G = 1 - \frac{0.1}{0.5} \cdot \frac{A_s + \beta \cdot b \cdot t}{360 \cdot h \cdot t \cdot \frac{I_f}{b^3 \cdot t}} \tag{39}$$

6.2 CURVED PANELS

Curved panels are covered by N.A.C.A. 2661 Sections 8-10 (Ref.2) and E.S.D.U. 77018 (Ref.12). S.O.R. 51 (Ref.6) also covers curved panels. This section follows the presentation of Ref.12 but uses the detail and equations from Ref.2 to clarify the analysis. The curved panel analysis in Ref.6 has not been considered.

Due to a lack of information on curved shear panels, some of the flat panel analysis has been adapted to curved panels to present an analysis that attempts to cover the same criteria. Some tentative assumptions have had to be made which have not been verified by test results.

6.2.1 Diagonal Tension Factor

The diagonal tension factor, k_d, of a curved web from Ref.2 is similar to that for a plane web but includes a factor for the panel radius:

$$k_d = \tanh\left[\left(0.5 + 0.3 \cdot \frac{t \cdot d}{R \cdot h}\right) \cdot \log_{10}\left(\frac{\tau}{\tau_b}\right)\right] \quad (40)$$

In the above equation, d/h is the greater of d/h and h/d but not exceeding 2. For some unexplained reason, Ref.12 applies the conservative restriction that d≥2h, modifying the above to:

$$k_d = \tanh\left[\left(0.5 + 0.6 \cdot \frac{t}{R}\right) \cdot \log_{10}\left(\frac{\tau}{\tau_b}\right)\right] \quad (41)$$

The latter expression is used and this is presented in Fig.23, a representation of Ref.12 Figure 1.

6.2.2 Angle of Diagonal Tension

The angle of diagonal tension, α, is calculated from the strains acting on the flange, stiffener and web (ε_f, ε_s, ε respectively). These are obtained from the Ref.2 equations 44, 45, 50, 51. In these equations, stiffener replaces flange and ring replaces upright and the angle α is measured from the stiffener, not the flange. The expression for ε_f allows for a stringer shared between adjacent panels, so A_s replaces $2 \cdot A_f$. The expressions for ε_s and ε remain unchanged:

$$\frac{E \cdot \varepsilon_s}{\tau} = -\frac{k_d \cdot \cot\alpha}{\frac{A_s}{h \cdot t} + 0.5 \cdot (1 - k_d)} \quad (42)$$

$$\frac{E \cdot \varepsilon_f}{\tau} = -\frac{k_d \cdot \tan\alpha}{\frac{A_f}{d \cdot t} + 0.5 \cdot (1 - k_d)} \quad (43)$$

$$\frac{E \cdot \varepsilon}{\tau} = \frac{2 \cdot k_d}{\sin 2\alpha} + (1 - k_d) \cdot (1 + \nu) \cdot \sin 2\alpha \quad (44)$$

When the ring or frame is floating (i.e. with cutouts through which the stiffeners pass), the $0.5\cdot(1\text{-}k_d)$ in equation (43) is omitted.

An additional allowance for panel curvature is included in the expression for tanα. This allowance depends on the panel aspect ratio:

d/h≥1:
$$\tan^2\alpha = \frac{\varepsilon - \varepsilon_s}{\varepsilon - \varepsilon_f + \frac{1}{24}\cdot\frac{E}{\tau}\cdot\left(\frac{h}{R}\right)^2} \tag{45}$$

d/h<1:
$$\tan^2\alpha = \frac{\varepsilon - \varepsilon_s}{\varepsilon - \varepsilon_f + \frac{1}{8}\cdot\frac{E}{\tau}\cdot\left(\frac{h}{R}\right)^2\cdot\tan\alpha} \tag{46}$$

Equations (40)-(46) are used to produce Figs.24a-e which represent Ref.12 Figures 2-6 precisely.

6.2.3 Frame Strength

Two calculations are performed for the frame. Under the action of induced end-load and inward radial loading in conjunction with the applied end-load, critical fibres are checked. The inter-atttachment buckling to the panel is then considered.

6.2.3.1 Maximum Applied Frame Stress

The average applied compressive stress in the frame due to diagonal tension is obtained from equation (43). The lower part of the expression represents the effective frame area:

$$A_{fe} = A_f + 0.5\cdot(1-k_d)\cdot a\cdot t \tag{47}$$

$$\sigma_f = -\frac{k_d\cdot\tau\cdot a\cdot t\cdot\tan\alpha}{A_{fe}} \tag{48}$$

Note that $A_{fe} = A_f$ for a frame attached to the stringer crown without a direct connection to the panel.

The inward lateral load/unit length applied to the frame, from Ref.12 equation 6, is:

$$w_f = \frac{k_d\cdot\tau\cdot t\cdot b}{2\cdot R} \tag{49}$$

The frame strength is found by iteration of equations (47), (48) and (49) together with the appropriate diagonal tension coefficient until a shear stress is reached where the frame fibre is at its maximum allowable stress. The maximum frame fibre stress is given by:

$$\sigma_{fr} = \frac{k_d\cdot\tau\cdot t\cdot a\cdot\tan\alpha}{A_{fe}} + \frac{w_f\cdot b^2}{12}\cdot\frac{x_{fre}}{I_{fe}} + \frac{|P_f|}{A_{fe}}\cdot\frac{\tau}{\tau_{app}} \tag{50}$$

P_f is positive for compression. The absolute value is taken to allow for an applied compressive load of the same magnitude in the opposite edge member when the externally-applied load is tension.

This expression includes the components due to end-load and bending due to diagonal tension and applied end-load. The moment expression represents the end of a clamped beam, $w \cdot l^2/12$. This represents a compromise with that for the middle of a clamped beam, $w \cdot l^2/24$, and the middle of a simply-supported beam, $w \cdot l^2/8$. The assumption that this is valid is made without reference to any external source. It is done in the absence of any explicit expression for the bending moment in the frame, such as that in equation (19) for the flat panel.

6.2.3.2 Frame Inter-Attachment Buckling

Ref.12 does not consider inter-attachment buckling. This is found using the same allowable inter-attachment buckling stress as for the flat panel (see 6.1.3.1). The applied stress with which to compare this is similar to the maximum applied edge member stress expression in equation (50) but with the bending component removed. Again, an iteration is performed on the applied loads until this stress reaches the allowable value.

6.2.4 Panel Strength

6.2.4.1 Panel Stress at Centre of Panel

Ref.12 Figure 8 presents curves of maximum applied principal stress in the centre of the panel. From a consideration of strain energies, expressions for the stresses along and across the diagonal tension buckles are given by Ref.2 equations (28). The expression for the shear stress is relative to the diagonal tension buckles:

$$\sigma_x = \frac{2 \cdot k_d \cdot \tau}{\sin 2\alpha} + \tau \cdot (1 - k_d) \cdot \sin 2\alpha \tag{51}$$

$$\sigma y = -\tau \cdot (1 - k_d) \cdot \sin 2\alpha \tag{52}$$

$$\tau_{xy} = \tau \cdot (1 - k_d) \cdot \sin\left(2 \cdot \left(\frac{\pi}{4} - \alpha\right)\right) \tag{53}$$

From the above stresses it is possible to calculate the principal stresses in the web using the standard plane stress relationships:

$$\sigma_1, \sigma_2 = \tfrac{1}{2} \cdot (\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4 \cdot \tau_{xy}{}^2} \tag{54}$$

$$\theta = \tfrac{\pi}{4} - \tfrac{1}{2} \cdot \tan^{-1}\left[\frac{2 \cdot \tau_{xy}}{\sigma_x - \sigma_y}\right] \tag{55}$$

The angle θ is measured from the flange or stringer in the same direction as α. The applied stress at the centre of the panel is iterated for applied shear stress until the maximum principal stress reaches the material allowable. Equations (54) and (55) are the mathematical representations of Figs.8 and 9 in Ref.12. They are given here in Fig.25 and Fig.26.

6.2.4.2 Panel Stress near Frame

This follows the procedure for the strength at the centre of the panel, but the local stress in the frame is included in the panel stress components. The diagonal tension is not fully developed near the edges of the panel so this procedure may tend to be conservative. However, Ref.11 gives some indication that the summation of all the stress components in this way may be suitable.

The stresses in the mid-panel are modified as follows:

$$\begin{aligned} \sigma_x &\to \sigma_x - \frac{P_F}{A_{fe}} \cdot \sin\alpha \\ \sigma_y &\to \sigma_y + \frac{P_F}{A_{fe}} \cdot \cos\alpha \\ \tau_{xy} &\to \tau_{xy} \end{aligned} \qquad (56)$$

These are then combined as described in equations (54) and (55).

Note that compression *relieves* the diagonal tension. The frame under tension will be the most critical in the assembly. The applied frame load, P_F, in the above equation needs to be factored with the applied shear in the iteration to maintain the same loading ratio.

6.2.5 Stringer Strength

For the flat panel, several expressions for applied stiffener stress were given relating to particular locations in its section and along its length. The curved panel analysis provides only one expression for stiffener stress. Considering the different nature of loading on the curved panel stiffener, it is thought that it would not be appropriate to use the flat panel analysis to provide the more-detailed stiffener stresses. The effect of the diagonal tension on the stringer in a curved panel creates not only compression but also an inward-acting radial load. Both forms of loading are then taken in conjunction with externally-applied end-loads. The lateral loading is expressed as a uniform lateral load/unit length along the central portion of the stiffener, w_s, and as a total lateral load, W_s.

Generally, the same design criteria are applied as for flat panel stiffeners. However, some of the overall modes have to be considered with lateral loading. Some other modes have been adopted from the flat panel analysis due to their absence from available curved panel references. Forced crippling is such an example.

6.2.5.1 Average Stringer Stress

The applied stringer stress is obtained from equation (42). The lower part of the expression represents the effective stringer area:

$$A_{se} = A_s + 0.5 \cdot (1 - k_d) \cdot b \cdot t \qquad (57)$$

$$\sigma_s = -\frac{k_d \cdot \tau \cdot b \cdot t \cdot \cot\alpha}{A_{se}} \qquad (58)$$

6.2.5.2 Stringer Uniform Lateral Load

Due to curvature, a lateral load is applied to the stringer. The diagonal tension waves in the panel each side of the stringer tend to flatten each panel forming a polygonal cross-section. The diagonal tension from each panel is resolved at the stringer through half the included angle between them. The loading is uniform over the central portion of the stringer and is given by:

$$w_s = \frac{k_d \cdot \tau \cdot t \cdot b \cdot \tan\alpha}{R} \qquad (59)$$

6.2.5.3 Stringer Total Lateral Load

The total lateral load is given in Ref.12 Figure 7. The precise derivation of these curves is not known. However, it has been possible to fit expressions to them which match to within a fraction of a percent (see Fig.27):

$$\frac{W_S \cdot R}{k_d \cdot \tau \cdot t \cdot b \cdot a} = \tan\alpha - 10^{\left(-0.813 \cdot \log_{10}\left(\frac{a}{b}\right) - 0.39\right)} \qquad (60)$$

6.2.5.4 Stringer Flexural Instability

Equation (58) is used in conjunction with a modified secant formula to determine the flexural instability allowable. The method of incorporating lateral load with end load follows the procedure adopted in Ref.18. This includes two further components with the eccentricity: a deflection component and a bending component derived from moment divided by end-load. These additional components are based on simply-supported ends. It would be possible to consider the ends clamped but as the curved panel analysis does not present any reduction in effective strut length, the allowance for clamping cannot be determined. The assumption of simple supports is thus conservative but also necessary. The modified secant formula is:

$$f_{all} = f_{fi} \cdot \left[1 + \left(\frac{e}{c} + \left(\frac{w_s}{f_{li} \cdot A}\right) \cdot f_{fi} \cdot \frac{A}{c} \cdot \frac{5}{384} \cdot \frac{l^4}{E \cdot I} + \left(\frac{w_s}{f_{li} \cdot A}\right) \cdot \frac{1}{c} \cdot \frac{l^2}{24}\right) \cdot \sec\left(\frac{l_e}{2 \cdot k} \cdot \sqrt{\frac{f}{E_T}}\right)\right] \qquad (61)$$

Within the brackets beginning e/c, the components due to deflection and bending respectively can be identified. These begin with the factor ($w_s/f_{fi} \cdot A$) which is a constant representing the ratio of lateral load to end load. The component due to deflection thus increases with f_{fi}^2 and the component due to bending increases with f_{fi}.

For a given shear stress, a given set of section constants and factor $w_s/f_{fi} \cdot A$ is found. For a given $w_s/f_{fi} \cdot A$, f_{li} is found. This is compared with f_{all} and thus the shear stress is iterated until the flexural instability stress is reached.

The flexural instability analysis uses stringer/panel properties identical to the flat panel analysis. These properties are normal to the panel. In reality, the curvature affects the section properties contributed by the panel, but this is not considered here as it is assumed that the panel curvature is too small to make it significant.

6.2.5.5 Stringer Torsional Instability

Equation (58) is used in conjunction with the formula for torsional instability (equation (30)) to determine the torsional instability allowable. This folows the same procedure as Section 6.1.5.5.

6.2.5.6 Stringer Local Instability

Section 6.1.5.6 gives details of the allowable local instability stresses. As there is no available expression to determine local applied stress, the modified secant formula is used taking the local instability allowable at the attached flange. Equation (58) is used in conjunction with the secant formula to determine the local instability allowable. This folows the same procedure as 6.2.5.4 but replacing f_{all} with the local instability stress.

6.2.5.7 Stringer Forced Crippling

The analysis from the flat panel is used (Section 6.1.5.7). This is identical apart from the determination of diagonal tension factor and the angle of diagonal tension, including the peak stress factor. The results of this part of the analysis should be treated with caution as the applicability of the analysis can be challenged from many aspects. In particular, the lateral loading is not included in this analysis.

6.2.5.8 Stringer Inter-Attachment Buckling

This follows the same procedure as the local instability analysis, being centred around the attached flange. The allowable inter-attachmentt buckling stresses are the same as for the flat panel analysis in Section 6.1.5.8.

6.2.6 Attachment Strength

Apart from the inter-attachment buckling of the frame, the analyses for the attachments come from the flat panel analysis. These are all likely to contain inaccuracies but no other methods are currently available at this time.

6.2.6.1 Frame Attachment

The frame strength is found in the same way as for the flat panel, using an iteration based on equation (34). Again, in the absence of any specific calculation for curved panels, it is assumed that the flat panel analysis is adequate.

6.2.6.2 Stiffener/Frame Load

This load is only applicable at the end of a bay where the stringer runs out. Otherwise, the load would carry across into the next bay. The flat panel analysis provides for a reduction in the stringer end-load which would be based on stringer stress and effective stringer area (see Section 6.1.6.2). This reduction is a maximum of 25%. The validity of this reduction for the curved panel analysis is unclear. Therefore, to remain conservative, the full value is used.

6.2.6.3 Stringer Attachment Tension Criteria

The stringer attachment tension criteria are assumed to be as valid for the curved panel as for the flat panel. The analysis thus follows 6.1.6.3.

6.2.7 Panel Stiffness

There is a panel stiffness calculation for flat panels in 6.1.7. Due to inward radial movement, it is thought that any estimate based on this would not be appropriate for curved panels.

7.0 LIMITATIONS TO ANALYSIS

It is assumed that the panels are loaded uniformly in pure shear and only edge members and stiffeners, with effective panel contributions, carry additional end load.

The analysis applies to panels with aspect ratio, $0.2 \leq h/d \leq 1$ with a well-developed tension field, $\tau/\tau_b > 2$. Stresses lower than this are unpredictable due to the effect of initial irregularities.

Although similar materials are normally assumed for flanges, stiffeners and webs, a certain amount of allowance can be made for small differences.

The analysis is primarily applicable to the elastic range. Some allowance for plasticity may be made in certain areas, but these are of a tentative nature.

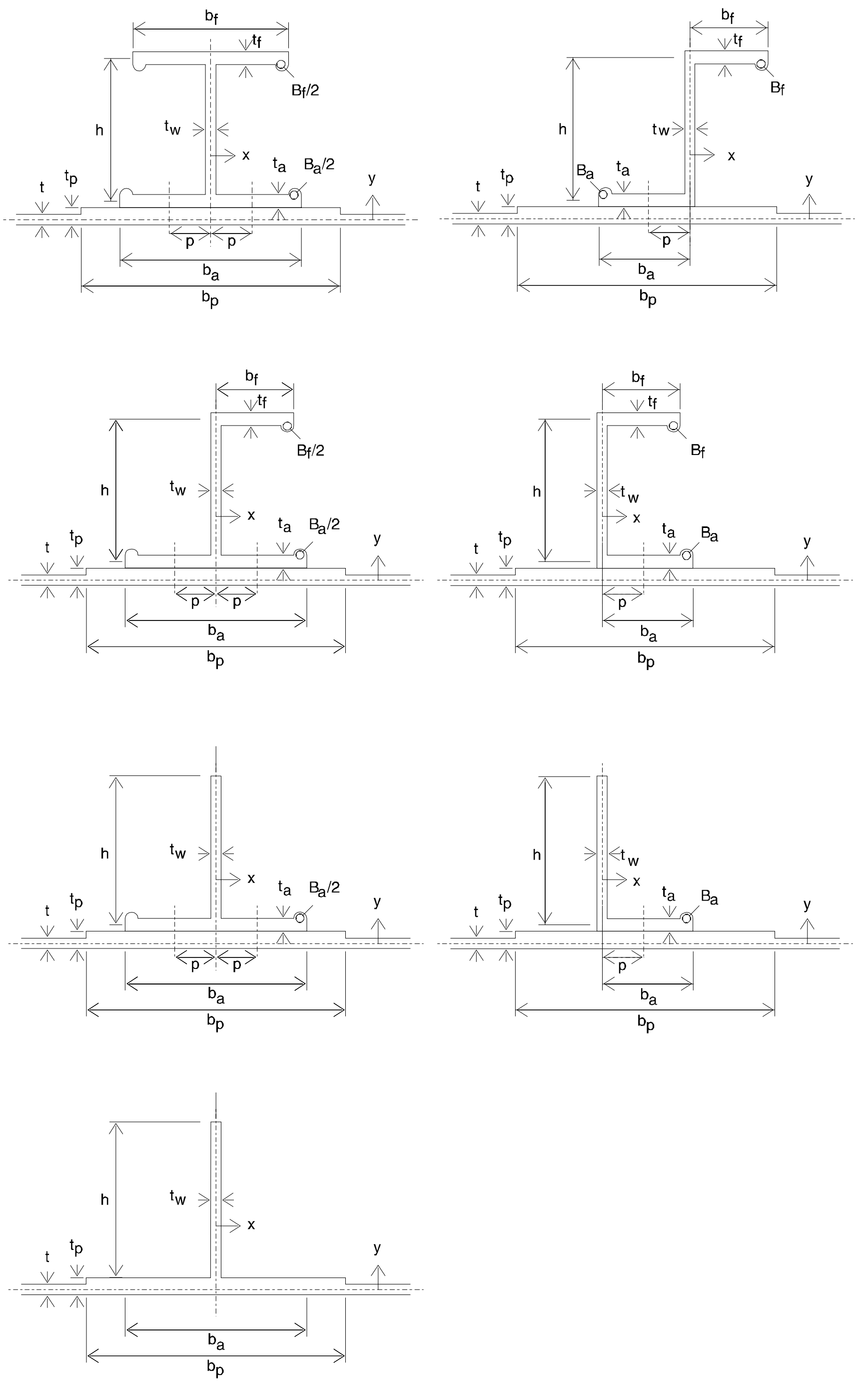


Fig.1: Stiffener and Panel Section Geometry (standard notation)

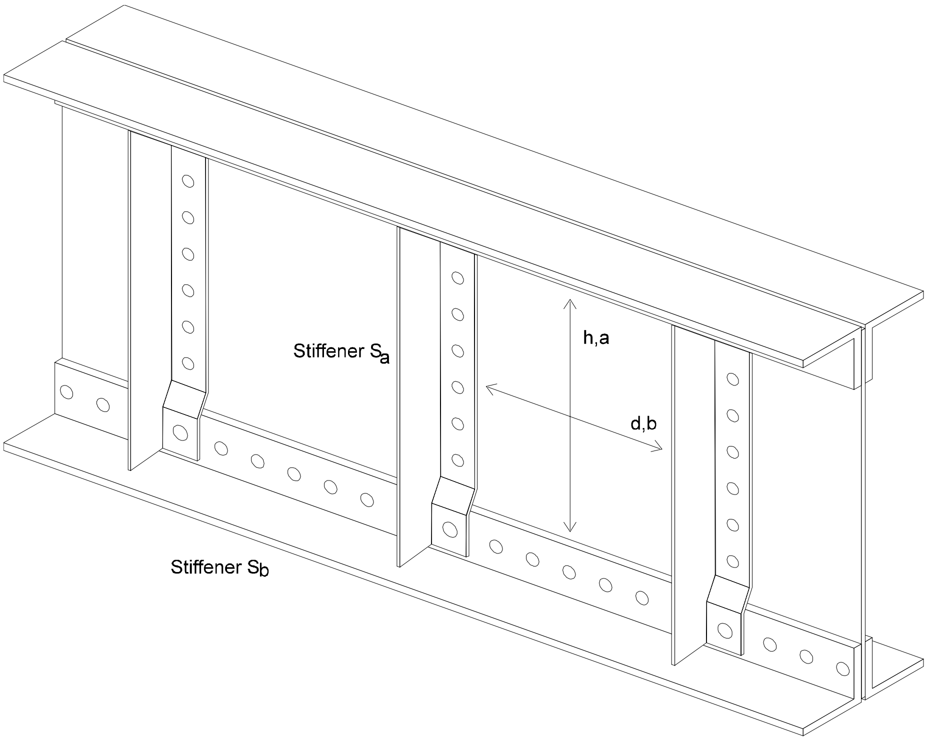


Fig.2: Flat Panel Configuration

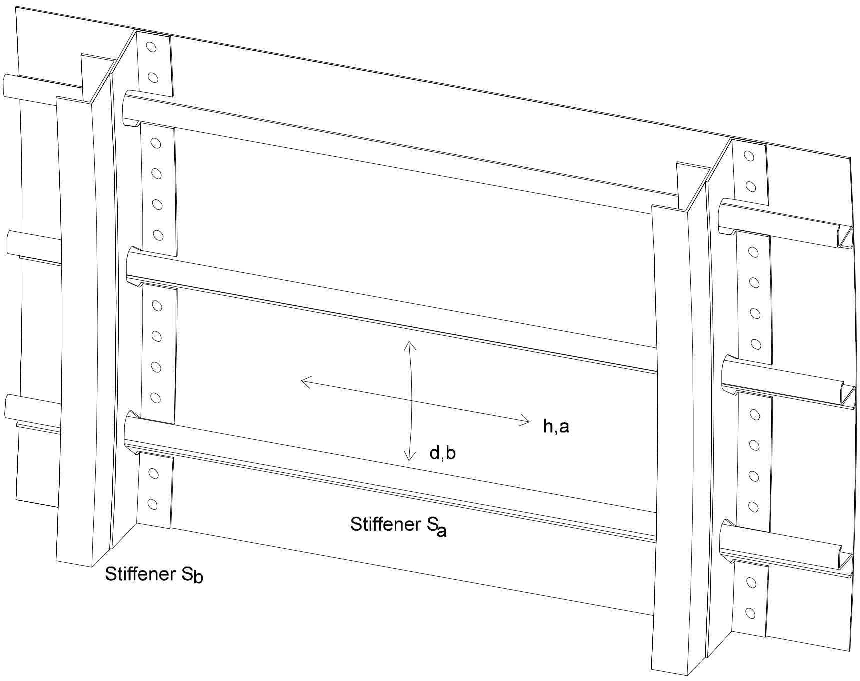


Fig.3: Curved Panel Configuration

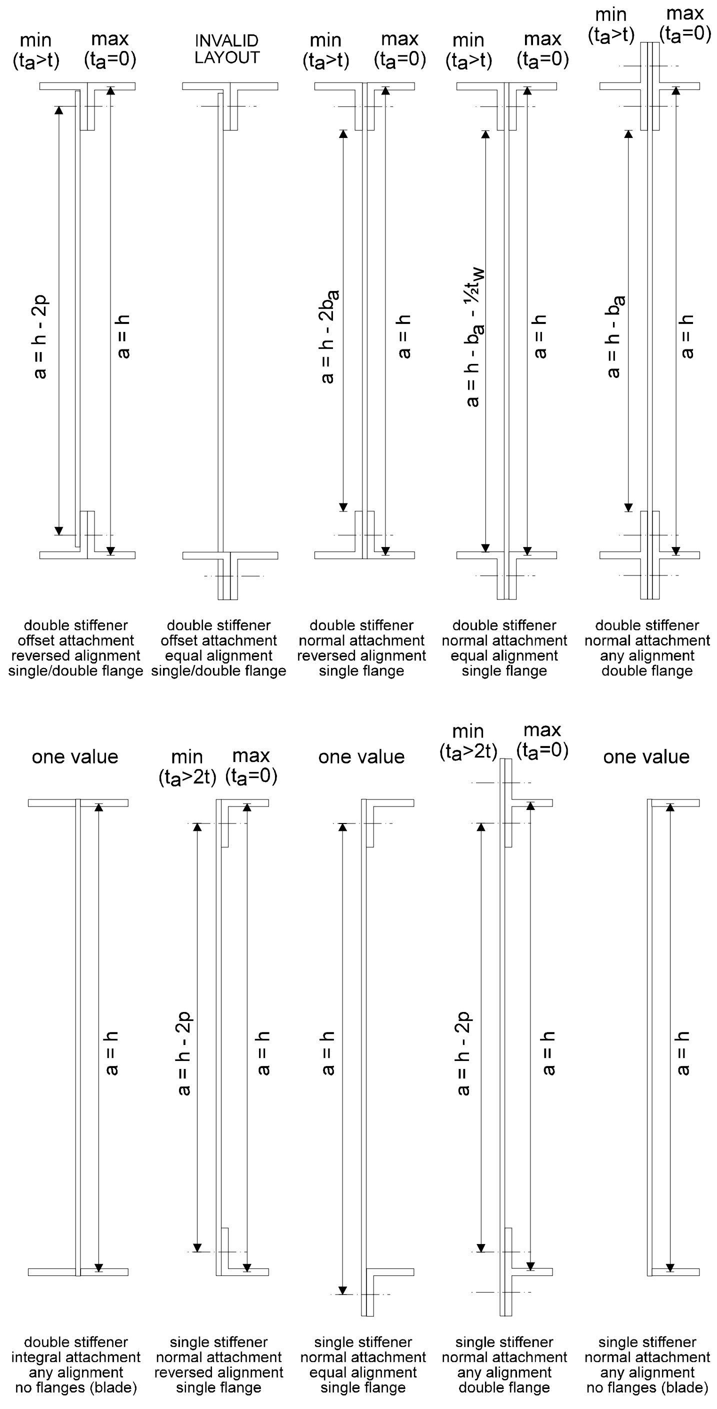


Fig.4: Definition of Plain Panel Width

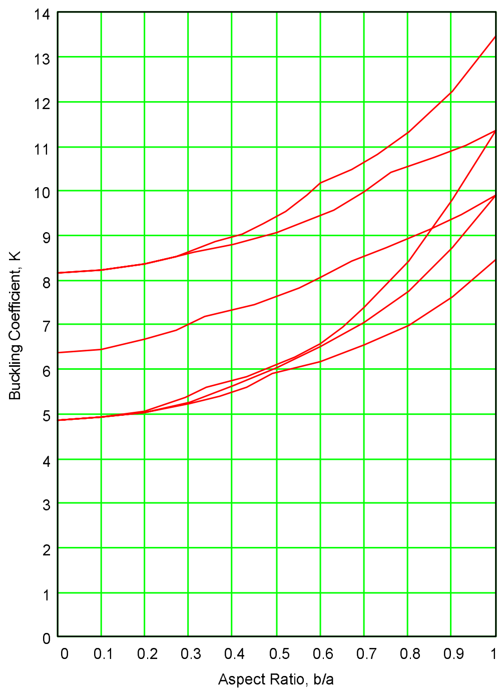


Fig.5: Buckling Coefficients for a Single Panel
(Ref. ESDU 71005)

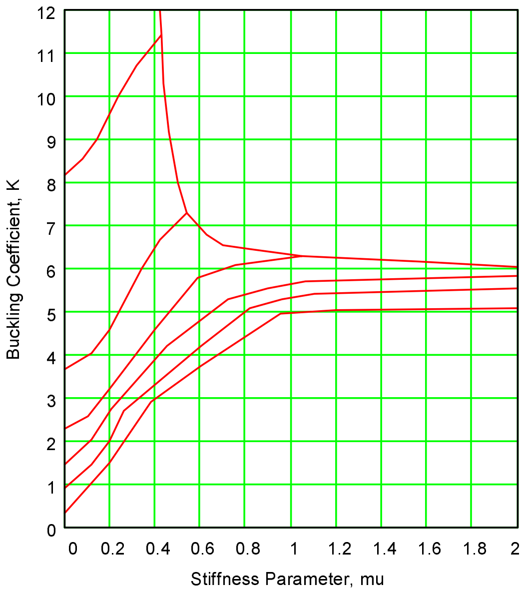


Fig.6a: Buckling Coefficients for Panels with Transverse Stiffeners
(Ref. ESDU 02.03.02, longitudinal edges clamped, no torsional stiffness, J = 0)

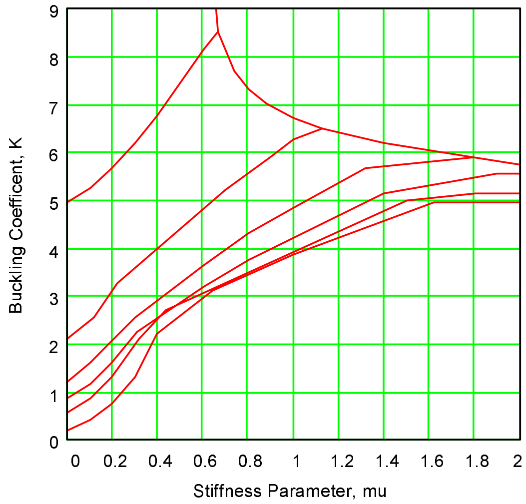
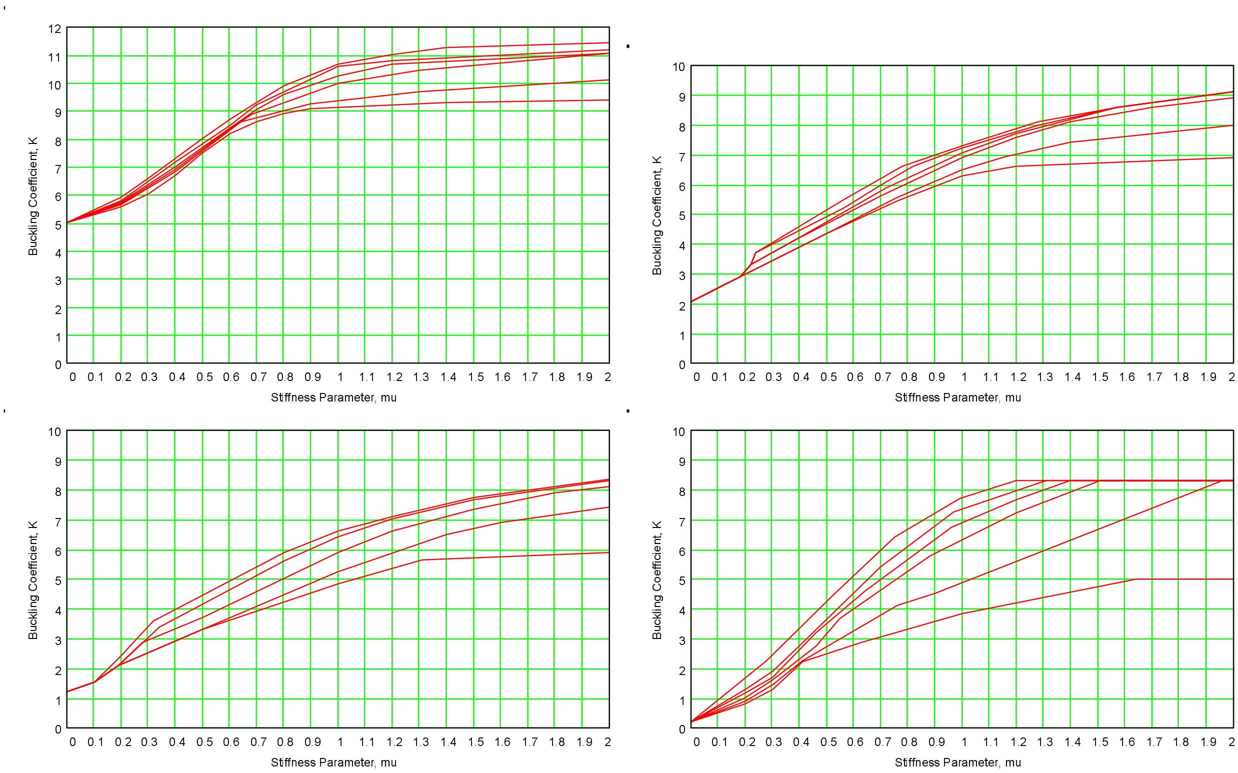
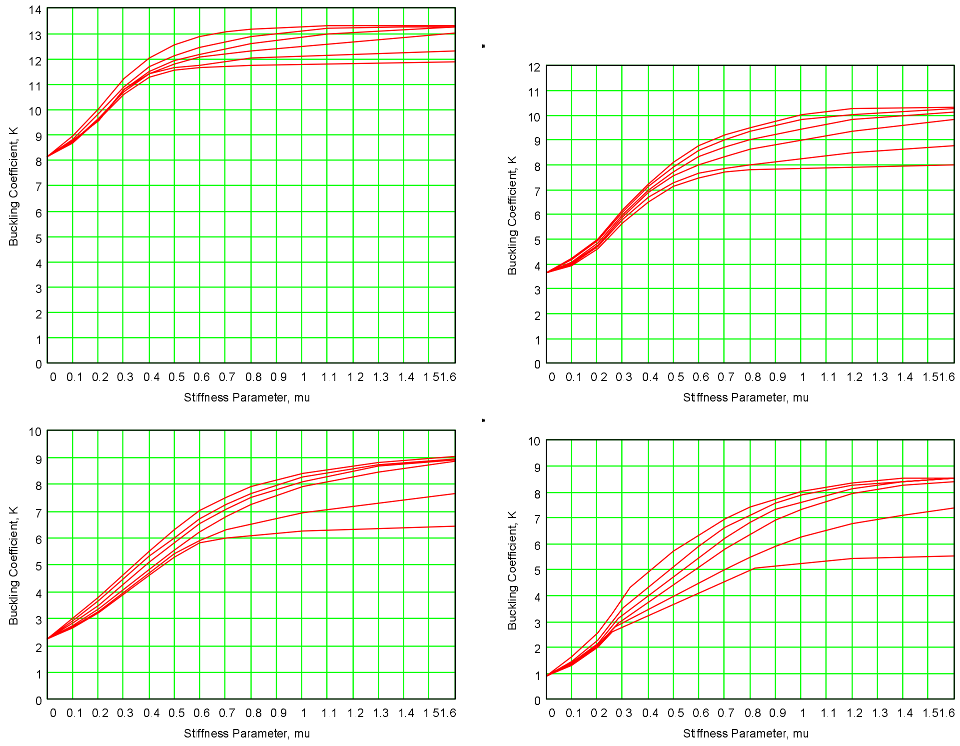


Fig.6b: Buckling Coefficient for Panels with Transverse Stiffeners
(Ref. ESDU 02.03.02 , longitudinal edges simply-supported, no torsional stiffness, J = 0)



Figs.6c-6f: Buckling Coefficient for Panels with Transverse Stiffeners
(Ref. ESDU 02.03.02, longitudinal edges simply-supported, a/b = 1.0, 1.5, 2.0, 5.0 respectively)



Figs.6g-6j: Buckling Coefficient for Panels with Transverse Stiffeners
(Ref. ESDU 02.03.02, longitudinal edges clamped, a/b = 1.0, 1.5, 2.0, 3.0 respectively)

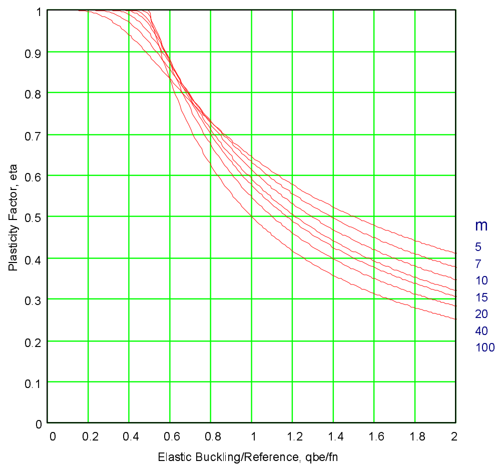


Fig.7: Plasticity Correction
(Ref. ESDU 71005 Fig.2, calculated from ESDU 83044)

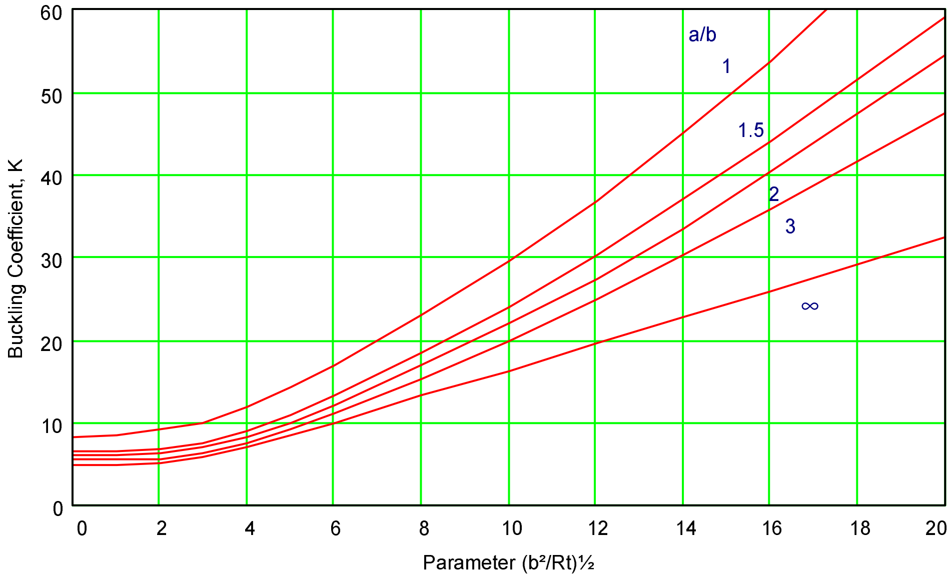


Fig.8: Curved Panel Buckling Coefficients
(Ref. ESDU 02.03.18)

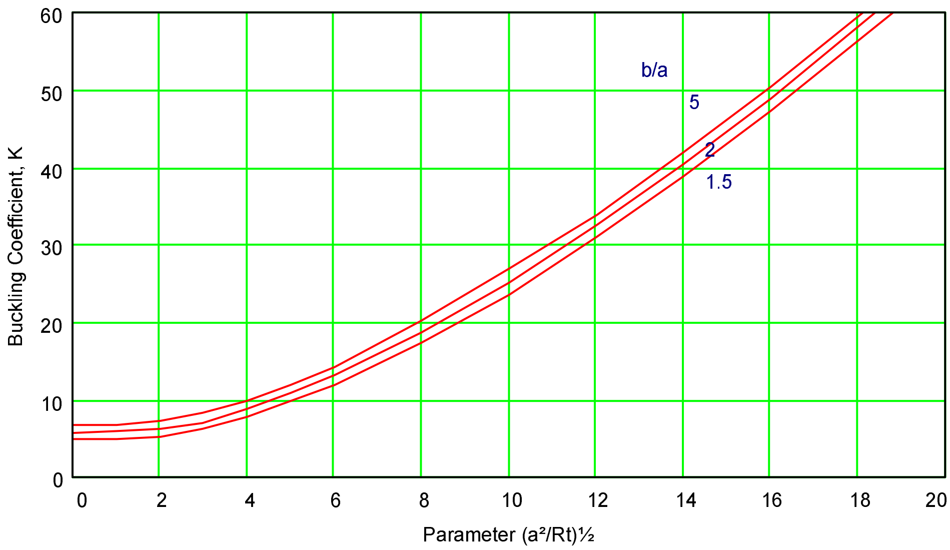
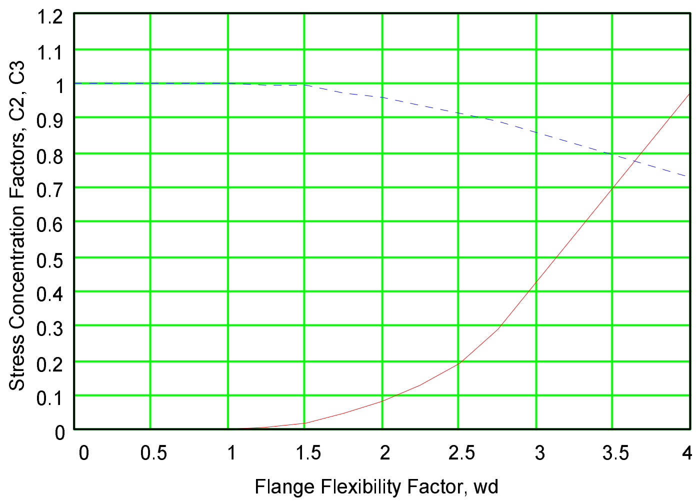
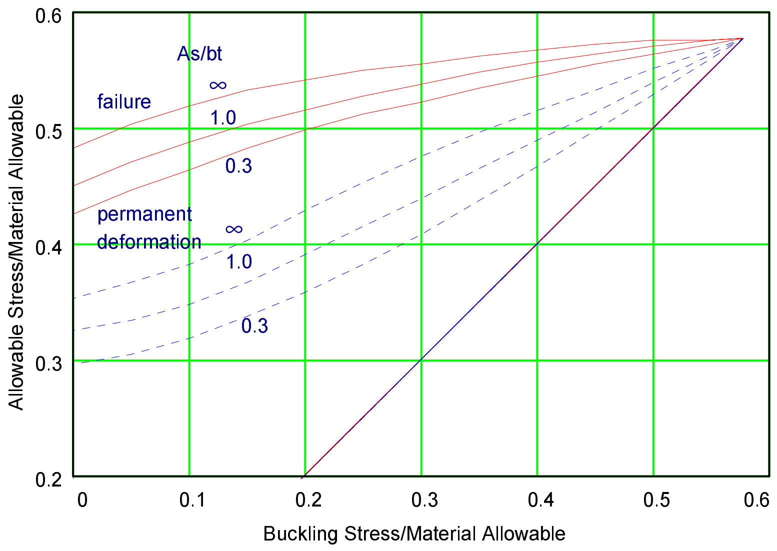


Fig.9: Curved Panel Buckling Coefficients
(Ref. ESDU 02.03.19)



Factor C2
Factor C3

Fig.10: Stress Concentration Factors
(Ref. NACA TN2661 Figure 18)



Failure
Permanent Buckling

Fig.11: Allowable Web Stress in Centre of Panel
(Ref. ESDU 77014 Fig.1)

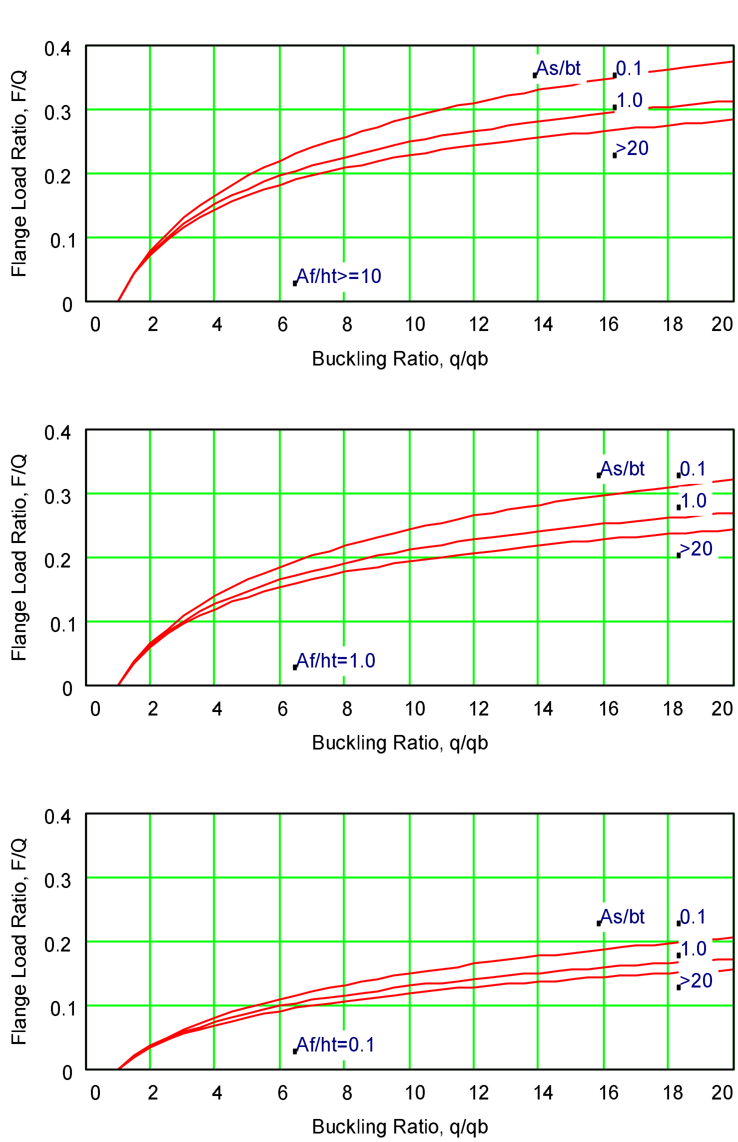


Fig.12: End Load in Edge Member
(These do not exactly match ESDU 77014 Figs.10-12, see text)

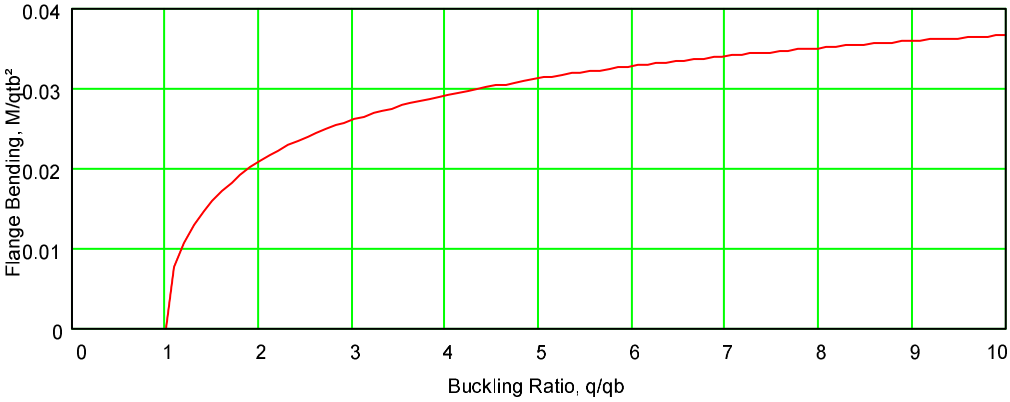


Fig.13: Bending Moment in Edge Member
(Ref. ESDU 77014 Fig.13)

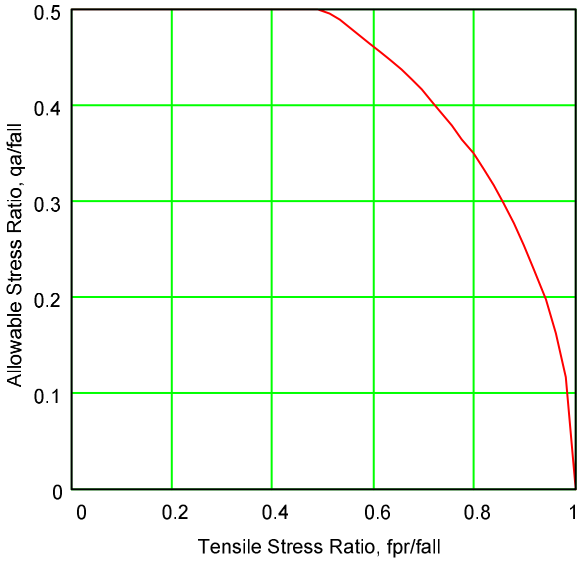


Fig.14: Allowable Stress in Panel near Edge Member
(Ref. ESDU 77014 Fig.2)

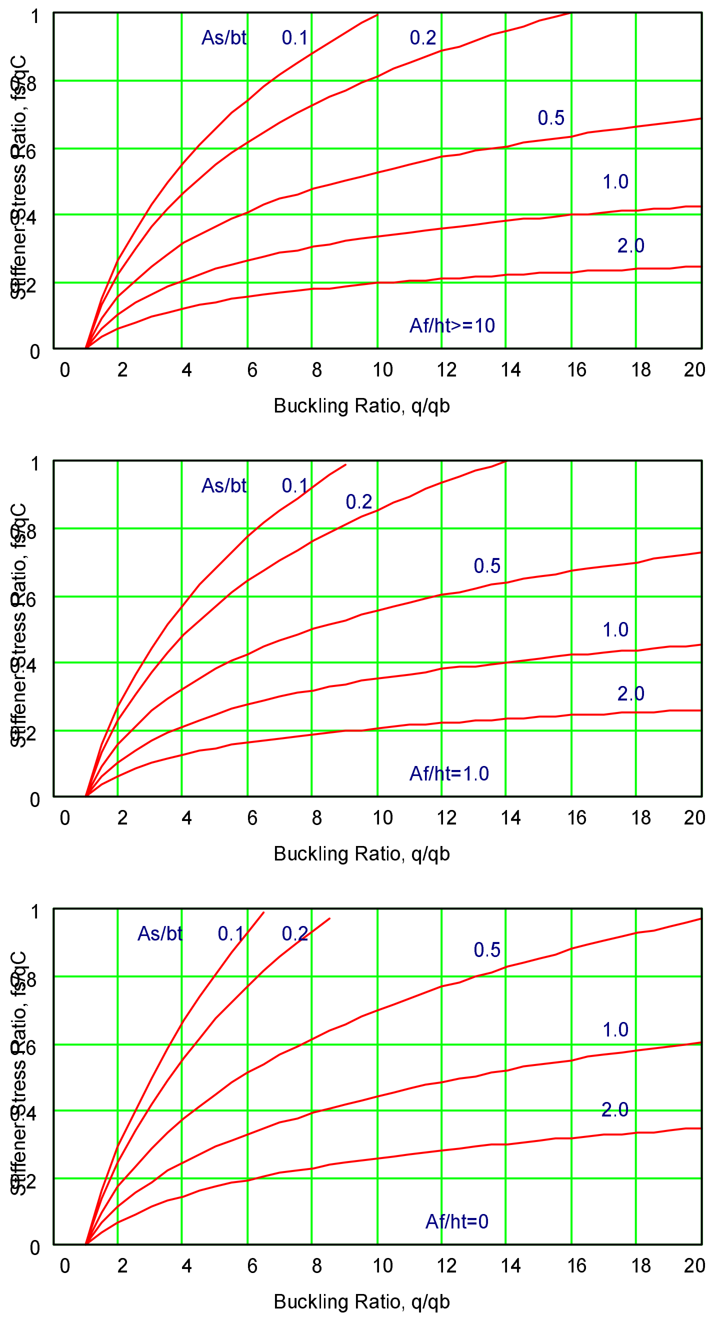


Fig.15: Stress in Stiffener
(These do not exactly match ESDU 77014 Figs.3-5, see text)

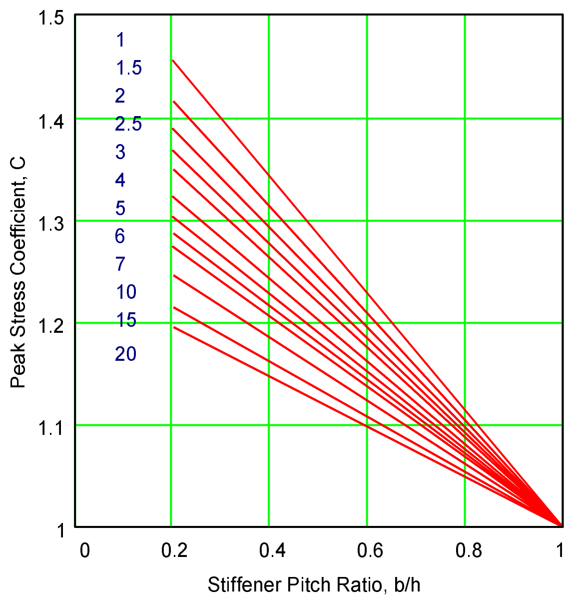


Fig.16: Stiffener Peak Stress Coefficient (Ref. ESDU 77014 Fig.6)

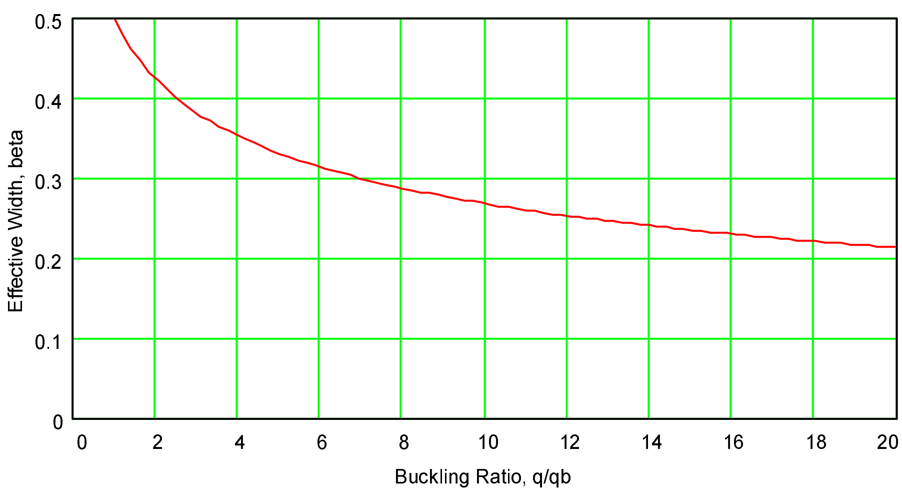


Fig.17: Effective Panel Width for Stiffener Flexural Instability (Ref. ESDU 77014 Fig.7)

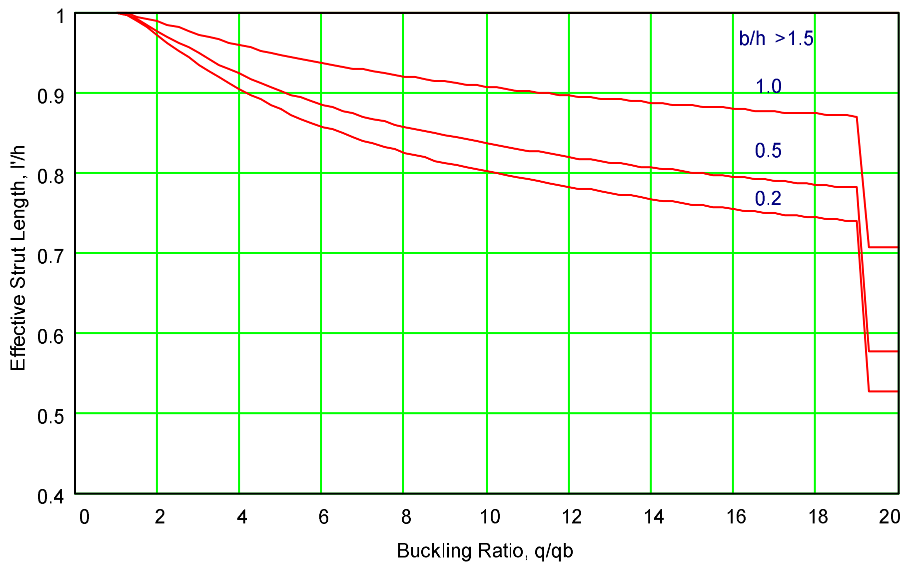


Fig.18: Effective Strut Length for Stiffener Flexural Instability
(Ref. ESDU 77014 Fig.8)

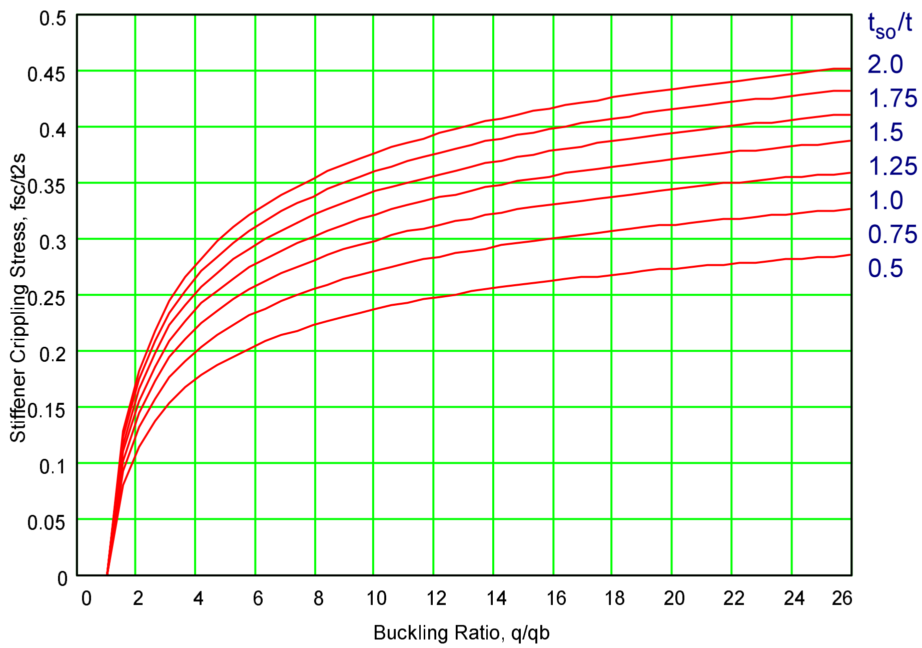


Fig.19: Stiffener Crippling Stress
(Ref. ESDU 77014 Fig.9)

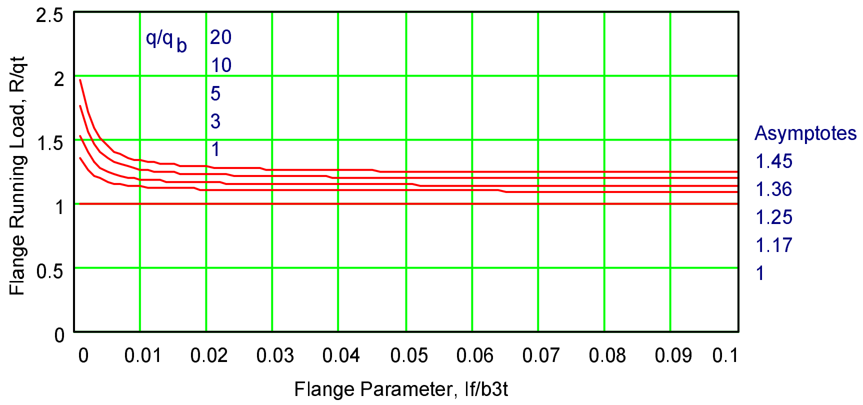


Fig.20: Edge Member/Panel Attachment Running Load
(Ref. ESDU 77014 Fig.14)

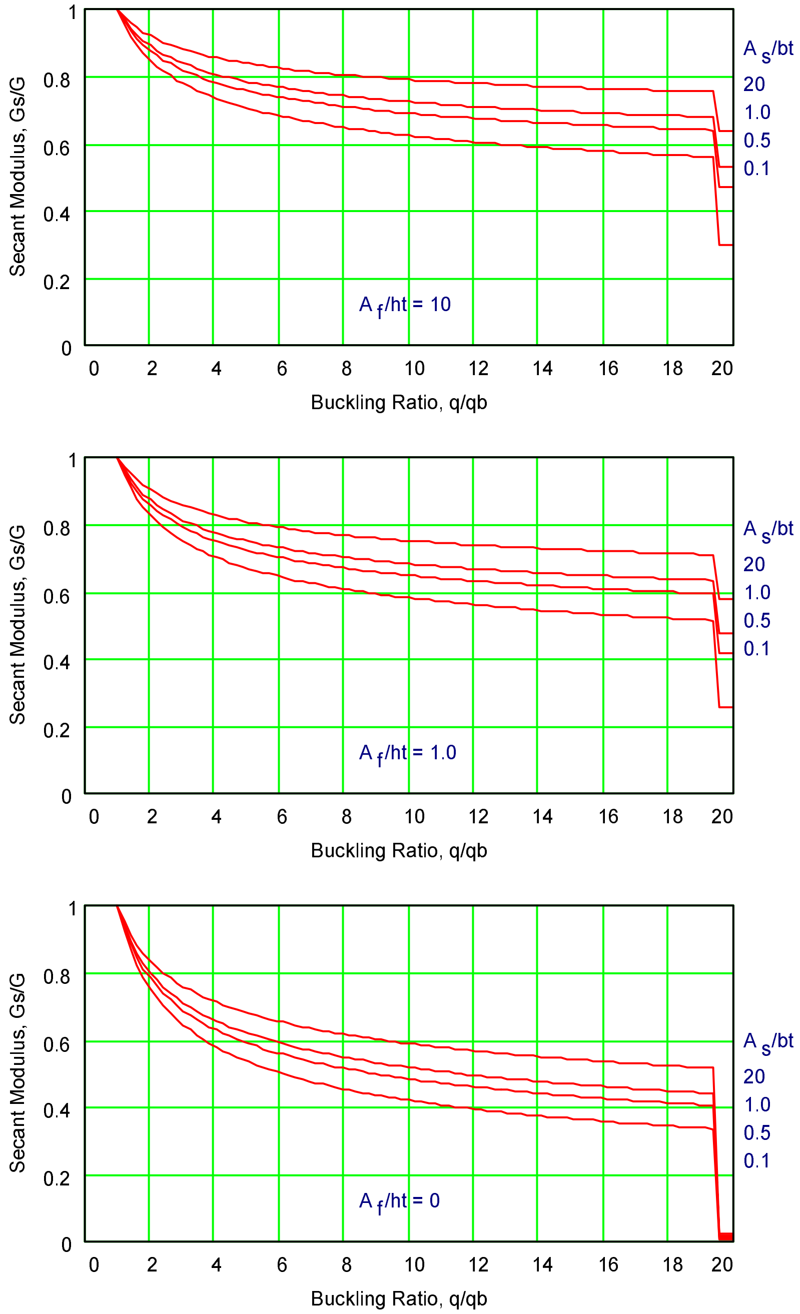
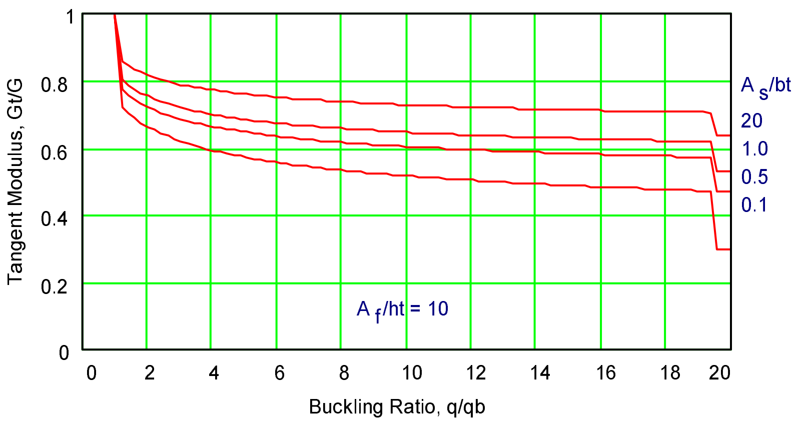
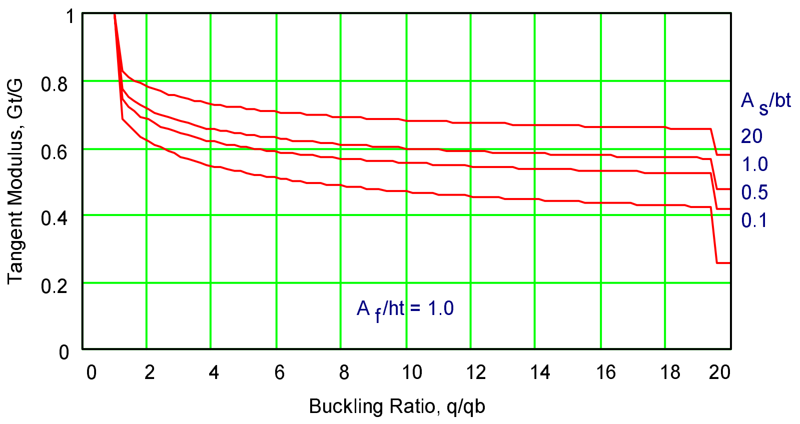


Fig.21: Panel Secant Modulus Reduction
(Ref. ESDU 77014 Figs.15-17)





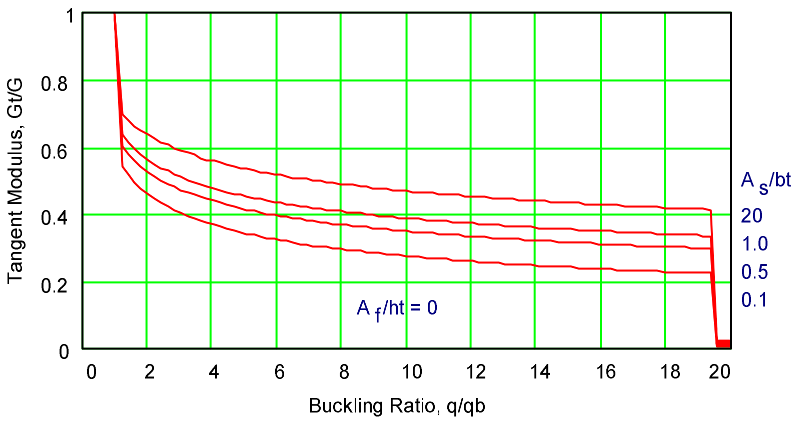


Fig.22: Panel Tangent Modulus Reduction
(Ref. ESDU 77014 Figs.18-20)

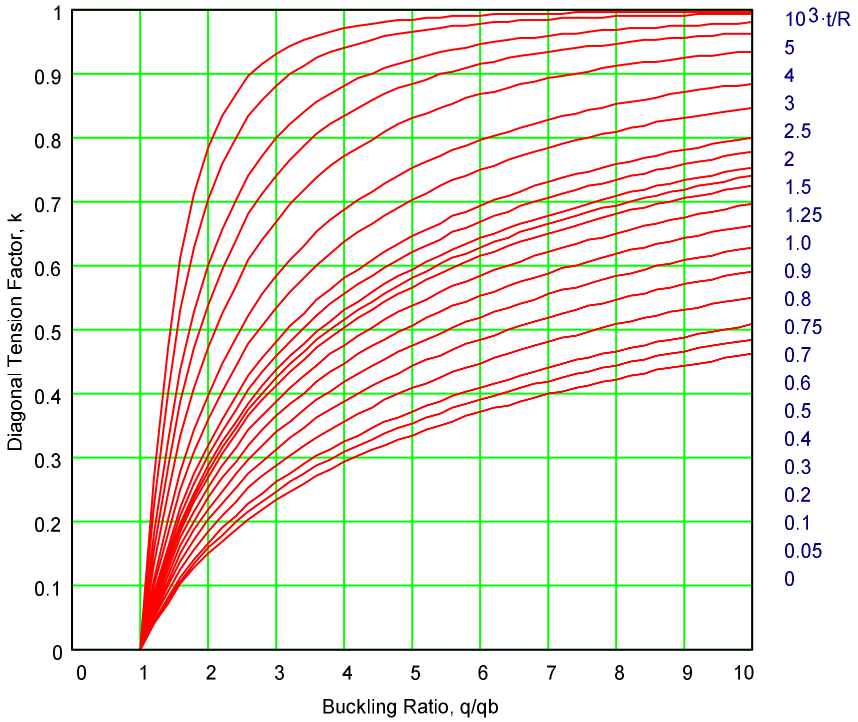


Fig.23: Diagonal Tension Factor for Curved Panels
(Ref. ESDU 77018 Fig.1)

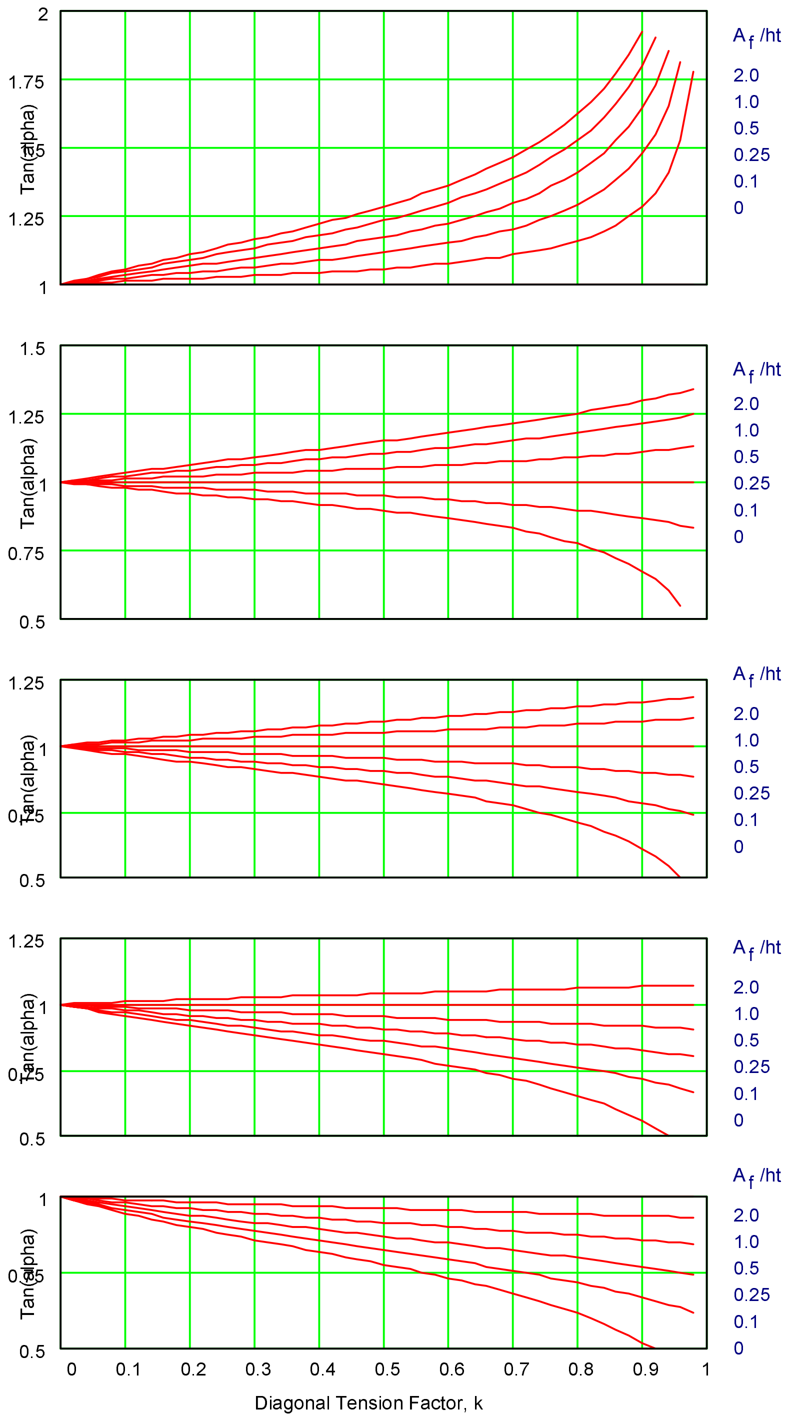


Fig.24a: Angle of Diagonal Tension for Curved Panels, (E/q)(b/R)²=0
(Ref. ESDU 77018 Fig.2)

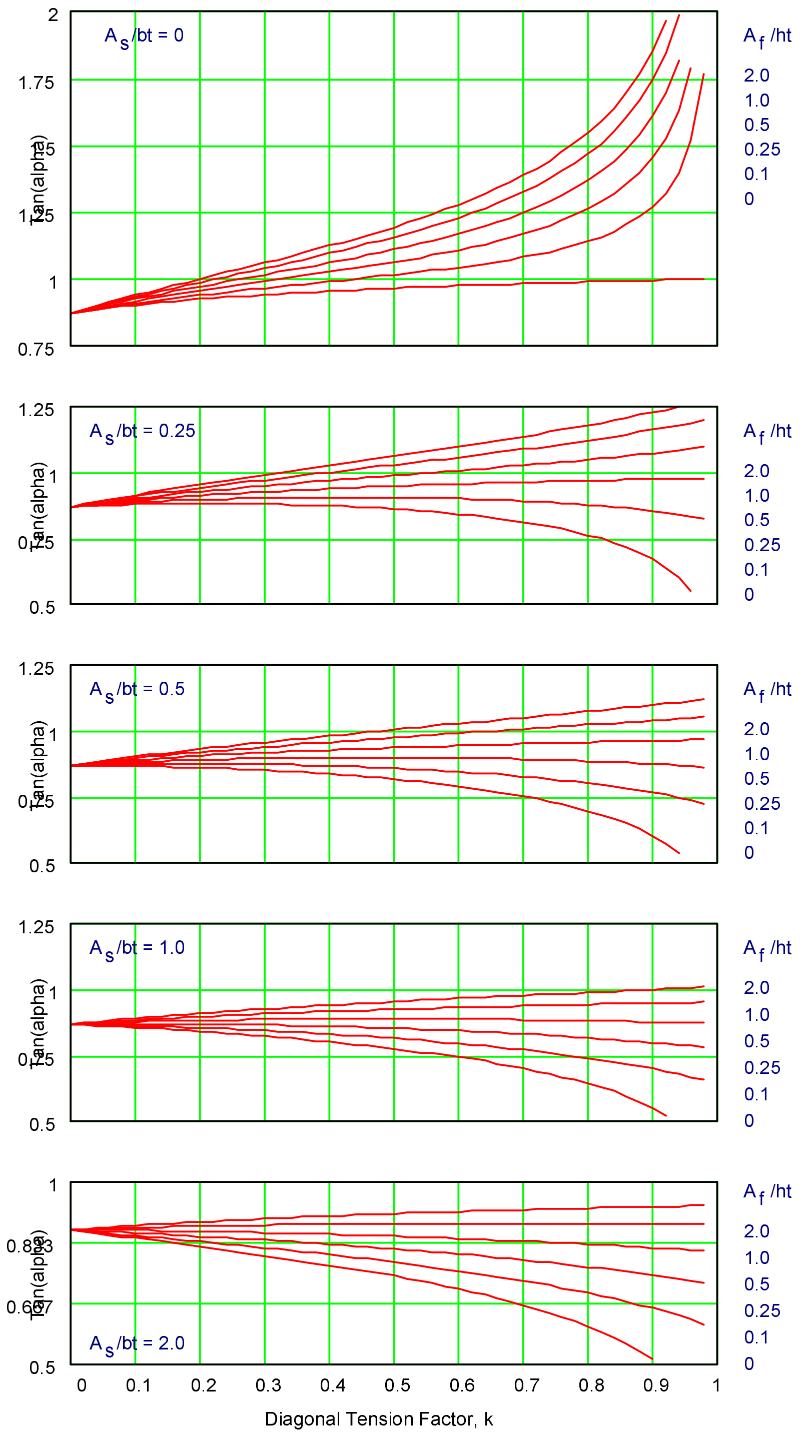


Fig.24b: Angle of Diagonal Tension for Curved Panels, (E/q)(b/R)²=10
(Ref. ESDU 77018 Fig.3)

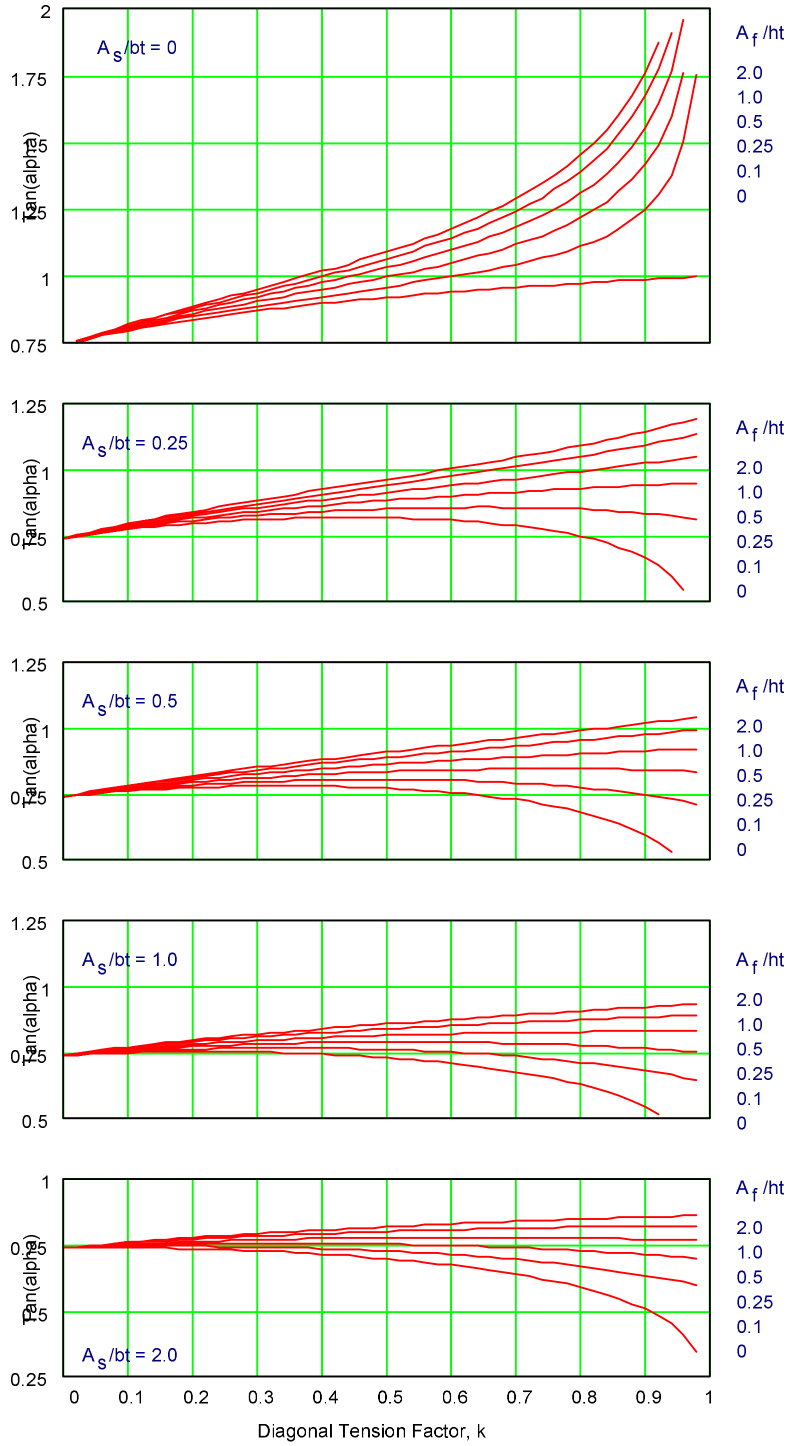


Fig.24c: Angle of Diagonal Tension for Curved Panels, (E/q)(b/R)²=25
(Ref. ESDU 77018 Fig.4)

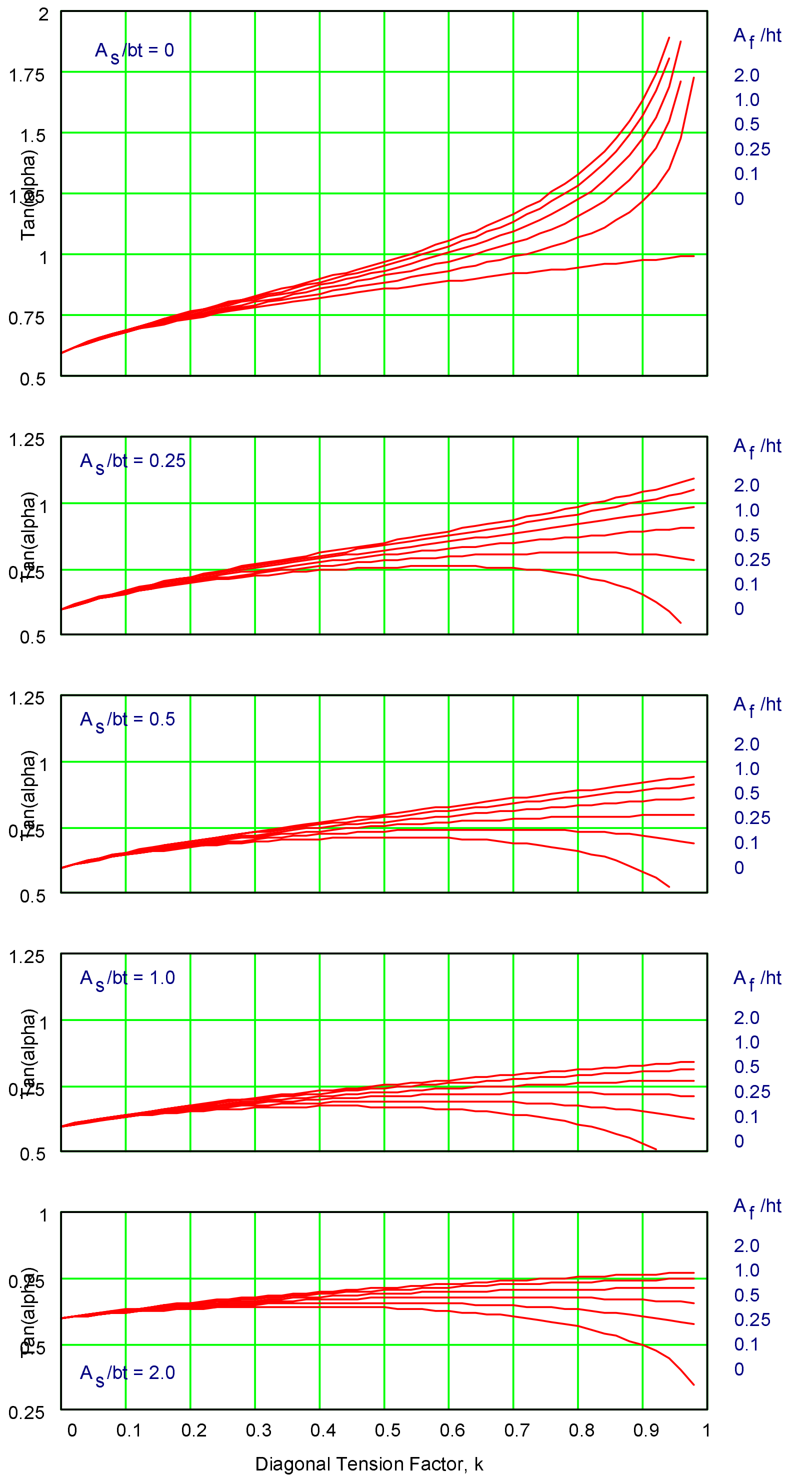


Fig.24d: Angle of Diagonal Tension for Curved Panels, $(E/q)(b/R)^2=50$
(Ref. ESDU 77018 Fig.5)

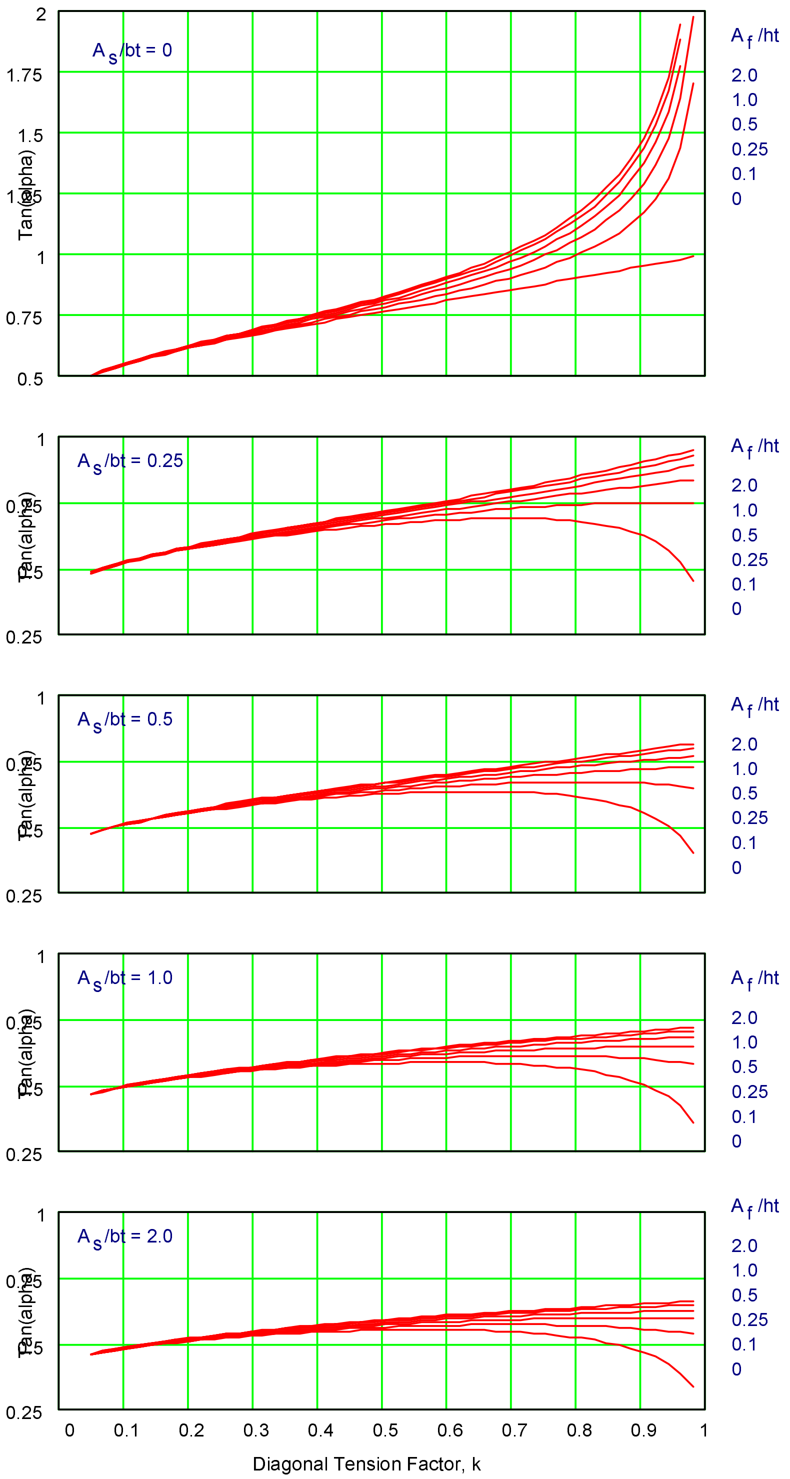


Fig.24e: Angle of Diagonal Tension for Curved Panels, $(E/q)(b/R)^2=100$
(Ref. ESDU 77018 Fig.6)

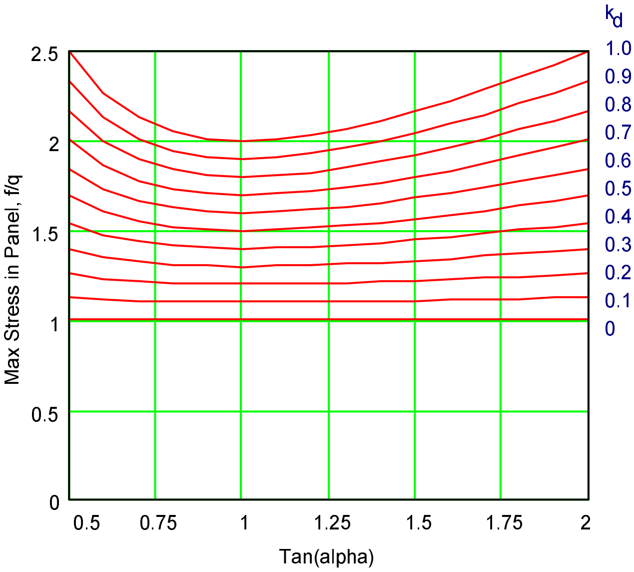


Fig.26: Maximum Tensile Stress in Centre of Panel
(Ref. ESDU 77018 Fig.8)

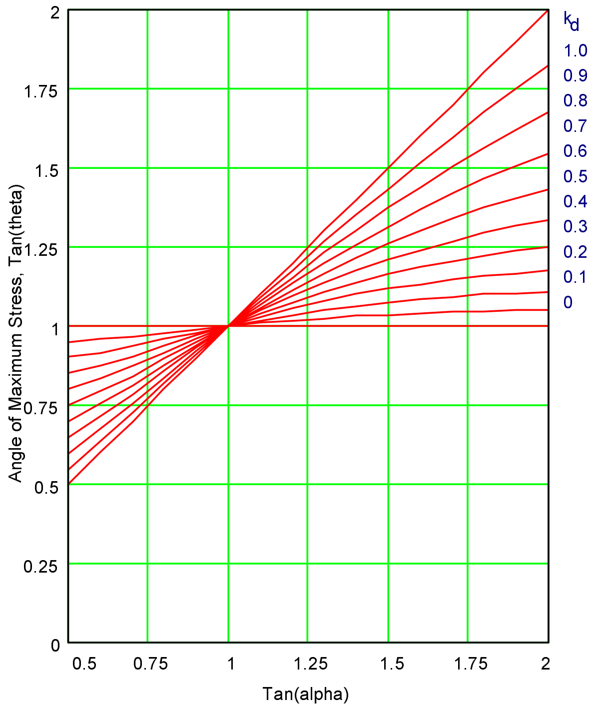


Fig.27: Angle of Maximum Tensile Stress in Centre of Panel
(Ref. ESDU 77018 Fig.9)

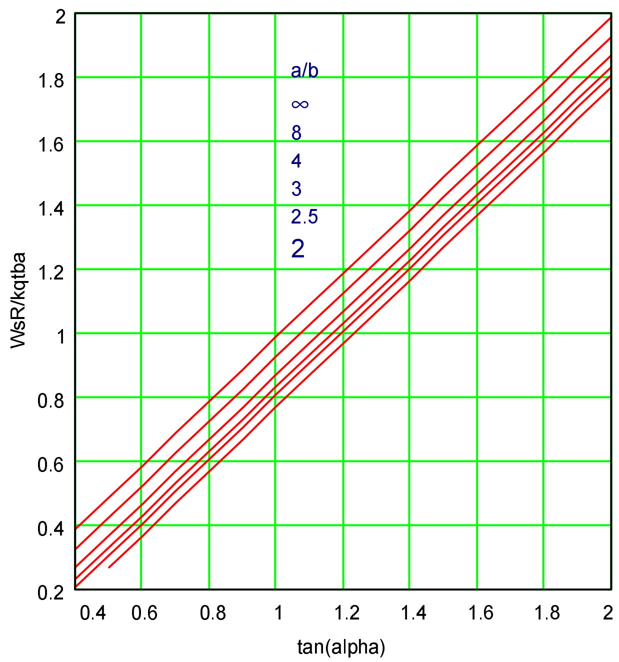


Fig.25: Total Lateral Load on Stiffener
(Ref. ESDU 77018 Fig.7)

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APA115 Volume 3, Appendix A: EXAMPLE HAND CALCULATION

Table of Contents

A1.0 INTRODUCTION

This appendix presents several hand calculations to verify the operation of program APA115. These calculations cover the three methods available. Most of the examples are based on the original examples in the references. Some of these hand calculations do not entirely agree with the original examples. Reasons and justifications have been provided in such cases.

It is not intended to verify the program in comparison with test results in this appendix. Neither is it intended to be a comparative study of the individual methods used for shear panel analysis in the program. The purpose of this appendix is to verify manually these methods on an individual basis using examples given with each method. Only the output relevant to each particular analysis is included.

The hand calculations may be long and repetitive. To facilitate the calculations, the spreadsheet MathCAD Version 6+ has been used. This provides for the automated solution of sets of equations and the interpolation and presentation of figures and graphs where appropriate.

A2.0 E.S.D.U. ANALYSIS

The E.S.D.U. analysis is the preferred analysis method of APA115. Two examples are given here - one for flat panels and one for curved panels. These examples come from the two E.S.D.U. data sheets dealing with post-buckled analysis.

The analysis performed by APA115 goes into far greater detail than the E.S.D.U. examples, producing true reserve factors. This requires some lengthy iterations which are not suitable for reproducing in full in this document.

A2.1 E.S.D.U. 77014 Flat Panel Example

This example is taken from E.S.D.U. data sheet 77014 Amendment A. As mentioned in the main text, there are disagreements with some of the curves presented in E.S.D.U. In this hand-worked example, the suggested adjustments to the curves have been incorporated.

Other E.S.D.U. data sheets are referenced in E.S.D.U. 77014 so the hand-calculation for these sheets is also included. All the calculations have been performed on a single MathCAD spreadsheet (with included libraries). This has been edited and the relevant parts included in this section.

The geometric properties of the panel calculated here are the same as those derived by APA115. They differ from the data sheet in some cases. In particular, the torsion constants determined by E.S.D.U. are thought to be inaccurate.

The E.S.D.U. data sheet performs a more extensive analysis in its own example than that described in its main text. In particular, the stiffener stability is checked using methods from other data sheets. These checks are fairly simple and do not cover all possibilities. APA115 has extended these to provide a more comprehensive analysis.

A2.1.1 Basic Data

A2.1.1.1 Material Properties

The material properties E, f_n, m were taken from the data sheet. However, the quoted values of t_2 and f_{all} were not consistent with these if the standard method for derivation for Aluminium alloys were to be used. A default Poisson's ratio of 0.3 is used in APA115. On comparison with the values for shear modulus obtained with this assumption, it appears to be consistent with the data sheet.

The values of 0.2% proof stress were found using the following expression:

$$\mathbf{t_2}(E, f_n, m) := f_n \cdot \left(\frac{E \cdot 0.002 \cdot m}{f_n}\right)^{\frac{1}{m}}$$

This gave 342.9, 332.7, 332.7N/mm² for the panel, stiffener and edge member respectively.

A2.1.1.2 Stiffener and Flange Section Constants

The following constants are specific to this example. The flange is a double L-section and the stiffener is a single L-section. Note that I_{yse} for single attachment stiffeners does not include A·x components as panel and stiffener are not considered to be linked.

The geometry for the basic data is identical to the E.S.D.U. data sheet. The section constants are mostly the same but the calculated stiffener torsion constant in the data sheet does not include the aspect ratio correction and is thus a little too high. The value for the torsion constant including the skin on the data sheet is unachievable unless an unrealistic effective panel width of 115·t is used. The data sheet value is 200mm⁴ and the program value is 127mm⁴.

A2.1.1.2.1 Basic stiffener section constants (L-section)

$$A_s := \left(h_{ss} + \frac{t_{as}}{2}\right) \cdot t_{ws} + \left(b_{as} - \frac{t_{ws}}{2}\right) \cdot t_{as} \qquad A_s = 125.44$$

$$x_s := \frac{1}{A_s} \cdot \left(b_{as}^2 - \frac{t_{ws}^2}{4}\right) \cdot \frac{t_{as}}{2} \qquad x_s = 9.796$$

$$y_s := \frac{1}{A_s} \cdot \left(h_{ss}^2 - \frac{t_{as}^2}{4}\right) \cdot \frac{t_{ws}}{2} + \frac{t_{as}}{2} + t_{ps} - \frac{t}{2} \qquad y_s = 11.196$$

$$t_s := \frac{A_s}{h_{ss} + b_{as}} \text{ (ref. only)} \qquad t_s = 1.6$$

$$I_{xs} := \left(h_{ss} + \frac{t_{as}}{2}\right)^3 \cdot \frac{t_{ws}}{12} + \left(b_{as} - \frac{t_{ws}}{2}\right) \cdot \frac{t_{as}^3}{12} \ldots$$
$$+ \left(h_{ss} + \frac{t_{as}}{2}\right) \cdot t_{ws} \cdot \left(\frac{h_{ss}}{2} + \frac{t_{as}}{4} + t_{ps} - \frac{t}{2} - y_s\right)^2 \ldots$$
$$+ \left(b_{as} - \frac{t_{ws}}{2}\right) \cdot t_{as} \cdot \left(y_s - t_{ps} - \frac{t_{as}}{2} + \frac{t}{2}\right)^2$$

$$I_{xs} = 20102.176$$

$$I_{ys} := \left(h_{ss} + \frac{t_{as}}{2}\right) \cdot \frac{t_{ws}^3}{12} + \left(b_{as} - \frac{t_{ws}}{2}\right)^3 \cdot \frac{t_{as}}{12} \ldots$$
$$+ \left(h_{ss} + \frac{t_{as}}{2}\right) \cdot t_{ws} \cdot x_s^2 \ldots$$
$$+ \left(b_{as} - \frac{t_{ws}}{2}\right) \cdot t_{as} \cdot \left(\frac{b_{as}}{2} + \frac{t_{ws}}{4} - x_s\right)^2$$

$$I_{ys} = 20102.176$$

$$J_s := \frac{b_{as} \cdot t_{as}^3}{3} \cdot \left[1 - 0.63 \cdot \frac{t_{as}}{b_{as}} \cdot \left[1 - \frac{1}{12} \cdot \left(\frac{t_{as}}{b_{as}}\right)^4\right]\right] \ldots$$
$$+ \frac{h_{ss} \cdot t_{ws}^3}{3} \cdot \left[1 - 0.63 \cdot \frac{t_{ws}}{h_{ss}} \cdot \left[1 - \frac{1}{12} \cdot \left(\frac{t_{ws}}{h_{ss}}\right)^4\right]\right]$$

$$J_s = 104.29$$

A2.1.1.2.2 Stiffener + panel section constants

$$A_{se} := A_s + b_{as} \cdot t + b_{ps} \cdot \left(t_{ps} - t\right) \qquad A_{se} = 172.48$$

$$x_{se} := \frac{1}{A_{se}} \cdot \left[\left(A_{se} - A_s\right) \cdot \frac{b_{as}}{2} + A_s \cdot x_s\right] \qquad x_{se} = 12.47$$

$$y_{se} := \frac{1}{A_{se}} \cdot \left[A_s \cdot y_s + b_{ps} \cdot \left(t_{ps} - t\right) \cdot \frac{t_{ps}}{2}\right] \qquad y_{se} = 8.142$$

$$I_{xse} := I_{xs} + A_s \cdot \left(y_{se} - y_s\right)^2 + \left(A_{se} - A_s\right) \cdot y_{se}^2 \ldots$$
$$+ b_{ps} \cdot \left(t_{ps} - t\right) \cdot \left(y_{se} - \frac{t_{ps}}{2}\right)^2 + \left(A_{se} - A_s\right) \cdot \frac{t^2}{12} \ldots$$
$$+ b_{ps} \cdot \frac{\left(t_{ps} - t\right)^3}{12}$$

$$I_{xse} = 24396.11$$

$$I_{yse} := I_{ys} + \mathbf{min}\left(b_{as}, b_{ps}\right)^3 \cdot \frac{t_{ps}}{12} + \mathbf{max}\left(b_{as} - b_{ps}, 0\right)^3 \cdot \frac{t}{12} \qquad I_{yse} = 26125.805$$

$$J_{se} := J_s + \mathbf{min}\left(b_{as}, b_{ps}\right)\cdot\frac{t_{ps}^{\ 3}}{3} + \mathbf{max}\left(b_{as} - b_{ps}, 0\right)\cdot\frac{t^3}{3} \qquad J_{se} = 126.869$$

x_{es} is the shear centre, required for the polar moment of inertia in the torsional instability analysis:

$$x_{es} := \frac{-1}{I_{xse}}\cdot\left[\left(y_{se} - \frac{t_{as}}{2} - t_{ps} + \frac{t}{2}\right)\cdot b_{as}\cdot b_{as}\cdot\frac{t_{as}}{2}\cdot\left(\frac{t_{as}}{2} + t_{ps} - \frac{t}{2}\right) \ldots \atop + \left(y_{se} - \frac{t_{ps}}{2} + \frac{t}{2}\right)\cdot\frac{t_{ps}}{2}\cdot b_{ps}\cdot b_{as}\cdot\left(\frac{t_{ps}}{2} - \frac{t}{2}\right)\right] \qquad x_{es} = -0.476$$

A2.1.1.2.3 Basic flange section constants (double L-section)

$$A_f := 2\cdot\left[\left(h_{sf} + \frac{t_{af}}{2}\right)\cdot t_{wf} + \left(b_{af} - \frac{t_{wf}}{2}\right)\cdot t_{af}\right] \qquad A_f = 2000$$

$$x_f := \frac{2}{A_f}\cdot\left(b_{af}^{\ 2} - \frac{t_{wf}^{\ 2}}{4}\right)\cdot\frac{t_{af}}{2} \qquad x_f = 10.641$$

y'_f is for a single L-section. For a double L-section, $y_f = 0$:

$$y'_f := \frac{2}{A_f}\cdot\left(h_{sf}^{\ 2} - \frac{t_{af}^{\ 2}}{4}\right)\cdot\frac{t_{wf}}{2} + \frac{t_{af}}{2} + t_{pf} - \frac{t}{2} \qquad y'_f = 22.384 \qquad y_f := 0$$

$$t_f := \frac{A_f}{2\cdot\left(h_{sf} + b_{af}\right)} \quad \text{(ref. only)} \qquad t_f = 8.865$$

I'_{xf} is for a single L-section and I_{xf} is for a double L-section:

$$I'_{xf} := \begin{bmatrix} \left(h_{sf} + \frac{t_{af}}{2}\right)^3\cdot\frac{t_{wf}}{12} + \left(b_{af} - \frac{t_{wf}}{2}\right)\cdot\frac{t_{af}^{\ 3}}{12} \ldots \\ + \left(h_{sf} + \frac{t_{af}}{2}\right)\cdot t_{wf}\cdot\left(\frac{h_{sf}}{2} + \frac{t_{af}}{4} + t_{pf} - \frac{t}{2} - y'_f\right)^2 \ldots \\ + \left(b_{af} - \frac{t_{wf}}{2}\right)\cdot t_{af}\cdot\left(y'_f - t_{pf} - \frac{t_{af}}{2} + \frac{t}{2}\right)^2 \end{bmatrix} \qquad I'_{xf} = 412278.677$$

$$I_{xf} := 2\cdot\left[\left(h_{sf}+\frac{t_{af}}{2}\right)^3\cdot\frac{t_{wf}}{12}+\left(b_{af}-\frac{t_{wf}}{2}\right)\cdot\frac{t_{af}^3}{12}\ldots\right.$$
$$+\left(h_{sf}+\frac{t_{af}}{2}\right)\cdot t_{wf}\cdot\left(\frac{h_{sf}}{2}+\frac{t_{af}}{4}+t_{pf}-\frac{t}{2}-y_f\right)^2\ldots$$
$$\left.+\left(b_{af}-\frac{t_{wf}}{2}\right)\cdot t_{af}\cdot\left(y_f-t_{pf}-\frac{t_{af}}{2}+\frac{t}{2}\right)^2\right]\ldots$$
$$+A_f\cdot y_f^2$$

$I_{xf} = 1826644.267$

$$I_{yf} := 2\cdot\left[\left(h_{sf}+\frac{t_{af}}{2}\right)\cdot\frac{t_{wf}^3}{12}+\left(b_{af}-\frac{t_{wf}}{2}\right)^3\cdot\frac{t_{af}}{12}\ldots\right.$$
$$+\left(h_{sf}+\frac{t_{af}}{2}\right)\cdot t_{wf}\cdot x_f^2\ldots$$
$$\left.+\left(b_{af}-\frac{t_{wf}}{2}\right)\cdot t_{af}\cdot\left(\frac{b_{af}}{2}+\frac{t_{wf}}{4}-x_f\right)^2\right]$$

$I_{yf} = 523831.337$

$$J_f := 2\cdot\left[\frac{b_{af}\cdot t_{af}^3}{3}\cdot\left[1-0.63\cdot\frac{t_{af}}{b_{af}}\cdot\left[1-\frac{1}{12}\cdot\left(\frac{t_{af}}{b_{af}}\right)^4\right]\right]\ldots\right.$$
$$\left.+\frac{h_{sf}\cdot t_{wf}^3}{3}\cdot\left[1-0.63\cdot\frac{t_{wf}}{h_{sf}}\cdot\left[1-\frac{1}{12}\cdot\left(\frac{t_{wf}}{h_{sf}}\right)^4\right]\right]\right]$$

$J_f = 48373.019$

A2.1.1.2.4 Flange + panel section constants

$$A_{fe} := A_f + b_{af}\cdot t + b_{pf}\cdot\left(t_{pf}-t\right)$$

$A_{fe} = 2062.16$

$$x_{fe} := \frac{1}{A_{fe}}\cdot\left[\left(A_{fe}-A_f\right)\cdot\frac{b_{af}}{2}+A_f\cdot x_f\right]$$

$x_{fe} = 11.101$

y'_{fe} is for a single L-section. For a double L-section, $y_{fe} = 0$:

$$y'_{fe} := \frac{1}{A_{fe}-0.5\cdot A_f}\cdot\left[0.5\cdot\left(A_f\cdot y'_f\right)+b_{pf}\cdot\left(t_{pf}-t\right)\cdot\frac{t_{pf}}{2}\right]$$

$y'_{fe} = 21.074$

$y_{fe} := 0$

I'_{xfe} is for a single L-section and I_{xfe} is for a double L-section:

$$I'_{xfe} := I'_{xf} + 0.5\cdot A_f\cdot\left(y'_{fe}-y'_f\right)^2+\left(A_{fe}-A_f\right)\cdot y'^2_{fe}\ldots$$
$$+b_{pf}\cdot\left(t_{pf}-t\right)\cdot\left(y'_{fe}-\frac{t_{pf}}{2}\right)^2+\left(A_{fe}-A_f\right)\cdot\frac{t^2}{12}+b_{pf}\cdot\frac{\left(t_{pf}-t\right)^3}{12}$$

$I'_{xfe} = 441608.33$

$$I_{xfe} := I_{xf} + A_f\left(y_{fe} - y_f\right)^2 + \left(A_{fe} - A_f\right)\cdot y_{fe}^2 \ldots$$
$$+ b_{pf}\left(t_{pf} - t\right)\cdot\left(y_{fe} - \frac{t_{pf}}{2}\right)^2 + \left(A_{fe} - A_f\right)\cdot\frac{t^2}{12} + b_{pf}\frac{\left(t_{pf} - t\right)^3}{12}$$

$I_{xfe} = 1826651.726$

$$I_{yfe} := I_{yf} + \mathbf{min}\left(b_{af}, b_{pf}\right)^3\cdot\frac{t_{pf}}{12} + \mathbf{max}\left(b_{af} - b_{pf}, 0\right)^3\cdot\frac{t}{12}$$

$I_{yfe} = 537730.521$

$$J_{fe} := J_f + \mathbf{min}\left(b_{af}, b_{pf}\right)\cdot\frac{t_{pf}^3}{3} + \mathbf{max}\left(b_{af} - b_{pf}, 0\right)\cdot\frac{t^3}{3}$$

$J_{fe} = 48402.856$

x'_{ef} is the shear centre of a single L-section:

$$x'_{ef} := \frac{-1}{I'_{xfe}}\cdot\left[\begin{array}{l}\left(y'_{fe} - \frac{t_{af}}{2} - t_{pf} + \frac{t}{2}\right)\cdot b_{af}\, b_{af}\frac{t_{af}}{2}\cdot\left(\frac{t_{af}}{2} + t_{pf} - \frac{t}{2}\right) \ldots \\ + \left(y'_{fe} - \frac{t_{pf}}{2} + \frac{t}{2}\right)\cdot\frac{t_{pf}}{2}\cdot b_{pf}\, b_{af}\left(\frac{t_{pf}}{2} - \frac{t}{2}\right)\end{array}\right]$$

$x'_{ef} = -1.842$

A2.1.1.3 Torsion-related section constants:

$$I_{es} := I_{xs} + I_{ys} + A_s\cdot\left[\left(x_s - x_{es}\right)^2 + y_s^2\right]$$

$I_{es} = 69162.647$

$$I_{ef} := I_{xf} + I_{yf} + A_f\left[\left(x_f - x'_{ef}\right)^2 + y_f^2\right]$$

$I_{ef} = 2662105.717$

$$\Gamma_s := \frac{1}{36}\cdot\left[\begin{array}{l}\left(b_{as}\cdot t_{as}\right)^3 + \left(h_{ss}\cdot t_{ws}\right)^3 \ldots \\ + \left[b_{ps}\cdot\left(t_{ps} - t\right)\right]^3 + \left(b_{as}\cdot t\right)^3\end{array}\right]$$

$\Gamma_s = 16598.444$

$$\Gamma_f := \frac{1}{36}\cdot\left[\begin{array}{l}\left(b_{af}\, t_{af}\right)^3 + \left(h_{sf}\, t_{wf}\right)^3 \ldots \\ + \left[b_{pf}\left(t_{pf} - t\right)\right]^3 + \left(b_{af}\, t\right)^3\end{array}\right]$$

$\Gamma_f = 7561729.386$

A2.1.1.4 Plain Panel Size and Effective Panel Size

Here, r_s and r_f are 0 for single and 1 for double sections. The plain panel size is the panel between the stiffener attached flanges. The effective panel size is between stiffener centroids.

$$b := d - r_s\cdot 2\cdot b_{as}$$
$b = 283$

$$a := h - r_f\, 2\cdot b_{af}$$
$a = 448$

$$d_e := d - r_s\cdot 2\cdot x_s$$
$d_e = 283$

$$h_e := h - r_f\, 2\cdot x_f$$
$h_e = 530.318$

A2.1.1.5 Applied Shear Stress

The applied shear stress is based on the effective height of the panel, between edge member centroids.

$$\tau_{app} := \frac{Q}{h_e \cdot t} \qquad \tau_{app} = 78.412$$

A2.1.1.6 Edge Member Critical Fibres

The critical fibres are for edge member bending within the panel plane. The fibre distances are referred to the edge member neutral axis. For an L-section, they are considered to be at each edge of the attached flange. The allowable fibre stresses are the material allowables for the edge member:

$$x_{f1} := \left|x_f\right| \qquad x_{f1} = 10.641$$

$$x_{f2} := \left|b_{af} - x_f\right| \qquad x_{f2} = 41.159$$

$$f_{e1} := f_{allf}$$

$$f_{e2} := f_{allf}$$

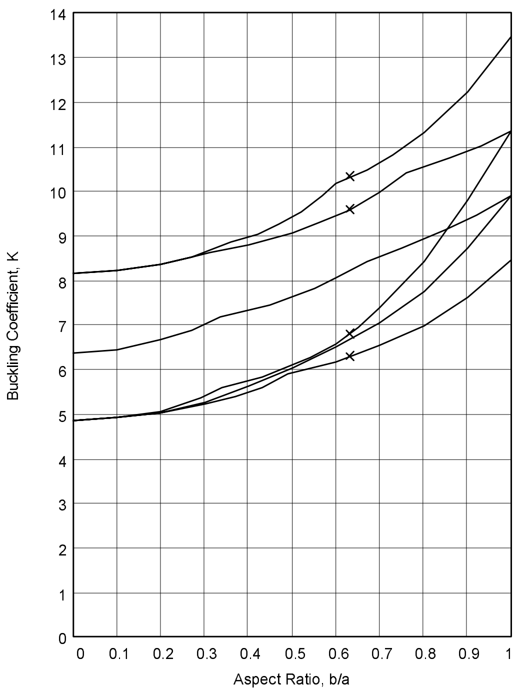
A2.1.2 Initial Panel Buckling

The initial panel buckling stress is eventually obtained from E.S.D.U. 02.03.02. However, APA115 also provides the buckling stress as calculated from E.S.D.U. 71005 for comparison. Both are calculated here.

A2.1.2.1 E.S.D.U. 71005 Initial Panel Buckling

The buckling coefficient from Fig.1 is found. Several coefficients dependent on edge conditions are obtained.

$$ba := \frac{b}{a} \qquad ba = 0.632$$



ESDU 71005 Figure 1

$K_1 = 6.281 \quad K_{1a} = 9.579 \quad K_{1b} = 6.785 \quad K_{1c} = 10.316$

These are for all pinned, side A (stiffeners) clamped, side B (edge members) clamped and all clamped respectively.

A2.1.2.2 E.S.D.U. 02.03.02 Initial Panel Buckling

Four buckling coefficients are found based on longitudinal edges clamped or simply-supported and whether stiffener torsion is included or not. APA115 analyses both stiffener and edge-member as if they were transverse stiffeners.

The correct inertias to use in the parameters μ are those of the stiffener about the mid-panel (required by 02.03.02, but not used by 77014). These are determined as follows:

$$I'_{xs} := I_{xs} + A_s \cdot y_s^2 \qquad I'_{xs} = 35825.903$$

$$I'_{xf} := I_{xf} + A_f \cdot y_f^2 \qquad I'_{xf} = 1826644.267$$

$$\mu_s := \sqrt{\frac{E_s \cdot I'_{xs} \cdot b}{E_p \cdot a^2 \cdot t^3}} \qquad \mu_s = 4.988$$

$$\mu_f := \sqrt{\frac{E_f \cdot I'_{xf} \cdot a}{E_p \cdot b^2 \cdot t^3}} \qquad \mu_f = 70.937$$

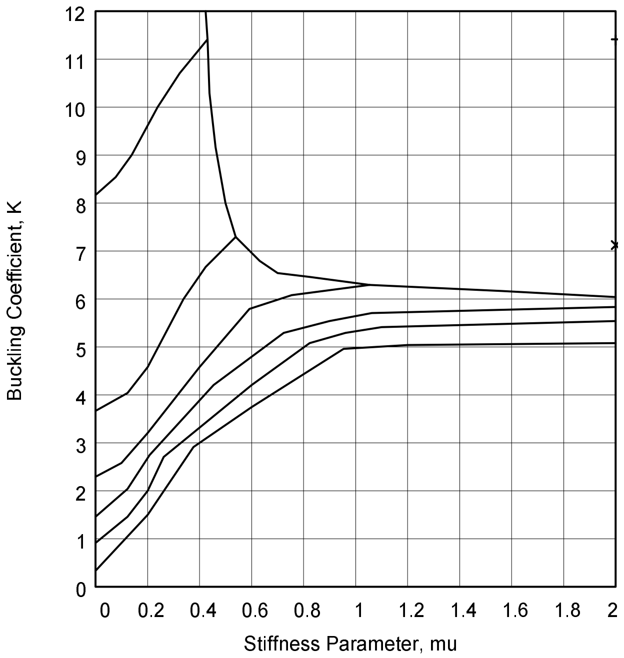
Additionally, the following parameters are required:

$$ab := \frac{a}{b} \qquad ab = 1.583$$

$$GJEI_s := \frac{1}{2.6} \cdot \frac{J_s}{I'_{xs}} \qquad GJEI_s = 0.0011$$

$$GJEI_f := \frac{1}{2.6} \cdot \frac{J_f}{I'_{xf}} \qquad GJEI_f = 0.0102$$

Fig.1: K vs. μ, longitudinal edges clamped, J=0

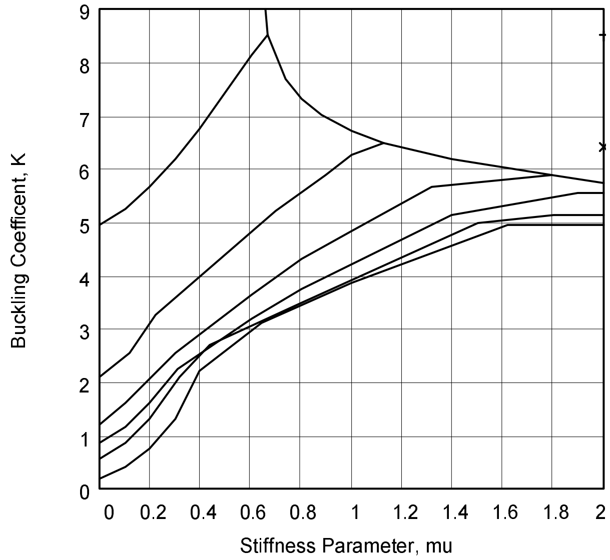


ESDU 02.03.02 Figure 1

$\mu_{ccs} = 0.57$ $\mu_{ccf} = 0.43$ $K_{c0s} = 7.134$ $K_{c0f} = 11.4$

Note that b/a is limited to 1 and μ is limited to 2. K is limited to the value on the μ_{crit} curve and not extrapolated.

Fig.2: K vs. m, longitudinal edges simply-supported, J=0

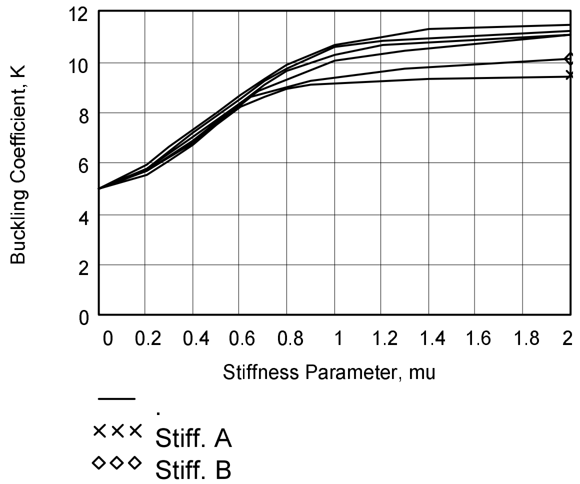


ESDU 02.03.02 Figure 2

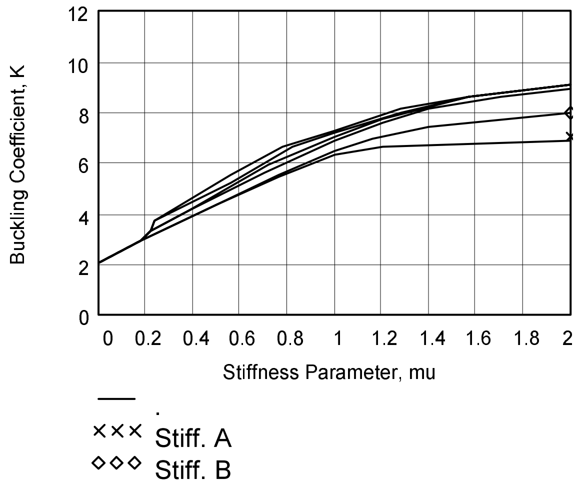
$\mu_{cps} = 1.22$ $\mu_{cpf} = 0.67$ $K_{p0s} = 6.4$ $K_{p0f} = 8.5$

Note that b/a is again limited to 1 and μ is limited to 2 and K is limited to the value on the μ_{crit} curve and not extrapolated. Note also that b/a is limited to 1 and μ is limited to 2.

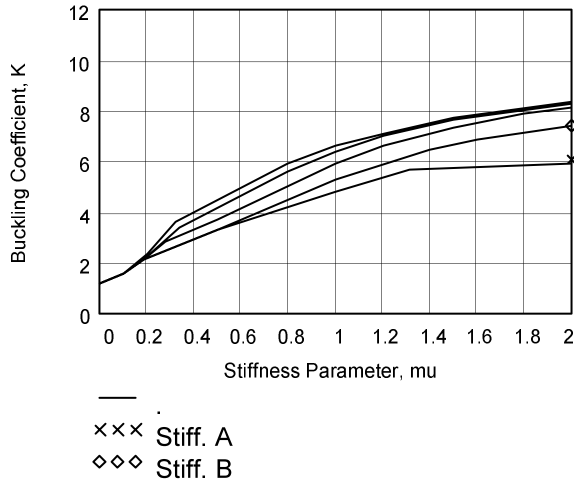
Figs.3-6: K vs. μ, longitudinal edges clamped



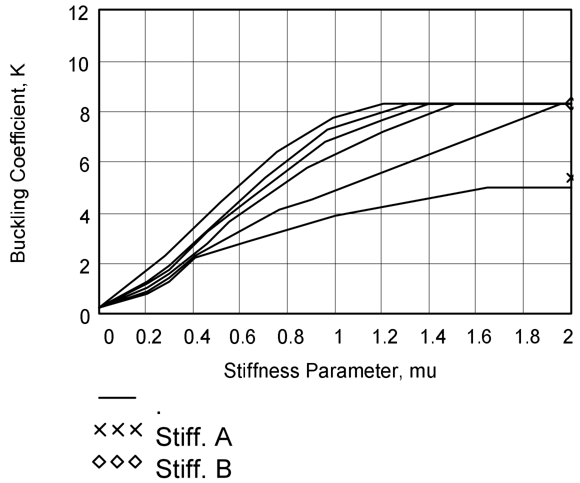
ESDU 02.03.02 Figure 3



ESDU 02.03.02 Figure 4



ESDU 02.03.02 Figure 5

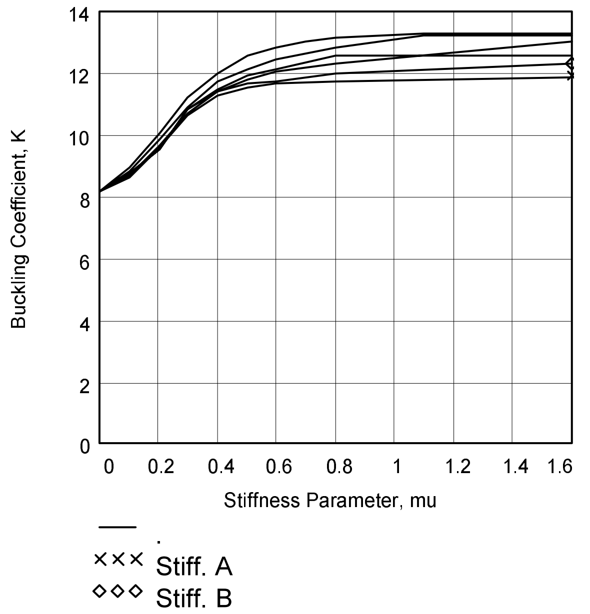


ESDU 02.03.02 Figure 6

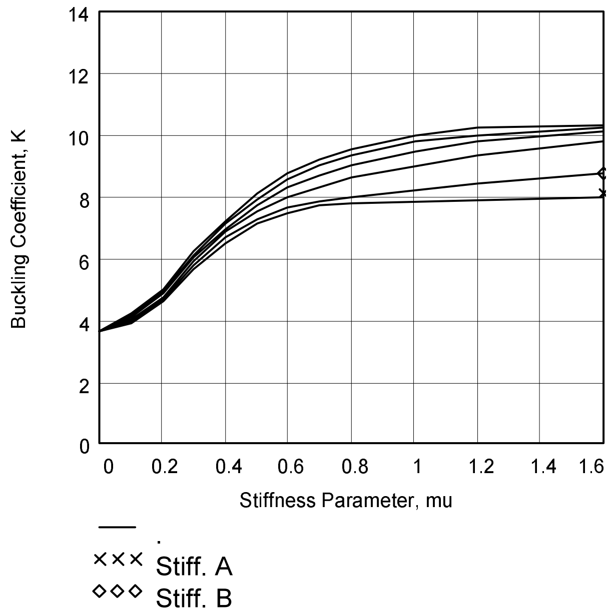
$K_{ps} = 6.865 \quad K_{pf} = 10.104$

Note that b/a is limited to 1 and μ is limited to 2.

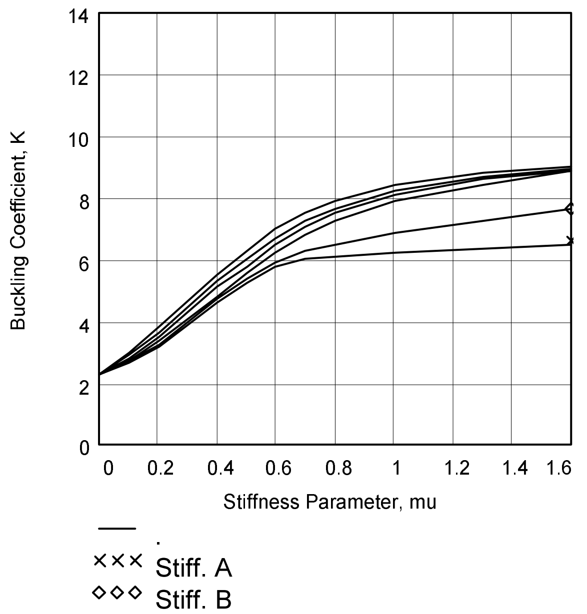
Figs.7-10: K vs. μ, longitudinal edges clamped



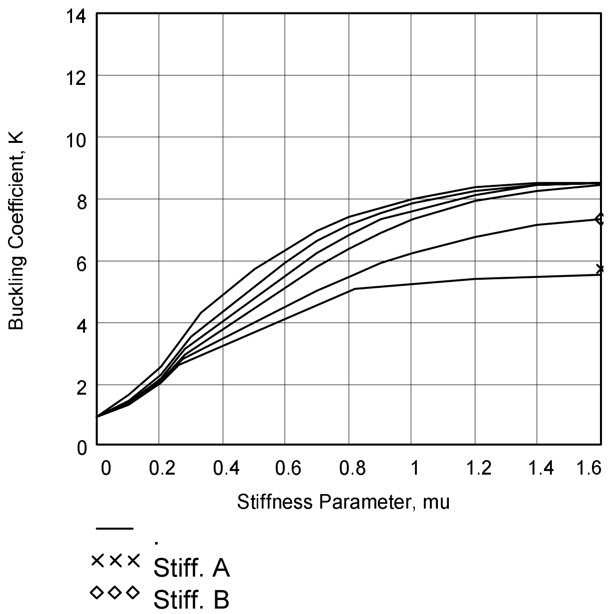
ESDU 02.03.02 Figure 7



ESDU 02.03.02 Figure 8



ESDU 02.03.02 Figure 9



ESDU 02.03.02 Figure 10

K_{cs} = 7.835 K_{cf} = 12.303

Note that μ is limited to 2.

A2.1.2.3 Buckling Across Stiffeners

These criteria are for comparison with the method of S.O.R. 51. The required inertia of the stiffener (and edge member) are based on the ratio of μ_{crit} to μ. When the stiffeners have this inertia, they provide simple support only. There is no additional restraint nor any buckling across them. Below this value, buckling across stiffeners occurs.

$$I_{xrs} := \left(I_{xs} + A_s \cdot y_s^2\right) \cdot \frac{\mu_{cps}}{\mu_s} \qquad I_{xrs} = 8760.63 \quad (I_{xs} = 20102.176 \quad)$$

$$I_{xrf} := \left(I_{xf} + A_f y_f^2\right) \cdot \frac{\mu_{cpf}}{\mu_f} \qquad I_{xrf} = 17252.602 \quad (I_{xf} = 1826644.267 \quad)$$

In this case, no buckling occurs across the stiffeners.

A2.1.2.4 Increased Effective Panel Width

Also to compare with S.O.R.51 is the increased effective panel width. The panel width of a simply-supported panel with the same buckling coefficient as the actual panel is found from the square root of the ratio of buckling coefficients of the actual panel (E.S.D.U. 02.03.02) to the simply-supported panel (E.S.D.U. 71005).

$$a_e := a \cdot \sqrt{\frac{K_1}{K_{pf}}} \qquad a_e = 353.212 \quad (a = 448 \quad)$$

$$b_e := b \cdot \sqrt{\frac{K_1}{K_{ps}}} \qquad b_e = 270.703 \quad (b = 283 \quad)$$

In this case, the effective panel widths are less than the actual panel widths indicating that both pairs of stiffeners are providing more than simple support.

A2.1.2.5 Initial Elastic Panel Buckling Stress

The assumed buckling coefficient (K_2) for this particular example is K_{cs}. This is because the longitudinal edges are effectively clamped by a double L-section.

$$K_2 := K_{cs} \qquad K_2 = 7.835$$

$$\tau_{be} := K_2 \cdot E_p \cdot \left(\frac{t}{b}\right)^2 \qquad \tau_{be} = 10.495$$

A2.1.2.6 Plasticity Correction

E.S.D.U. 71005 Fig.2 gives a set of plasticity correction curves for shear buckling. This is basically a secant modulus correction. The curves may be obtained from the material stress-strain curve using twice the elastic shear stress and dividing the plastic shear stress by 2. A MathCAD function for detemining plastic stress is used and the actual value plotted on a representation of the E.S.D.U. figure derived from this function:

The MathCAD function, a programmed iteration loop, is:

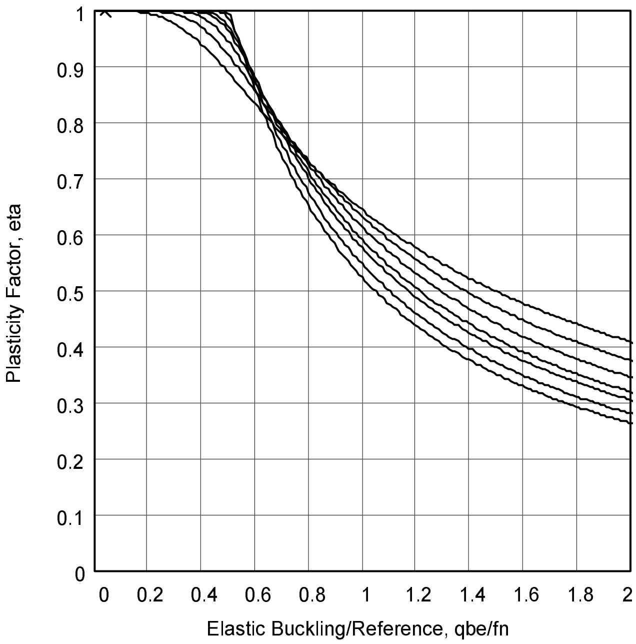
$$\mathbf{f}\left(E, f_n, m, \varepsilon\right) \equiv \left| \begin{array}{l} \text{break} \quad \text{if} \quad \varepsilon \le 0 \\ fe \leftarrow \varepsilon \cdot \dfrac{E}{f_n} \\ f \leftarrow \varepsilon^{\frac{1}{m}} \\ \delta f \leftarrow 1 \\ \text{while} \quad \left|\delta f\right| > 0.000001 \\ \quad \left| \begin{array}{l} \delta f \leftarrow \dfrac{1 - \dfrac{fe}{f + \dfrac{1}{m} \cdot f^m}}{1 + (m-1) \cdot \dfrac{f^{m-1}}{m + f^{m-1}}} \\ f \leftarrow f \cdot (1 - \delta f) \end{array} \right. \\ f \cdot f_n \end{array} \right.$$

In this example, the panel buckles elastically.

$$qbefn := \frac{\tau_{be}}{f_{np}} \qquad qbefn = 0.034$$

$$\tau_b := \frac{1}{2} \cdot \mathbf{f}\left(E_p, f_{np}, m_p, 2 \cdot \frac{\tau_{be}}{E_p}\right) \qquad \tau_b = 10.495$$

$$\eta := \frac{\tau_b}{\tau_{be}} \qquad \eta = 1$$



ESDU 71005 Figure 2

A2.1.3 Post Buckling Analysis

This is based on E.S.D.U. 77014 and its references.

A2.1.3.1 Initial Parameters

The following are used for all the post-buckled analysis.

$$Afht := \frac{A_f}{h_e \cdot t} \cdot \frac{E_f}{E_p} \qquad Afht = 2.675$$

$$Asbt := \frac{A_s}{b \cdot t} \cdot \frac{E_s}{E_p} \qquad Asbt = 0.314$$

$$Asebt := \frac{Asbt}{1 + \frac{A_s \cdot y_s^2}{I_{xs}}} \qquad Asebt = 0.176$$

$$bh := \frac{b}{h_e} \qquad bh = 0.534$$

$$x_{fre} := p_f - x_{fe} \qquad x_{fre} = 26.899$$

Under the applied load, the buckling ratio, diagonal tension factor and angle of diagonal tension, α, are determined with reference to functions. The calculation for α finds the solution of four equations (see 6.1.2 in the main text) by expressing these in a single equation for which MathCAD finds the root.

$$\tau\tau_b := \frac{\tau_{app}}{\tau_b} \qquad \tau\tau_b = 7.471$$

$$k := \mathbf{k}\left(\tau\tau_b\right) \qquad k = 0.411$$

$$\alpha := \alpha\,(k, Afht, Asbt, 0.3, 0.25 \cdot \pi) \qquad \alpha \cdot \frac{180}{\pi} = 41.212$$

The MathCAD functions that calculate k and α are:

$$\mathbf{k}(q) \equiv \tanh(0.5 \cdot \log(q))$$

$$\alpha\,(k, Afht, Asbt, \mu, \alpha) \equiv root\left[\left[\frac{2 \cdot k}{\sin(2 \cdot \alpha)} + (1-k) \cdot (1+\mu) \cdot \sin(2 \cdot \alpha) + \frac{k}{\tan(\alpha) \cdot (2 \cdot Afht + .5 \cdot (1-k))}\right] \ldots, \alpha \atop + -\left[\frac{2 \cdot k}{\sin(2 \cdot \alpha)} + (1-k) \cdot (1+\mu) \cdot \sin(2 \cdot \alpha) + \frac{k \cdot \tan(\alpha)}{Asbt + .5 \cdot (1-k)}\right] \cdot \tan(\alpha)^2\right]$$

A2.1.3.2 Post-Buckled Loading

The post buckled loading is referred to the applied load and the above initial parameters.

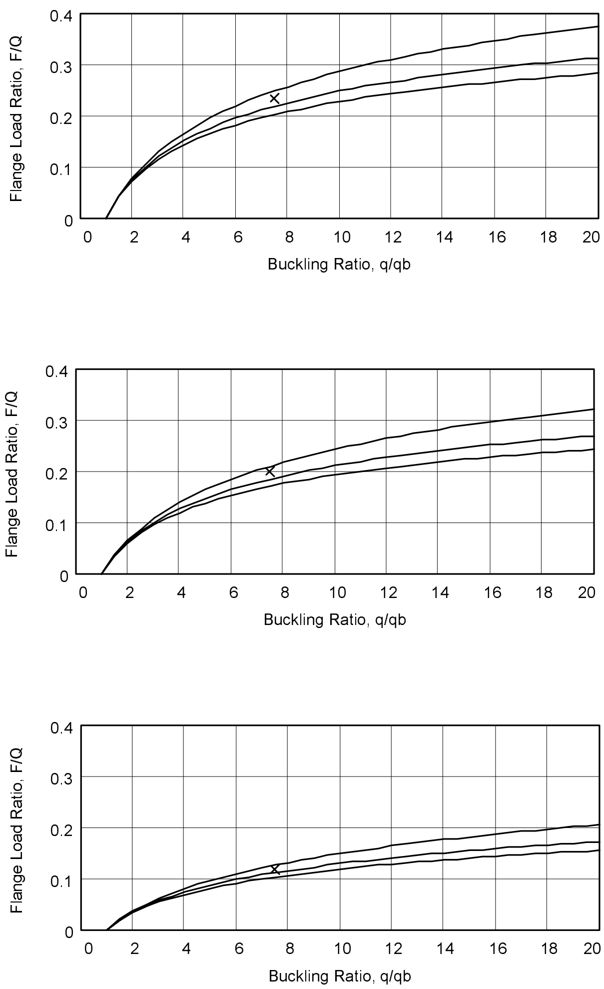
A2.1.3.2.1 Edge member end load, E.S.D.U. 77014 Figs.10-12

These curves are derived from the original equations. The following MathCAD function is used:

$$\mathbf{Fig10C}(k, \alpha, Afht) \equiv \frac{Afht \cdot k \cdot \cot(\alpha)}{2 \cdot Afht + .5 \cdot (1 - k)}$$

$$FQ := \mathbf{Fig10C}(k, \alpha, Afht) \qquad FQ = 0.222$$

$$F := FQ \cdot Q \qquad F = 11095$$



ESDU 77014 Figures 10-12

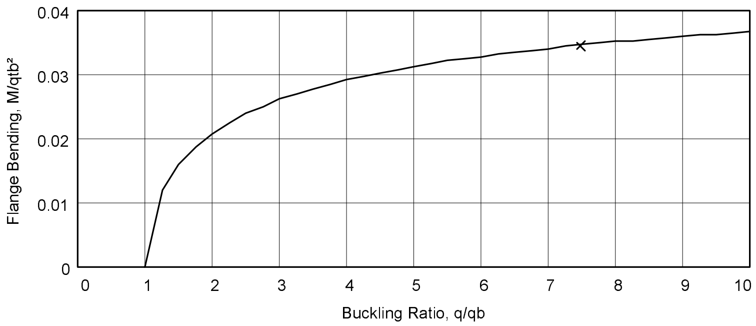
A2.1.3.2.2 Edge member bending moment, E.S.D.U. 77014 Fig.13

This curve is derived from the orginal equation. The following MathCAD function is used:

$$\mathbf{Fig13C}(k) \equiv 0.054 \cdot \sqrt{k}$$

$$Mqtb2 := \mathbf{Fig13C}(k) \qquad Mqtb2 = 0.035$$

$$M := Mqtb2 \cdot \tau_{app} \cdot t \cdot b^2 \qquad M = 260857$$



ESDU 77014 Figure 13

A2.1.3.2.3 Edge member running load, E.S.D.U. 77014 Fig.14

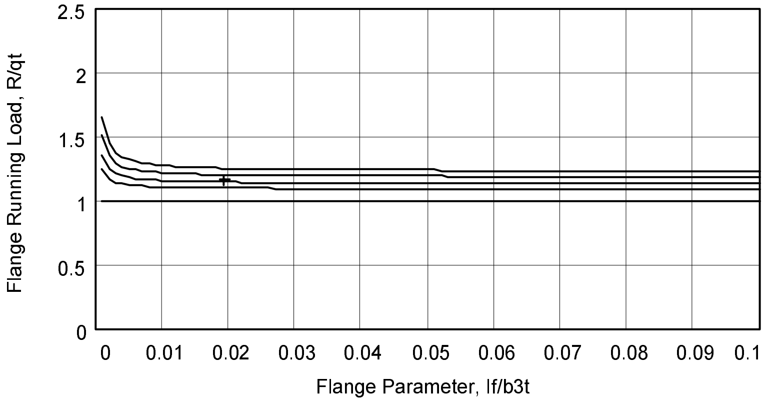
These curves are derived from the original equations. The following MathCAD function is used:

$$\mathbf{Fig14C}(b, h_e, I_{fe}, t, k) \equiv \left[1 + k \cdot \mathbf{Fig18}\left[0.7 \cdot b \cdot \left(\frac{t}{2 \cdot I_{fe} \cdot h_e}\right)^{.25}\right]\right] \cdot (1 + .414 \cdot k)$$

$$Ifb3t := \frac{I_{yf}}{b^3 \cdot t} \qquad Ifb3t = 0.019$$

$$Rqt := \mathbf{Fig14C}(b, h_e, I_{yfe}, t, k) \qquad Rqt = 1.176$$

$$R := Rqt \cdot \tau_{app} \cdot t \qquad R = 110.662$$



ESDU 77014 Figure 14

A2.1.3.2.4 Average stiffener and stiffener heel stress, E.S.D.U. 77014 Figs.3-5

These curves are derived from the original equations. The following MathCAD function is used:

$$\mathbf{Fig3C}(k, \alpha, Asbt) \equiv \frac{k \cdot \tan(\alpha)}{Asbt + .5 \cdot (1 - k)}$$

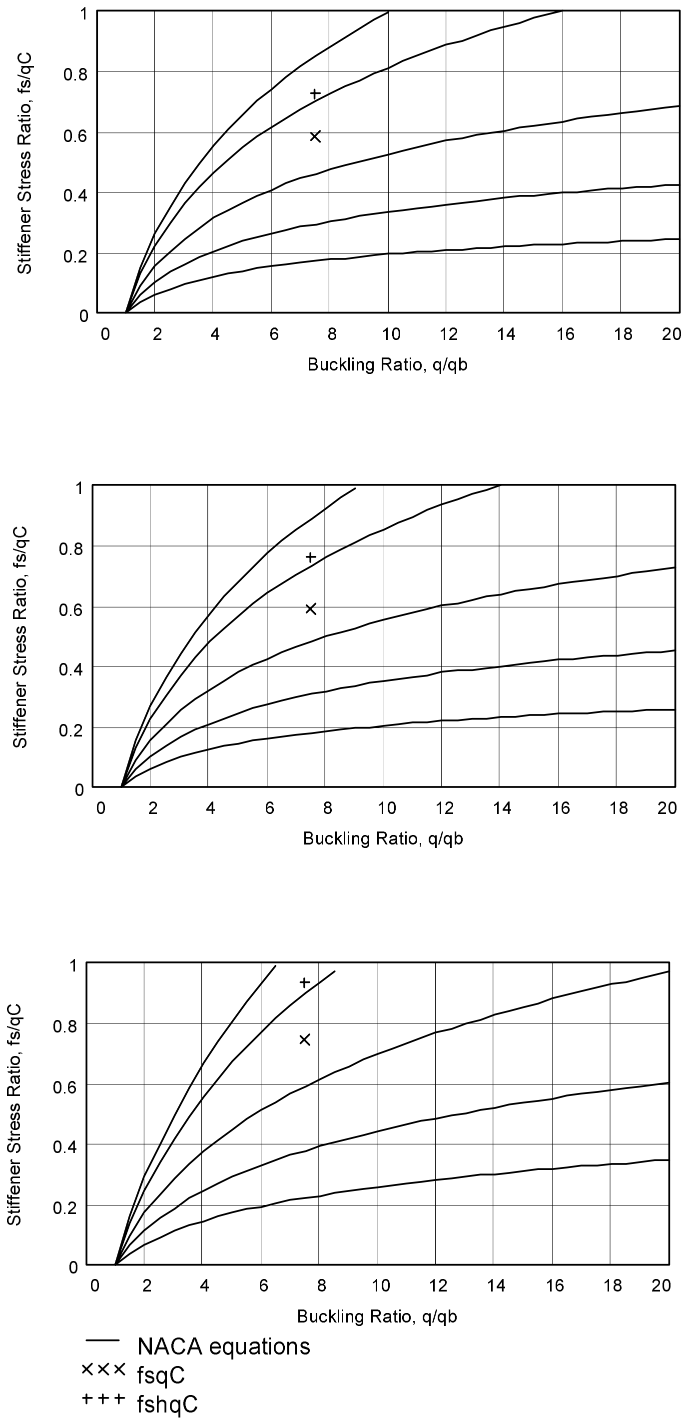
The stiffener heel stress uses the peak stress factor C calculated next.

$fsqC := \mathbf{Fig3C}(k, \alpha, Asbt)$ $\qquad fsqC = 0.591$

$fshqC := \mathbf{Fig3C}(k, \alpha, Asebt)$ $\qquad fshqC = 0.764$

$f_{sav} := fsqC \cdot \tau_{app}$ $\qquad f_{sav} = 46.344$

(The stiffener heel stress uses the peak stress factor C calculated next.)



ESDU 77014 Figures 3-5

A2.1.3.2.5 Peak stiffener stress, E.S.D.U. 77014 Fig.6

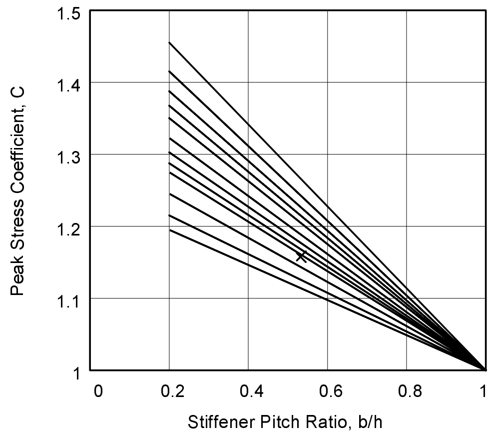
These curves are derived from the original equations. The following MathCAD function is used:

$$\mathbf{Fig6C}(k, bh) \equiv 1 + .57\cdot(1 - k)\cdot(1 - bh)$$

$$C := \mathbf{Fig6C}(k, bh) \qquad C = 1.157$$

$$f_s := C\cdot f_{sav} \qquad f_s = 53.601$$

$$f_{sh} := fshqC\cdot C\cdot \tau_{app} \qquad f_{sh} = 69.304$$



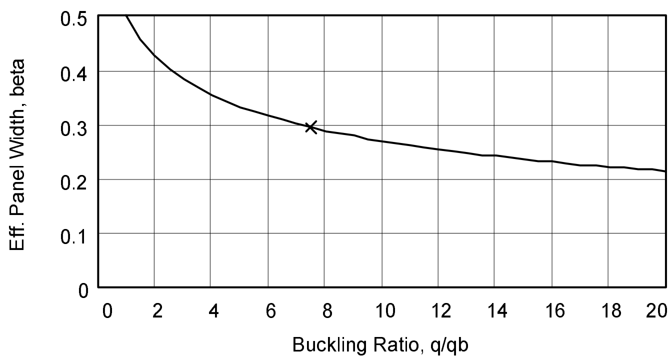
ESDU 77014 Figure 6

A2.1.3.2.6 Effective panel width acting with stiffener, E.S.D.U. 77014 Fig.7

This curve is derived from the original equation. The following MathCAD function is used:

$$\mathbf{Fig7C}(k) \equiv 0.5\cdot(1 - k)$$

$$\beta := \mathbf{Fig7C}(k) \qquad \beta = 0.295$$



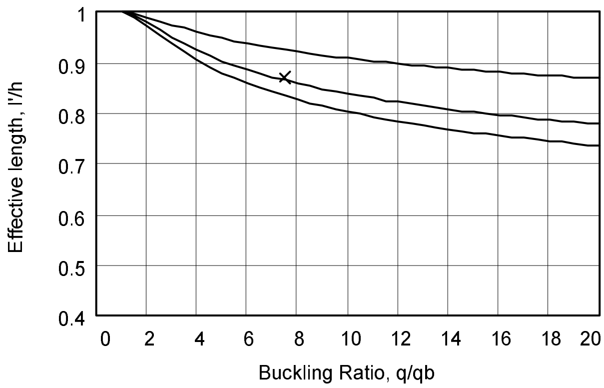
ESDU 77014 Figure 7

A2.1.3.2.7 Effective length of stiffener, E.S.D.U. 77014 Fig.8

These curves are derived from the original equations. The following MathCAD function is used:

$$\mathbf{Fig8C}(k, bh) \equiv \text{if}\left[bh < 1.5, \frac{1}{\sqrt{1 + k^2 \cdot (3 - 2 \cdot bh)}}, 1\right]$$

$lh := \mathbf{Fig8C}(k, bh)$ $\qquad lh = 0.868$



ESDU 77014 Figure 8

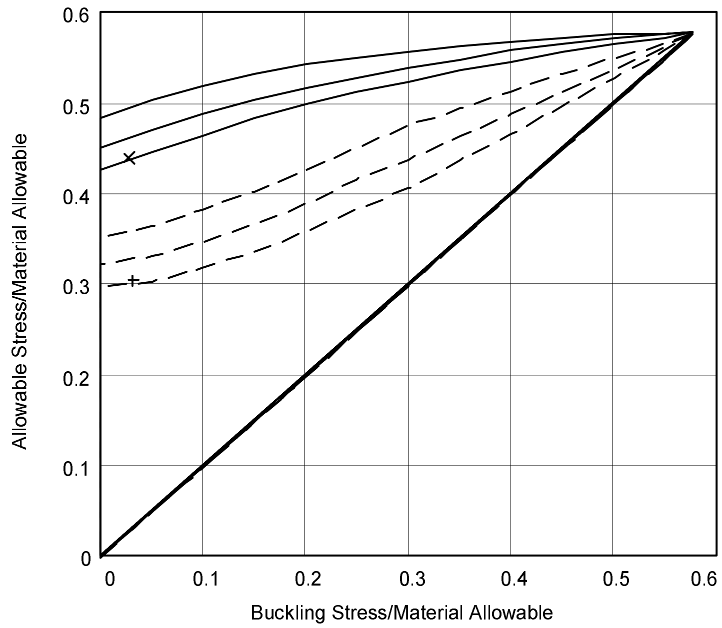
A2.1.3.2.8 Load on edge member/stiffener joint, E.S.D.U. 77014 para 7.8.4.2

$P_{fs} := \mathbf{P_{fs}}\left(f_s, A_s, \tau\tau_b\right)$ $\qquad P_{fs} = 5349.374$

A2.1.3.3 Allowable Panel Stresses

A2.1.3.3.1 Allowable Stresses in Mid-Panel, E.S.D.U. 77014 Fig.1

$$qbf2 := \frac{\tau_b}{t_{2p}}, \quad qbftp := \frac{\tau_b}{f_{allp}} \qquad qbf2 = 0.031 \quad , qbftp = 0.027$$



ESDU 77014 Figure 1

The curve does not have an analytical solution and is solved by linear interpolation.

$$qaf2 := \mathbf{Fig1C}_{\mathbf{pd}}(qbf2, Asbt), \quad qaftp := \mathbf{Fig1C}_{\mathbf{fl}}(qbftp, Asbt) \qquad qaf2 = 0.304 \quad , \quad qaftp = 0.438$$

$$\tau_{lpd} := qaf2 \cdot t_{2p}, \ \tau_{fl} := qaftp \cdot f_{allp}, \ \tau_{gpd} := qaftp \cdot t_{2p} \qquad \tau_{lpd} = 104.2, \tau_{fl} = 169.2, \ \tau_{gpd} = 150.4$$

$$RF_{lpd} := \frac{\tau_{lpd}}{\tau_{app}} \cdot PF \qquad RF_{lpd} = 1.993$$

$$RF_{fl} := \frac{\tau_{fl}}{\tau_{app}} \qquad RF_{fl} = 2.158$$

$$RF_{gpd} := \frac{\tau_{gpd}}{\tau_{app}} \cdot PF \qquad RF_{gpd} = 2.876$$

A2.1.3.3.2 Allowable Shear Stress near Edge Member, E.S.D.U. 77014 Fig.2

Due to the net applied load in the edge member being a function of the applied end-load and the end-load induced by shear, an iterative procedure is required. A set of seven equations are solved using MathCAD's solve loop. The MathCAD functions have been defined earlier.

The first two equations are the diagonal tension factor and the angle of diagonal tension:

$$k_{ej} = \mathbf{k}\left(\frac{\tau_a}{\tau_b}\right) \qquad \alpha_{ej} = \alpha\left(k_{ej}, Afht, Asbt, 0.3, \alpha_{ej}\right)$$

The second two equations are the loads in the edge member:

$$FQ = \mathbf{Fig10C}\left(k_{ej}, \alpha_{ej}, Afht\right) \qquad Mqtb2 = \mathbf{Fig13C}\left(k_{ej}\right)$$

The third two equations are the stress in the edge member at the rivet line, which is then converted to a stress in the panel:

$$f_{fr} = \frac{1}{A_{fe}} \cdot \left(\left|P_f\right| \cdot \frac{\tau_a}{\tau_{app}} + FQ \cdot \tau_a \cdot h \cdot t\right) + Mqtb2 \cdot \tau_a \cdot t \cdot b^2 \cdot \frac{x_{fre}}{I_{yfe}} \qquad f_{pr} = f_{fr} \cdot \frac{E_p}{E_f}$$

The final equation represents the interation curve in E.S.D.U. 77014 Fig.2

$$\sqrt{\frac{1}{\left(\frac{f_{pr}}{f_{allp}}\right)^2 + 3 \cdot \left(\frac{\tau_a}{f_{allp}}\right)^2}} = 1$$

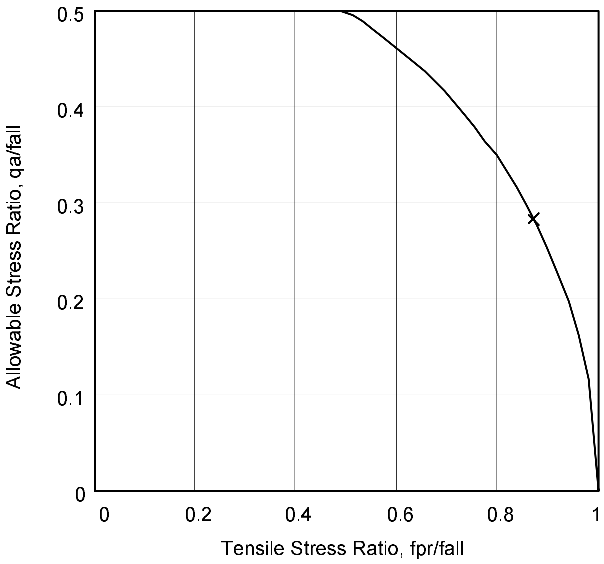
The following solution is obtained:

$$fprftp := \frac{f_{pr}}{f_{allp}} \qquad fprftp = 0.87$$

$$qaftp := \frac{\tau_a}{f_{allp}} \qquad qaftp = 0.284$$

$$\tau_a = 109.743$$

$$RF_{ej} := \frac{\tau_a}{\tau_{app}} \qquad RF_{ej} = 1.4$$



ESDU 77014 Figure 2

A2.1.3.4 Allowable Stiffener Stresses

The allowable stiffener stresses for each failure mode are generally solved using a MathCAD solve loop. Taking a large initial τ and finding all related parameters as start values to begin each loop.

A2.1.3.4.1 Flexural instability stress, E.S.D.U. 77014 Figs.3-5,6,7,8

Note that this mode of failure has a variable allowable based on a changing effective panel width. The iteration effectively involves solving twelve equations. The solution procedure is as follows:

The first two equations are the diagonal tension factor and the angle of diagonal tension:

$$k_{fi} = \mathbf{k}\left(\frac{\tau_{fi}}{\tau_b}\right) \qquad \alpha_{fi} = \alpha\left(k_{fi}, Afht, Asbt, 0.3, 0.25\cdot\pi\right)$$

The next three equations are the effective panel width, the effective strut length and the peak stress factor:

$$\beta_{fi} = \mathbf{Fig7C}\left(k_{fi}\right) \qquad lh_{fi} = \mathbf{Fig8C}\left(k_{fi}, bh\right) \qquad C_{fi} = \mathbf{Fig6C}\left(k_{fi}, bh\right)$$

The next three equations are the section constants for the effective strut:

$$A_{sfi} = A_s + \beta_{fi}\cdot b\cdot t\cdot\frac{E_p}{E_s} \qquad y_{sfi} = A_s\cdot\frac{y_s}{A_{sfi}} \qquad I_{sfi} = I_{xs} + A_s\cdot\left(y_s - y_{sfi}\right)^2 + \beta_{fi}\cdot b\cdot t\cdot\frac{E_p}{E_s}\cdot\left(y_{sfi}^{\,2} + \frac{t^2}{12}\right)$$

The next three equations determine the allowable stress based on a secant formula analysis. The MathCAD function $\mathbf{f}_{fi}$ is a programming loop that solves the secant formula.
The minimum allowable based on stiffener or panel criteria is used:

$$f_{fip} = \mathbf{f_{fi}}\left(E_p, f_{np}, m_p, f_{allp}, A_{sfi}, I_{sfi}, -y_{sfi}, lh_{fi}\cdot h_e, -y_{sfi} - .001\cdot h_e\right)$$

$$f_{fis} = \mathbf{f_{fi}}\left[E_s, f_{ns}, m_s, f_{alls}, A_{sfi}, I_{sfi}, h_{ss} + .5\cdot\left(t + t_{as}\right) - y_{sfi}, lh_{fi}\cdot h_e, -y_{sfi} - .001\cdot h_e\right]$$

$$f_{fi} = \mathbf{min}\left(f_{fip}, f_{fis}\right)$$

The secant formula for the strut analysis is:

$$f_{all} := f\cdot\left(1 + \frac{e}{c}\cdot\sec\left(\frac{L_e}{2\cdot k}\cdot\sqrt{\frac{f}{E_T}}\right)\right)$$

The iterative loop which performs the secant formula analysis is as follows:

$$
\mathbf{f}_{\mathbf{fi}}\left(E, f_n, m, f_{all}, A, I, y, L, e\right) \equiv
\begin{array}{|l}
fe0 \leftarrow \left(\dfrac{\pi}{L}\right)^2 \cdot \dfrac{I}{A} \\
f0fn \leftarrow \dfrac{\mathbf{f'}\left(E, f_n, m, fe0\right)}{f_n} \\
ffn \leftarrow \mathrm{if}\left[1 + e\cdot\dfrac{A\cdot y}{I} = 0, -999, \dfrac{f_{all}}{f_n}\cdot\left(1 + e\cdot\dfrac{A\cdot y}{I}\right)^{-1}\right] \\
ffn \leftarrow \mathrm{if}\left(\left|f0fn\right| < \left|ffn\right|, f0fn\cdot\dfrac{\left|ffn\right|}{ffn}, ffn\right) \\
\delta ffn \leftarrow ffn \\
ffn \leftarrow 0 \\
\text{while } \left|\delta ffn\right| > 0.000001 \\
\quad \begin{array}{|l}
\delta ffn \leftarrow 0.5\cdot\delta ffn \\
ffn \leftarrow ffn + \delta ffn \\
fe \leftarrow \dfrac{f_n}{E}\cdot\left[\left|ffn\right| + \left(\left|ffn\right|\right)^m\right] \\
\text{if } fe < fe0 \\
\quad \begin{array}{|l}
ffn0 \leftarrow \dfrac{f_{all}}{f_n}\cdot\left(1 + e\cdot\dfrac{A\cdot y}{I}\cdot\sec\left(\dfrac{\pi}{2}\cdot\sqrt{\dfrac{fe}{fe0}}\right)\right)^{-1} \\
ffn \leftarrow ffn + \delta ffn \quad \text{if } \left|ffn0\right| > \left|ffn\right|
\end{array} \\
ffn \leftarrow ffn - \delta ffn
\end{array} \\
f \leftarrow \left|ffn\right|\cdot f_n
\end{array}
$$

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In the secant formula loop is included the function **f'** which finds the plastic stress from f/E_t which comes from the following equation:

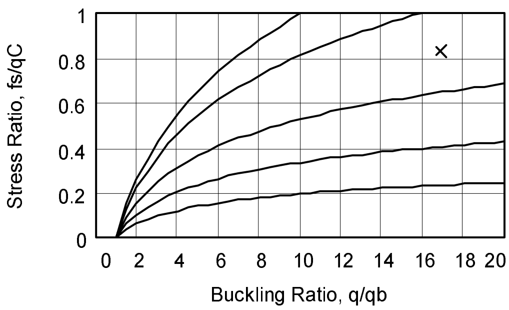
$$\frac{f}{E_t}\cdot\frac{E}{f_n} := \frac{f}{f_n} + \left(\frac{f}{f_n}\right)^m$$

The actual MathCAD function is:

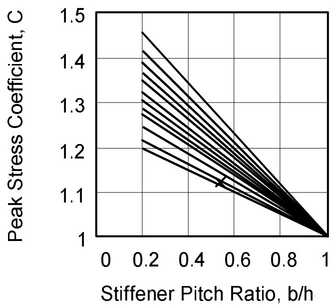
$$\mathbf{f'}\left(E, f_n, m, fE_t\right) \equiv \left|\begin{array}{l} \text{break} \quad \text{if} \quad fE_t \le 0 \\ fe \leftarrow fE_t \cdot \dfrac{E}{f_n} \\ f \leftarrow fE_t^{\frac{1}{m}} \\ \delta f \leftarrow 1 \\ \text{while} \quad \left|\delta f\right| > 0.000001 \\ \quad \left|\begin{array}{l} \delta f \leftarrow \dfrac{1 - \dfrac{fe}{f + f^m}}{1 + (m-1)\cdot\dfrac{f^{m-1}}{1 + f^{m-1}}} \\ f \leftarrow f\cdot(1 - \delta f) \end{array}\right. \\ f \cdot f_n \end{array}\right.$$

Finally, the allowable flexural instability stress is set equal to the applied stress:

$$f_{fi} = \mathbf{Fig3C}\left(k_{fi}, \alpha_{fi}, Asbt\right)\cdot\tau_{fi}\cdot C_{fi}$$

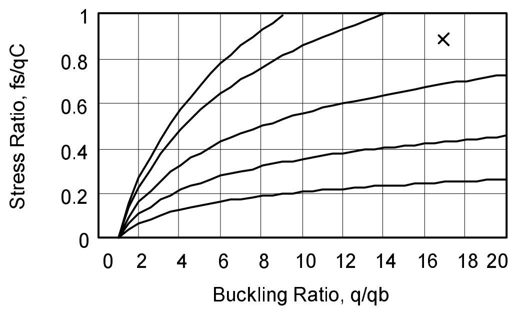


ESDU 77014 Figure 3

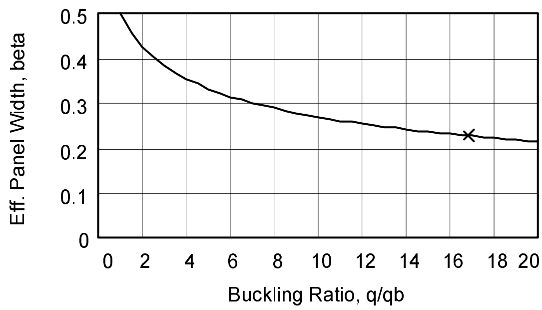


ESDU 77014 Figure 6

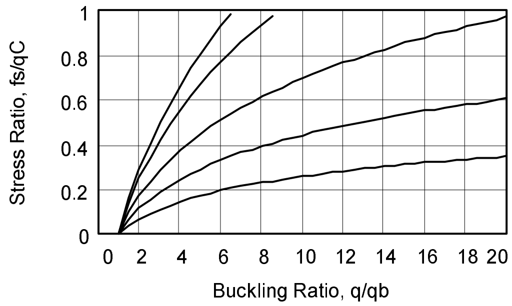
DRAFT



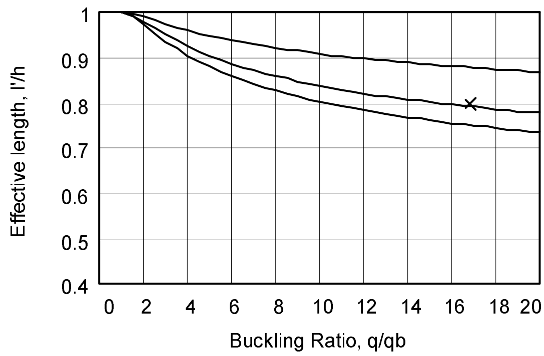
ESDU 77014 Figure 4



ESDU 77014 Figure 7



ESDU 77014 Figure 5



ESDU 77014 Figure 8

Results of flexural instability analysis

$k_{fi} = 0.546$ $\alpha_{fi} = 0.701$ $\beta_{fi} = 0.227$ $lh_{fi} = 0.796$ $C_{fi} = 1.121$

$A_{sfi} = 215.98$ $y_{sfi} = 6.503$ $I_{sfi} = 26704$ $f_{fip} = 222.9$ $f_{fis} = 168.4$ $f_{fi} = 168.4$ $\tau_{fi} = 176.6$

$$\tau fi_{\tau b} := \frac{\tau_{fi}}{\tau_b} \qquad \tau fi_{\tau b} = 16.826$$

$$RF_{fi} := \frac{\tau_{fi}}{\tau_{app}} \qquad RF_{fi} = 2.252$$

A2.1.3.4.2 Torsional instability stress, E.S.D.U. 77014 Figs.3-5,6

The elastic allowable torsional instability stress is as follows, corrected for plasticity by function **f'**:

$$fE_t := \frac{1}{I_{es}} \cdot \left[\frac{J_s}{2.6} + \Gamma_s \cdot \left(\frac{\pi}{h} \right)^2 \right] \qquad f_{ti} := \mathbf{f'}\left(E_s, f_{ns}, m_s, fE_t\right)$$

The iteration effectively involves solving four equations. The solution procedure is as follows:
The first two equations are the diagonal tension factor and the angle of diagonal tension:

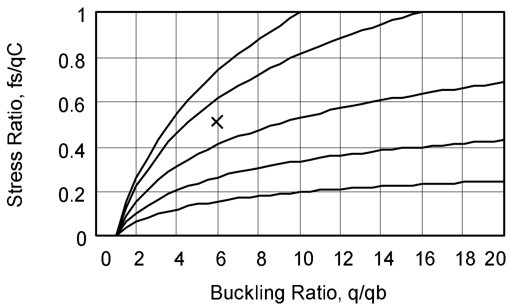
$$k_{ti} = \mathbf{k}\left(\frac{\tau_{ti}}{\tau_b}\right) \qquad \alpha_{ti} = \alpha\left(k_{ti}, Afht, Asbt, 0.3, 0.25 \cdot \pi\right)$$

The next equation is the peak stress factor:

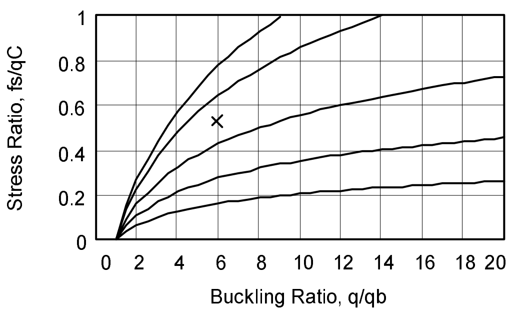
$$C_{ti} = \mathbf{Fig6C}\left(k_{ti}, bh\right)$$

Finally, the allowable torsional instability stress is set equal to the applied stress:

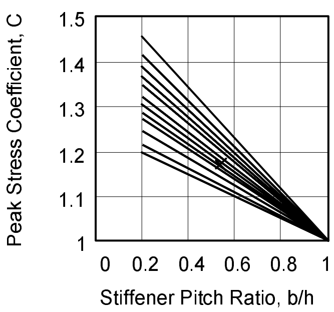
$$f_{ti} = \mathbf{Fig3C}\left(k_{ti}, \alpha_{ti}, Asbt\right) \cdot \tau_{ti} \cdot C_{ti}$$



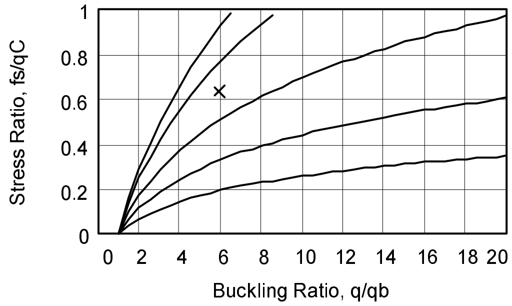
ESDU 77014 Figure 3



ESDU 77014 Figure 4



ESDU 77014 Figure 6



ESDU 77014 Figure 5

Results of torsional instability analysis

$k_{ti} = 0.367$ $\alpha_{ti} = 0.725$ $C_{ti} = 1.168$ $f_{ti} = 37.3$ $\tau_{ti} = 61.8$

$$\tau ti_{\tau b} := \frac{\tau_{ti}}{\tau_{b}} \qquad \tau ti_{\tau b} = 5.89$$

$$RF_{ti} := \frac{\tau_{ti}}{\tau_{app}} \qquad RF_{ti} = 0.788$$

A2.1.3.4.3 Local instability stress, E.S.D.U. 77014 Figs.3-5,6

Firstly, the elastic local instability stress is calculated (using f/E_t). This is purely dependent on geometry. It is then corrected for plasticity.

$$fE_t := 0.58 \cdot \left(\frac{t_{as}}{b_{as}}\right)^2 \qquad f_{li} := \mathbf{f'}\left(E_s, f_{ns}, m_s, fE_t\right)$$

The iteration effectively involves solving four equations. The solution procedure is as follows

The first two equations are the diagonal tension factor and the angle of diagonal tension:

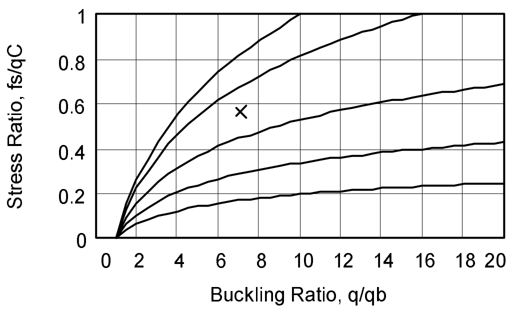
$$k_{li} = \mathbf{k}\left(\frac{\tau_{li}}{\tau_b}\right) \qquad \alpha_{li} = \alpha\left(k_{li}, Afht, Asebt, 0.3, 0.25 \cdot \pi\right)$$

The next equation is the peak stress factor:

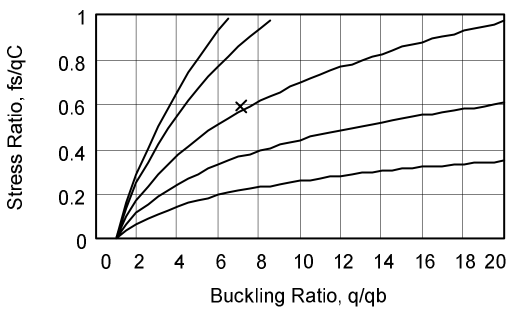
$$C_{li} = \mathbf{Fig6C}\left(k_{li}, bh\right)$$

Finally, the allowable local instability stress is set equal to the applied stress:

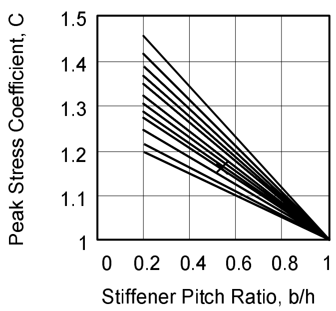
$$f_{li} = \mathbf{Fig3C}\left(k_{li}, \alpha_{li}, Asebt\right) \cdot \tau_{li} \cdot C_{li}$$



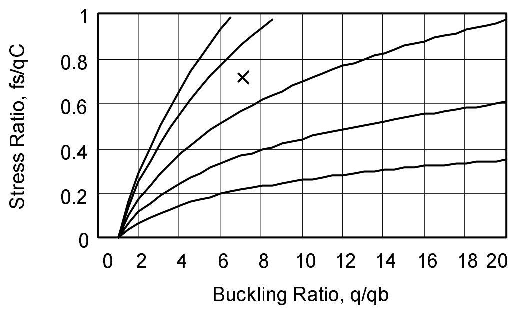
ESDU 77014 Figure 3



ESDU 77014 Figure 4



ESDU 77014 Figure 6



ESDU 77014 Figure 5

Results of local instability analysis

$k_{li} = 0.4$ $\alpha_{li} = 0.705$ $C_{li} = 1.159$ $f_{li} = 61.3$ $\tau_{li} = 73.9$

$$\tau li_{\tau b} := \frac{\tau_{li}}{\tau_b} \qquad \tau li_{\tau b} = 7.039$$

$$RF_{li} := \frac{\tau_{li}}{\tau_{app}} \qquad RF_{li} = 0.942$$

A2.1.3.4.4 Forced crippling stress, E.S.D.U. 77014 Figs.3-5,8,9

The iteration effectively involves solving four equations. The solution procedure is as follows:
The first two equations are the diagonal tension factor and the angle of diagonal tension:

$$k_{sc} = \mathbf{k}\left(\frac{\tau_{sc}}{\tau_b}\right) \qquad \alpha_{sc} = \alpha\left(k_{sc}, Afht, Asbt, 0.3, 0.25\cdot\pi\right)$$

The next equation is the peak stress factor:

$$C_{sc} = \mathbf{Fig6C}\left(k_{sc}, bh\right)$$

The forced crippling stress is obtained from Fig.9 is:

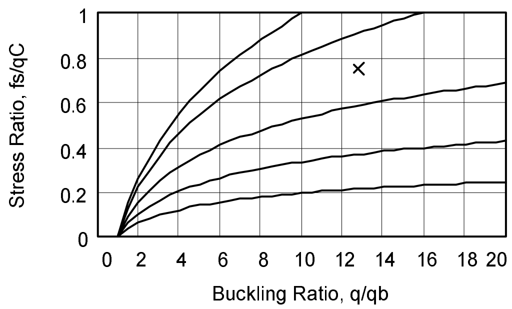
$$f_{sc} = \mathbf{Fig9C}\left(k_{sc}, tsot\right)\cdot t_{2s} \qquad \text{where} \qquad tsot := \frac{t_{as}}{t}$$

and the analytical expression is:

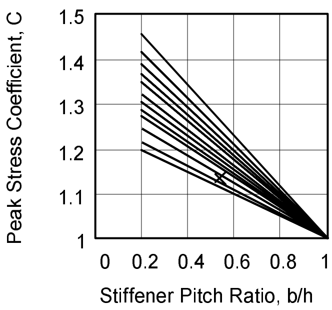
$$\mathbf{Fig9C}(k, tsot) \equiv .5\cdot k^{\frac{2}{3}}\cdot tsot^{\frac{1}{3}}$$

Finally, the allowable forced crippling stress is set equal to the applied stress:

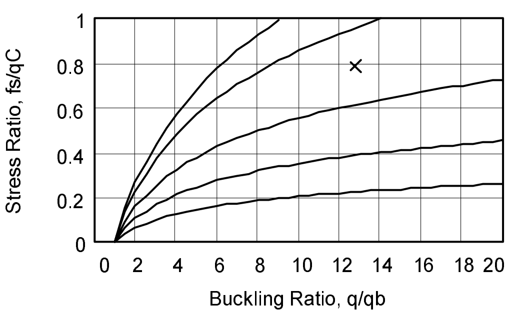
$$f_{sc} = \mathbf{Fig3C}\left(k_{sc}, \alpha_{sc}, Asbt\right)\cdot\tau_{sc}\cdot C_{sc}$$



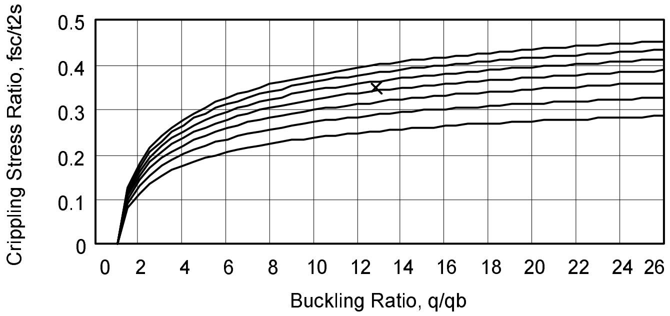
ESDU 77014 Figure 3



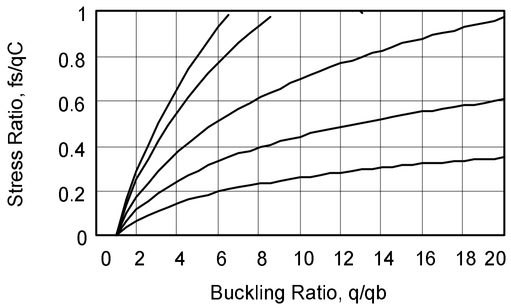
ESDU 77014 Figure 6



ESDU 77014 Figure 4



ESDU 77014 Figure 9



ESDU 77014 Figure 5

Results of forced crippling analysis

$k_{sc} = 0.503 \quad \alpha_{sc} = 0.707 \quad C_{sc} = 1.132 \quad f_{sc} = 115.8 \quad \tau_{sc} = 134.1$

$$fsct2s := \frac{f_{sc}}{t_{2s}} \qquad fsct2s = 0.348$$

$$\tau sc_{\tau b} := \frac{\tau_{sc}}{\tau_{b}} \qquad \tau sc_{\tau b} = 12.778$$

$$RF_{sc} := \frac{\tau_{sc}}{\tau_{app}} \qquad RF_{sc} = 1.71$$

A2.1.3.4.5 Inter-rivet buckling stress, E.S.D.U. 77014 Figs.3-5,6

For mushroom-headed rivets, the inter-rivet buckling coefficient, K_r, is 3. The elastic inter-rivet buckling stress (based on f/E_t) is found for both panel and stiffener. It is then corrected for plasticity and the minimum value used:

$$fE_{tp} := \frac{K_r}{12} \cdot \left(\frac{\pi \cdot t}{s_s}\right)^2 \qquad fE_{ts} := \frac{K_r}{12} \cdot \left(\frac{\pi \cdot t_{as}}{s_s}\right)^2 \qquad \text{where} \quad K_r := 3$$

$$f_{irs} := \mathbf{min}\left(\mathbf{f'}\left(E_p, f_{np}, m_p, fE_{tp}\right), \mathbf{f'}\left(E_s, f_{ns}, m_s, fE_{ts}\right)\right)$$

The iteration effectively involves solving four equations. The solution procedure is as follows

The first two equations are the diagonal tension factor and the angle of diagonal tension:

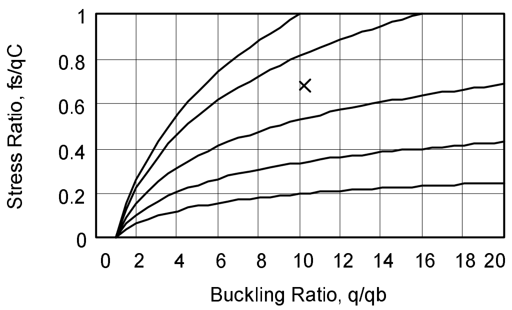
$$k_{irs} = \mathbf{k}\left(\frac{\tau_{irs}}{\tau_b}\right) \qquad \alpha_{irs} = \alpha\left(k_{irs}, Afht, Asebt, 0.3, 0.25 \cdot \pi\right)$$

The next equation is the peak stress factor:

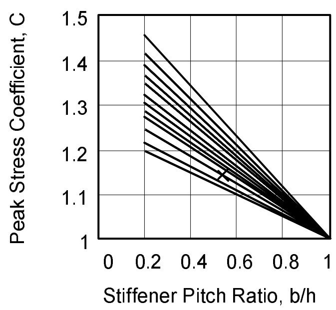
$$C_{irs} = \mathbf{Fig6C}\left(k_{irs}, bh\right)$$

Finally, the inter-rivet buckling stress is set equal to the applied stress:

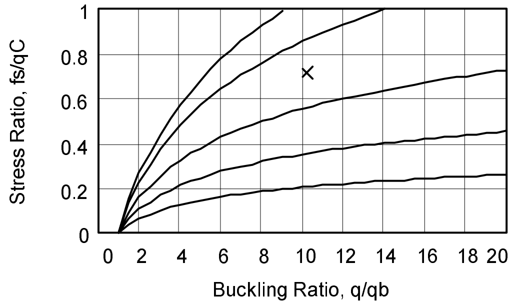
$$f_{irs} = \mathbf{Fig3C}\left(k_{irs}, \alpha_{irs}, Asebt\right) \cdot \tau_{irs} \cdot C_{irs}$$



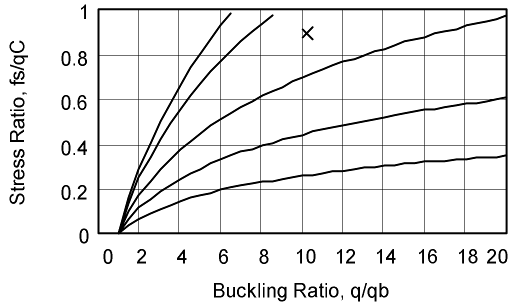
ESDU 77014 Figure 3



ESDU 77014 Figure 6



ESDU 77014 Figure 4



ESDU 77014 Figure 5

Results of inter-rivet buckling analysis

$k_{irs} = 0.465$ $\alpha_{irs} = 0.694$ $C_{irs} = 1.142$ $f_{irs} = 105.9$ $\tau_{irs} = 106.5$

$$RF_{irs} := \frac{\tau_{irs}}{\tau_{app}} \qquad RF_{irs} = 1.358$$

$$\tau irs_{\tau b} := \frac{\tau_{irs}}{\tau_{b}} \qquad \tau irs_{\tau b} = 10.149$$

A2.1.3.5 Allowable Edge Member Stresses

A2.1.3.5.1 Inter-rivet Buckling

For mushroom-headed rivets, the inter-rivet buckling coefficient, K_r, is 3. The elastic inter-rivet buckling stress (based on f/E_t) is found for both panel and edge member. It is then corrected for plasticity and the minimum value used:

$$fE_{tp} := \frac{K_r}{12} \cdot \left(\frac{\pi \cdot t}{s_f}\right)^2 \qquad fE_{tf} := \frac{K_r}{12} \cdot \left(\frac{\pi \cdot t_{af}}{s_f}\right)^2$$

$$f_{irf} := \mathbf{min}\left(\mathbf{f'}\left(E_p, f_{np}, m_p, fE_{tp}\right), \mathbf{f'}\left(E_f, f_{nf}, m_f, fE_{tf}\right)\right)$$

The iteration effectively involves solving four equations. The solution procedure is as follows

The first two equations are the diagonal tension factor and the angle of diagonal tension:

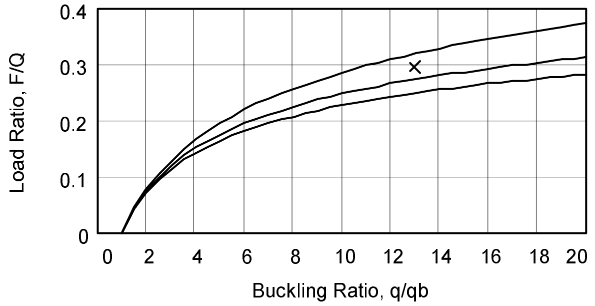
$$k_{irf} = \mathbf{k}\left(\frac{\tau_{irf}}{\tau_b}\right) \qquad \alpha_{irf} = \alpha\left(k_{irf}, Afht, Asbt, 0.3, 0.25 \cdot \pi\right)$$

The next equation provides the additional end-load due to shear:

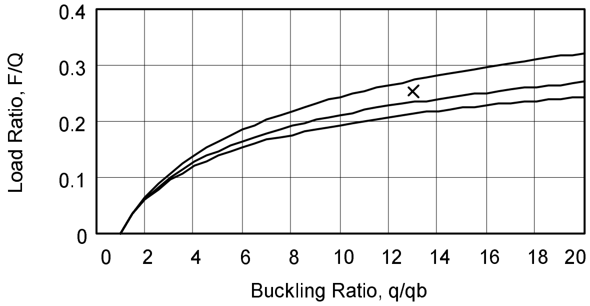
$$FQ_{irf} = \mathbf{Fig10C}\left(k_{irf}, \alpha_{irf}, Afht\right)$$

Finally, the inter-rivet buckling stress is set equal to the applied stress. This is the average stress at the edge member neutral axis based on applied end-load and end-load due to shear. Bending moment is not considered for the stress at the neutral axis:

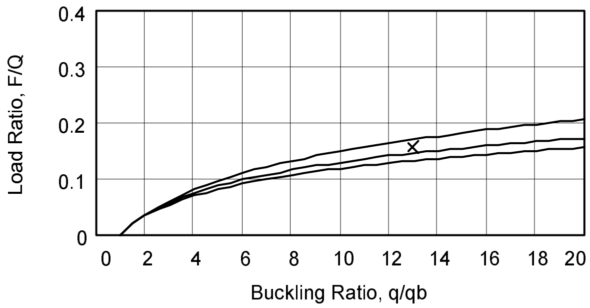
$$f_{irf} = \frac{FQ_{irf} \cdot \tau_{irf} \cdot h_e \cdot t}{A_{fe}} + \frac{P_f}{A_{fe}} \cdot \frac{\tau_{irf}}{\tau_{app}}$$



ESDU 77014 Figure 10



ESDU 77014 Figure 11



ESDU 77014 Figure 12

Results of edge-member inter-rivet buckling analysis

$k_{irf} = 0.505$ $\alpha_{irf} = 0.706$ $FQ_{irf} = 0.283$ $f_{irf} = 330$ $\tau_{irf} = 135.7$

$$RF_{irf} := \frac{\tau_{irf}}{\tau_{app}}$$ $RF_{irf} = 1.731$

$$\tau irf_{\tau b} := \frac{\tau_{irf}}{\tau_b}$$ $\tau irf_{\tau b} = 12.934$

A2.1.3.5.2 Edge Member Critical Fibre Stress 1

The iteration effectively involves solving five equations. The solution procedure is as follows. The first two equations are the diagonal tension factor and the angle of diagonal tension:

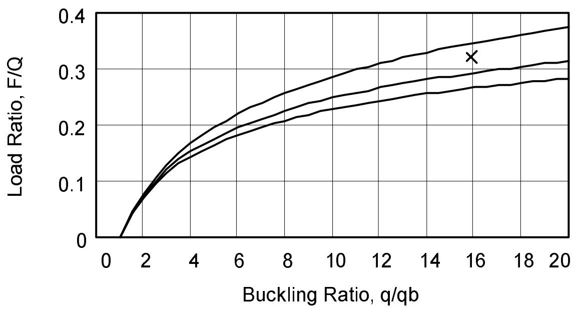
$$k_{e1} = \mathbf{k}\left(\frac{\tau_{e1}}{\tau_b}\right) \qquad \alpha_{e1} = \alpha\left(k_{e1}, Afht, Asbt, 0.3, 0.25\cdot\pi\right)$$

The next two equations provide the additional end-load due to shear and the induced bending moment due to shear:

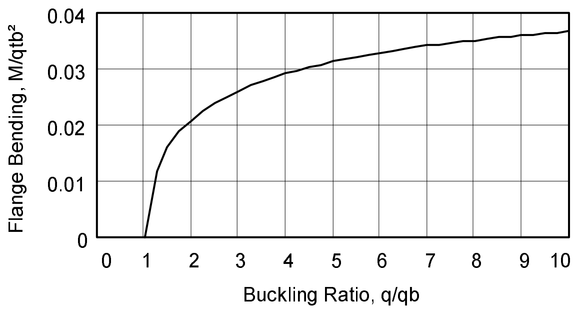
$$FQ_{e1} = \mathbf{Fig10C}\left(k_{e1}, \alpha_{e1}, Afht\right) \qquad Mqtb2_{e1} = \mathbf{Fig13C}\left(k_{e1}\right)$$

Finally, the allowable stress in the edge member is set equal to the applied stress. This is the stress due to applied end-load and end-load due to shear plus the stress due to bending:

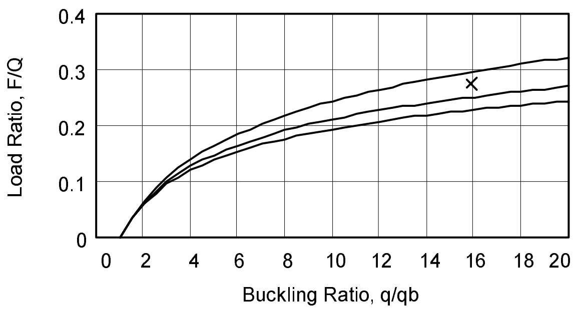
$$f_{e1} = \frac{FQ_{e1}\cdot\tau_{e1}\cdot h_e\cdot t}{A_{fe}} + \frac{P_f}{A_{fe}}\cdot\frac{\tau_{e1}}{\tau_{app}} + Mqtb2_{e1}\cdot\tau_{e1}\cdot t\cdot b^2\cdot\frac{x_{f1}}{I_{yfe}}$$



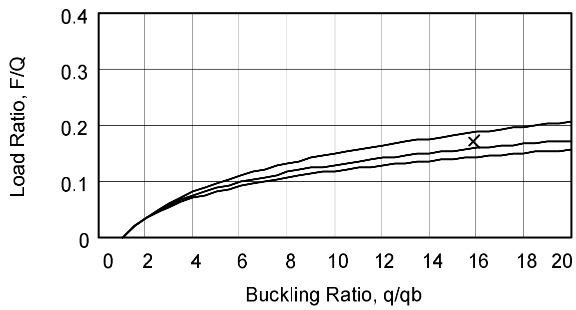
ESDU 77014 Figure 10



ESDU 77014 Figure 13



ESDU 77014 Figure 11



ESDU 77014 Figure 12

Results of max. fibre stress analysis:

$k_{e1} = 0.537$ $\alpha_{e1} = 0.702$ $FQ_{e1} = 0.304$ $Mqtb2_{e1} = 0.04$ $f_{e1} = 417$ $\tau_{e1} = 165.9$

$$\tau e1_{\tau b} := \frac{\tau_{e1}}{\tau_{b}} \qquad \tau e1_{\tau b} = 15.811$$

$$RF_{e1} := \frac{\tau_{e1}}{\tau_{app}} \qquad RF_{e1} = 2.116$$

A2.1.3.5.3 Edge Member Critical Fibre Stress 2

This is identical to the analysis for the critical fibre 1. The iteration effectively involves solving five equations. The solution procedure is as follows:

The first two equations are the diagonal tension factor and the angle of diagonal tension:

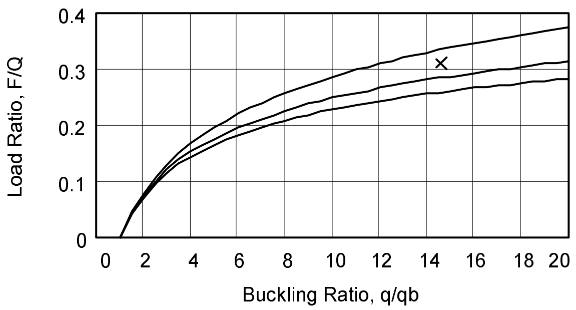
$$k_{e2} = \mathbf{k}\left(\frac{\tau_{e2}}{\tau_b}\right) \qquad \alpha_{e2} = \alpha\left(k_{e2}, Afht, Asbt, 0.3, 0.25\cdot\pi\right)$$

The next two equations provide the additional end-load due to shear and the induced bending moment due to shear:

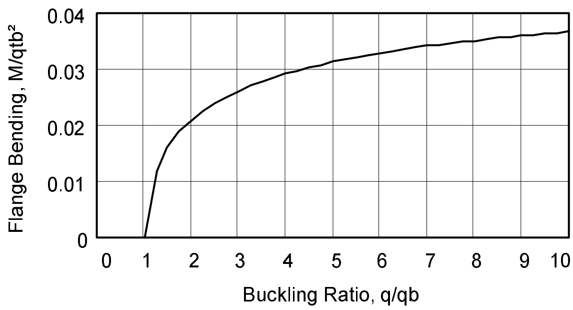
$$FQ_{e2} = \mathbf{Fig10C}\left(k_{e2}, \alpha_{e2}, Afht\right) \qquad Mqtb2_{e1} = \mathbf{Fig13C}\left(k_{e1}\right)$$

Finally, the allowable stress in the edge member is set equal to the applied stress. This is the stress due to applied end-load and end-load due to shear plus the stress due to bending:

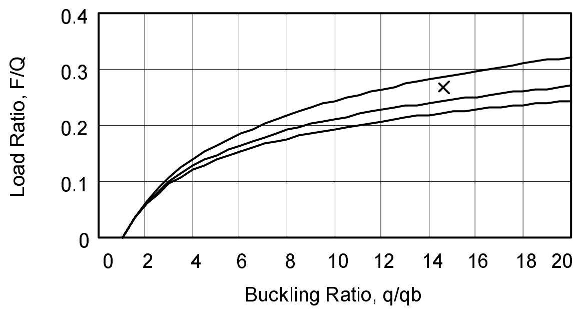
$$f_{e2} = \frac{FQ_{e2}\cdot\tau_{e2}\cdot h_e\cdot t}{A_{fe}} + \frac{P_f}{A_{fe}}\cdot\frac{\tau_{e2}}{\tau_{app}} + Mqtb2_{e2}\cdot\tau_{e2}\cdot t\cdot b^2\cdot\frac{x_{f2}}{I_{yfe}}$$



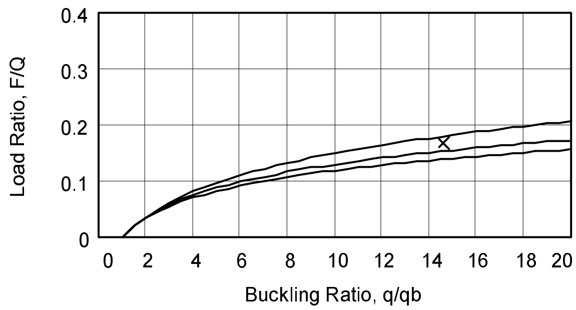
ESDU 77014 Figure 10



ESDU 77014 Figure 13



ESDU 77014 Figure 11



ESDU 77014 Figure 12

Results of max. fibre stress analysis:

$k_{e2} = 0.524$ $\alpha_{e2} = 0.704$ $FQ_{e2} = 0.296$ $Mqtb2_{e2} = 0.039$ $f_{e2} = 417$ $\tau_{e2} = 153.2$

$$\tau e2_{\tau b} := \frac{\tau_{e2}}{\tau_b} \qquad \tau e2_{\tau b} = 14.593$$

$$RF_{e2} := \frac{\tau_{e2}}{\tau_{app}} \qquad RF_{e2} = 1.953$$

A2.1.3.6 Attachment Analysis

A2.1.3.6.1 Rivet Tension Criterion

Single stiffeners, required strength:

$$R_{ten} := 0.22 \cdot f_{alls} \cdot t \qquad R_{ten} = 110.088$$

$$N_{alls} := \frac{P_{ults}}{s_s} \qquad N_{alls} = 20$$

$$RF_{ten} := \frac{N_{alls}}{R_{ten}} \qquad RF_{ten} = 0.182$$

A2.1.3.7 Stiffness Analysis

A2.1.3.7.1 Secant and Tangent Modulus Reduction

The MathCAD functions calculating these shear moduli are as follows. The secant modulus is found directly but the tangent modulus is found from the slope of the secant modulus curve:

$$GsG := \mathbf{Fig15C}(k, \alpha, Afht, Asbt, 0.3) \qquad GsG = 0.681$$

$$GtG := \mathbf{Fig18C}(k, \alpha, Afht, Asbt, 0.3) \qquad GtG = 0.576$$

A2.1.3.7.2 Further Reduction due to Edge Member Flexibility

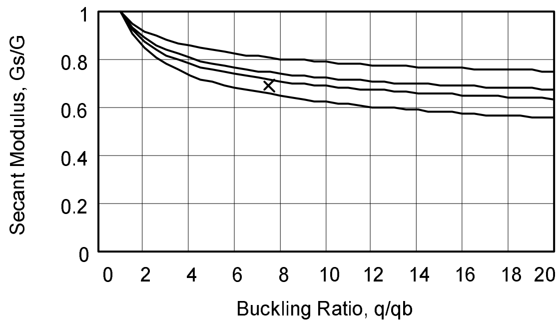
Factor G_f based on 10% reduction for factor = 0.5. This is a tentative calculation.

$$G_f := \frac{A_s + \beta \cdot b \cdot t}{360 \cdot Ifb3t \cdot h_e \cdot t} \qquad G_f = 0.051$$

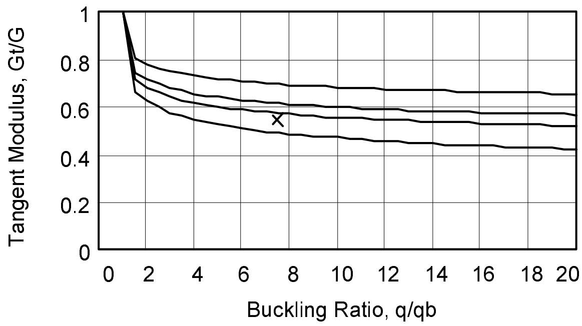
$$GsG' := \left(1 - \frac{0.1}{0.5} \cdot G_f\right) \cdot GsG \qquad GsG' = 0.674$$

$$GtG' := \left(1 - \frac{0.1}{0.5} \cdot G_f\right) \cdot GtG \qquad GtG' = 0.57$$

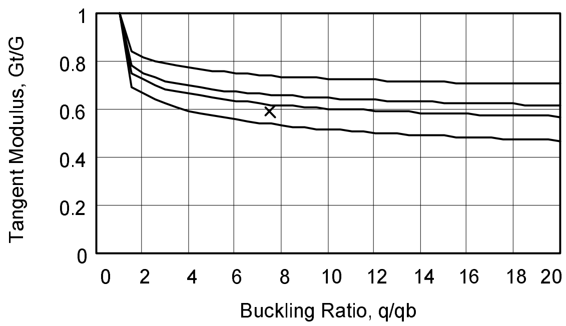
DRAFT



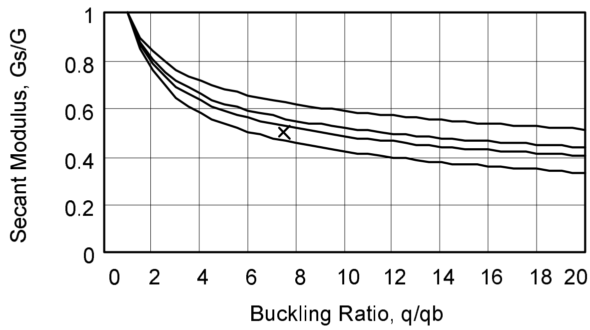
ESDU 77014 Figure 15



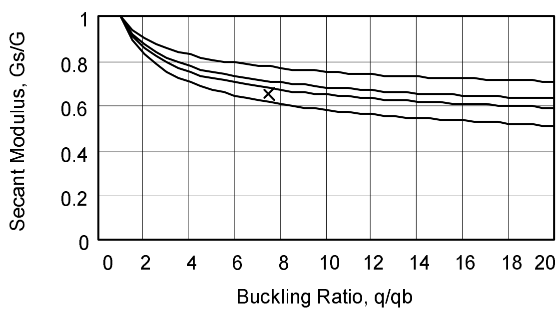
ESDU 77014 Figure 19



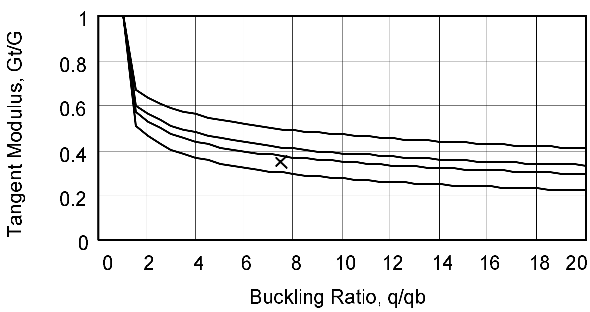
ESDU 77014 Figure 18



ESDU 77014 Figure 17



ESDU 77014 Figure 16



ESDU 77014 Figure 20

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Project/Standard: Development version                      Time: 12:16    1
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Title: APA115 program verification
Case : ESDU 77014 example

Input Data Summary                                       Units =      Nmm

   Material Properties                          Panel       Sa          Sb
   -------------------                                   (stiff.)    (E.M.)
   Youngs modulus                       E =    74500.0    63400.0     63400.0
   Reference stress                    fn =      305.0      296.0       296.0
   Material characteristic              m =     19.000     17.000      17.000
   Allowable stress                  fall =      386.0      417.0       417.0

   Panel Geometry
   --------------
   Panel thickness, overall                       t =       1.20
   Panel size (pitch Sa,Sb)                     d,h =     283.00      551.60
   Width of stiffener pad at Sa,Sb               bp =       0.00        0.00
   Panel thickness at Sa,Sb                      tp =       1.20        1.20

   Stiffener Geometry                                         Sa          Sb
   ------------------                                     (stiff.)    (E.M.)
   Section type                                sect =   L-section  DL-section
   Height between flanges                        hs =      39.20       61.00
   Free flange width (total for I)               bf =       0.00        0.00
   Attached flange width (total for H,I,J,T)     ba =      39.20       51.80
   Free flange bulb area (total for I)           Bf =       0.00        0.00
   Attached flange bulb area (total for I,J,T)   Ba =       0.00        0.00
   Web thickness                                 tw =       1.60        9.60
   Free flange thickness                         tf =       0.00        0.00
   Attached flange thickness                     ta =       1.60        8.00

   Attachment Geometry                                        Sa          Sb
   -------------------                                    (stiff.)    (E.M.)
   Attachment type                              att =   mush head   mush head
   Attachment pitch                               s =      50.00       12.50
   Attachment offset from stiffener web           p =      20.00       38.00
   Attachment diameter                           ds =       4.80        4.80
   Attachment tensile strength                 Pult =      1000.       1000.
   Attachment shear strength                  Psult =         0.          0.
   Allowable load in stiff./edge mem. joint   SFall =         0.

   Applied Loads
   -------------
   Applied shear load/Proof factor             Q,PF =     49900.       1.500
   Applied load in Sa,Sb (tension +ve)           Ps =         0.     379000.

   Analysis and Output Options
   ---------------------------
   Type of analysis/output selected:                     ESDU       extended
-----------------------------------------------------------------------------
User alterations to analysis:
   Stiffeners B are edge members
   Stiffeners B are orientated asymmetrically
```

Table A2.1 APA115 output: E.S.D.U. 77014 Example

```
BAe Airbus Ltd. Filton File: APA115 Version 1.0          Page:
AIRBUS APA115  Iss: 1.0/Mar 98     Name: M.J.H. Austin      Date: 07-APR-98
Project/Standard: Development version                      Time: 12:16   2
-----------------------------------------------------------------------------
Input File Name : D:\BAE\APA115\PROGRA~1\FORTRA~1\ds77014.inp

Title: APA115 program verification
Case : ESDU 77014 example

Initial Panel Data                                        Units =      Nmm

   Material Properties                            Panel        Sa          Sb
   -------------------                                      (stiff.)    (E.M.)
   0.2% proof stress                    t2 =      342.9      332.7       332.7

   Panel and Stiffener Geometry                               Sa          Sb
   ----------------------------                             (stiff.)    (E.M.)
   Plain panel size (to support at Sa,Sb)        b,a =      283.00      448.00
   Effective panel size (centroids Sa,Sb)      de,he =      283.00      530.32

   Stiffener area                                 As =      125.44     2000.00
   Stiffener centroid from mid-web                xs =        9.80       10.64
   Stiffener centroid from mid-panel              ys =       11.20        0.00
   Stiffener mean thickness                       ts =        1.60        8.87
   Stiffener 2nd M. of A. about n.a.             Ixs =      20102.    1826644.
   Stiffener 2nd M. of A. about n.a.             Iys =      20102.     523831.
   Stiffener torsion constant                     Js =        104.      48373.
   Stiffener polar 2nd M. of A.                  Ies =      69163.    2662106.
   Stiffener torsion-bending constant             GG =      16598.    7561730.

   Stiffener/panel area                          Ase =      172.48     2062.16
   Stiffener/panel centroid from mid-web         xse =       12.47       11.10
   Stiffener/panel centroid from mid-panel       yse =        8.14        0.00
   Stiffener/panel 2nd M. of A. about n.a.      Ixse =      24396.    1826652.
   Stiffener/panel 2nd M. of A. about n.a.      Iyse =      26126.     537731.
   Stiffener/panel torsion constant              Jse =        127.      48403.

   Nominal applied shear stress (Q/he/t)                 tauapp =        78.4
-----------------------------------------------------------------------------
6 warnings reported:
   Material moduli not within 10%: Ep, Es = 0.7450E+050.6340E+05
   Web size not 4 < h/tw < 20: h/tw = 0.2450E+02
   Rivet pitch not 3.5D < s < 6.0D: s/d = 0.1042E+02
   Att.flange size not 2< ba/ta <10: ba/ta = 0.2450E+02
   Material moduli not within 10%: Ep, Es = 0.7450E+050.6340E+05
   Rivet pitch not 3.5D < s < 6.0D: s/d = 0.2604E+01
```

Table A2.1 APA115 output: E.S.D.U. 77014 Example, cont.

```
BAe Airbus Ltd. Filton File: APA115 Version 1.0          Page:
AIRBUS APA115  Iss: 1.0/Mar 98     Name: M.J.H. Austin      Date: 07-APR-98
Project/Standard: Development version                     Time: 12:16    3
-----------------------------------------------------------------------------
Input File Name : D:\BAE\APA115\PROGRA~1\FORTRA~1\ds77014.inp

Title: APA115 program verification
Case : ESDU 77014 example

Initial Panel Buckling                                    Units =      Nmm

   Simple Flat Panel, ESDU 71005
   -----------------------------
   Aspect ratio                                            b/a =     0.632
   Buckling coefficient (fully clamped, Fig.1)             K1c =    10.316
   Buckling coefficient (side A clamped, Fig.1)            K1a =     9.579
   Buckling coefficient (side B clamped, Fig.1)            K1b =     6.785
   Buckling coefficient (simply-supported, Fig.1)           K1 =     6.281

   Stiffened Flat Panel, ESDU 02.03.02
   -----------------------------------                  Stiff.A    Stiff.B
   Aspect ratio                                 a/b =     1.583      0.632
   Stiffness parameter                            u =     4.988     70.937
   Torsional/flexural ratio                   GJ/EI =     0.001      0.010
   Longitudinal edges clamped:
     Critical stiffness parameter (Fig.1)        uc =     0.570      0.430
     Buckling coefficient (J=0, Fig.1)          Kc0 =     7.134     11.400
     Buckling coefficient (Figs.7-10)            Kc =     7.835     12.303
   Longitudinal edges simply-supported:
     Critical stiffness parameter (Fig.2)        uc =     1.220      0.670
     Buckling coefficient (J=0, Fig.2)          Kp0 =     6.400      8.500
     Buckling coefficient (Figs.3-6)             Kp =     6.865     10.104
   Equivalent flat panel (stiffener pitch)    be,ae =    270.70     353.21
   Stiffener-only inertia about mid-plane        Ix =    35826.   1826644.
   Required inertia to prevent buckling across  Ixr =     8761.     17253.

   Flat panel buckling coefficient selected                 K2 =     7.835
   Elastic flat panel buckling stress                      qb0 =      10.5
   Plasticity reduction factor (Ref.ESDU 71005 & 83044)   eta1 =     1.000
   Plastic flat panel buckling stress                       qb =      10.5
-----------------------------------------------------------------------------
6 warnings reported:
   ESDU 020302 analysis applies only to stiffener A
   ESDU 020302 a/b=1 assumed to analyse stiffener B
   ESDU 020302 Fig.1: u limited for K, stiffener A
   ESDU 020302 Fig.2: u limited for K, stiffener A
   ESDU 020302 Figs.3-6: u limited, stiffener A
   ESDU 020302 Figs.7-10: u limited, stiffener A
```

Table A2.1 APA115 output: E.S.D.U. 77014 Example, cont.

```
BAe Airbus Ltd. Filton File: APA115 Version 1.0            Page:
AIRBUS APA115  Iss: 1.0/Mar 98     Name: M.J.H. Austin     Date: 07-APR-98
Project/Standard: Development version                      Time: 12:16   4
-----------------------------------------------------------------------------
Input File Name : D:\BAE\APA115\PROGRA~1\FORTRA~1\ds77014.inp

Title: APA115 program verification
Case : ESDU 77014 example

Flat Panel Post-Buckled Analysis                           Units =      Nmm

   Initial Parameters
   ------------------
   Edge member area ratio (factored by modulus ratio)      Af/ht =     2.675
   Stringer area ratio (factored by modulus ratio)         As/bt =     0.314
   Applied buckling ratio                               Tau/Taub =     7.471

   Post-Buckled Loading on Edge Member
   -----------------------------------
   Edge member end load, ESDU 77014 Figs.10-12 (corrected)
     Edge member end load ratio                              F/Q =     0.222
     Edge member load                                          F =    11096.
   Edge member bending moment, ESDU 77014 Fig.13
     Edge member bending moment ratio                     M/qtb2 =     0.035
     Edge member bending moment                                M =   260857.
   Edge member running load, ESDU 77014 Fig.14 (corrected)
     Edge member bending parameter                        If/b3t =     0.019
     Rivet running load parameter                           R/qt =     1.176
     Running load along rivet line                             R =    110.66

   Post-Buckled Loading on Stiffener
   ---------------------------------
   Average stiffener stress, ESDU 77014 Figs.3-5 (corrected)
     Stiffener stress ratio                                fs/qC =     0.591
     Average stiffener stress                               fsav =      46.3
     Stiffener heel stress ratio                          fsh/qC =     0.764
     Stiffener heel stress                                   fsh =      69.3
   Peak stress factor, ESDU 77014 Fig.6
     Panel aspect ratio                                      b/h =     0.534
     Stiffener peak stress factor                              C =     1.157
     Peak stiffener stress                                    fs =      53.6
   Load on edge member/stiffener joint, ESDU 77014 para 7.8.4.2
     Interpolated value, fs*As*(88 + Tau/Taub)/120           Pfs =     5349.

   Panel Analysis
   --------------
   Mid-Panel deformation, ESDU 77014 Fig.1 lower curves
     Buckling stress/0.2% proof stress                  Taub/t2p =     0.031
     Allowable stress/0.2% proof stress               Taulpd/t2p =     0.304
     Local permanent deformation stress                   Taulpd =     104.2
     Reserve factor, local perm.def.                       RFlpd =     1.993
   Mid-panel failure, ESDU 77014 Fig.1 upper curves
     Buckling stress/ultimate stress                  Taub/fallp =     0.027
     Allowable stress/ultimate stress                Taufl/fallp =     0.438
     Allowable shear stress, panel failure                 Taufl =     169.2
     Reserve factor, failure                                RFfl =     2.158
     General permanent deformation stress                 Taugpd =     150.4
     Reserve factor, general perm.def.                     RFgpd =     2.876
   Edge panel failure, ESDU 77014 Fig.2
     Applied tensile stress/ultimate stress            fpr/fallp =     0.870
     Allowable stress/ultimate stress                Tauej/fallp =     0.285
     Stress at failure near edge member                    Tauej =     109.8
     Reserve factor, panel near edge member                 RFej =     1.401
-----------------------------------------------------------------------------
```

Table A2.1 APA115 output: E.S.D.U. 77014 Example, cont.

```
BAe Airbus Ltd. Filton File: APA115 Version 1.0          Page:
AIRBUS APA115  Iss: 1.0/Mar 98     Name: M.J.H. Austin      Date: 07-APR-98
Project/Standard: Development version                    Time: 12:16    5
-----------------------------------------------------------------------------
Input File Name : D:\BAE\APA115\PROGRA~1\FORTRA~1\ds77014.inp

Title: APA115 program verification
Case : ESDU 77014 example

Flat Panel Post-Buckled Analysis, cont.                  Units =       Nmm

   Stiffener Analysis
   ------------------
   Flexural instability, ESDU 77014 Figs.3-5,6
     Effective panel width factor                          beta =     0.227
     Effective strut length factor                         le/h =     0.796
     Flexural instability stress (Secant formula)           ffi =     168.4
     Buckling ratio at flexural instability          taufi/taub =    16.829
     Shear stress at flexural instability                 taufi =     176.6
     Reserve factor for flexural instability               RFfi =     2.252
   Torsional instability, ESDU 77014 Figs.3-5,6
     Torsional instability stress                           fti =      37.3
     Buckling ratio at torsional instability         tauti/taub =     5.894
     Shear stress at torsional instability                tauti =      61.9
     Reserve factor for torsional instability              RFti =     0.789
   Local instability, ESDU 77014 Figs.3-5,6
     Local instability stress (att. flange)                 fli =      61.3
     Buckling ratio at local instability             tauli/taub =     7.040
     Shear stress at local instability                    tauli =      73.9
     Reserve factor for local instability                  RFli =     0.942
   Forced crippling, ESDU 77014 Fig.9
     Attached flange thickness/panel thickness            tso/t =     1.333
     Forced crippling stress/0.2% proof stress           fsc/t2s =     0.348
     Forced crippling stress                                fsc =     115.8
     Buckling ratio at forced crippling              tausc/taub =    12.779
     Shear stress at forced crippling                     tausc =     134.1
     Reserve factor for forced crippling                   RFsc =     1.710
   Inter-rivet buckling, ESDU 77014 Figs.3-5,6
     Inter-rivet buckling stress                           firs =     105.9
     Buckling ratio at inter-rivet buckling         tauirs/taub =    10.149
     Shear stress at inter-rivet buckling                tauirs =     106.5
     Reserve factor for inter-rivet buckling               RFirs =     1.358
-----------------------------------------------------------------------------
```

Table A2.1 APA115 output: E.S.D.U. 77014 Example, cont.

```
BAe Airbus Ltd. Filton File: APA115 Version 1.0              Page:
AIRBUS APA115  Iss: 1.0/Mar 98     Name: M.J.H. Austin       Date: 07-APR-98
Project/Standard: Development version                        Time: 12:16   6
-----------------------------------------------------------------------------
Input File Name : D:\BAE\APA115\PROGRA~1\FORTRA~1\ds77014.inp

Title: APA115 program verification
Case : ESDU 77014 example

Flat Panel Post-Buckled Analysis, cont.                    Units =      Nmm

   Edge Member Analysis
   --------------------
   Inter-rivet buckling
     Inter-rivet buckling stress                            firf =     330.0
     Buckling ratio at inter-rivet buckling          tauirf/taub =    12.935
     Shear stress at inter-rivet buckling                 tauirf =     135.8
     Reserve factor for inter-rivet buckling               RFirf =     1.731
   Critical fibre stress, position 1
     Fibre offset from neutral axis                          xf1 =     10.64
     Fibre allowable stress                               falle1 =     417.0
     Buckling ratio at allowable fibre stress         taue1/taub =    15.811
     Shear stress at fibre stress                          taue1 =     165.9
     Reserve factor for fibre allowable                     RFe1 =     2.116
   Critical fibre stress, position 2
     Fibre offset from neutral axis                          xf2 =     41.16
     Fibre allowable stress                               falle2 =     417.0
     Buckling ratio at allowable fibre stress         taue2/taub =    14.593
     Shear stress at fibre stress                          taue2 =     153.2
     Reserve factor for fibre allowable                     RFe2 =     1.953

   Attachment Analysis
   -------------------
   Stiffener rivet tension criterion
     Allowable tensile strength/unit length                 Nall =     20.00
     Required tensile strength/unit length         0.22*falls*t =    110.09
     Nominal reserve factor (ref.only)                           =  (   0.182)

   Stiffness Analysis
   ------------------
   Secant & tangent modulus reduction, ESDU 77014 Figs.15-17,18-20
     Reduction for secant modulus                           Gs/G =     0.681
     Reduction, allowing for edge member flex.             Gs/G` =     0.674
     Reduction for tangent modulus                          Gt/G =     0.576
     Reduction, allowing for edge member flex.             Gt/G` =     0.570
-----------------------------------------------------------------------------
```

Table A2.1 APA115 output: E.S.D.U. 77014 Example, cont.

APA115 Volume 3, Appendix B: ESDU SHEAR ANALYSIS

Table of Contents

B1.0 INTRODUCTION

This appendix gives a brief description of the E.S.D.U. data sheets used by APA115, together with an explanation of the derivation of some of the graphical data and the inter-relation of the various data sheets. In order to do this, reference was made to the N.A.C.A. reports TN2661 and TN2662 from which much of the theory originates. Further reference has occasionally been made to books by Kuhn (the author of the N.A.C.A. reports) and Hawker Siddeley report S.O.R.51.

The E.S.D.U. data sheets provide a regularly-updated industry standard. Some of the original data sheets, notably the R.Ae.S series, have been superseded. The empirical data on some of the later sheets has been based on the older series. Also, some of the buckling curves have been obtained by extensive iterations of energy equations. Such data is not derived analytically in APA115 but presented as graphical data within the relevant subroutines.

The graphical data used in the APA115 analysis has been directly extracted from the Fortran subroutines and used to plot the figures in this appendix. Thus, although these figures appear to be reproductions of the original E.S.D.U. figures, they serve as verification of the data in the subroutines.

The figures which have an analytical solution have been prepared on MathCAD 6+ using the same equations as the Fortran subroutines to verify the procedures.

B2.0 E.S.D.U. 71005

This data sheet analyses simple shear webs under a combination of specific edge conditions - either simply-supported or clamped. The buckling coefficients are theoretical and are represented in program APA115 as data.

Of particular note in this data sheet is the plasticity correction of E.S.D.U. 71005 figure 2. E.S.D.U. 83044 provides plasticity correction factors for a range of instability modes, including one for shear. However, E.S.D.U. 71005 does not use this but derives the curves from E.S.D.U. 76016 in the following manner. Firstly, a factor of 2 is applied to the elastic buckling stress. Secondly, at this elastic stress, the plasticity correction factor, η_1 is found. This is then applied to the original stress. It is not known why this is done, but it matches all the curves precisely.

The curves of buckling coefficient are given in Fig.B1. These are taken from the subroutine data. The plasticity correction factor is derived analytically within the subroutine as detailed above. Fig.B2 shows the results of these calculations.

B3.0 E.S.D.U. 020302

This data sheet analyses long shear panels with transverse stiffeners. The edge members may be considered as simply-supported or clamped and the stiffeners may have torsional stiffness, providing a partly-clamped contribution to the panel stiffness.

The derivation of these coefficients is based on some very lengthy and obscure calculations performed by, amongst others, Cook and Rockey. It was not possible to determine if this data sheet is an accurate representation of this work. The buckling coefficients are thus represented in the program as data and reproduced in Figs.B3-B6.

The criterion of buckling across stiffeners is provided by this data sheet. A contribution from the stiffeners is included in the panel stiffness and this provides a range of buckling modes. These are reflected in discontinuities in the curves. The onset of buckling coefficient is given by the μ_c curve such that simple support is provided above and to the right of this curve and buckling occurs to the left and bottom, where the curves for constant aspect ratio are presented.

The curves may be related to E.S.D.U. 71005 at specific locations (see Figs.B7-B9). Fig.B7 identifies points A-H on the E.S.D.U. 71005 curves at aspect ratios, b/a = 0, 1/2, 1/1.5 and 1 for simply-supported and clamped panels. These same points are located on Fig.B8 and Fig.B9 on the E.S.D.U. 02.03.02 curves. Noting that the aspect ratio used for E.S.D.U. 02.03.02 is presented as a/b, the lower points on the μ = 0 axis can be factored down from the a/b = 1 value by $(a/b)^2$.

B4.0 E.S.D.U. 02.03.18 & 02.03.19

These data sheets simply provide buckling coefficients for curved panels. The first data sheet is for panels longer axially than circumferentially and the second is for panels longer circumferentially. The curves are taken from N.A.C.A. TN 2661 (figures 30a,b of that reference) and are shown here in Fig.B10 and Fig.B11. In the APA115 analysis, these curves have been combined in the same subroutine for use in the program. They may be plotted on the same graph by multiplying the x axis of E.S.D.U. 02.03.19 by b/a.

B5.0 E.S.D.U. 77014

This data sheet performs a post-buckled analysis on shear panels. It refers to other data sheets for initial panel buckling (E.S.D.U. 71005 and E.S.D.U. 02.03.02 for example) and the given example checks some stiffener criteria with other data sheets.

The figures in this data sheet have been based largely on equations from the N.A.C.A. reports (TN2661, TN2662). There have been several discrepancies found in some of these figures and E.S.D.U. were notified of these in April 1990, but have not responded. When prompted a couple of years later, apparently the person who compiled this data sheet had left and the original files had been lost.

Most of the figures in the data sheet may be derived analytically. However, one parameter required to derive some of the figures is not included in the data sheet. It is the stress concentration factor, C_2, from N.A.C.A. TN2661 figure 18. This is reproduced as Fig.B12 in this appendix from data in the Fortran subroutine.

A parameter required by most of the analysis is the coefficient of diagonal tension. This is related to the buckling ratio of the panel as follows (N.A.C.A. TN2661, equation 27, p18):

$$k = \tanh\left[0.5 \cdot \log_{10}\left(\frac{q}{q_b}\right)\right] \tag{1}$$

The angle of diagonal tension, α, is required for many of the calculations. This is obtained as for the N.A.C.A. method, but the notation differs: A_s/bt replaces A_{Ue}/dt and μ = 0.3. The following expression (from N.A.C.A. TN2661, equations 30a-30d, pp19-20) is solved by iteration for α:

$$\tan^2\alpha = \left[\frac{\dfrac{2\cdot k}{\sin(2\cdot\alpha)} + (1-k)\cdot(1+\mu)\cdot\sin(2\cdot\alpha) + \dfrac{k\cdot\cot\alpha}{2\cdot\dfrac{A_f}{h\cdot t} + 0.5\cdot(1-k)}}{\dfrac{2\cdot k}{\sin(2\cdot\alpha)} + (1-k)\cdot(1+\mu)\cdot\sin(2\cdot\alpha) + \dfrac{k\cdot\tan\alpha}{\dfrac{A_s}{b\cdot t} + 0.5\cdot(1-k)}}\right] \tag{2}$$

B5.1 Panel Failure and Permanent Deformation

E.S.D.U. 77014 figure 1 gives criteria for failure and permanent deformation in the centre of the panel, i.e. away from the effect of the edge members. It is understood from E.S.D.U. that this figure was derived originally from some old data and the documentation is no longer available. Each curve had no more than 3 points. This figure is stored as data in the relevant subroutine.

For values of A_s/bt less than 0.3, a linear interpolation performed to the $q_a = q_b$ line where A_s/bt is assumed to be zero, is thought too be too conservative. It is difficult to come up with a realistic interpolation when the parameters on the curves are 0.3, 1.0 and infinity! Therefore a linear interpolation for bt/A_s is performed. The parameters for bt/A_s are 3.33, 1.0 and 0.

This figure is reproduced in Fig.B13 from data in the Fortran subroutine.

B5.2 Allowable Shear Stress near Flange

The interaction of shear and direct stress in E.S.D.U. 77014 figure 2, representing a panel criterion near the edge member, is derived from the following:

$$\left(\frac{f_{pr}}{f_{all}}\right)^2 + 3\cdot\left(\frac{q_a}{f_{all}}\right)^2 = 1 \tag{3}$$

such that:

$$\text{if } \frac{f_{pr}}{f_{all}} > 0.5,\quad \frac{q_a}{f_{all}} = \sqrt{\frac{1}{3}\cdot\left[1-\left(\frac{f_{pr}}{f_{all}}\right)^2\right]} \qquad \text{if } \frac{f_{pr}}{f_{all}} \le 0.5,\quad \frac{q_a}{f_{all}} = 0.5 \tag{4}$$

These expressions are plotted in Fig.B14, reproducing the E.S.D.U. figure. In these expressions, f_{pr} is the total stress in the flange which includes that due to diagonal tension and thus a function of q_a. Therefore, this figure requires an iterative solution. In the E.S.D.U. example, this figure is read using the value for q_a directly and is in error unless $f_{pr}/f_{all} < 0.5$.

B5.3 Average Stiffener Stress

E.S.D.U. 77014 figures 3-5 should be obtained by the following expression (N.A.C.A. TN2661, equation 30a, p19). However, it appears that the data sheet is in error, over-estimating f_s by between 8.6% and 12.8%. This is erring on the conservative side. The N.A.C.A. expression has been used by APA115 and the results are given in Fig.B15, which is a representation of what the E.S.D.U. figures should look like.

$$\frac{f_s}{q\cdot C} = \frac{k\cdot\tan\alpha}{\frac{A_s}{b\cdot t} + 0.5\cdot(1-k)} \tag{5}$$

B5.4 Peak Stiffener Stress

There is an expression which can be used to derive the peak stiffener stress factor of E.S.D.U. 77014 figure 6. This has been obtained from B.A.C. S.D.D. 35.1.10.

$$C = 1 + 0.57\cdot(1-k)\cdot\left(1-\frac{b}{h}\right) \tag{6}$$

However, a similar expression is to be found from N.A.C.A. TN2661, where its own figure 15 may expressed as follows:

$$C = 1 + 0.775\cdot(1-k)\cdot\left(1-\frac{b}{1.2\cdot h}\right) \tag{7}$$

These two expressions are compared in Fig.B16. The E.S.D.U. factor is always less than the N.A.C.A. factor, by up to 11.5%, making it the more optimistic of the two.

B5.5 Effective Plate Width

The effective plate width factor, β, in E.S.D.U. figure 7 is the same as N.A.C.A. TN2661 (equation 29, p19) and the same as the curve in B.A.C. S.D.D. 35.2.12. It is shown in Fig.B17 and is defined as follows:

$$\beta = 0.5 \cdot (1 - k) \tag{8}$$

B5.6 Effective Stiffener Strut Length

The effective stiffener strut length, given in E.S.D.U. figure 8, is taken from N.A.C.A. TN2661 equation 35. However, N.A.C.A. say that this is valid for double stiffeners only and for single stiffeners, l/h = 0.5 should be used. E.S.D.U. uses this curve for single and double stiffeners with eccentric loading for data sheet 01.01.01. N.A.C.A., S.O.R. 51 and Kuhn use 0.5·h and no eccentricity. The E.S.D.U. figure is reproduced in Fig.B18 using the following:

$$\text{if } \frac{b}{h} < 1.5, \quad \frac{l}{h} = \frac{1}{\sqrt{1 + k^2 \cdot \left(3 - 2 \cdot \frac{b}{h}\right)}}, \qquad \text{if } \frac{b}{h} \geq 1.5, \quad \frac{l}{h} = 1 \tag{9}$$

B5.7 Stiffener Crippling Stress

For stiffener crippling, N.A.C.A. TN2661 gives equations 37a, 37b for materials 24S-T3 and 75S-T6. Noting the value of t_2 for these materials, these equations are seen to be equivalent to the E.S.D.U. figure. However, N.A.C.A. say that this is for single stiffeners only - double stiffeners are 0.8 of this value (N.A.C.A. equations 36a, 36b). The E.S.D.U. figure is reproduced in Fig.B19 using the following:

$$\frac{f_{SC}}{t_{2S}} = 0.5 \cdot k^{\frac{2}{3}} \cdot \left(\frac{t_{SO}}{t}\right)^{\frac{1}{3}} \tag{10}$$

B5.8 Flange End Loads

E.S.D.U. figures 10-11 should be obtained by the following expression (N.A.C.A. TN2661, equation 30b, p20). However, similar to figures 3-5, it appears that the data sheet is in error, over-estimating F by between 0.7% and 11% for A_f/ht = 1 or above. More significantly, E.S.D.U. figure 12 (for A_f/ht = 0) should not exist at all because the expression for F/Q becomes zero. The N.A.C.A. expression has been used by APA115 and the results are given in Fig.B20, which is a representation of what the E.S.D.U. figures should look like.

$$\frac{F}{Q}=\frac{\frac{A_f}{h\cdot t}\cdot k\cdot\cot\alpha}{2\cdot\frac{A_f}{h\cdot t}+0.5\cdot(1-k)} \qquad (11)$$

B5.9 Flange Bending Parameter

The flange bending parameter in E.S.D.U. figure 13 is the same as that given in S.O.R. 51 figure 14b and reproduced in Fig.B21 using the following:

$$\frac{M}{q\cdot t\cdot b^2}=0.054\cdot\sqrt{k} \qquad (12)$$

B5.10 Flange Joint Load/Unit Length

The flange joint load per unit length in E.S.D.U. figure 14 should match N.A.C.A. TN2661 equations 33a and 34 combined:

$$\frac{R}{q\cdot t}=(1+k\cdot C_2)\cdot(1+0.414\cdot k) \qquad (13)$$

where C_2 is obtained from Fig.B12 using the flange flexibility factor, ω_d, as follows:

$$\omega_d=1.25\cdot 0.7\cdot\sqrt[4]{\frac{I_f}{b^3\cdot t}} \qquad (14)$$

In the above expression, 1.25 is the Wagner factor from N.A.C.A. TN2661 equation 19a. The results of this calculation are given in Fig.B22. These deviate slightly from E.S.D.U. figure 14 in that the asymptotes are different by a small amount.

B5.11 Plate Shear Stiffness

The plate shear stiffness, presented in E.S.D.U. 77014 figures 15-20 in the form of secant moduli and tangent moduli, may be obtained from Kuhn pp56-57. The equations are as follows where G_{IDT}/G is equivalent to the E.S.D.U. G_s/G:

$$\frac{E}{G_{DT}} = \frac{4}{\sin^4\alpha} + \frac{\tan^2\alpha}{\frac{A_s}{b\cdot t} + 0.5\cdot(1-k)} + \frac{\cot^2\alpha}{2\cdot\frac{A_f}{h\cdot t} + 0.5\cdot(1-k)}$$

$$G_{DT} = \frac{2\cdot(1+\mu)}{\frac{E}{G_{DT}}} \qquad (15)$$

$$\frac{G_{IDT}}{G} = \frac{1}{1 + k\cdot\left(\frac{1}{G_{DT}} - 1\right)}$$

These are given in Fig.B23. The tangent moduli may be obtained numerically by calculating the change in secant modulus over a small range of buckling ratio:

$$\left(\frac{G_T}{G}\right)_i = \frac{(G_s/G)_{i+1}\cdot(q/q_b)_{i+1} - (G_s/G)_i\cdot(q/q_b)_i}{(q/q_b)_{i+1} - (q/q_b)_i} \qquad (16)$$

Fig.B24 reproduces E.S.D.U. 77014 figures 18-20 using this expression. There is, however, some slight deviation. APA115 uses these expressions, but they do not form part of the strength analysis.

B6.0 E.S.D.U. 77018

This data sheet performs a post-buckled analysis on curved shear panels. It refers to other data sheets for initial panel buckling (02.03.18, 02.03.19 for example) with the restriction that the panel has a well-developed tension field which is contained between stiffeners. Also, the straight sides of the panel should be at least twice as long as the curved sides.

Most of the figures in this data sheet have been based on equations from the N.A.C.A. reports (TN2661, TN2662). These may thus be derived analytically. A further figure has been fitted mathematically to simplify and quicken the analysis.

This analysis is not so comprehensive as the flat panel analysis. No allowances are made for continuous panels with transverse stiffeners or for the analysis of the stiffeners, or edge members, themselves. The maximum panel stress is a theoretical value at mid-thickness.

B6.1 Diagonal Tension Factor

This is a developed form of the flat panel diagonal tension factor. It is simplified from N.A.C.A. TN2661, equation 50, p69 and is restricted to d/h > 2:

$$k = \tanh\left[\left(0.5 + 600 \cdot \frac{t}{R}\right) \cdot \log_{10}\left(\frac{q}{q_b}\right)\right] \qquad (17)$$

This expression provides Fig.B25, representing E.S.D.U. 77018 figure 1. The full N.A.C.A. expression allows for any d/h:

$$k = \tanh\left[\left(0.5 + 300 \cdot \frac{t}{R} \cdot \frac{d}{h}\right) \cdot \log_{10}\left(\frac{q}{q_b}\right)\right] \qquad (18)$$

where d/h = 2 if d/h > 2

B6.2 Angle of Diagonal Tension

The angle of diagonal tension, α, is given in E.S.D.U. 77018 figures 2-6. This is obtained as in the N.A.C.A. method, but the notation differs: A_s/bt replaces A_f/ht , A_f/at replaces A_s/bt and μ = 0.3. The following expression (derived from N.A.C.A. TN2661, equation 44, p64) is solved by iteration for α:

$$\tan^2\alpha = \left[\frac{\dfrac{2 \cdot k}{\sin(2 \cdot \alpha)} + (1-k) \cdot (1+\mu) \cdot \sin(2 \cdot \alpha) + \dfrac{k \cdot \cot\alpha}{\dfrac{A_s}{b \cdot t} + 0.5 \cdot (1-k)}}{\dfrac{2 \cdot k}{\sin(2 \cdot \alpha)} + (1-k) \cdot (1+\mu) \cdot \sin(2 \cdot \alpha) + \dfrac{k \cdot \tan\alpha}{\dfrac{A_f}{a \cdot t} + 0.5 \cdot (1-k)} + \dfrac{1}{24} \cdot \dfrac{E}{q} \cdot \left(\dfrac{b}{R}\right)^2}\right] \qquad (19)$$

Note that the 2 is dropped from A_s/bt. This is because the stringer is shared between two panels. The E.S.D.U. figures are matched exactly by the solution of expression. They are reproduced in Figs.B26-B30.

B6.3 Inward Radial Load on Stiffener

E.S.D.U. 77018 figure 7 provides for calculation on the inward radial load acting on the stiffener. The derivation of this figure is not known. However, an expression to represent it was found which resulted in accuracy of between -0.6% and 1%. The expression used is as follows:

$$\frac{W_s \cdot R}{k \cdot q \cdot t \cdot b \cdot a} = \tan\alpha - 10^{-0.813\log_{10}\left(\frac{a}{b}\right) - 0.390} \qquad (20)$$

Fig.B31 shows the the reproduction of the E.S.D.U. figure using the above expression.

B6.4 Maximum Tensile Stress in Plate

E.S.D.U. figure 8 gives the maximum tensile stress in the plate and figure 9 gives its angle. To obtain these figures, the stress distribution due to simple shear (proportional to 1-k) is combined with that from the distribution due to diagonal tension (proportional to k). The stresses are resolved along the diagonal (σ_x):

$$\begin{aligned} \sigma_x &= (1-k)\cdot \sin(2\cdot\alpha) + \frac{2\cdot k}{\sin(2\alpha)} \\ \sigma_y &= (1-k)\cdot \sin(2\cdot\alpha) \\ \tau_{xy} &= (1-k)\cdot \sin\left(\frac{\pi}{2} - 2\cdot\alpha\right) \end{aligned} \qquad (21)$$

From these stresses the maximum principle stress and its angle, θ, are found:

$$\begin{aligned} \sigma_1 &= \tfrac{1}{2}\cdot(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\cdot\tau_{xy}^{\;2}} \\ \vartheta &= \frac{\pi}{4} - \tfrac{1}{2}\cdot\tan^{-1}\left[\frac{2\cdot\tau_{xy}}{(\sigma_x - \sigma_y)}\right] \end{aligned} \qquad (22)$$

These two expressions produce Fig.B32 and Fig.B33, reproductions of the E.S.D.U. 77018 figures.

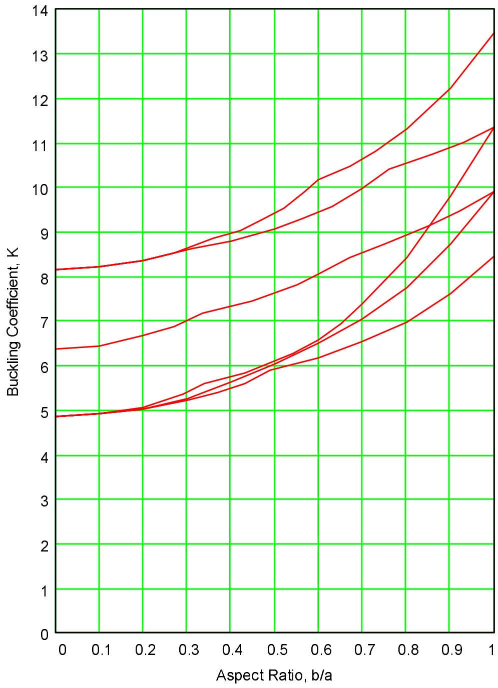


Fig.B1: ESDU 71005 Figure 1 - Buckling Coefficients for a Single Panel

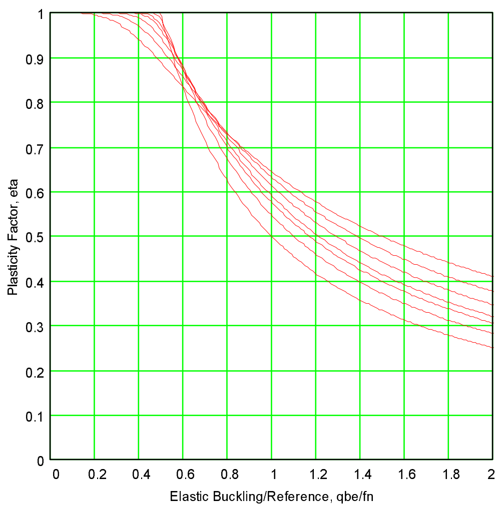


Fig.B2: ESDU 71005 Figure 2 - Plasticity Reduction Factors

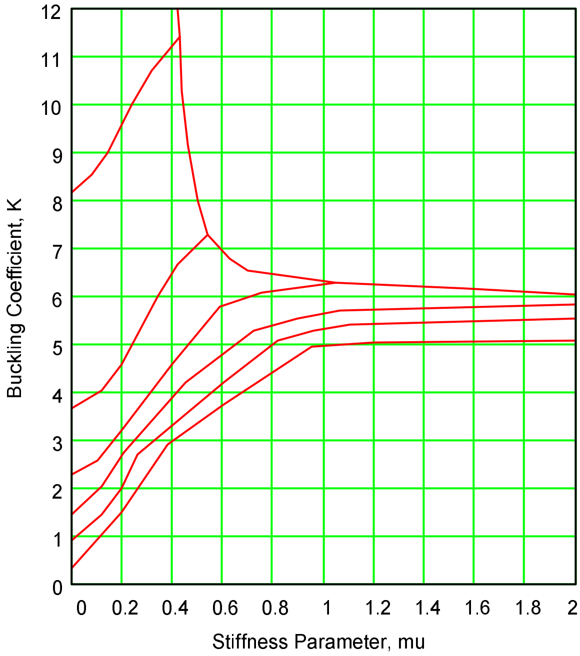


Fig.B3: ESDU 02.03.02 Figure 1 - Buckling Coefficients Panels with Transverse Stiffeners (longitudinal edges clamped, no torsional stiffness, J = 0)

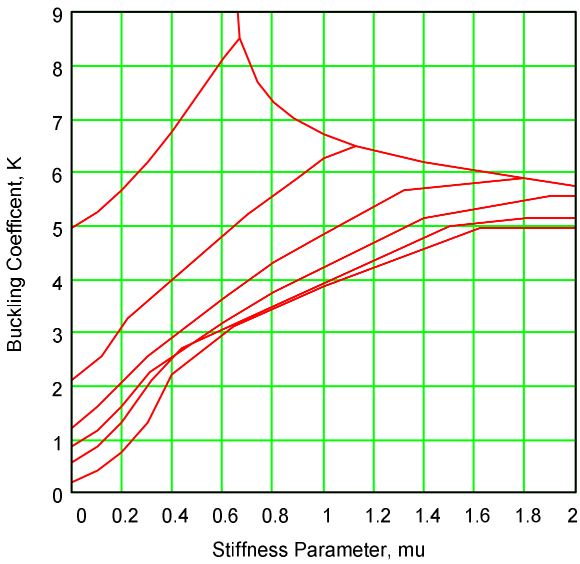


Fig.B4: ESDU 02.03.02 Figure 2 - Buckling Coefficient Panels with Transverse Stiffeners (longitudinal edges simply-supported, no torsional stiffness, J = 0)

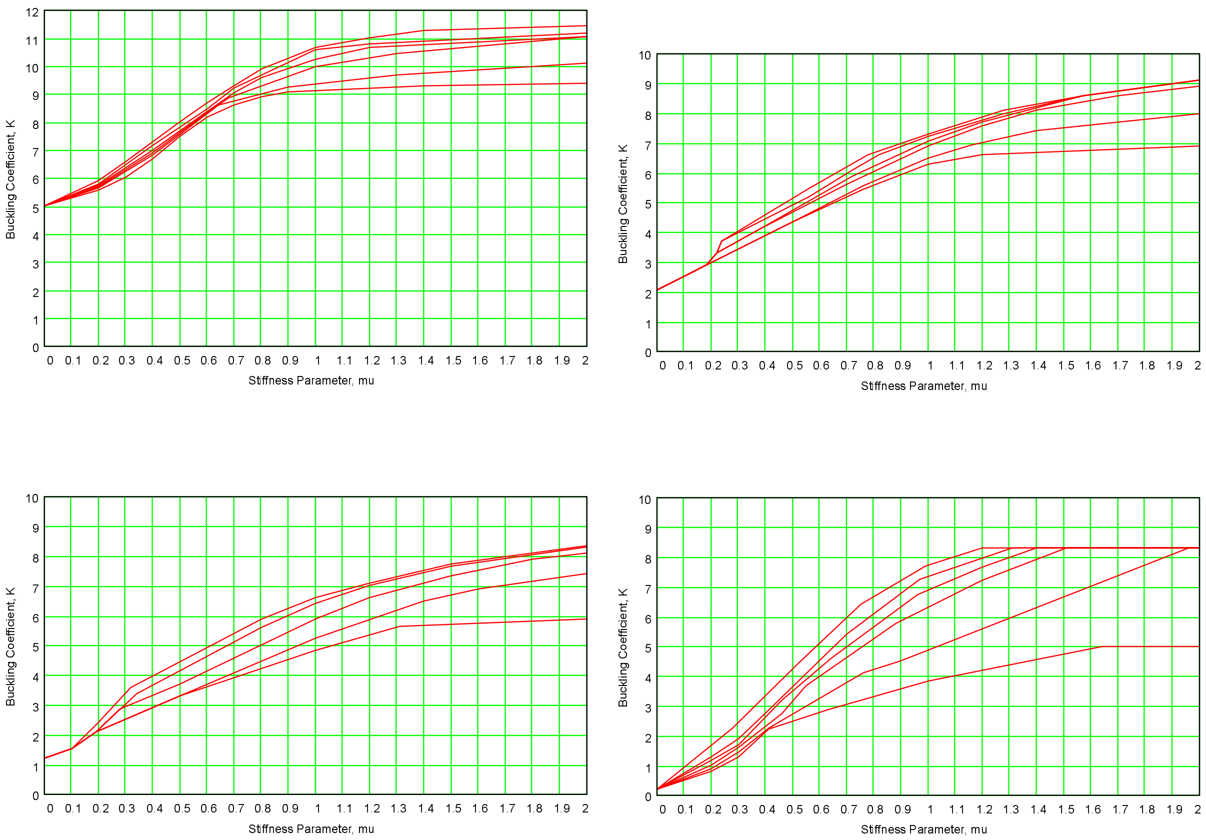


Fig.B5: ESDU 02.03.02 Figures 3-6 - Buckling Coefficient Panels with Transverse Stiffeners (longitudinal edges simply-supported, a/b = 1.0, 1.5, 2.0, 5.0 respectively)

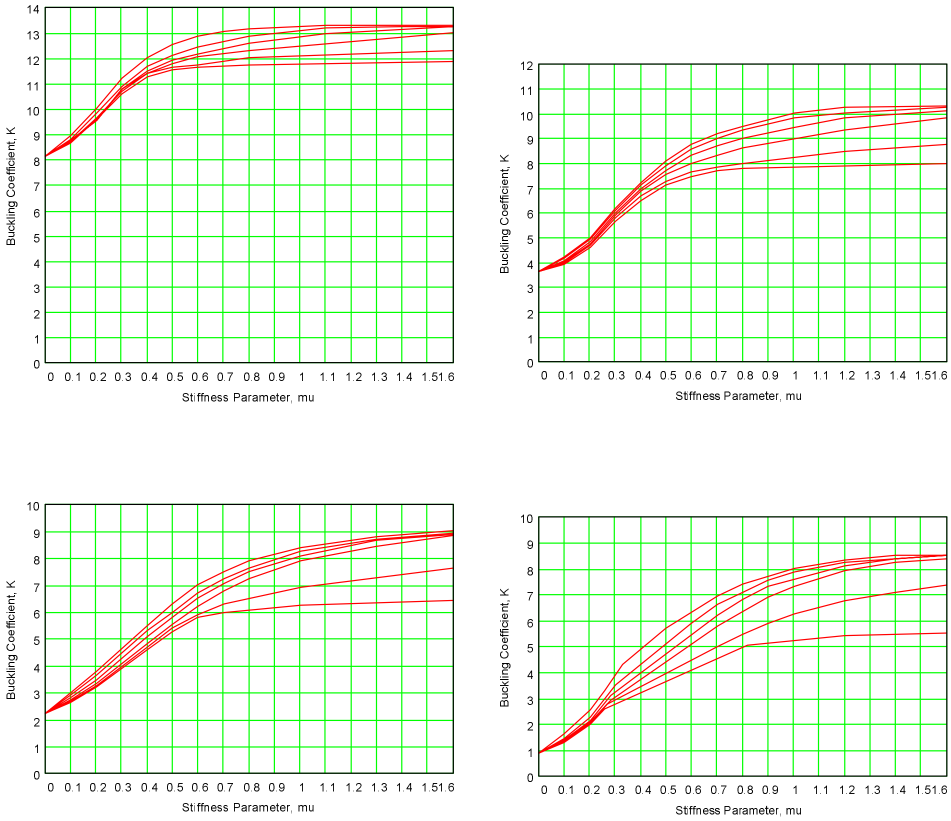


Fig.B6: ESDU 02.03.02 Figures 7-10 - Buckling Coefficient Panels with Transverse Stiffeners (longitudinal edges clamped, a/b = 1.0, 1.5, 2.0, 3.0 respectively)

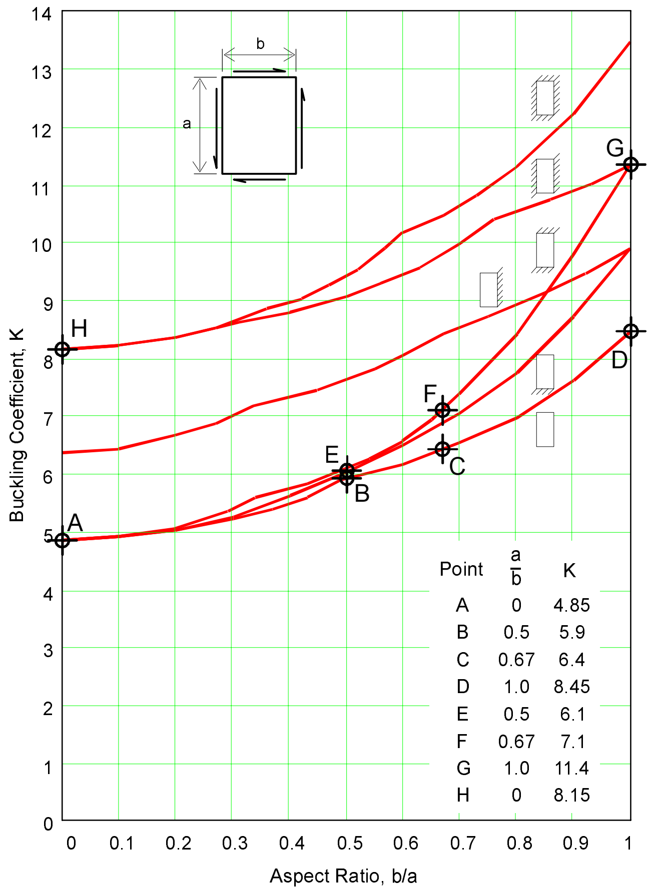


Fig.B7: ESDU 71005 Figure 1 - Comparison with ESDU 02.03.02

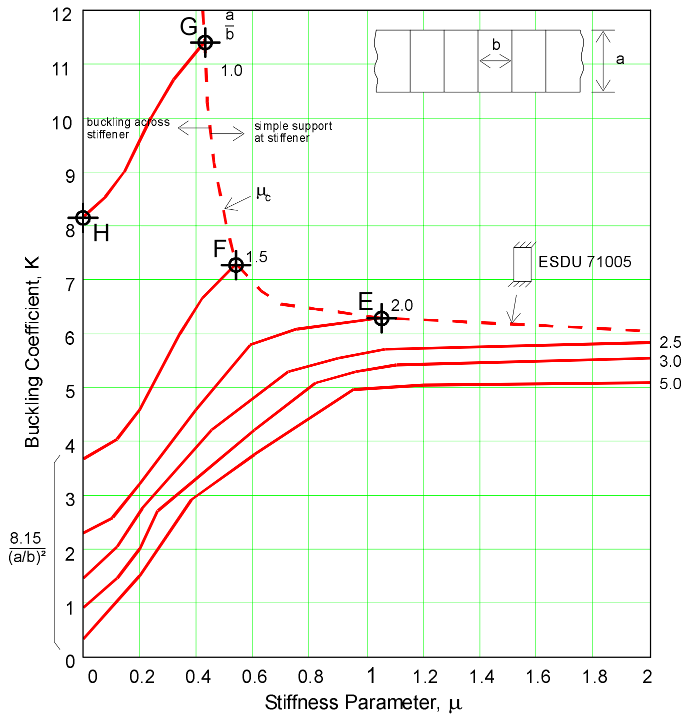


Fig.B8: ESDU 02.03.02 Figure 1 - Comparison with ESDU 71005

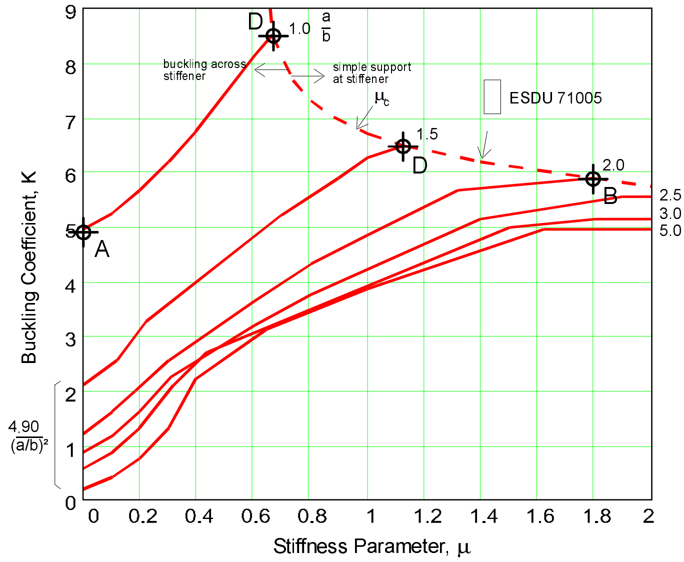


Fig.B9: ESDU 02.03.02 Figure 2 - Comparison with ESDU 71005

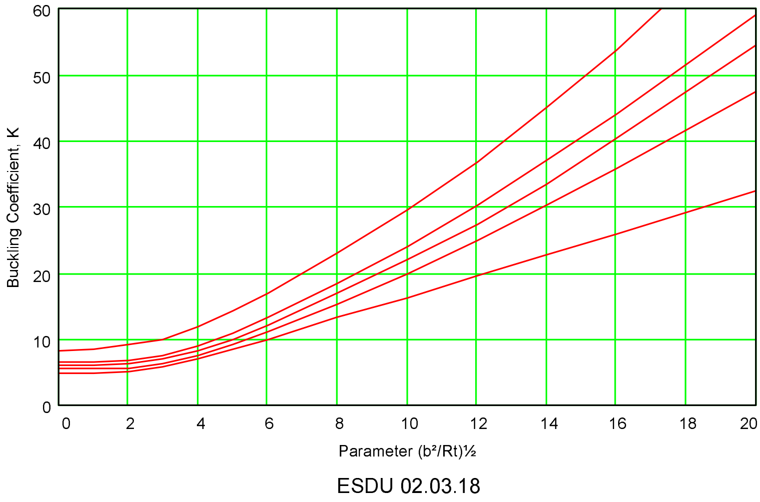


Fig.B10: ESDU 02.03.18 - Buckling Coefficients for Long Curved Panels

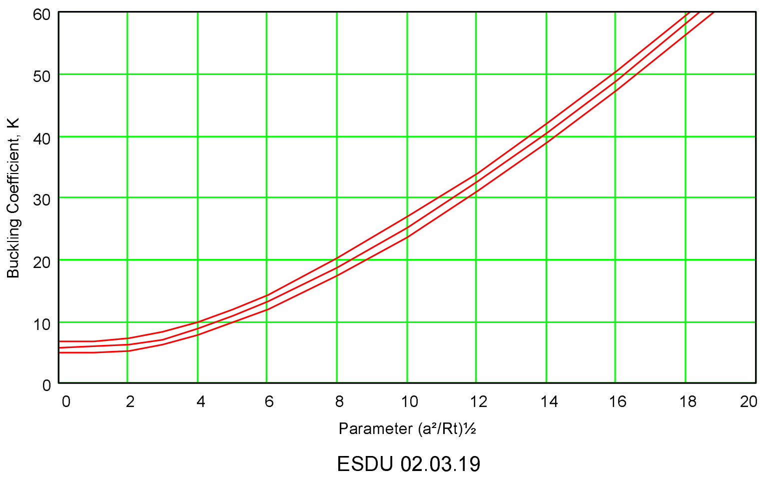


Fig.B11: ESDU 02.03.19 - Buckling Coefficients for Short Curved Panels

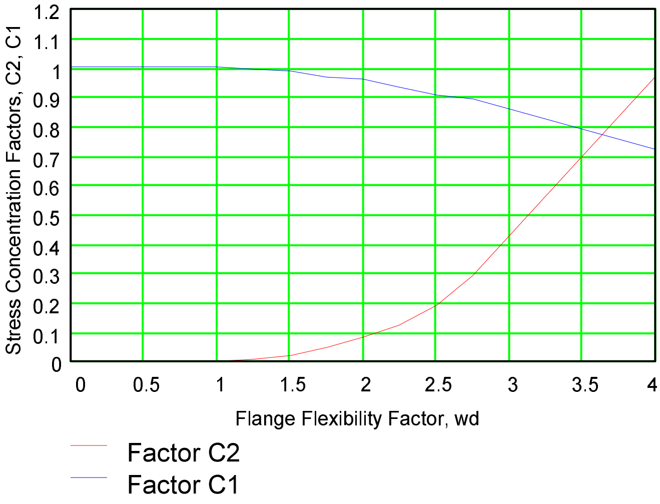


Fig.B12: NACA TN2661 Figure 18 - Stress Concentration Factors

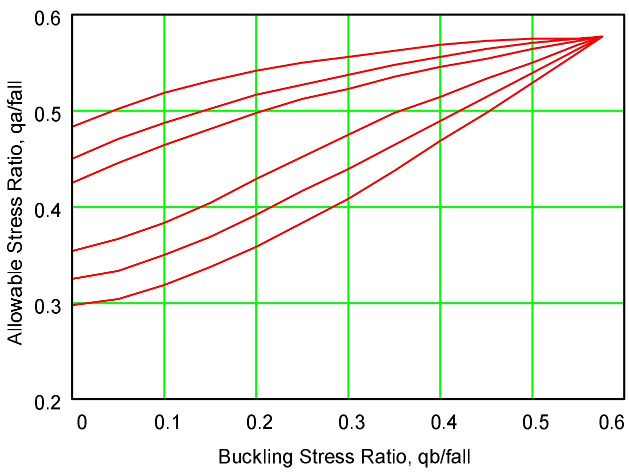


Fig.B13: ESDU 77014 Figure 1 - Failure and Permanent Deformation at Panel Centre

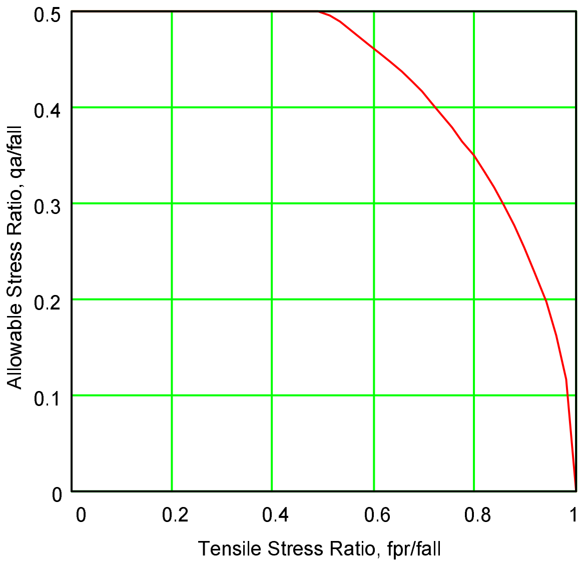


Fig.B14: ESDU 77014 Figure 2 - Allowable Stress near Flange

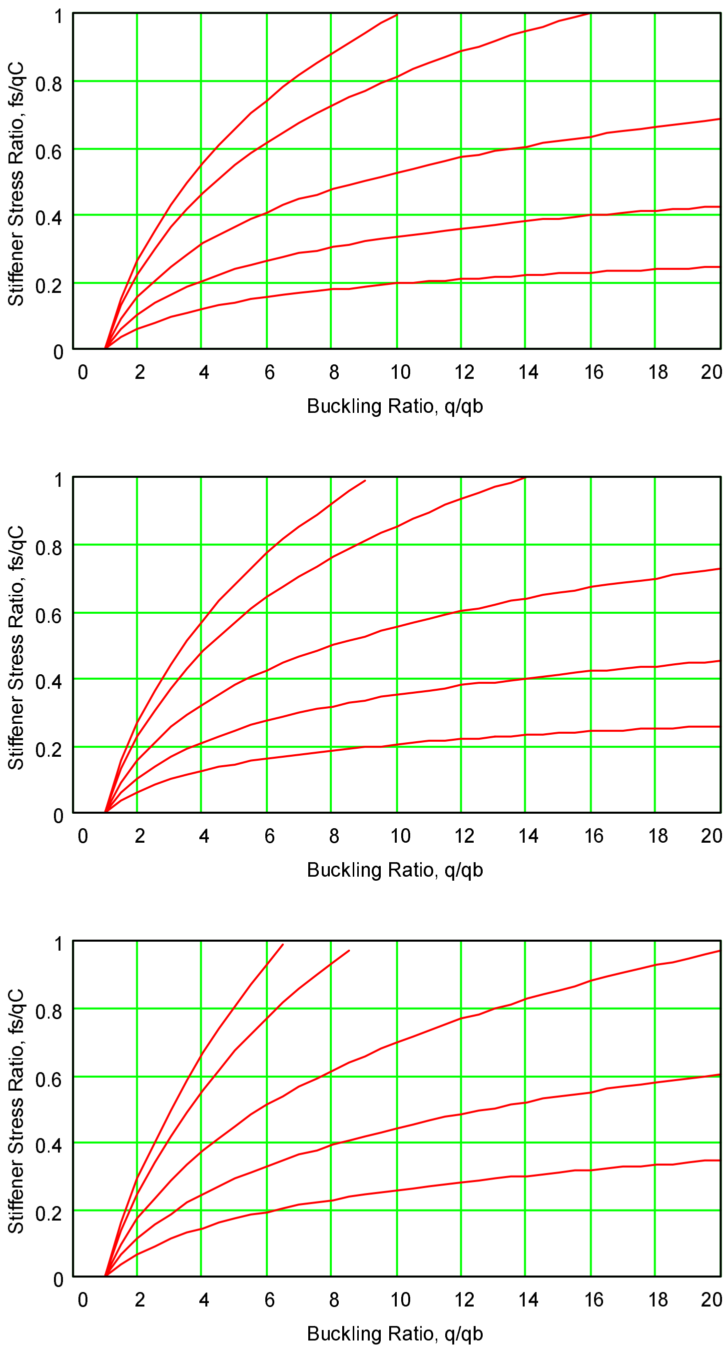
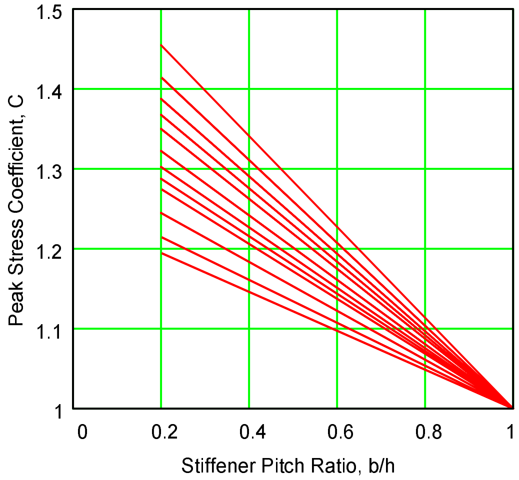


Fig.B15: ESDU 77014 Figures 3-5 - Average Stiffener Stresses
(A_f/ht >= 10, A_f/ht = 1, A_f/ht = 0)



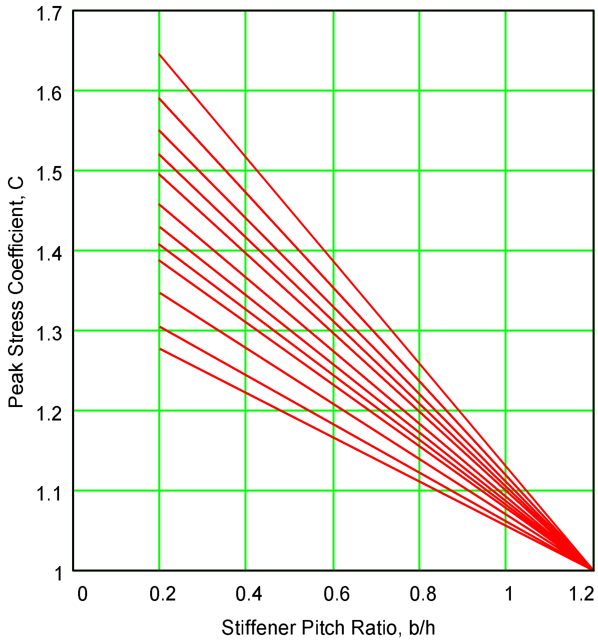


Fig.B16: ESDU 77014 Figure 6 - Peak Stiffener Stress
(compared with NACA TN2661 figure 15, bottom)

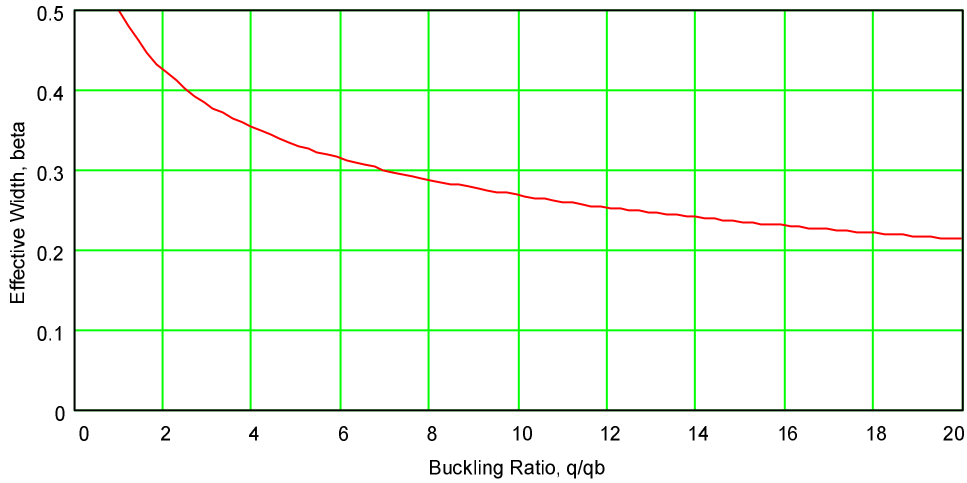


Fig.B17: ESDU 77014 Figure 7 - Effective Plate Width

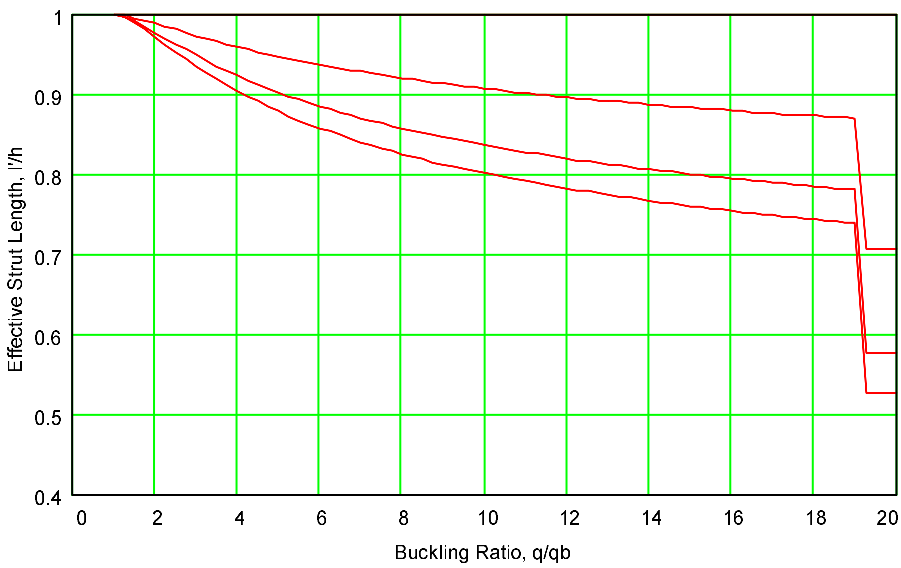


Fig.B18: ESDU 77014 Figure 8 - Effective Stiffener Strut Length

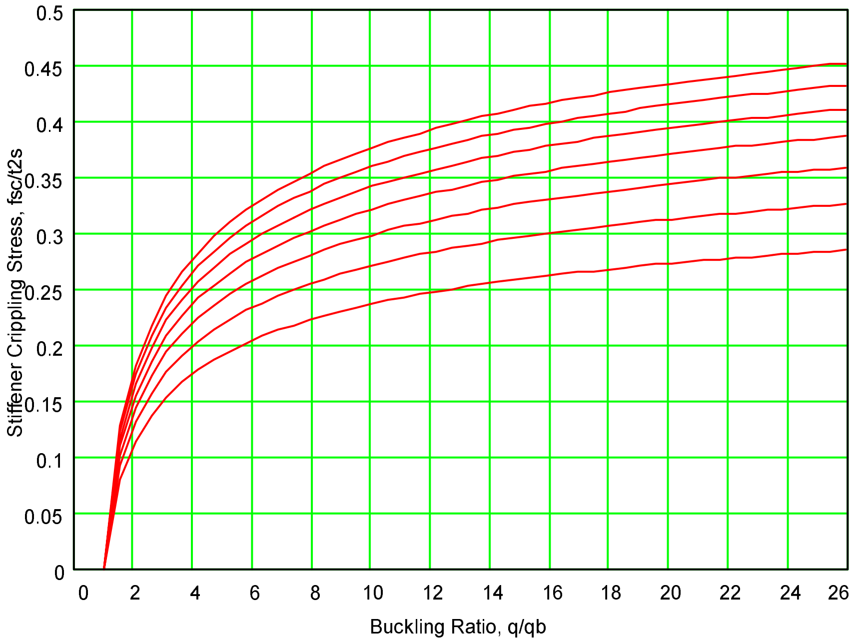


Fig.B19: ESDU 77014 Figure 9 - Stiffener Crippling Stress

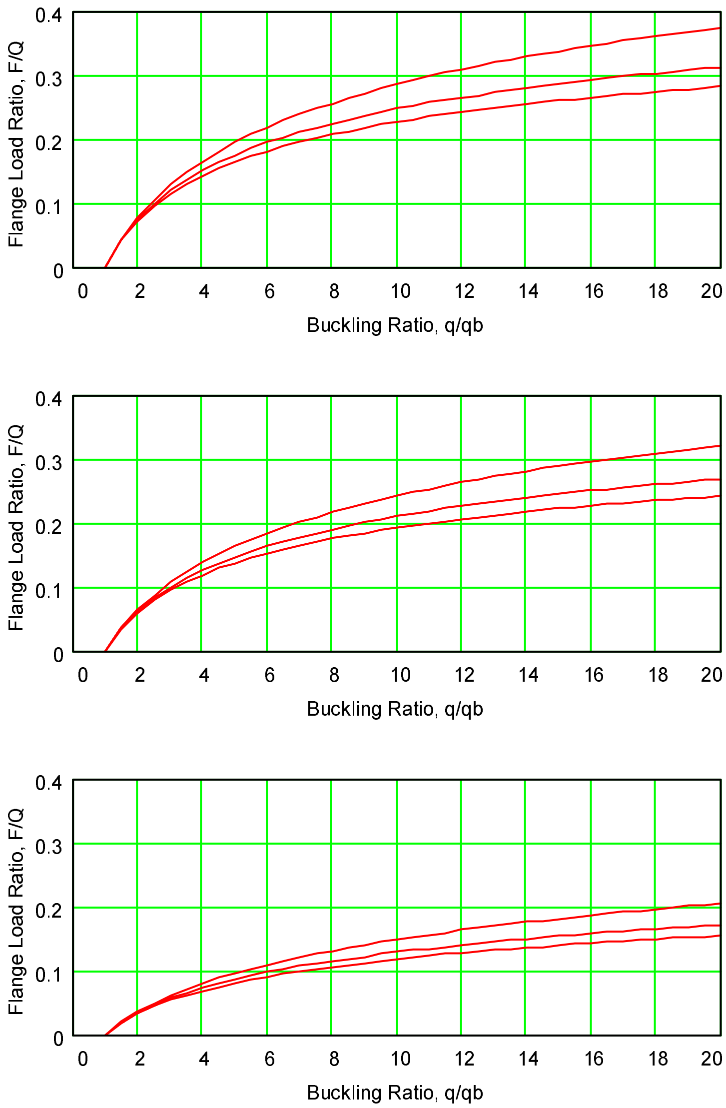


Fig.B20: ESDU 77014 Figures 10-12 - Flange End Load
(A_f/ht >=10, A_f/ht =1, A_f/ht =0.2)

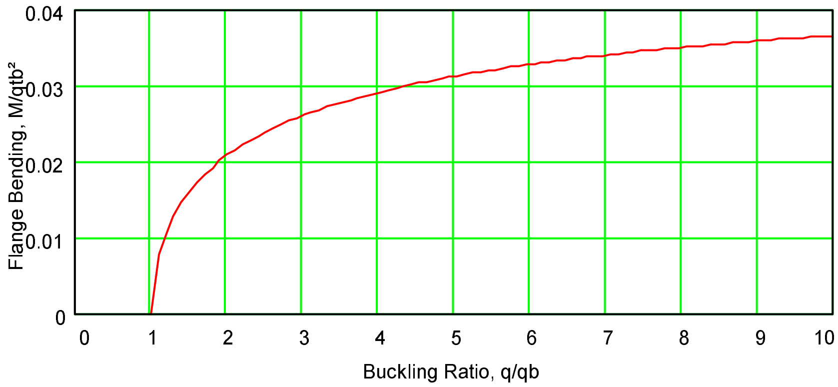


Fig.B21: ESDU 77014 Figure 13 - Flange Bending Parameter

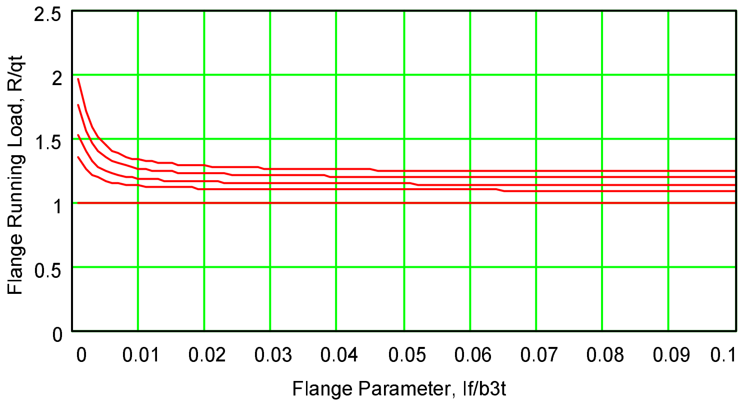


Fig.B22: ESDU 77014 Figure 14 - Flange Joint Load / Unit Length

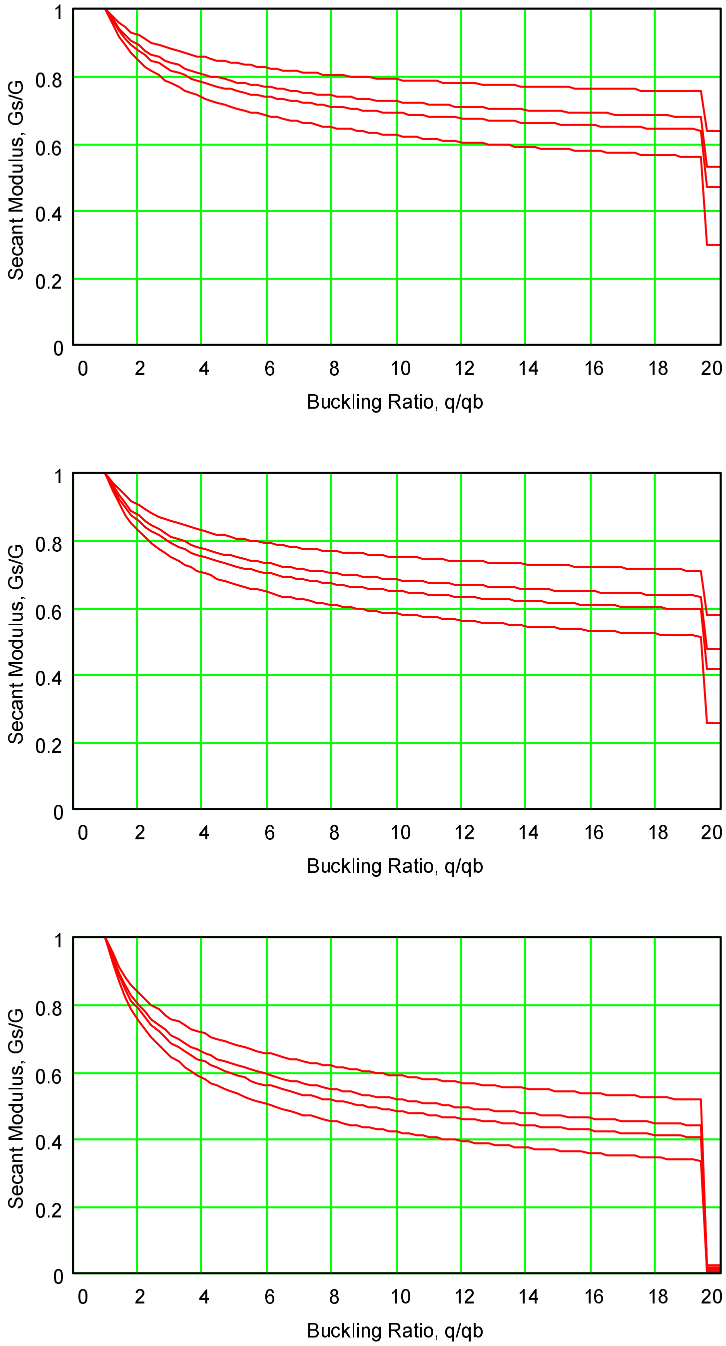


Fig.B23: ESDU 77014 Figures 15-17 - Effective Panel Secant Modulus
(A_f/ht >=10, A_f/ht =1, A_f/ht =0)

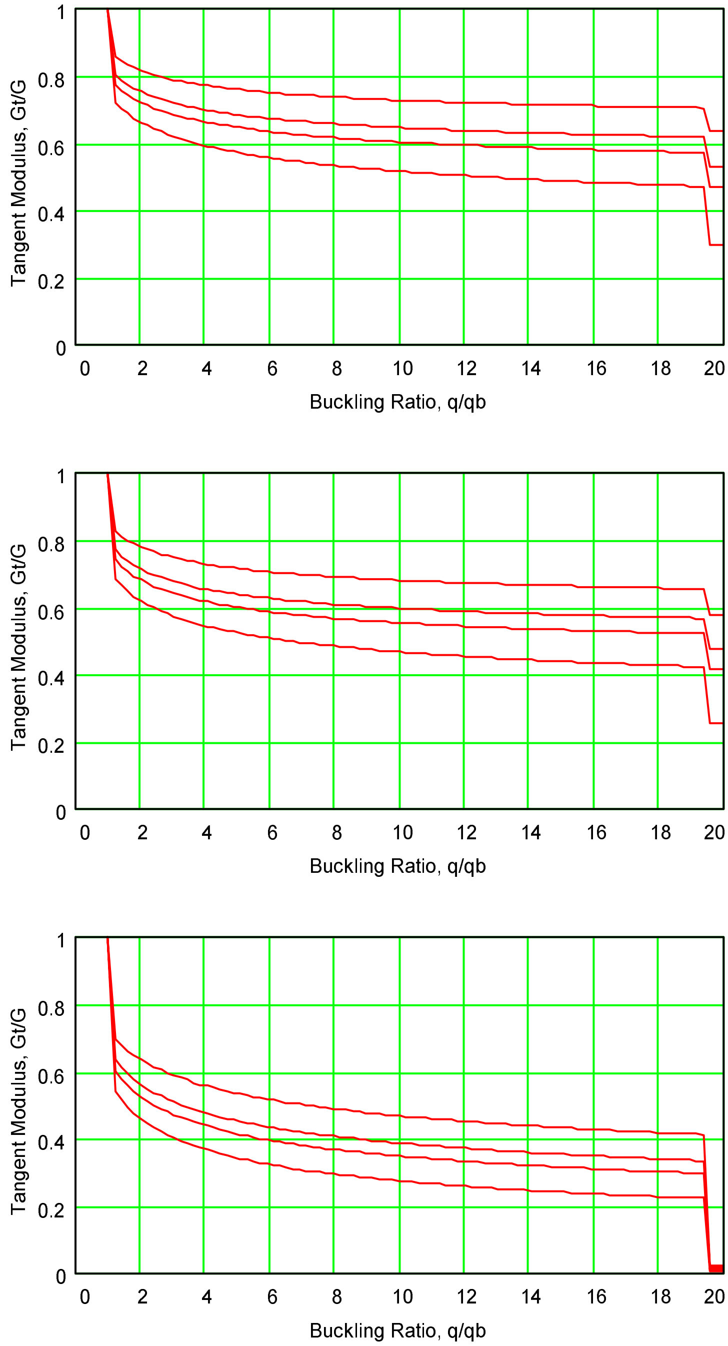


Fig.B24: ESDU 77014 Figures 18-20 - Effective Panel Tangent Modulus
(A_f/ht >=10, A_f/ht =1, A_f/ht =0)

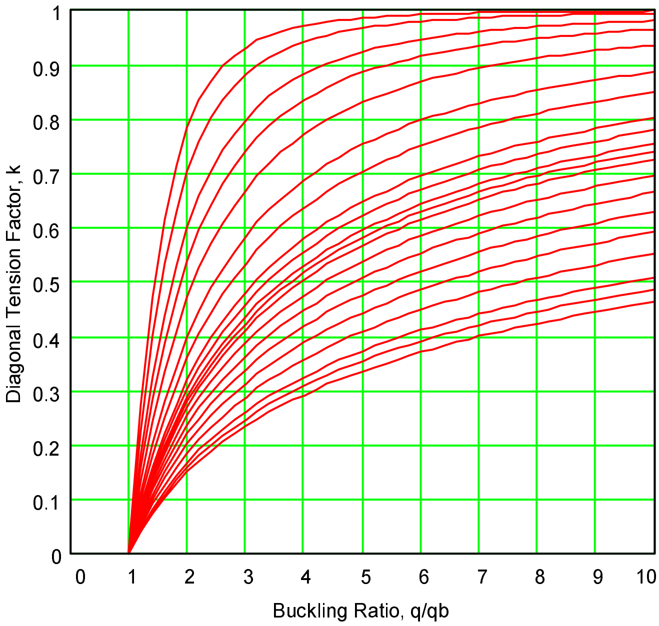


Fig.B25: ESDU 77018 Figure 1 - Diagonal Tension Coefficient

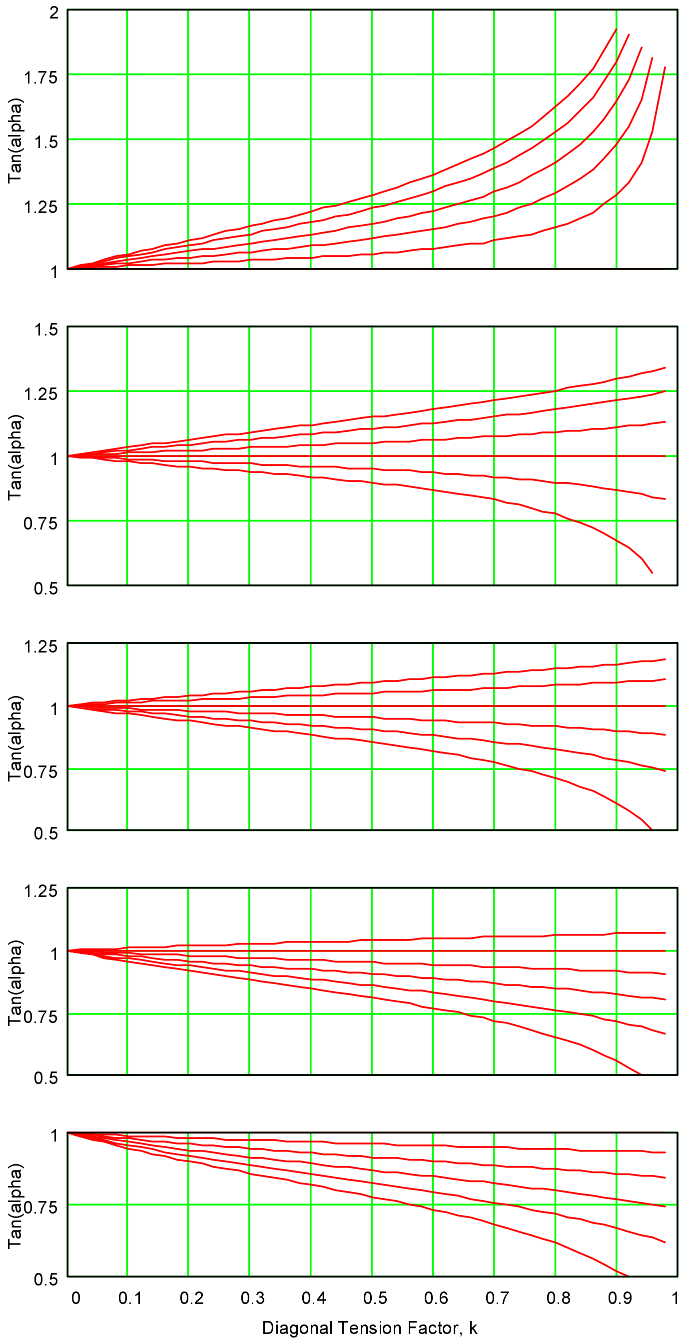


Fig.B26: ESDU 77018 Figure 2 - Angle of Diagonal Tension, $(E/q)(b/R)^2=0$

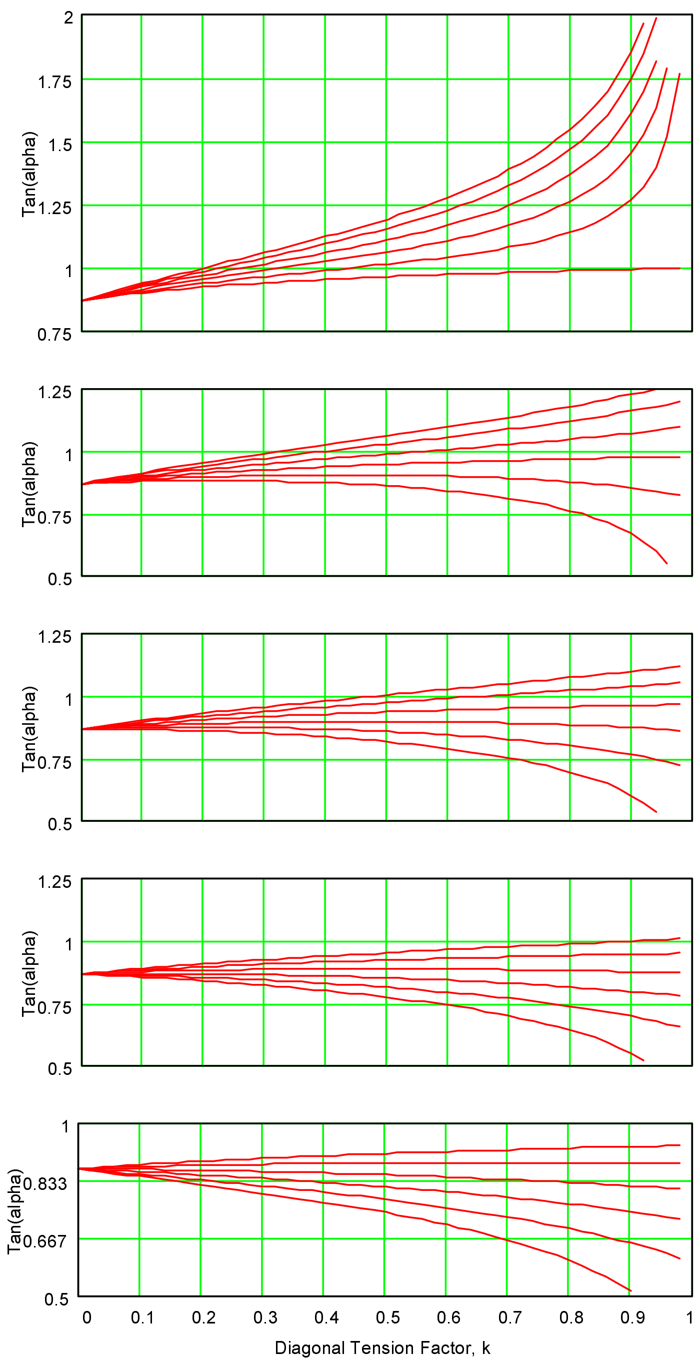


Fig.B27: ESDU 77018 Figure 3 - Angle of Diagonal Tension, $(E/q)(b/R)^2=10$

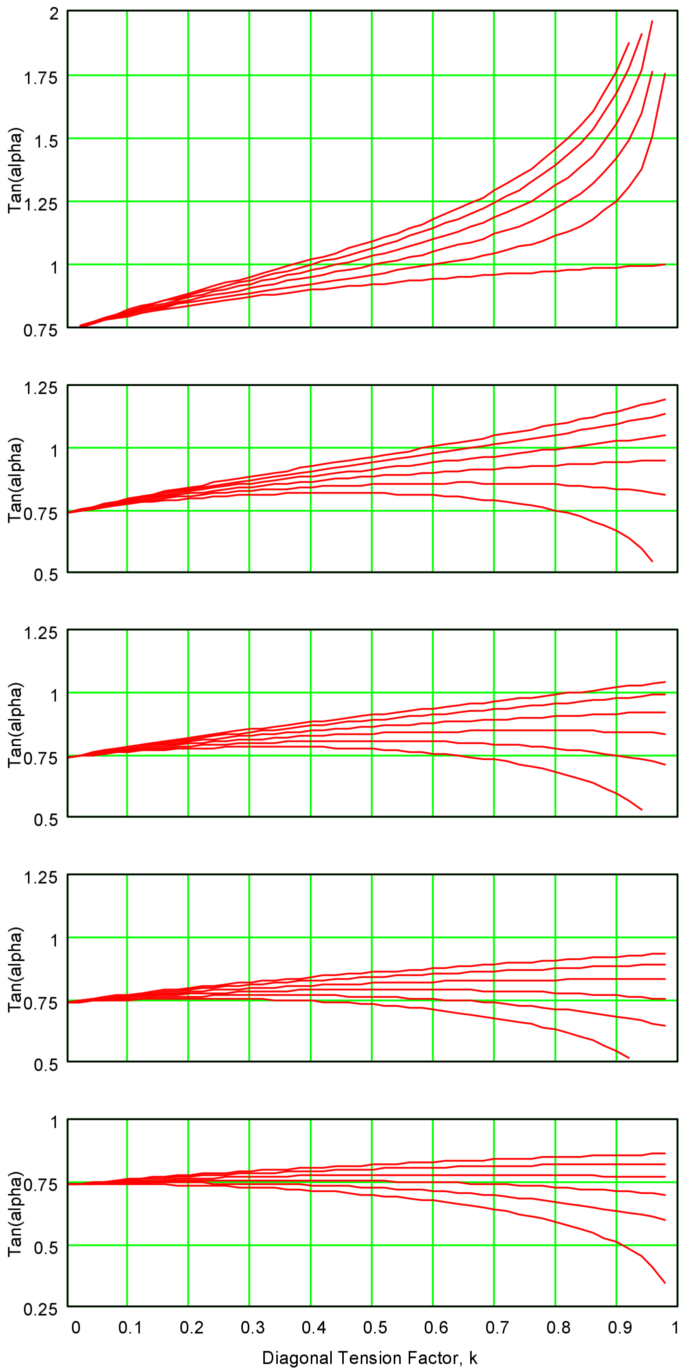


Fig.B28: ESDU 77018 Figure 4 - Angle of Diagonal Tension, $(E/q)(b/R)^2=25$

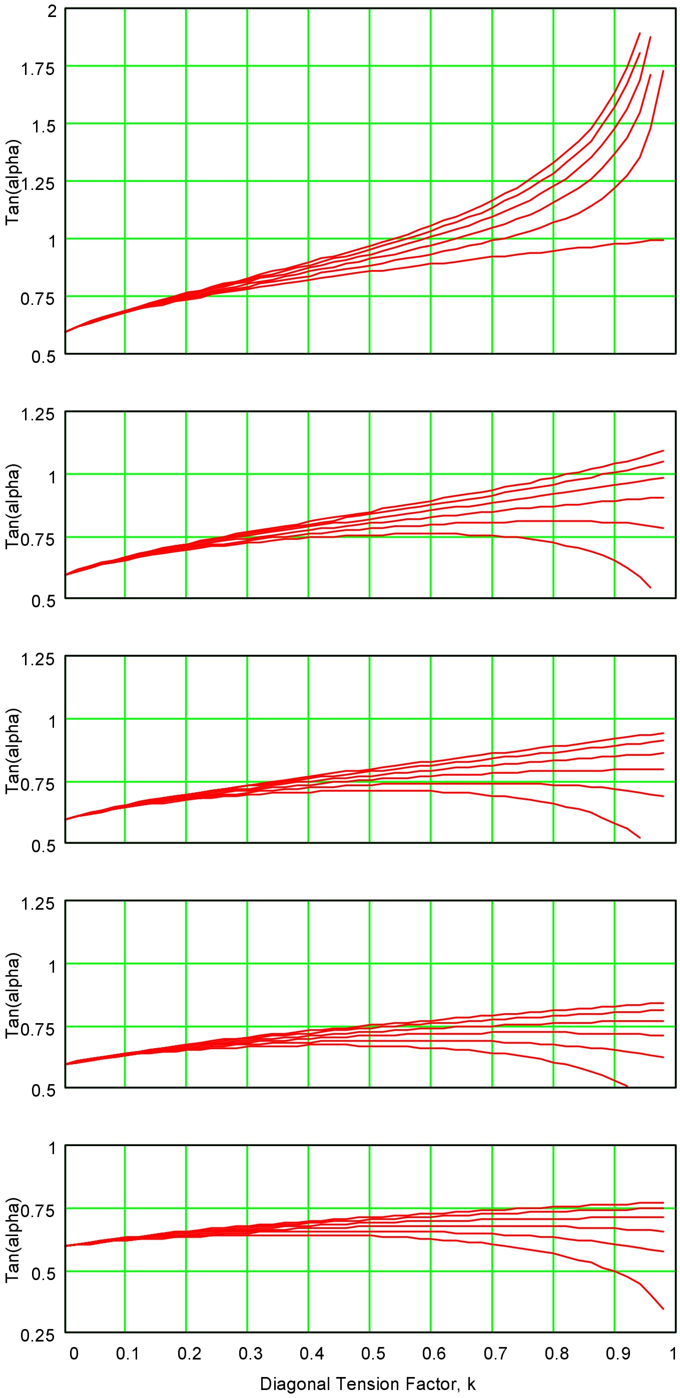


Fig.B29: ESDU 77018 Figure 5 - Angle of Diagonal Tension, $(E/q)(b/R)^2=50$

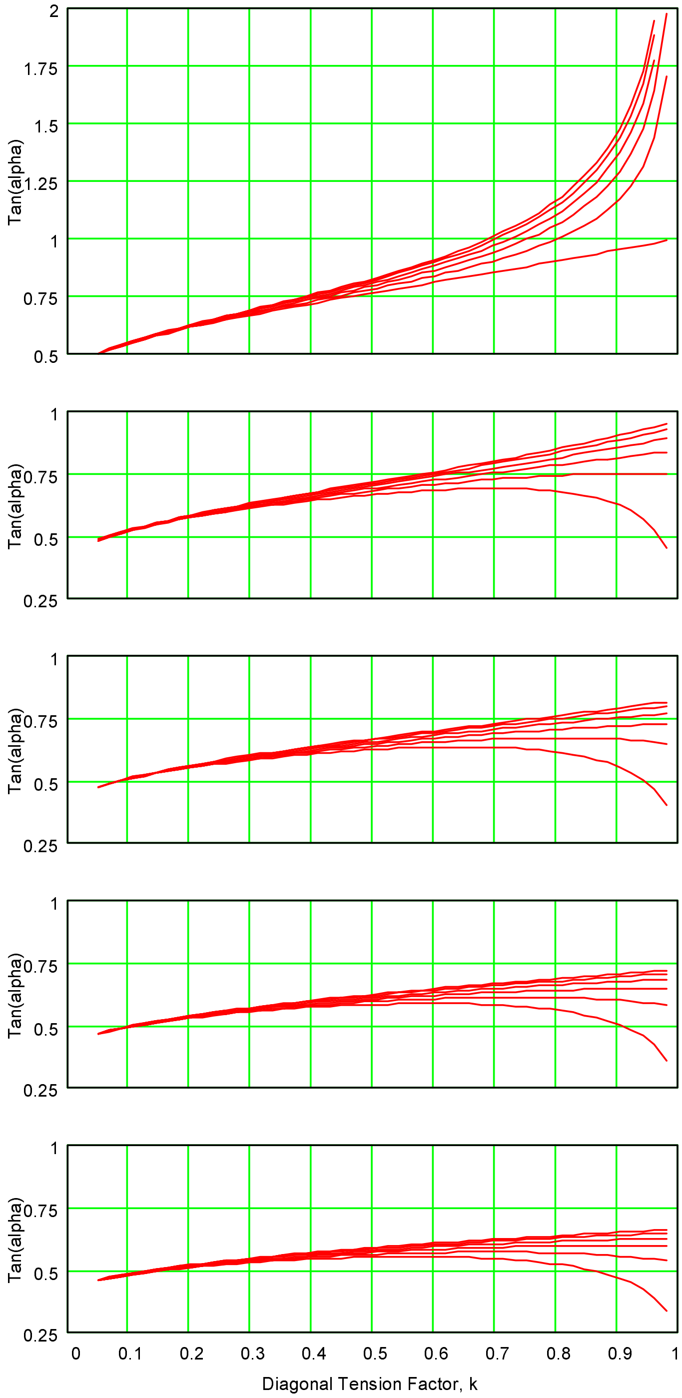


Fig.B30: ESDU 77018 Figure 6 - Angle of Diagonal Tension, $(E/q)(b/R)^2=100$

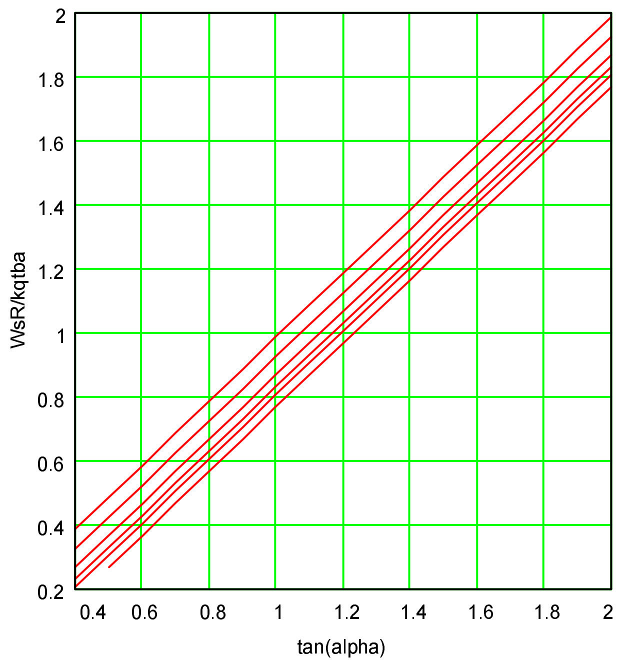


Fig.B31: ESDU 77018 Figure 7 - Inward Radial Load on Stiffener

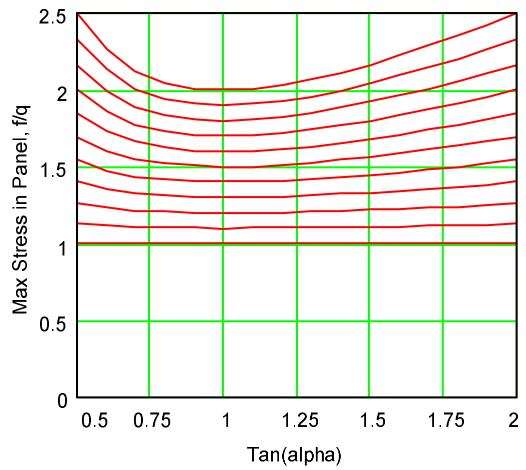


Fig.B32: ESDU 77018 Figure 8 - Maximum Principal Stress in Panel

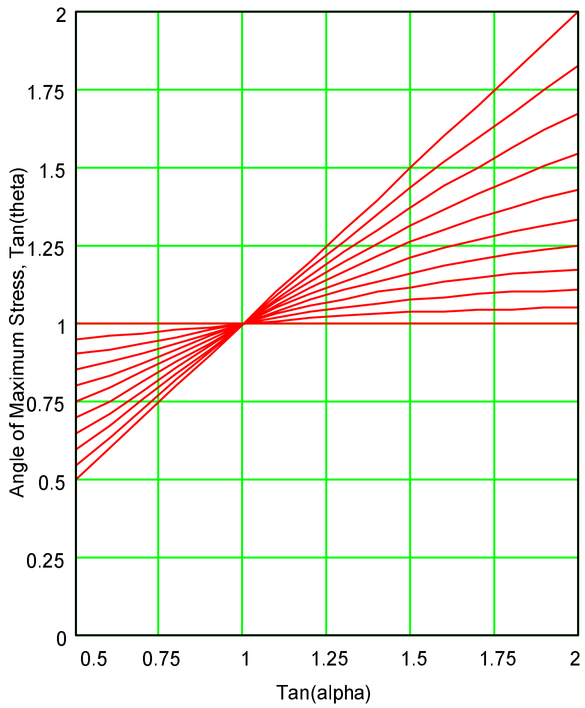


Fig.B33: ESDU 77018 Figure 9 - Angle of Maximum Principal Stress

APA115 Volume 3, Appendix C: NACA SHEAR ANALYSIS

Table of Contents

C1.0 INTRODUCTION

This appendix gives a brief description of the N.A.C.A. method of shear analysis covered by N.A.C.A. technical notes TN2661 and TN2662. The emphasis is on rationalisation and simplification of the equations and figures in the report and present them in the order they are implemented in program APA115. The N.A.C.A. report TN2661 itself is to be recommended for a detailed explanation of the behaviour of panels under shear in both a technical and descriptive sense.

The two N.A.C.A. reports date back to 1952, and provide the groundwork on which many post-buckled analyses of shear panels have been based. They were part of a research program to develop accurate methods of analysis for types of shear panels which were currently in use. These were predominantly fabricated assemblies with thin webs and consequently high buckling ratios.

The first note, TN2661, provides the theoretical and empirical relationships which were developed from the test results with a detailed description of panel behaviour. The second note, TN 2662, documents the test specimens and results and has not been directly referred to in APA115.

N.A.C.A. TN2661 is divided into two sections. The first section covers flat panels and the second section covers curved panels. There is some correlation between the two but, generally, the flat panel analysis goes into greater depth.

The graphical data used in the APA115 analysis has been directly extracted from the Fortran subroutines and used to plot the figures in this appendix. Thus, although these figures appear to be reproductions of the original N.A.C.A. figures, they serve as verification of the data in the subroutines.

The figures which have an analytical solution have been prepared on MathCAD 6+ using the same equations as the Fortran subroutines to verify the procedures.

C2.0 FLAT PANEL ANALYSIS

This essentially follows the order in the section 4 summary of N.A.C.A. TN2661 although the original equations are referenced to their original locations in the report.

C2.1 EFFECTIVE AREA OF UPRIGHT

The effective area of the upright (stiffener) is reduced due to its offset from the web. The expression for the effective area (from TN2661 equation 22, page 13) is:

$$A_{U_e} = \frac{A_U}{1+\left(\frac{e}{\rho}\right)^2} \tag{1}$$

A_U is the area of the stiffener without contribution from the panel; e is the offset of its centroid from the mid-plane of the panel; ρ is the radius of gyration about its own neutral axis.

C2.2 INITIAL PANEL BUCKLING

The critical shear stress, at which initial panel buckling occurs, is based on a theoretical curve for a simply-supported panel which is modified by restraint factors derived empirically. These factors are related to the type of sections used for the uprights or flanges and the thickness of the leg in contact with the web. The net result may be an increase or decrease in the critical shear stress. The assumption is that the restraint is independent of the outstanding leg (e.g. stiffener web normal to the panel). The elastic shear stress is given by TN2661 equation 32, page 26:

$$\tau_{cr,elastic} = k_{ss} \cdot E \cdot \left(\frac{t}{d_c}\right) \cdot \left[R_h + \frac{1}{2} \cdot (R_d - R_h) \cdot \left(\frac{d_c}{h_c}\right)^3\right] \tag{2}$$

In the above, h_c and d_c are the clear panel dimensions. It assumes $d_c<h_c$. If $h_c<d_c$, they should be exchanged. k_{ss}, the simply-supported buckling coefficient is obtained from TN2661 figure 12a and the restraint factors, R_d, R_h, from TN2661 figure 12b. These are reproduced in Fig.C1. k_{ss} is the same as the simply-supported curve in E.S.D.U. 71005.

N.A.C.A. corrects the elastic shear stress for plasticity from two material curves (24S-T3 and Alclad 75S-T6) and no allowance is made for other materials. Therefore, APA115 performs its own correction based on E.S.D.U. (effectively, the plasticity curves in data sheet 76016). This does not match the N.A.C.A. curves particularly well, but it does provide a general solution. Fig.C2 shows the comparison of TN2661 figure 12c and E.S.D.U.

Following normal practice (not stated in TN2661), the critical shear stress is limited to $t_2/\sqrt{3}$.

C2.3 DIAGONAL TENSION FACTOR AND ANGLE OF DIAGONAL TENSION

The diagonal tension factor is given by the following expression, (TN2661 equation 27, page 18). This is also displayed as the lowest curve in Fig.C3, representing TN2661 figure 13:

$$k = \tanh\left[0.5 \cdot \log_{10}\left(\frac{\tau}{\tau_{cr}}\right)\right] \tag{3}$$

In this equation, the nominal web shear stress, τ, is based on the effective depth of the beam measured between the centroids of the flanges. For an unbuckled panel, when $\tau<\tau_{cr}$, k=0.

The angle of diagonal tension, α, comes from TN2661 equations 30a-30d, pages 19-20. These are rearranged as follows and solved by iteration:

$$\tan^2\alpha = \left[\frac{\dfrac{2 \cdot k}{\sin(2\cdot\alpha)} + (1-k)\cdot(1+\mu)\cdot\sin(2\cdot\alpha) + \dfrac{k\cdot\cot\alpha}{2\cdot\dfrac{A_f}{h\cdot t} + 0.5\cdot(1-k)}}{\dfrac{2 \cdot k}{\sin(2\cdot\alpha)} + (1-k)\cdot(1+\mu)\cdot\sin(2\cdot\alpha) + \dfrac{k\cdot\tan\alpha}{\dfrac{A_{Ue}}{d\cdot t} + 0.5\cdot(1-k)}}\right] \tag{4}$$

For an unbuckled panel, α is set to a nominal 45°. When $1<\tau/\tau_{cr}<2$, there is a simplified expression that can be used. This would be helpful in a hand calculation, but the program uses the full expression above.

C2.4 STRESSES IN UPRIGHTS

The average stress along the length of the uprights is given by TN2661 equation 30a, p19:

$$\sigma_U = -\frac{k \cdot \tau \cdot \tan\alpha}{\frac{A_{U_e}}{d \cdot t} + 0.5 \cdot (1-k)} \tag{5}$$

This equation is represented in Fig.C4, corresponding to TN2661 figure 14.

The maximum stress in the uprights occurs at mid-height and is given by a custom fit to TN2661 figure 15. This is plotted in Fig.C5. The expression used is:

$$\frac{\sigma_{U_{max}}}{\sigma_U} = 0.775 \cdot (1-k) \cdot \left(1 - \frac{d}{1.2 \cdot h}\right) + 1 \tag{6}$$

TN2661 figure 16a provides a means of obtaining the angle of diagonal tension from the ratio σ_U/τ calculated above. A discrepancy has been noted with this figure. No curve can exist at k = 0. Also, $\sigma_U/\tau<2k/(1-k)$ because $\tan\alpha<1$ and the k = 0.5 curve should not reach $\sigma_U/\tau=2$. APA115 uses the analytical solution for this calculation and the graphical errors are avoided. In addition, Fig.16a appears to be in error on the $2A_F/ht$ = 1 curve above A_{Ue}/dt = 0.8. Fig.C6 gives a corrected representation of the figure.

C2.5 MAXIMUM WEB STRESS

The maximum web stress is calculated using concentration factors C_1 and C_2. C_1 is given by the following expression (TN2661 equation 33a, p27) and given in Fig.C7:

$$C_1 = \frac{1}{\sin(2 \cdot \alpha)} - 1 \tag{7}$$

The concentration factors C_2 and C_3 are given in Fig.C8. There are obtained experimentally and are represented as data. The maximum web stress is given by TN2661 equation 33a, page 27:

$$\tau'_{max} = \tau \cdot (1 + k^2 \cdot C_1) \cdot (1 + k \cdot C_2) \tag{8}$$

C2.6 ALLOWABLE WEB STRESSES

N.A.C.A. provides the basic allowable web stresses for two materials only (24S-T3 and Alclad 75S-T6) in figures 19a and 19b. These are subject to conditions (e.g. the panel connections should be snug bolts and washers, or sandwiched between flange angles, or fastened with loosened rivets). It was possible to 'non-dimensionalise' the curves by relating them to the t_2 stresses of 52 ksi and 65 ksi respectively (see Appendix G for a full proof of this). The value τ_{all} at k = 0 was found to be:

$$\tau_{all_{k=0}} = \frac{1}{1.1} \cdot \frac{t_2}{\sqrt{3}} \qquad (9)$$

Although obtained by inspection of the figures, this value is reasonable based on the normal ratio of direct to shear stress of √3 and a 10% factor allowing for a scatter band from test results (page 29 of TN2661). A polynomial fit for the curve at α=45° was found to be within an error of +0.4%, -1.4% as follows:

$$\frac{\tau_{all}}{\tau_{all_{k=0}}} = -0.377 \cdot k^3 + 0.893 \cdot k^2 - 0.716 \cdot k + 1 \qquad (10)$$

The correction for angle of diagonal tension at k = 2 is:

$$\frac{\tau_{all_{k=2,\alpha}}}{\tau_{all_{k=2,\alpha=45}}} = 1 - \sin(2 \cdot \alpha) \qquad (11)$$

Noting that $\tau_{all(k=2)} = 0.8 \cdot \tau_{all(k=0)}$, the overall expression for a complete set of generalised curves is as follows:

$$\frac{\tau_{all}}{\sigma_{all}} = \frac{1}{1.1 \cdot \sqrt{3}} \cdot \left[-0.377 \cdot k^3 + 0.893 \cdot k^2 - 0.716 \cdot k + 1 - 0.8 \cdot k \cdot (1 - \sin(2 \cdot \alpha))\right] \qquad (12)$$

where σ_{all} is taken as t_2, the 0.2% proof tensile stress.
The generalised curves are given in Fig.C9. However, program APA115 uses the curve at α = 45° in conjunction with a value for τ_{max} using C_1 and C_2, following TN2661 section 4.7, page 44.

C2.7 EFFECTIVE COLUMN LENGTH OF UPRIGHT

The effective column length of the upright (TN2661 equation 35, p46) is:

$$d < h_U : \quad L_e = \frac{h_U}{\sqrt{1 + k^2 \cdot \left(3 - 2 \cdot \frac{d}{h_U}\right)}}, \qquad d \geq h_U : \quad L_e = h_U \qquad (13)$$

h_U is the length of the upright measured between the centroids of upright-to-flange rivet patterns.

C2.8 ALLOWABLE UPRIGHT STRESSES

C2.8.1 Allowable Crippling Stress

The allowable crippling stresses are given in TN2661 for two materials only. These are different for double and single uprights. It has been found that the allowable crippling stresses can be related numerically directly to the 0.2% proof stress of the material. As this criterion is usually qualitatively related in some way to 0.2% proof stress, it is thought reasonable to make this assumption for all materials. The expression, derived from TN2661 equations 36a,b and 37a,b is as follows:

$$\text{single uprights:}\quad \sigma_o = \tfrac{1}{2}\cdot t_2 \cdot k^{\frac{2}{3}} \cdot \left(\frac{t_U}{t}\right)^{\frac{1}{3}}, \qquad \text{double uprights:}\quad \sigma_o = 0.8\cdot \tfrac{1}{2}\cdot t_2 \cdot k^{\frac{2}{3}} \cdot \left(\frac{t_U}{t}\right)^{\frac{1}{3}} \quad (14)$$

This is an elastic stress which is corrected for plasticity in TN2661 by the factor E_{sec}/E. This is the same as the E.S.D.U., or Ramberg-Osgood, method. Following normal practice, APA1115 limits the crippling stress to t_2. TN2661 does not do this.

C2.8.2 Allowable Column Buckling Stress

The allowable column buckling stress is considered to be the Euler stress (see TN2661 pages 46,47). For double uprights, the column buckling stress is based on the slenderness ratio using the effective column length, L_e/ρ For single uprights it is based on half the geometric slenderness ratio, i.e. $h_U/2\rho$. These are corrected for plasticity and limited to t_2. The expressions for the allowable elastic column buckling stress are thus:

$$\text{single uprights:}\quad \sigma_{all} = E\cdot\left(\frac{2\cdot\pi\cdot\rho}{h_U}\right)^2, \qquad \text{double uprights:}\quad \sigma_{all} = E\cdot\left(\frac{\pi\cdot\rho}{L_e}\right)^2 \quad (15)$$

C2.9 ALLOWABLE JOINT LOADS

C2.9.1 Web to Flange Rivets

TN2661 equation 34, page 14 gives the following expression for the applied rivet load per unit length:

$$R'' = \frac{S_W}{h_R}\cdot(1+0.414\cdot k) \quad (16)$$

C2.9.2 Upright to Flange Rivets

TN2661 equation 39, page 48 gives the applied upright to flange rivet load as simply the end load in the upright, neglecting the gusset effect:

$$P_U = \sigma_U \cdot A_{U_e} \quad (17)$$

For double uprights, $A_{Ue} = A_U$.

C2.9.3 Upright to Web Rivets

TN2661 equation 40, page 49 gives the required rivet shear strength (i.e. allowable shear load) for 24S-T3 material, in imperial units, only as follows:

$$R_R = \frac{100\cdot Q\cdot h_U}{b\cdot L_e}\ \text{kips} \quad (18)$$

where Q is the second moment of area of the upright through the panel mid-plane, b is the width of the outstanding leg of the upright and h_U/L_e is as calculated above. As it stands, this expression is limited to one material. However, it is expected that R_R will be proportional to the 0.2% proof stress of the upright or the panel, whichever is less. Noting that the t_2 stress for 24S-T3 is 52 ksi, the following general expression is tentatively suggested:

$$R_R = 1.923 \cdot t_2 \cdot \frac{Q \cdot h_U}{b \cdot L_e} \text{ kips} \tag{19}$$

There is a tension requirement on the joint. This is not related to the applied load but to the allowable panel stress. It has been shown in the N.A.C.A. tests that if joints conform to this requirement then no joint failures occur but, of course, no reserve factors can be derived for this criterion. The required tensile strength per unit run is different for single or double uprights and is repeated from TN2661 equations 41,42, page 49:

$$\text{single uprights:} \quad tsr > 0.22 \cdot t \cdot \sigma_{ult}, \qquad \text{double uprights:} \quad tsr > 0.15 \cdot t \cdot \sigma_{ult} \tag{20}$$

The column buckling of double uprights may be affected by a loss of rivet strength. Fig.C10, a represention of TN2661 figure 21, provides a factor to allow for this.

C2.10 EFFECTIVE SHEAR MODULUS

TN2661 figure 22a, page 17 provide the effective shear modulus for the panel. The expression for this is derived from TN2661 equations 31a,b pages 21,22:

$$\frac{E}{G_{IDT}} = \frac{4}{\sin^2(2 \cdot \alpha)} + \frac{\tan^2 \alpha}{\frac{A_{U_e}}{d \cdot t} + 0.5 \cdot (1-k)} + \frac{\cot^2 \alpha}{2 \cdot \frac{A_F}{h \cdot t} + 0.5 \cdot (1-k)} \tag{21}$$

$$\frac{G_{IDT}}{G} = \frac{1}{1 + k \cdot \left(\frac{1}{2 \cdot (1+\mu)} \frac{E}{G_{IDT}} - 1 \right)} \tag{22}$$

Fig.C11 plots the expression fo G_{IDT} and is the same as TN2661 figure 22a.

C2.11 SECONDARY STRESSES IN FLANGES

These take the form of end-load stress and bending stress.

C2.11.1 Stress due to End-Load

The compressive stress in the flange due to diagonal tension from TN2661 page 50 does not include any contribution from the effective panel and is adapted from equation 30b:

$$\sigma = \frac{k \cdot S_W \cdot \cot \alpha}{2 \cdot A_F} \tag{23}$$

The effective panel would add 0.5·(1-k)·h·t to the denominator of the above expression.

C2.11.2 Stress due to Bending Moment

The primary bending moment in the flange, over an upright, is adapted from TN2661 equation 18 as follows:

$$M_{max} = k \cdot C_3 \cdot \frac{S_W \cdot d^2 \cdot \tan\alpha}{12 \cdot h} \tag{24}$$

C_3 is obtained from Fig.C8. The bending moment between uprights is half the value at the top of the uprights.

C3.0 CURVED PANEL ANALYSIS

The curved panel analysis is more limited in scope than the flat panel analysis. Some of the parameters required are already covered in the flat panel analysis and will not be repeated here.

When curved panels buckle, they tend to approach a flat panel in shape in the limiting case under the effect of the diagonal tension waves. However, the initial stages of post-buckled behaviour are not directly predictable. Also, subject to the restraint of circular frames, the panel will not necessarily be able to adopt a flat shape.

The curved panel analysis considers two panel configurations which are characterised by the eventual mode the diagonal tension waves take. These have different expressions for the angle of diagonal tension, on which the analysis of shear panels is largely based. Long panels, with the short edges curved, tend to adopt a plane as if the short edges were flat. Short panels tend to adopt an hyperboloid of revolution such that each consecutive diagonal tension fold undergoes some small rotation round the axis of the cylinder, pulling only the centre of the panel inwards.

C3.1 INITIAL PANEL BUCKLING

The buckling coefficients for curved panels have been analytically derived and supported by tests going back to 1935. N.A.C.A. TN2661 figures 30a,b provide the buckling coefficient. Apart from the specific parameters plotted and the log-log scale, these figures are identical to those adopted by E.S.D.U. in data sheets 02.03.18/19. Fig.C12 and Fig.C13 show these curves plotted from the data in the Fortran subroutine. The correction for plasticity follows the same procedure as for flat panels.

C3.2 DIAGONAL TENSION FACTOR AND ANGLE OF DIAGONAL TENSION

The diagonal tension factor is obtained from an extension to that used for flat panels, given in TN2661 equation 50, page 69:

$$k = \tanh\left[\left(0.5 + 300 \cdot \frac{t}{R} \cdot \frac{d}{h}\right) \cdot \log_{10}\left(\frac{\tau}{\tau_{cr}}\right)\right] \tag{25}$$

In the above expression, d/h is replaced by h/d if h>d, the maximum value being limited to 2. Fig.C3 provides a graphical representation of this, the same as TN2661 figure 13.

The angle of diagonal tension takes two forms, both for closely-spaced stiffeners where h/R<1/3. For long panels and d<h:

$$\tan^2\alpha = \frac{\dfrac{2\cdot k}{\sin(2\cdot\alpha)} + (1-k)\cdot(1+\mu)\cdot\sin(2\cdot\alpha) + \dfrac{k\cdot\cot\alpha}{\dfrac{A_{ST}}{h\cdot t} + 0.5\cdot(1-k)}}{\dfrac{2\cdot k}{\sin(2\cdot\alpha)} + (1-k)\cdot(1+\mu)\cdot\sin(2\cdot\alpha) + \dfrac{k\cdot\tan\alpha}{\dfrac{A_{RG}}{d\cdot t} + 0.5\cdot(1-k)} + \dfrac{1}{24}\cdot\dfrac{E}{\tau}\cdot\left(\dfrac{h}{R}\right)^2} \qquad (26)$$

For short panels where h<d:

$$\tan^2\alpha = \frac{\dfrac{2\cdot k}{\sin(2\cdot\alpha)} + (1-k)\cdot(1+\mu)\cdot\sin(2\cdot\alpha) + \dfrac{k\cdot\cot\alpha}{\dfrac{A_{ST}}{h\cdot t} + 0.5\cdot(1-k)}}{\dfrac{2\cdot k}{\sin(2\cdot\alpha)} + (1-k)\cdot(1+\mu)\cdot\sin(2\cdot\alpha) + \dfrac{k\cdot\tan\alpha}{\dfrac{A_{RG}}{d\cdot t} + 0.5\cdot(1-k)} + \dfrac{1}{8}\cdot\dfrac{E}{\tau}\cdot\left(\dfrac{h}{R}\right)^2} \qquad (27)$$

Note that, apart from the expression related to radius, these equations are identical to those for a flat panel where A_{Ue}/dt is replaced by A_{RG}/dt and $2\cdot A_F$/ht is replaced by A_{ST}/ht. (As the stiffener is shared between panels, the factor 2 is removed from A_F).

C3.3 STRESSES IN STIFFENERS AND RINGS

As part of the determination of α, the average stress in the stiffener may be found as follows (from TN2661 equation 51, page 69):

$$\sigma_{ST} = -\frac{k\cdot\tau\cdot\cot\alpha}{\dfrac{A_{ST}}{h\cdot t} + 0.5\cdot(1-k)} \qquad (28)$$

The maximum stress in the stiffener is found using the same expression as for the flat panel (see Fig.C5).

For the ring, TN2661 equation 52, page 69 gives:

$$\sigma_{RG} = -\frac{k\cdot\tau\cdot\tan\alpha}{\dfrac{A_{RG}}{d\cdot t} + 0.5\cdot(1-k)} \qquad (29)$$

If there is a floating ring (connected to the top of the stiffeners and not directly to the panel), there is no contribution from the panel and the 0.5·(1-k) factor is omitted.

C3.4 BENDING MOMENT IN STRINGERS AND RINGS

TN2661 page 79 gives the maximum bending in the stringers, equally applicable to the the stringer at the rings and halfway along the stringer:

$$M_{ST} = k \cdot \tau \cdot t \cdot \frac{h \cdot d^2}{24 \cdot R} \cdot \tan\alpha \qquad (30)$$

If the ring is floating there is a maximum bending moment at the junction with the stringer given by TN2661 page 80:

$$M_{RG} = k \cdot \tau \cdot t \cdot \frac{h^2 \cdot d}{12 \cdot R} \cdot \tan\alpha \qquad (31)$$

C3.5 ALLOWABLE WEB STRESSES

Using the angle of diagonal tension for the curved web, the same expression is used as for a flat panel (see Section C2.6). This gives the basic allowable shear stress, τ^*_{all}. Allowance for curvature is made following TN2661 equations 53,54 page 72 as follows:

$$\tau_{all} = \tau^*_{all} \cdot (0.65 + \Delta) \qquad (32)$$

where:

$$\Delta = 0.3 \cdot \tanh\left(\frac{A_{RG}}{d \cdot t}\right) + 0.1 \cdot \tanh\left(\frac{A_{ST}}{h \cdot t}\right) \qquad (33)$$

This value may be increased by 10% if the rivets are tight. This is allowed for in the relevant subroutine in APA115.

C3.6 INSTABILITIES IN STIFFENERS AND RINGS

Three categories of instability are considered.

C3.6.1 General Instability of Panel

TN2661 gives a figure, based on test results, for the general instability of the panel allowing for the effect of the stiffeners and rings. This is only for 24S-T3 material but, in line with other assumptions in this analysis, it has been assumed to be directly related to the 0.2% proof stress. In this way, the figure may be used for all materials by factoring by the ratio of 0.2% proof stresses. (The proof stress for 24S-T3 is 52ksi). Fig.C14 gives a representation of TN2661 figure 34.

As this is an elastic stress, correction for plasticity is made using the Ramberg-Osgood method. However, the correction applied is the tangent modulus ratio as opposed to the secant modulus ratio used for pure panel modes.

The non-dimensional parameter used on the abscissa is:

$$\frac{(\rho_{ST} \cdot \rho_{RG})^{\frac{7}{8}}}{(d \cdot h)^{\frac{1}{2}} \cdot R^{\frac{3}{4}}} \qquad (34)$$

where ρ is the radius of gyration of the stiffener or ring plus panel contribution.

C3.6.2 Stringer Crippling

The crippling stress is the same as that for the flat panel. Whereas the flat panel has the option of single or double stiffener analysis, this analysis considers only single stiffeners (not expecting a stiffener on the outside of the structure). Therefore, the 0.8 factor appropriate for double stiffeners is not included. This is corrected for plasticity using the tangent modulus ratio.
For stringer crippling, the applied stress considered should be the maximum value.

C3.6.3 Stringer Column Strength

TN2661 Section 9.7, page 74, suggests that fixed-end conditions can probably be assumed for the stringers. Therefore a straight forward Euler sum is performed, using $I_e = ½ \cdot I$ and corrected for plasticity using the tangent modulus.

C3.6.4 Ring Crippling

If the ring is continuously attached to the panel, it should be checked for crippling in the same manner as the stringer (Section C3.6.2).

C3.7 ALLOWABLE JOINT LOADS

C3.7.1 Stiffener to Web Joint

The required rivet shear strength per unit length is given by TN2661 equation 55, page 75:

$$R'' = q \cdot \left[1 + k \cdot \left(\frac{1}{\cos\alpha} - 1\right)\right] \qquad (35)$$

where q is the shear flow.

C3.7.2 Ring to Web Joint

The required rivet shear strength per unit length is given by replacing sinα with cosα in the above equation:

$$R'' = q \cdot \left[1 + k \cdot \left(\frac{1}{\sin\alpha} - 1\right)\right] \qquad (36)$$

C3.8 EFFECTIVE SHEAR MODULUS

This is calculated as for the flat panel in Section C2.10.

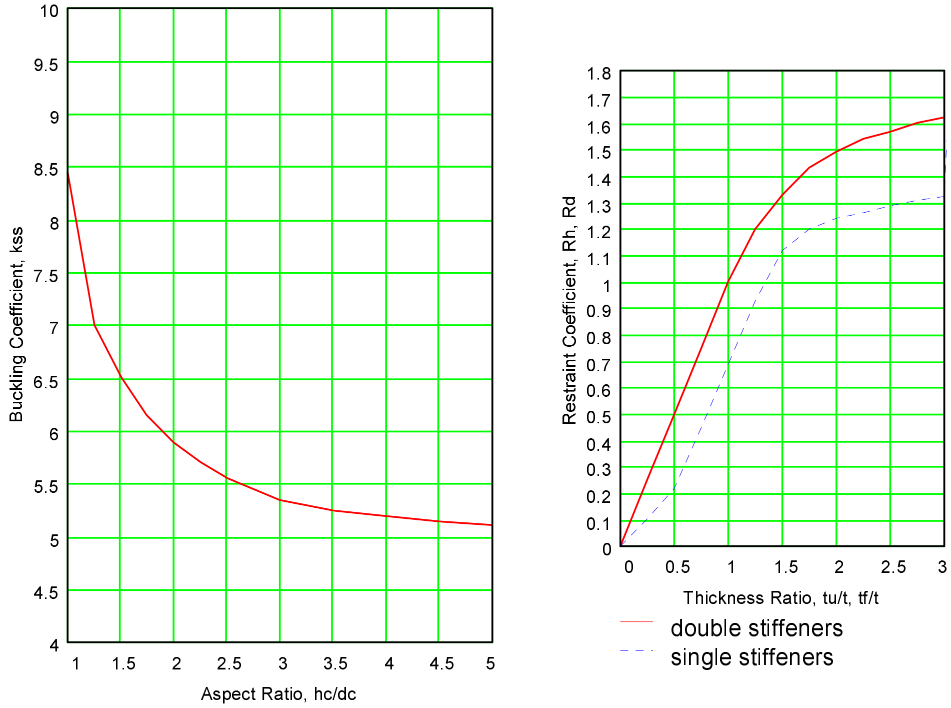


Fig.C1: NACA TN2661 Figures 12a,12b - Initial Panel Buckling

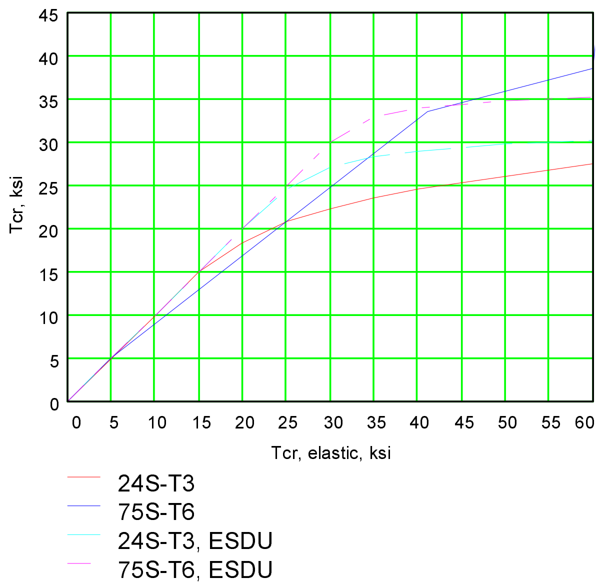


Fig.C2: NACA TN2661 Figure 12c - Plasticity Correction
(compared with ESDU, Ramberg-Osgood method)

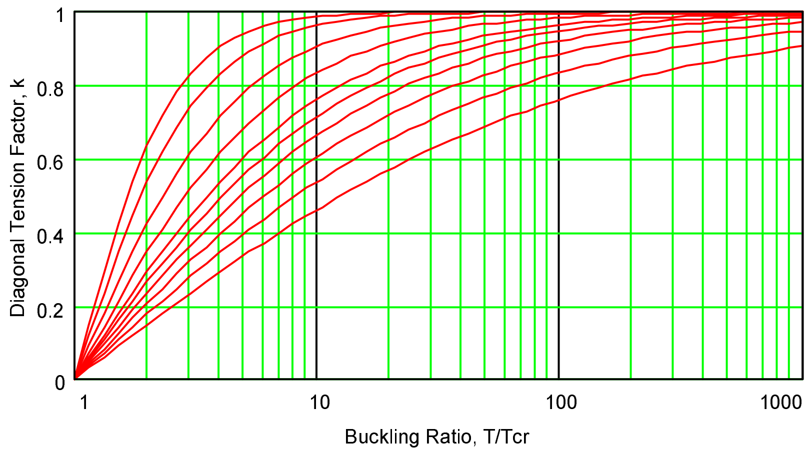


Fig.C3: NACA TN2661 Figure 13 - Diagonal Tension Coefficient

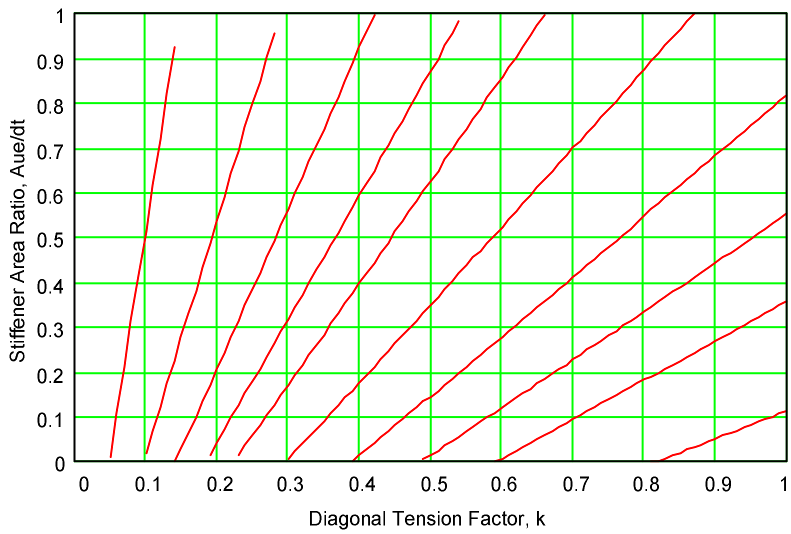


Fig.C4: NACA TN2661 Figure 14 - Diagonal Tension Analysis Chart

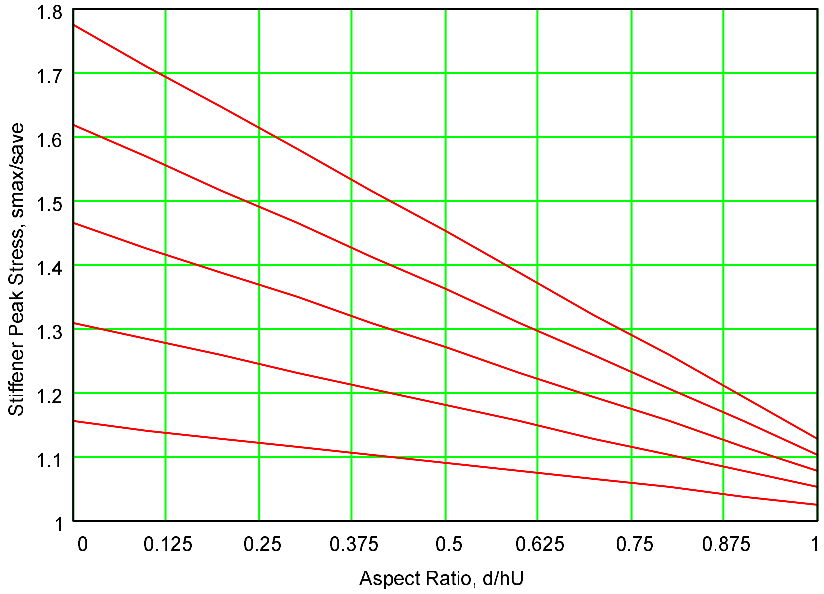


Fig.C5: NACA TN2661 Figure 15 - Maximum Stress in Stiffener

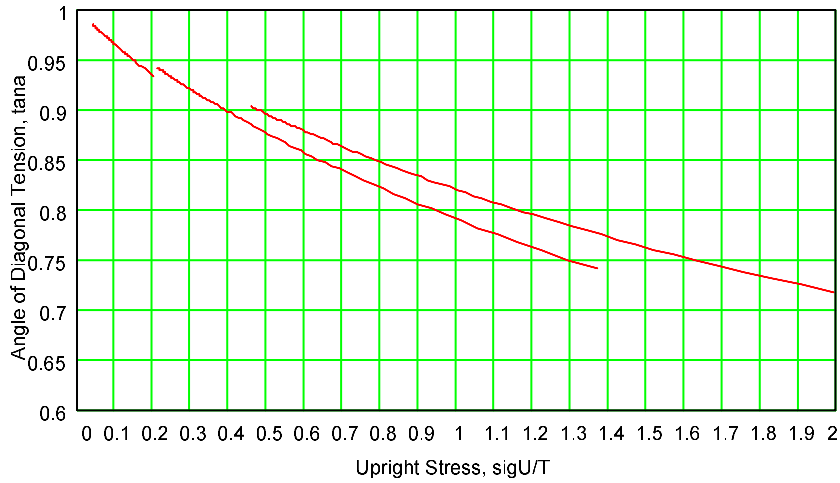


Fig.C6: NACA TN2661 Figure 16a - Incomplete Diagonal Tension, corrected

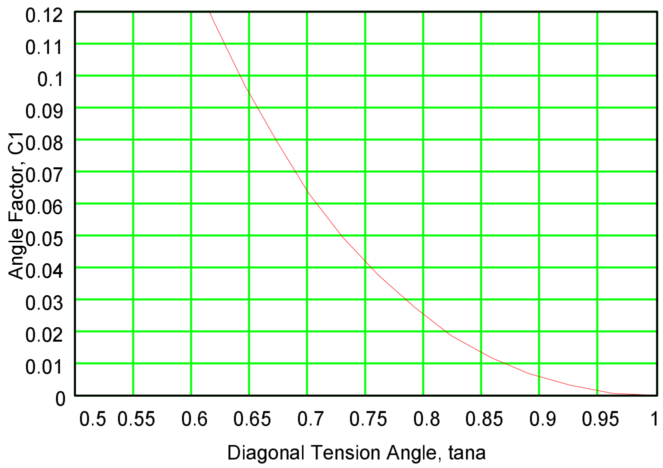


Fig.C7: NACA TN2661 Figure 17 - Angle Factor C_1

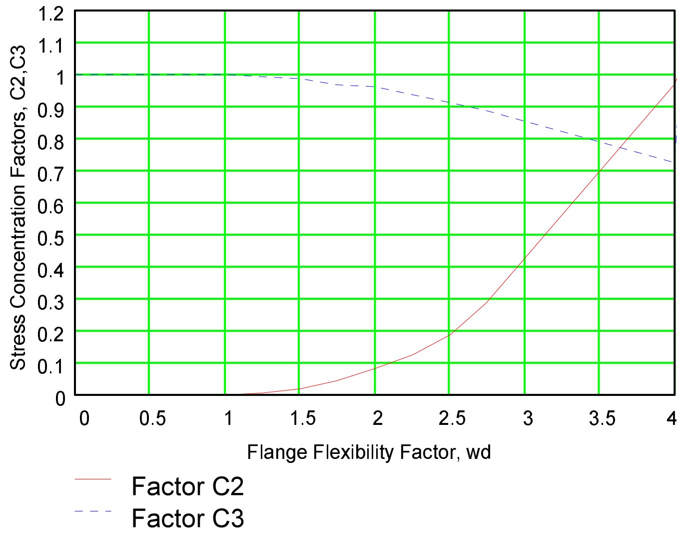


Fig.C8: NACA TN2661 Figure 18 - Stress Concentration Factors C2, C3

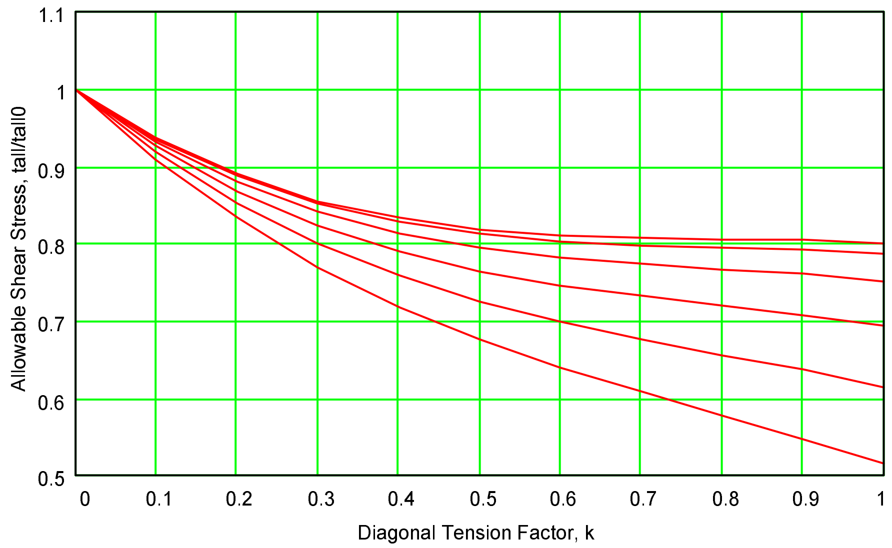


Fig.C9: NACA TN2661 Figures 19a,b - Generalised Allowable Stresses
(the allowable shear stress is related to the 0.2% proof stress as follows: $\tau_{all0} = t_2/(1.1\sqrt{3})$)

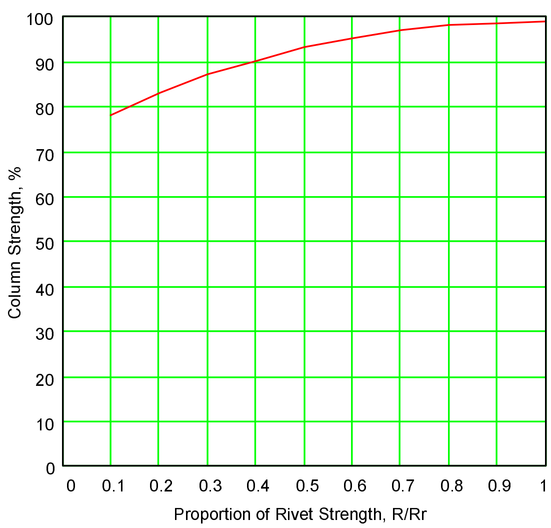


Fig.C10: NACA TN2661 Figure 21 - Strength of Riveted Columns

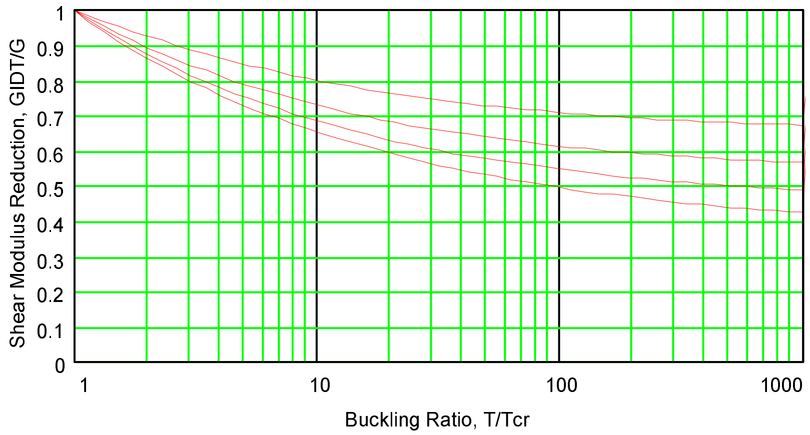


Fig.C11: NACA TN2661 Figure 22a - Modulus Ratio for Panel

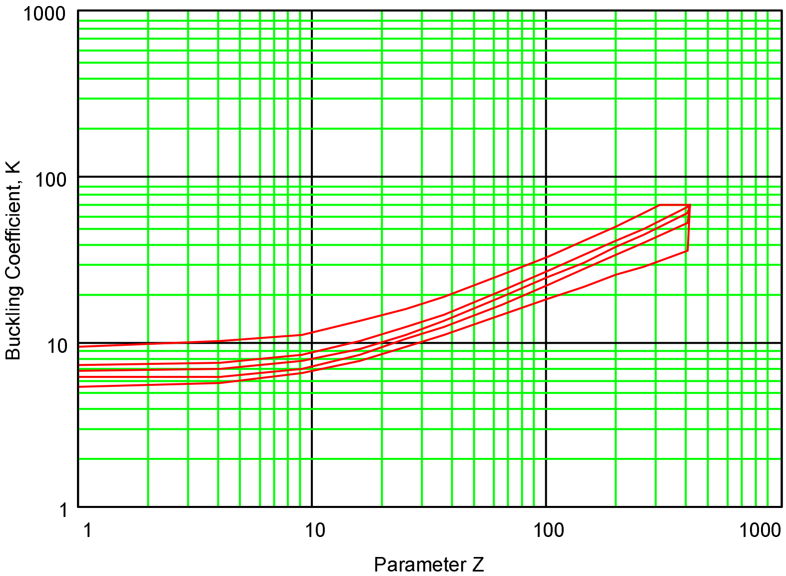


Fig.C12: NACA TN2661 Figure 30a - Initial Curved Panel Buckling, long panels

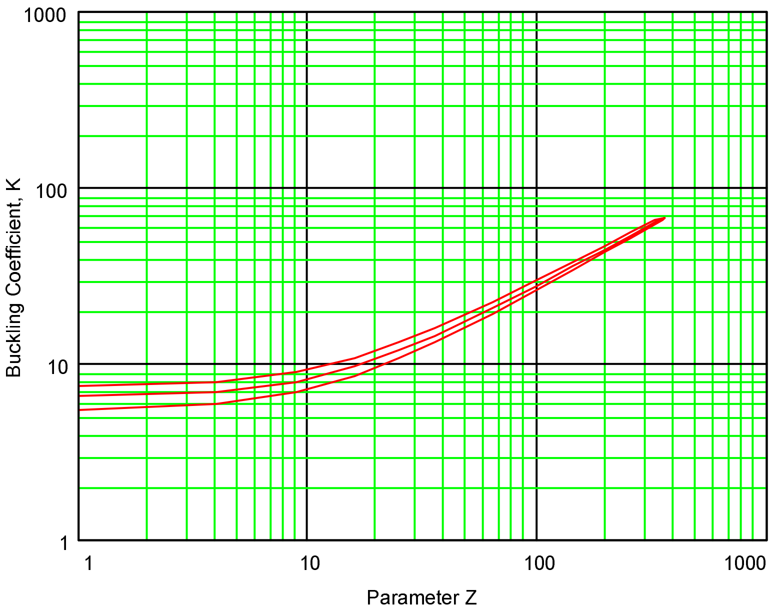


Fig.C13: NACA TN2661 Figure 30a - Initial Curved Panel Buckling, short panels

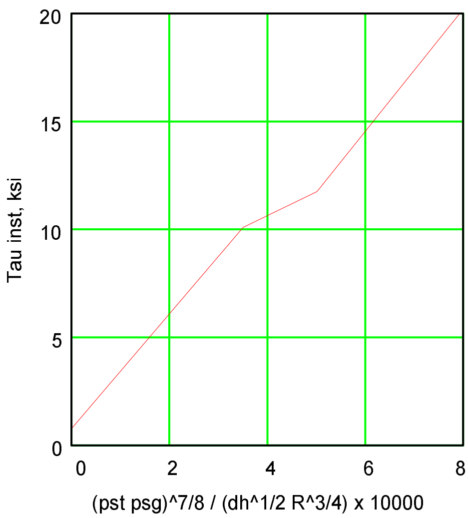


Fig.C14: NACA TN2661 Figure 34 - General Panel Instability

APA115 Volume 3, Appendix D: SOR51 SHEAR ANALYSIS

Table of Contents

D1.0 INTRODUCTION

This appendix gives a description of APA115's implementation of the flat shear panel analysis of Hatfield Report S.O.R. 51. The Hatfield report owes most of its derivation to research done by N.A.C.A. in 1952. Further information has been taken from various R.Ae.S and E.S.D.U. data sheets. These have also referenced the N.A.C.A. reports and a theoretical treatise by Leggett dating back to 1940. Some additional information was also included in S.O.R. 51 for which the references are not available.

S.O.R. 51 was compiled at Hatfield in April 1964 in a form intended to facilitate hand calculations and updated in January 1988. The report was difficult to follow and at least two unofficial re-writes of the flat-panel analysis have since been compiled to provide clarification (February 1987 and March 1992). There has also been a history of programs developing from S.O.R. 51: Hatfield HST-020-2, Filton FH101A and APA107, and an unofficial version of FH101A in 1987 from which the APA115 subroutines have been taken.

The relevant figures from S.O.R. 51 are reproduced in this appendix. Some have been directly extracted from the Fortran subroutines in APA115. Others, which are derived from theoretical relationships, are produced with the same equations as in the subroutines. Thus all these figures provide a check on the subroutines. The figures have been prepared on MathCAD 6+.

It is useful to look at Appendix F which compares the various analyses performed by APA115. In particular, the derivation of some of the figures and methods in S.O.R. 51 from E.S.D.U. data sheets is described. Appendix F shows inaccuracies in this derivation which have lead to the suggestion in the main report that the E.S.D.U. analysis is to be preferred.

D2.0 NOTATION

The notation used in this appendix is based mainly on S.O.R. 51. Further notation has been added where necessary. Input to APA107 deviates from this notation in some instances. Further detail on the APA107 notation may be found in Appendix E.

Figs.D1a-c shows the notation for the panel geometry.

NOTATION	DESCRIPTION

Panel Geometry

a	Distance between webs of stiffeners S_b (panel depth)
a'	Plain panel depth
a_e	Effective panel depth
a_e'	Increased effective panel depth
b	Distance between webs of stiffeners S_a (panel width)
b'	Plain panel width
b_e	Effective panel width
b_e'	Increased effective panel width
d	Pitch of stiffeners S_a
h	Pitch of stiffeners S_b
t	Panel web thickness

Stiffener Geometry

A_s	Area including effective skin under stiffener
A_s'	Effective area (= $A_s - (A_s y_s)^2/Ixs$)
A_s^*	2 x area of flanges in contact with web (= A_s if > A_s)
A_{sx}	Area including width s of web (= A_s + st)
d_1	Width of flange in contact with web
d_2	Width of lip of flange d_1
d_3	Free width of flange d_1 (i.e. beyond joint line)
h_p	Web width under closed section stiffener
I_s	2nd M. of A. about axis || to mid-plane thro' stiffener centroid
I_{xs}	2nd M. of A. about axis through mid-plane
I_s'	2nd M. of A. about axis || to mid-plane including width s of web through combined centroid
I_c	2nd M. of A. including 25t of skin about axis normal to panel if compression edge member
I_t	2nd M. of A. including 25t of skin about axis normal to panel if tension edge member
I_p	Polar moment of inertia about base of stiffener
J	Torsion constant
k_s	Radius of gyration
s	Effective width of skin acting with stiffener
t_s	Total thickness of web plus flange
t_s'	Thickness of flange only
w_s	Flange width of integral or reduxed stiffener
wh,wd	Flange parameter if edgemember (S_a, S_b respectively)
y_s	Distance of centroid of A_s from web mid-plane
y_{sx}	Distance of centroid of A_{sx} from web mid-plane
Γ	Warping constant

Material Properties

f_1	0.1% proof tensile stress
f_2	0.2% proof tensile stress
f_{ult}	Ultimate tensile stress
f_n	Reference stress
f	General stress
m	Material characteristic
E	Young's modulus
E_t	Tangent modulus
E_s	Secant modulus
G	Shear modulus
G_s	Secant shear modulus
μ	Poisson's ratio

Stresses

f	Direct stress, general
f_{app}	Applied direct stress
f_{av}	Stiffener average stress along length
f_{cr}	Stiffener forced crippling stress
f_{ed}	Edge member stress
f_{fail}	Stiffener failure stress with applied end load
f_{fi}	Stiffener flexural instability stress
f_i	Stiffener instability stress, general
f_{ir}	Stiffener inter-rivet buckling stress
f_{li}	Stiffener local instability stress, general
f_{lic}	Local instability stress of web under closed section
f_{lif}	Stiffener local instability stress, free flange
f_{lil}	Stiffener local instability stress, flange lip
f_{max}	Stiffener maximum stress along length
f_{pj}	Stiffener stress at failure of stiffener to panel joint
f_{se}	Stiffener stress at failure of stiffener to edge member
f_{ti}	Stiffener torsional instability stress
τ	General shear stress
τ_{app}	Applied shear stress
τ_{bo}	Elastic buckling stress
τ_b	Actual buckling stress
τ_{pb}	Basic permanent buckling stress
τ_{pb}'	Actual permanent buckling stress
τ_{all}	Basic permissible web stress
τ_{allej}	Failing shear stress based on edge member joint
τ_{allpj}	Failing shear stress based on stiffener joint
τ_{fail}	Shear stress at failure
τ_{fail}'	Shear stress at failure including edge member stress
τ_c	Panel shear stress at forced crippling of stiffener

τ_{fi}	Panel shear stress at stiffener flexural instability
τ_i	Panel shear stress at stiffener instability, general
τ_{ir}	Panel shear stress at stiffener inter-rivet buckling
τ_{pj}	Panel shear stress at stiffener to web failure
τ_{ti}	Panel shear stress at stiffener torsional instability

Miscellaneous Parameters

K	Buckling coefficient
C_s	Stiffener average stress coefficient
k_d	Diagonal tension factor
k_{fc}	Stiffener forced crippling factor
M_e	Edge member bending moment
PF	Ratio of ultimate to proof load
α	Edge member direct stress coefficient
β	Edge member bending moment coefficient
P_{se}	Allowable load for stiffener to edge member attachment

D3.0 PANEL ANALYSIS

The initial panel buckling is approached in stages. The panel is analysed as an equivalent simply-supported panel with modified panel sizes to allow for edge fixation due to stiffeners and edge members. The effect of stiffener torsional stiffness is covered by the use of effective panel sizes in S.O.R. 51 figure 1a (here reproduced in Fig.D3) and the effect of out-of-plane stiffener flexural stiffness is covered by the use of effective panel sizes in S.O.R. 51 figure 1c (here reproduced in Fig.D4). The latter includes the criterion of buckling across stiffeners.

D3.1 EFFECTIVE PANEL SIZE

The effective panel sizes are found from Fig.D3. Allowance is made for the stiffener pad (or flange) dimensions in this figure. This gives effective panel sizes, a_e and b_e, equal to or less than the dimensions a and b.

A modification to S.O.R. 51 made by program APA107 includes a modification to the original curves given in S.O.R. 51 figures 1a,1b, shown on Fig.D3 as a dashed line. The difference occurs at values of t_{sa}/t or t_{sb}/t below 2, and is intended to control reduction of effective panel size due to pad width when the total pad thickness is small. Originally, the effective panel size could be reduced when t_{sa}/t or t_{sb}/t was only infinitesimally greater than 1. The limit on panel size at t_{sa}/t or t_{sb}/t equal to 1 is now fixed at the distance between the stiffeners, i.e. a or b. It was decided to extend this limit to t_{sa}/t or t_{sb}/t equal to 1.5. The panel size is reduced linearly to meet the original curve at t_{sa}/t or t_{sb}/t equal to 2. The curve in this region is thus independent of the values w_{sa} and w_{sb} and cannot be defined on the same figure. In the range of t_{sa}/t or t_{sb}/t between 1.5 and 2, the effective panel size cannot be read from the curve and must be calculated.

The following additional dimensions (plain panel widths) are required for the analysis:

$$\begin{aligned} a' &= a - w_{sb} \\ b' &= b - w_{sa} \end{aligned} \qquad (1)$$

For panels with riveted or bolted stiffeners, Fig.D3 is still used. This replaces S.O.R. 51 figure 1b, but uses t_{sa} and t_{sb} and not t_{sa}' and t_{sb}'. For single rivet lines, the effective panel size remains the actual panel size.

D3.2 INCREASED EFFECTIVE PANEL SIZE

An allowance is made for the stiffener flexibility using Fig.D4. This gives increased effective panel sizes, a_e' and b_e', which are greater than or equal to a_e and b_e respectively. These dimensions are not to be considered as actual dimensions but rather as referring to imaginary panel dimensions which enable the panel to be analysed as if it were simply-supported. These dimensions may exceed the actual dimensions by a considerable amount when the stiffeners are weak.

For use in Fig.D4, the stiffener area must include the panel under the stiffener plus the effective widths a'/2 and b'/2. The stiffener areas input to APA107 do not include the panel under the stiffener whereas S.O.R. 51 does. As the APA115 subroutine SOR51P uses input in APA107 notation, this panel is added first (to give A_{sa}, y_{sa}, I_{sa}) and then the effective widths a'/2 and b'/2 are included. The net result in terms of stiffener constants is as follows:

$$I_{xsa} = I_{sa} + \frac{b' \cdot t^3}{24} + A_{sa} \cdot y_{sa}{}^2$$
$$I_{sa} = I_{xsa} - \frac{A_{sa}{}^2 \cdot y_{sa}{}^2}{A_{sa} + \frac{1}{2} \cdot b' \cdot t} \qquad (2)$$
$$A_{sa}' = A_{sa} - \frac{A_{sa}{}^2 \cdot y_{sa}{}^2}{I_{xsa}}$$

$$I_{xsb} = I_{sb} + \frac{a' \cdot t^3}{24} + A_{sb} \cdot y_{sb}{}^2$$
$$I_{sb} = I_{xsb} - \frac{A_{sb}{}^2 \cdot y_{sb}{}^2}{A_{sb} + \frac{1}{2} \cdot a' \cdot t} \qquad (3)$$
$$A_{sb}' = A_{sb} - \frac{A_{sb}{}^2 \cdot y_{sb}{}^2}{I_{xsb}}$$

Fig.D4 also gives the minimum stiffness requirement for the stiffeners to prevent buckling across them. Buckling across stiffeners should be considered as permanent deformation. As such it may undercut the permanent deformation allowable for the web stress (see Fig.D6). This is particularly true for integral stiffeners where the stiffeners are forced to follow the web buckles precisely. The extent of stiffener deformation cannot be determined from this analysis and it is possible that it would never become permanent. However, to be conservative, it is assumed that the stiffener depth is sufficient enough to cause proof stresses to be exceeded at the extreme fibres.

Failure is not necessarily implied by buckling across the stiffeners. If the increased effective panel size extends beyond the immediate stiffeners, the panel should be analysed using a larger panel defined by the next stiffeners in the structure. A long shear beam under constant shear with identical bays but inadequate stiffeners would collapse. A sufficiently reduced applied shear at adjacent bays in the beam would be the only way to allow buckling across the stiffeners in such a case.

In the past, there was an option in program APA107 (the program implementing S.O.R. 51) that allowed buckling across stiffeners. This contradicts S.O.R. 51 and its references and no theoretical basis for this assumption has been put forward. It has thus not been incorporated in APA115.

Fig.D4 is stored as data in the program. A precise analytical representation of this figure has not been found. However, comparisons with E.S.D.U. data sheets show a close representation can be derived which casts doubt on the accuracy of this figure. This is discussed in Appendix F.

D3.3 PANEL BUCKLING STRESS

The panel buckling coefficient is obtained from Fig.D5 (a representation of S.O.R. 51 figure 2) using the increased effective panel sizes. It may be the case that, whereas a was greater than b, b_e' could be greater than a_e' and the aspect ratio and buckling formula should exchange the panel dimensions to suit. The elastic buckling stress is:

$$\tau_{bo} = K \cdot E \cdot \left(\frac{t}{b_e'}\right)^2 \tag{4}$$

The plastic stress is determined by an iterative method using an equivalent direct stress equal to $\sqrt{3}\tau_{bo}$. The secant modulus ratio at this stress is used as a plasticity correction factor. Thus the plastic buckling stress is proportional to the secant modulus and is given by:

$$\tau_b = \frac{E_s}{E} \cdot \tau_{bo} = \frac{G_s}{G} \cdot \tau_{bo} \tag{5}$$

An upper limit of $f_2/\sqrt{3}$ is placed on the buckling stress (S.O.R. 51 originally used f_1 but most analyses use $f_2/\sqrt{3}$). The Ramberg-Osgood stress-strain curve parameters required for the calculation of the plasticity reduction are f_n and m, found from the proof stresses as follows:

$$m = \frac{\log\left(\frac{0.001}{0.002}\right)}{\log\left(\frac{f_1}{f_2}\right)}$$

$$f_n = f_1 \cdot \left(\frac{0.001 \cdot m \cdot E}{f_1}\right)^{\frac{1}{1-m}} \tag{6}$$

D3.4 ALLOWABLE WEB STRESS

There are several ways of presenting the allowable web stress depending on the location within the panel and the influence of the surrounding structure.

D3.4.1 Basic Allowable Web Stress

Fig.D6 (a representation of S.O.R. 51 figure 3f, taken largely from E.S.D.U.) provides allowables for the web away from the stiffeners and thus not allowing for edge member flexibility. This is based on empirical data and consequently there is no means of expressing it analytically. It gives the basic permanent buckling web stress and basic permissible web stress using the minimum value of A_{sa}/dt and A_{sb}/ht. Limited values of A_{sa}/dt and A_{sb}/ht are available at ∞, 1, 0.3. An interpolation between these values is difficult. An extrapolation is even more difficult. The procedure adopted is as follows. The A_{sa}/dt value is inverted to give 0, 1, 3.333 and a nominal value of 25 ($A_{sa}/dt = 0.04$) is given at the τ

= τ_b line. A linear interpolation is used with bt/A_{sa}. Originally, a linear interpolation with A_{sa}/bt down to the $\tau = \tau_b$ line was used but this was thought to be too conservative. The method adopted provides similar results to program APA107. Indeed, it was a requirement that the results of APA115 would not deviate greatly from the results of APA107. The allowable stresses are obtained from the curves as follows: τ_b/f_2 gives τ_b'/f_2 and thus τ_{pb}', τ_b/f_{ult} gives τ_{all}/f_{ult} and thus τ_{all}.
There is a further restriction on τ_{pb}' due to stiffener buckling where τ_{pb} is put equal to τ_b (see Section D3.4.2). In the case of flanged panels, failure is given by the lower curve in Fig.D6. This restriction would not have applied if buckling across stiffeners were allowed. This was the main attraction to providing this option in program APA107 (see Section 3.2) as it increases the apparent strength of the panel, sometimes significantly.

D3.4.2 Allowable Web Shear Stress allowing for Edge Member Flexibility

Fig.D7 (a representation of S.O.R. 51 figure 4a) gives the permanent buckling web stress and failure stress when edge member flexibility (but not end load) is included. It is derived with the aid of C_2, the variation of which is given in Fig.D2 (from N.A.C.A. TN2661 figure 16) and the value of diagonal tension factor, k_d (defined later in this section). The parameters w_d and w_h are based on edge member flexibility such that a value of zero implies a rigid edge member. Effectively, this means there is no edge member as such, but a continuous plate across a stiffener which provides very high in-plane stiffness.

edgemember S_a:
$$w_h = 0.7 \cdot h \cdot \sqrt[4]{\frac{t}{d \cdot (I_{ta} + I_{ca})}} \qquad (7)$$

edgemember S_b:
$$w_d = 0.7 \cdot d \cdot \sqrt[4]{\frac{t}{h \cdot (I_{tb} + I_{cb})}} \qquad (8)$$

In the case of two edge members, the minimum of w_d or w_h should be used. From Fig.D7, the allowable stresses are obtained as follows: τ_{pb}'/τ_b gives τ_{pb}/τ_{pb}' and thus τ_{pb}, τ_{all}/τ_b gives τ_{fail}/τ_{all} and thus τ_{fail}. Note Fig.D7 includes extra values of w_d at 5, 7, 10.
The equation deriving Fig.D7 is:

$$\frac{\tau_{fail}}{\tau_{all}} = \frac{1}{1 + k_d \cdot C_2} \qquad (9)$$

Where k_d is the diagonal tension factor given by:

$$k_d = \tanh\left[0.5 \cdot \log_{10}\left(\frac{\tau_{app}}{\tau_b}\right)\right] \qquad (10)$$

Fig.D7 is not applicable for flanged panels where τ_{fail} remains the same as τ_{all}.

D3.4.3 Allowable Web Stress at Panel Attachments

For fabricated panels, where the web to stiffener joints must be checked, Fig.D8 (a representation of S.O.R. 51 figure 4b) gives the allowable shear stress for particular values of joint allowables. It should be noted that this criterion is still web shear failure and not joint failure which is dealt with in the stiffener analysis.

The theoretical expression for Fig.D8 is:

$$\frac{r}{t} = \tau_{all} \cdot (1 + 0.414 \cdot k_d) \tag{11}$$

This is explicit for r/t but should be iterated to obtain a solution for τ_{all} based on a fixed r/t because k_d here is the factor at the shear stress τ_{all}, not at the nominal applied shear stress.

D3.4.4 Allowable Web Shear Stress including the Effects of Edge Member Stress

To allow for direct stress in the edge member the failure stress, τ_{fail}' and σ_{fail}' are obtained as follows, or from Fig.D9:

$$\begin{aligned} \frac{\tau_{fail}'}{\tau_{fail}} &= \frac{1}{\sqrt{1 + \left(\dfrac{\tau_{fail}}{l_{ult}} \dfrac{l_{app}}{\tau_{app}}\right)^2}} \\ \frac{\sigma_{fail}'}{\sigma_{fail}} &= \frac{1}{\sqrt{1 + \left(\dfrac{\tau_{fail}}{l_{ult}} \dfrac{l_{app}}{\tau_{app}}\right)^{-2}}} \end{aligned} \tag{12}$$

The reserve factors for the web may thus be quoted as follows:

$$\begin{aligned} RF_{proof} &= P_F \cdot \frac{\tau_{pb}}{\tau_{app}} \\ RF_{ultimate} &= \frac{\tau_{fail}}{\tau_{app}} \end{aligned} \tag{13}$$

where PF is the proof factor.

D4.0 STIFFENER ANALYSIS

The stiffener analysis remains basically the same as S.O.R. 51. In addition to the separate analysis for the stiffeners, buckling across the stiffeners must be considered (see Section D3.2).

Stiffener S_a will be used as an example, but stiffener S_b is similar with a',d, A_{sb}, A_{sb}', I_{sb}' replacing b', h, A_{sa}, A_{sa}', I_{sa}' respectively.

Against the different modes of stiffener failure must be related particular applied loads or stresses - average or maximum. These arise in dependence on the degree of shear buckling. In order to establish a reserve factor, it is thus necessary to find the value of the panel shear stress which causes the stiffener failure, even if this is beyond the panel allowable.

D4.1 AVERAGE STIFFENER STRESS

The average applied stiffener stress is related to the maximum applied stiffener stress according to the stiffener configuration. The maximum applied stiffener stress is to be compared with the lowest of the flexural instability and the local instability stresses, f_i. This should not exceed f_2. The average stiffener stress along the length of the stiffener is related to the applied shear stress.

The average stiffener stress, f_{av}, at a maximum stiffener stress equal to f_{max}, is found from Fig.D11 (a representation of S.O.R. 51 figure 8) except for all continuous stiffeners away from end bays and double stiffeners when the following expressions are used:

$$\text{for } \frac{h}{d}>1,\ f_{av}=\frac{f_{max}}{1+0.15\cdot\frac{h}{d}},\qquad \text{for } \frac{h}{d}\le 1,\ f_{av}=\frac{f_{max}}{1.15} \tag{14}$$

In this way, for a stiffener failure mode related to the maximum applied stress, the average stress in the stiffener is found. The shear stress required to generate this is found in the following section.

Note that two extra curves at h/d = 5 and 10 have been added to this figure. These are arbitrary curves which have been taken from program APA107 for compatibility.

D4.2 EQUIVALENT PANEL SHEAR STRESS

The average stress in the stiffener has been shown from tests to be proportional to the difference in the applied shear stress and the buckling stress. This has been expressed in a theoretical relationship which relates the stress to the value h/t (or d/t for stiffener S_b) together with the average stiffener stress coefficient, C_s, obtained from Fig.D10 (a represention of S.O.R. 51 figure 7).

$$f_{av}=C_s\cdot\frac{\tau-\tau_b}{\frac{A_{sa}'}{b'\cdot t}+\frac{s}{b'}} \tag{15}$$

where s is the smaller of 40·t and b'/1.5.

From this expression the shear stress at which instability occurs may be determined:

$$\tau_{inst} = \frac{f_{inst}}{C_s} \cdot \left[\frac{A_{sa}'}{b' \cdot t} + \frac{s}{b'} \right] + \tau_b \qquad (16)$$

Note that this directly solvable for τ_{inst} as none of the other parameters appear to depend on the applied shear stress, τ. This is contrary to the N.A.C.A. analysis where a non-linear relationship is used, requiring a solution by iteration. It is believed that the above method is an approximation to facilitate hand calculations.

D4.3 FLEXURAL INSTABILITY

The effective strut length for flexural instability, l_e, is taken from Fig.D12 (a representation of S.O.R. 51 figure 9). For single stiffeners or double stiffeners with ends fixed, this is always 0.5·l. Single stiffeners are thus considered to have the same effective length as a clamped strut under pure end load. No eccentricity is considered. The flexural instability stress is obtained from:

$$\frac{f_{fi}}{E} = \left[\pi \cdot \frac{k_s}{l_e} \right]^2 \qquad (17)$$

where k_s, the radius of gyration of the stiffener plus the effective skin about the combined neutral axis, is given by:

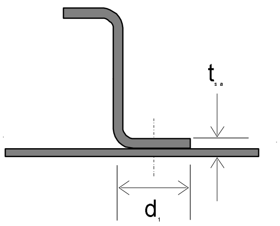
$$k = \sqrt{\frac{I_{xsa} - \frac{A_{sa}{}^2 \cdot y_{sa}{}^2}{A_{sa} + s \cdot t}}{A_{sa} + s \cdot t}} \qquad (18)$$

Plasticity correction is made with values of f_n and m calculated as in Section D3.3 (using different stiffener properties where appropriate) to give the flexural instability stress, f_{fi}.

D4.4 LOCAL INSTABILITY

Local instability may occur in several different modes in riveted or bolted stiffeners. In an integrally machined or reduxed panel, the modes are limited to a free flange mode and maybe a free flange mode of combined flange and web.

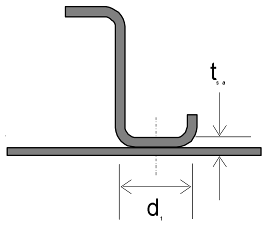
Three examples are given. For an unlipped stiffener attachment flange of length d_1:



The combined flange and web give:

$$f_{lif} = 0.58 \cdot E \cdot \left(\frac{t_{sa}}{d_1} \right)^2 \qquad (19)$$

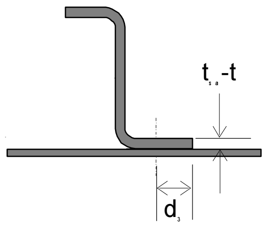
For the same flange with a lip:



The expression is modified:

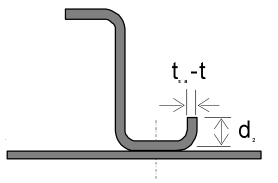
$$f_{lif} = 3.62 \cdot E \cdot \left(\frac{t_{sa}}{d_1} \right)^2 \qquad (20)$$

For a free flange width d_3 and thickness t_{sa} - t, such as that part of the stiffener flange protruding beyond a rivet line when there is no flange lip:



$$f_{lil} = 0.58 \cdot E \cdot \left(\frac{t_{sa} - t}{d_3} \right)^2 \qquad (21)$$

For a flange with a lip:



The local instability is given by the lesser of the following:

$$f_{lil} = 3.62 \cdot E \cdot \left(\frac{t_{sa} - t}{d_3} \right)^2 \qquad (22)$$

$$f_{lil} = 0.58 \cdot E \cdot \left(\frac{t_{sa} - t}{d_2}\right)^2 \qquad (23)$$

The minimum local instability stress, f_{li}, is found from the above stresses. This is corrected for plasticity by the tangent modulus using f_n and m as normal.

D4.5 INTER-ATTACHMENT BUCKLING

For inter-rivet buckling where p is the rivet pitch and t is the minimum thickness of stiffener flange or panel, the stability criterion applied is:

$$f_{ir} = K \cdot E \cdot \left(\frac{t}{p}\right)^2 \qquad (24)$$

The value of K depends on the type of rivets or bolts:

For bolted joints	K = 3.29
For spot welds	K = 2.88
For mushroom head rivets	K = 2.47
For countersunk or dimpled rivets	K = 1.23

This is corrected for plasticity by the tangent modulus using f_n and m as normal.

D4.6 COMBINED INTER-ATTACHMENT BUCKLING AND LOCAL INSTABILITY

When the inter-attachment buckling stress is less than the local instability stress, an "average" stress is taken as follows:

$$f_i = \frac{f_{li} \cdot A_{sa}' + f_{ir} \cdot s \cdot t}{A_{sa}' + s \cdot t} \qquad (25)$$

D4.7 FORCED CRIPPLING

Forced crippling of the stiffener is the result of shear buckles progressing through the stiffener attachment flange. In S.O.R. 51 this effect is considered for both riveted and bonded stiffeners. This applies regardless of the thickness of the attached flange and must therefore also be considered applicable for integral blades with or without integral pads underneath.

Integrally machined stiffeners could be considered equivalent to bonded stiffeners, but it was felt that, as such stiffeners could not separate from the panel, thus forced crippling is inappropriate. A similar approach could be suggested for bonded stiffeners if the bonding is very strong.

For riveted or bolted stiffeners, the forced crippling failure has been empirically determined and related to the value of k_{fc} obtained from Fig.D13 (representing S.O.R. 51 figure 12). The shear stress at forced crippling is:

$$\tau_{fc} = k_{fc} \cdot f_2 \cdot \left(\frac{A^*_{sa}}{b' \cdot t} + \frac{s}{b'} \right) \tag{26}$$

where A^*_{sa} is twice the total area of flanges or A_{sa} if less.

For lipped flanges where the lip is not less than five times the flange thickness, τ_{fc} is multiplied by 1.17. For closed section stiffeners, τ_{fc} is multiplied by 2.

Forced crippling only occurs after shear buckling has started and thus τ_{fc} cannot be lower than τ_b.

The equivalent compressive stress in the stiffener at the onset of forced crippling is:

$$f_{av} = C_s \cdot \frac{\tau_{fc} - \tau_b}{\frac{A_{sa}'}{b' \cdot t} + \frac{s}{b'}} \tag{27}$$

D4.8 TORSIONAL INSTABILITY

The average torsional stability stress, f_{ti}, is given by the following expression:

$$\frac{f_{ti}}{E} = \frac{1}{I_p} \cdot \left[J \cdot \frac{G}{E} + \Gamma \cdot \left(\frac{\pi}{h} \right)^2 \right] \tag{28}$$

In the above, E/G may be replaced by 2.6 for μ = 0.3. For plain stiffeners on integrally machined panels without bulbs or lips the warping constant is zero. This is a pessimistic expression which assumes that cleats do not provide axial restraint.

The torsional instability stress is corrected for plasticity using f_n and m as normal.

The equivalent elastic shear stress, τ_{ti}, at which instability occurs is then obtained as follows:

$$\tau_{ti} = \frac{f_{ti}}{C_s} \cdot \left(\frac{A_{sa}'}{b' \cdot t} + \frac{s}{b'} \right) + \tau_b \tag{29}$$

Open section cleated stiffeners are restrained axially at the cleats, halving the effective length, and the torsional instability stress is given by the modified equation:

$$\frac{f_{ti}}{E} = \frac{1}{I_p} \cdot \left[J \cdot \frac{G}{E} + \Gamma \cdot \left(\frac{2 \cdot \pi}{h} \right)^2 \right] \tag{30}$$

This, however, is not currently incorporated in the analysis.

D4.9 STIFFENER TO EDGE MEMBER ATTACHMENT

This attachment fails at a stiffener stress equal to:

$$f_{se} = \frac{P_{se}}{A_{sa}'} \tag{31}$$

The equivalent elastic shear stress, τ_{se}, at which failure occurs is then obtained as follows:

$$\tau_{se} = \frac{f_{se}}{C_s} \cdot \left(\frac{A_{sa}'}{b' \cdot t} + \frac{s}{b'} \right) + \tau_b \tag{32}$$

D4.10 STIFFENER TO WEB ATTACHMENT

The joint between the stiffener attachment and the web must meet the following strength requirement:

$$\text{Allowable running load} = 0.15 \cdot t \cdot f_{ult} \tag{33}$$

This is *not* an applied load. The attachments are designed to match the strength of the web.

D4.11 WEB UNDER CLOSED SECTION STIFFENER

The instability stress of that portion of the web under a closed section stiffener of width h_p is:

$$\frac{f_{lic}}{E} = 3.62 \cdot \left(\frac{t}{h_p} \right)^2 \tag{34}$$

This is corrected for plasticity using f_n and m as normal. It is not currently included in the analysis.

D4.12 EFFECT OF APPLIED DIRECT STRESS IN STIFFENER

Using the most critical instability or failure mode, allowance may be made for the applied direct stress in the stiffeners using Fig.D14 (representing S.O.R. 51 figure 13) or by iteration of the following equation:

$$\frac{\tau}{f} \cdot \frac{f_{app}}{\tau_{app}} = \frac{f_{fail}}{f_{app}} \cdot \left[1 - \frac{f_{fail}}{f_{all}} \right]^{-\frac{2}{3}} \tag{35}$$

where τ and f are the minimum stresses from the instability calculations and τ_{app} and f_{app} are the applied stresses. The stress, f_{fail}, should include shear and direct stress contributions. The original report didn't make this clear.

D5.0 EDGE MEMBER ANALYSIS

The edge member loads are determined using relationships from Ref.2. The diagonal tension factor, k_d, is required from equation (10). It will be assumed that S_b is an edge member as an example, but stiffener S_a is similar with b', h, A_{sa}, A_{sa}', I_{sa}' replacing a', d, A_{sb}, A_{sb}', I_{sb}' respectively.

The edge members are assumed heavy in this analysis. Consequently stability is not considered. Should stability be expected to be a problem, the edge member should be considered in the same manner as the stiffener.

D5.1 EDGE MEMBER PROPERTIES

The edge members are analysed as an extension to the stiffener analysis. The edge member is loaded by the tension field in compression and bending by half the panel. An effective section is required about an axis normal to the panel plane. The section taken is that due to the edge member itself plus 25t of the panel about the combined neutral axis.

D5.2 EDGE MEMBER DIRECT STRESS

The edge member direct stress is obtained as follows:

$$I_{ed} = \alpha \cdot \tau_{app} \qquad (36)$$

where α is found from Fig.D15 (representing S.O.R. 51 figure 14a) or alternatively from:

$$\alpha = \frac{k_d}{\frac{2}{d \cdot t} \cdot \left(A_{sb} + 25 \cdot t^2\right) + 0.5 \cdot \left(1 - k_d\right)} \qquad (37)$$

D5.3 EDGE MEMBER BENDING MOMENT

The edge member bending moment is obtained as follows:

$$M_e = \beta \cdot \tau_{app} \cdot t \cdot d^2 \qquad (37)$$

where β is found from Fig.D16 (representing S.O.R. 51 figure 14b) or alternatively from:

$$\beta = 0.054 \cdot \sqrt{k_d} \qquad (38)$$

D5.4 EDGE MEMBER STRESSES

The derivation of stresses in the edge member is somewhat arbitrary, as with the other analyses in program APA115. The main influence of the assumptions made are reflected in the panel area to be taken in conjunction with the edge member. This affects its contribution to the inertia and to the maximum fibre distance. Also, the theory is based on edge members whose dimensions are large in comparison with the panel thickness. In such a case, the edge member acts in more of an independent manner.

The assumption used here is that the panel associated with the stiffener, c, is the attached flange width (if any) plus 25t. This is considered to be located centrally under the edge member, adding a degree of conservatism to the analysis. The complete reinforced thickness of the panel is also added. Consistent with the net inertia, the extreme fibre distances are based on 12.5t and the attached flange size.

For reversed I,J,T sections:

$$\begin{aligned} y_{outer} &= \left|\min\left(-12.5 \cdot t, -0.5 \cdot b_a - x_s\right)\right| \\ y_{inner} &= \left|\max\left(-12.5 \cdot t, -0.5 \cdot b_a - x_s\right)\right| \\ y_{centre} &= \left|-x_s\right| \\ y_{rivet} &= \left|p - x_s\right| \end{aligned} \tag{40}$$

For other sections:

$$\begin{aligned} y_{outer} &= \left|\min\left(-12.5 \cdot t, -x_s\right)\right| \\ y_{inner} &= \left|\max\left(12.5 \cdot t, 0.5 \cdot b_a - x_s\right)\right| \\ y_{centre} &= \left|-x_s\right| \\ y_{rivet} &= \left|p - x_s\right| \end{aligned} \tag{41}$$

Here, fibres on the outermost and innermost fibres provide the highest stresses and the stress at the stiffener centre line and the attachment line are quoted for reference. It has been assumed that the attached flange on the edge member is orientated inwards towards the centre of the panel.

The derivation of the fibre stress comes from the externally applied stress plus the induced end-load stress and bending stress due to diagonal tension, due regard being made to the signs of all components:

$$\sigma = \sigma_{app} + f_{ej} + \frac{M_e \cdot y}{I} \tag{42}$$

This seems to be the best general approach that can be made for the edge member analysis. Not all cases will be suited by this method, in which case some engineering judgement is required.

APA115 has included an ALTER option for the user to select particular fibre offsets that overwrite the above values.

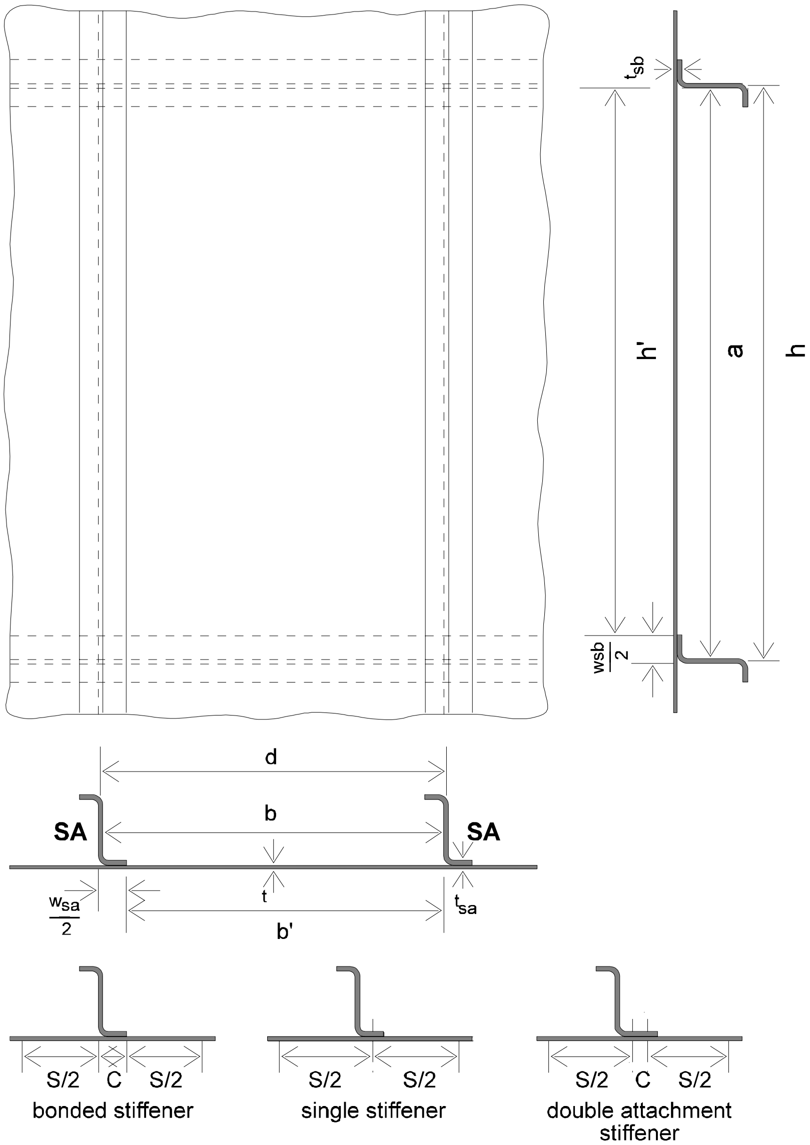


Fig.D1a: Panel Geometry for Z-Sections, SOR51 Notation

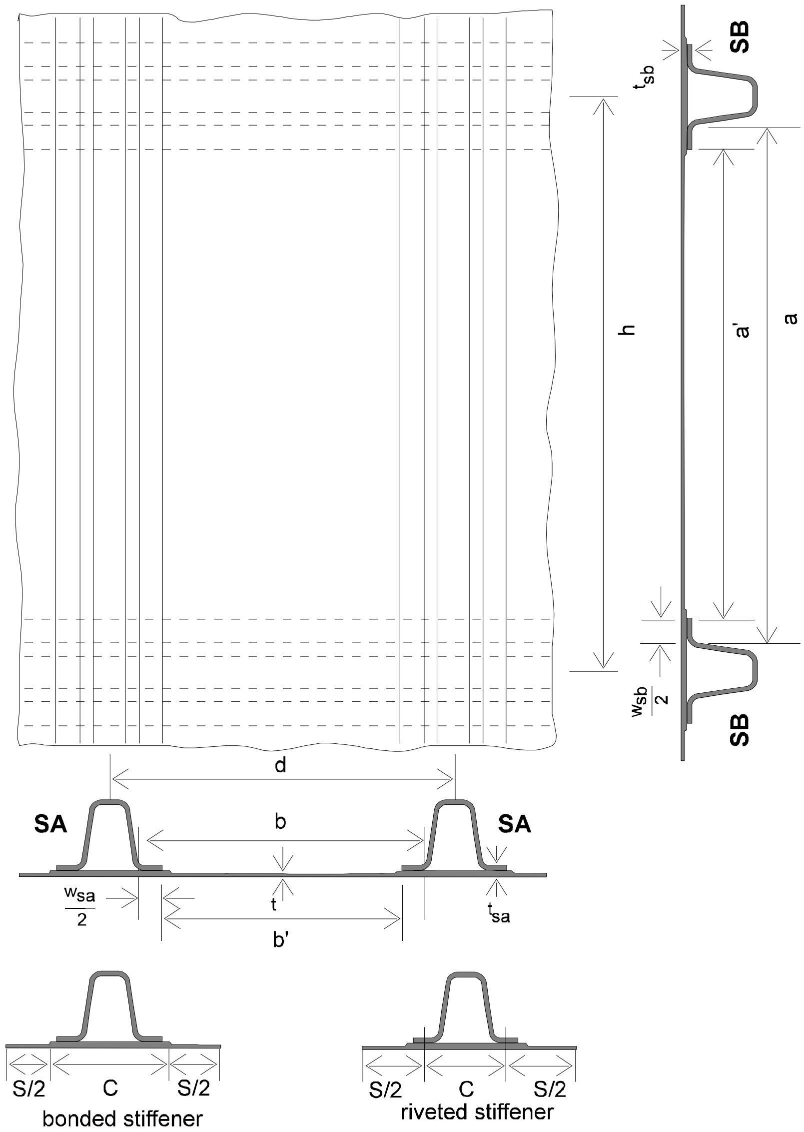


Fig.D1b: Panel Geometry for Hat Sections, SOR51 Notation

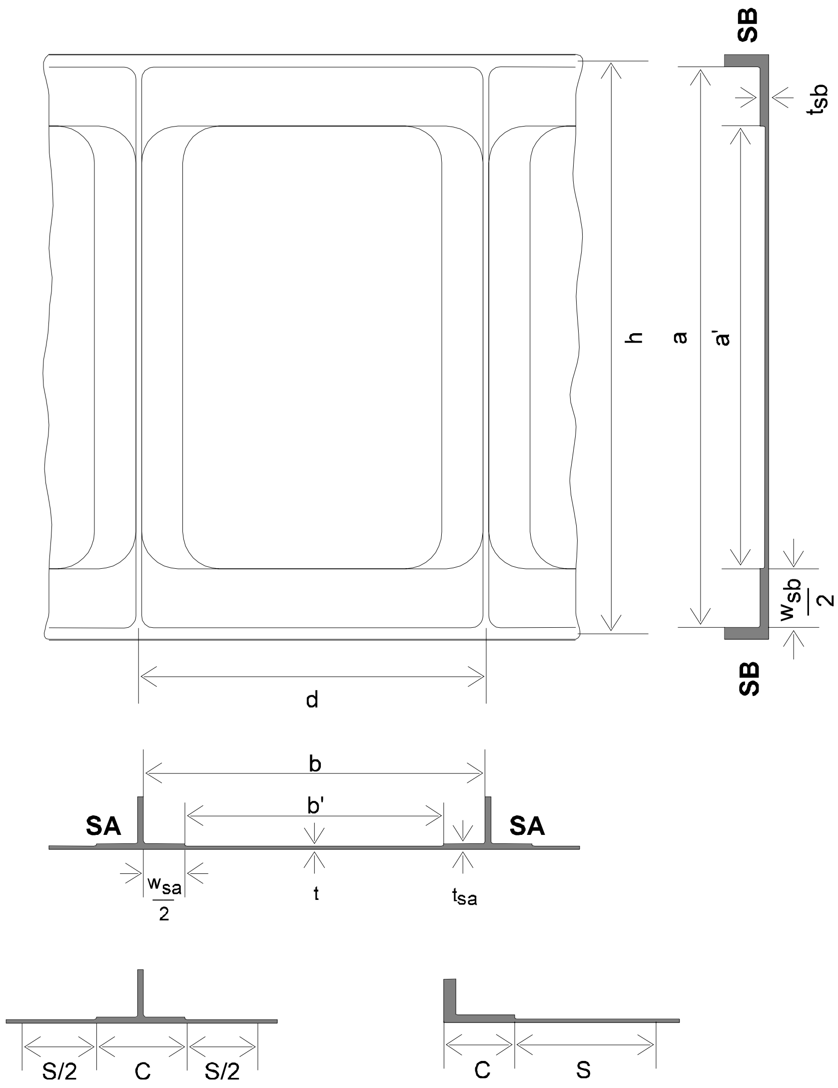


Fig.D1c: Panel Geometry for Integral Panels, SOR51 Notation

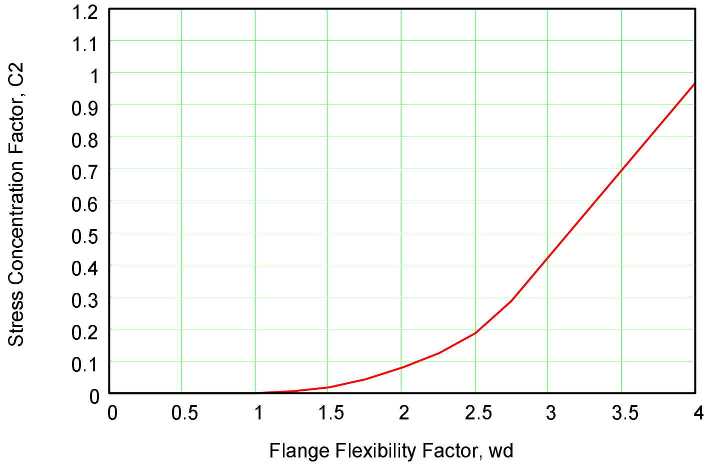


Fig.D2: NACA TN2661 Figure 18 - Stress Concentration Factors C2

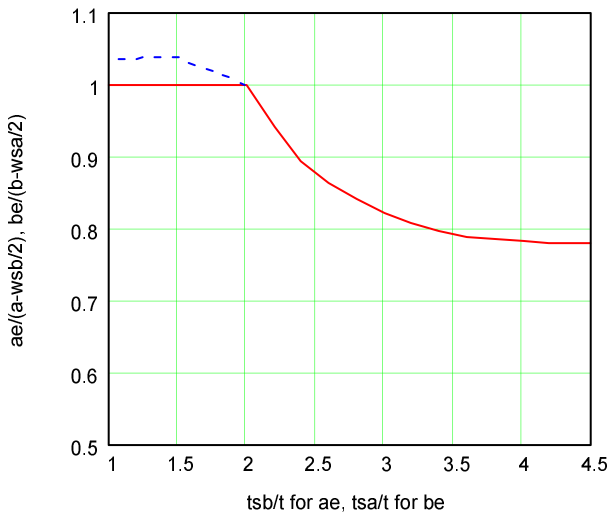


Fig.D3: SOR51 Figure 1a - Effective Panel Size for Buckling Stress
(The dashed line is a modification to APA107 such that a_e = a at t_{sb}/t = 1, b_e = b at t_{sa}/t = 1)

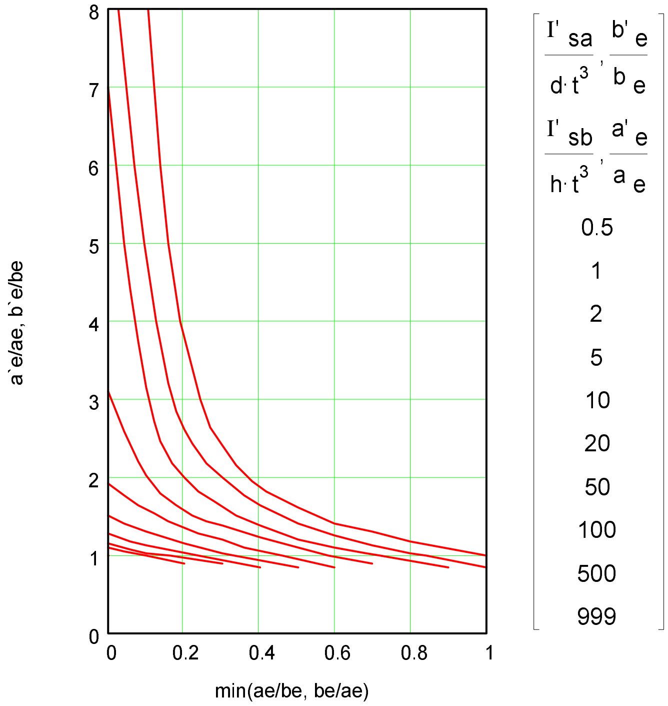


Fig.D4: SOR51 Figure 1c - Increased Effective Panel Size due to Stiffener Flexibility

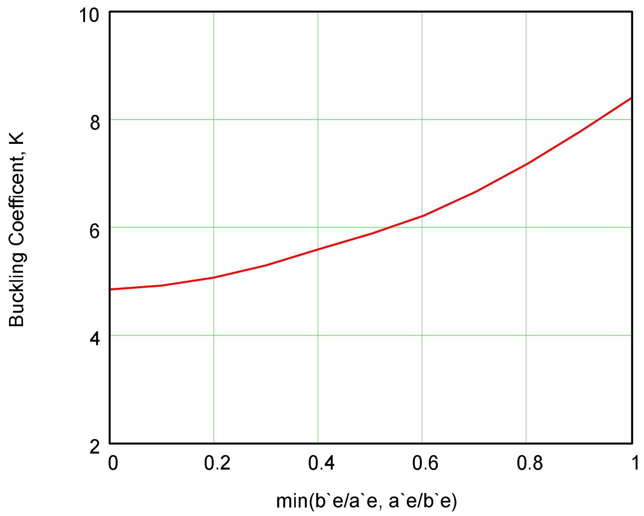


Fig.D5: SOR51 Figure 2c - Panel Buckling Stress Coefficient

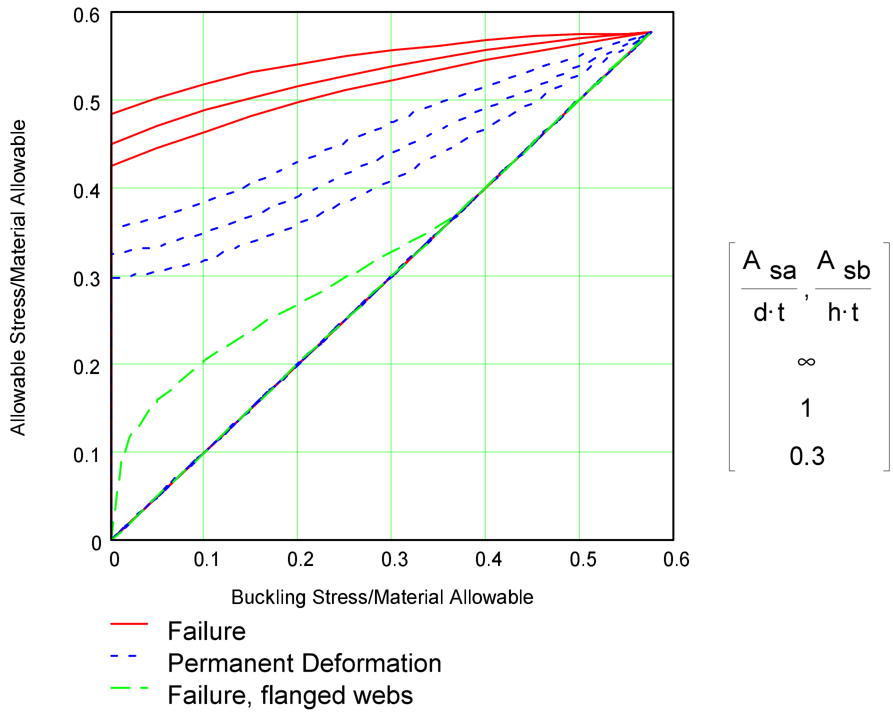


Fig.D6: SOR51 Figure 3f - Allowable Web Stress

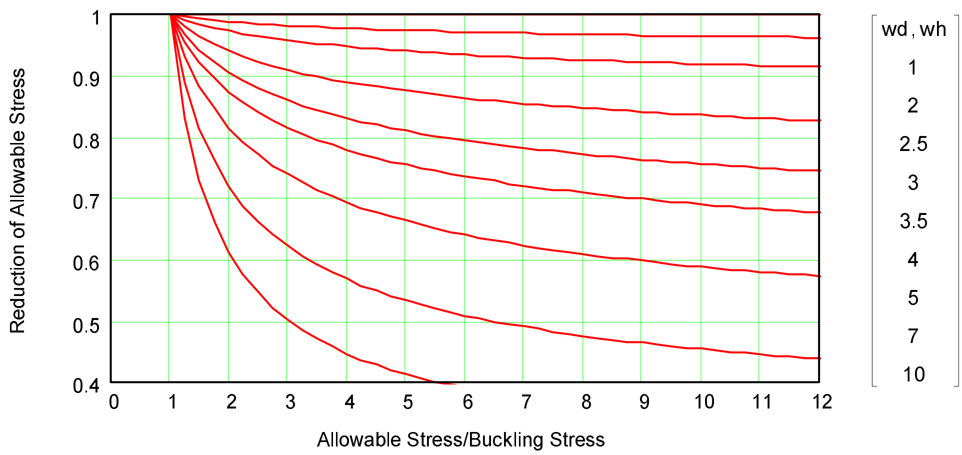


Fig.D7: SOR51 Figure 4a - Effect of Edge Member Flexibility on Allowable Web Stress

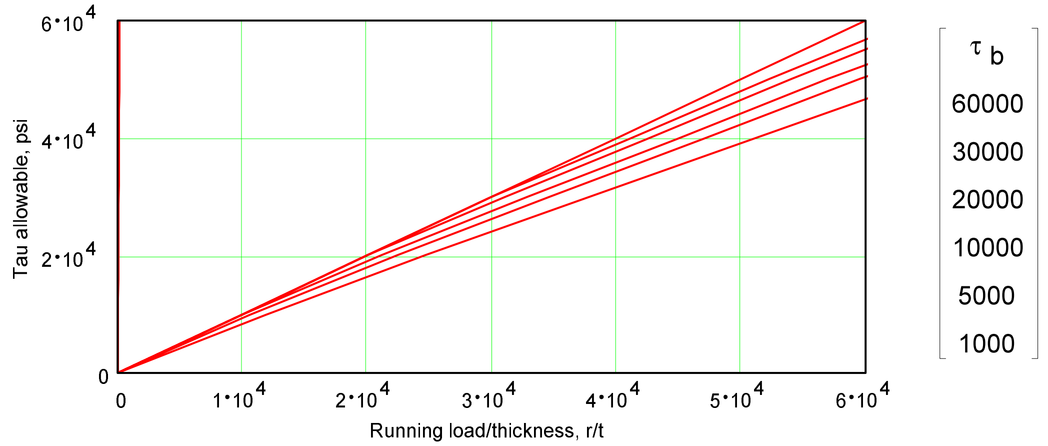


Fig.D8: SOR51 Figure 4b - Allowable Shear Stress Based on Joint Strength

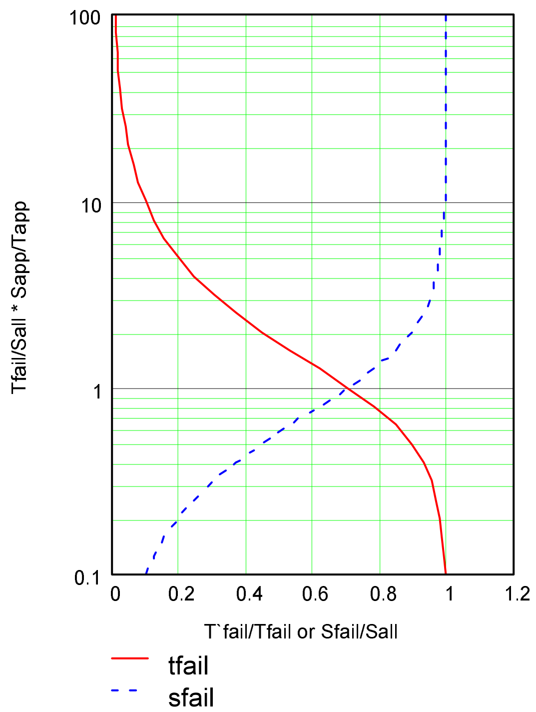


Fig.D9: SOR51 Figure 5 - Combined Shear and Direct Stress Based at Edge Member

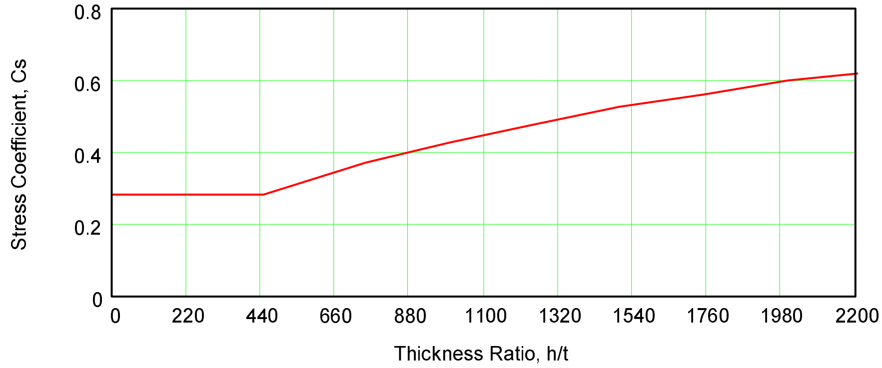


Fig.D10: SOR51 Figure 7 - Average Stiffener Stress Coefficient

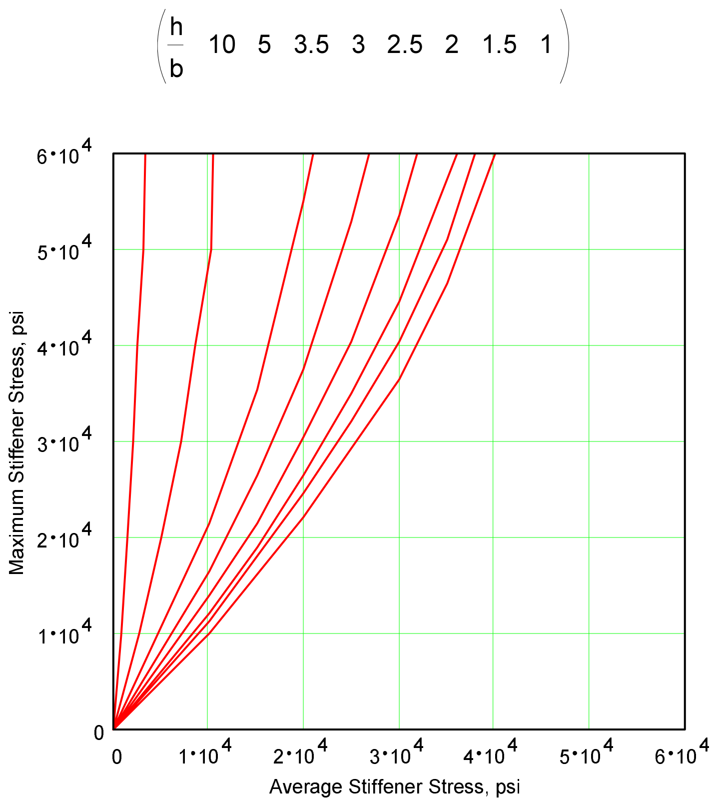


Fig.D11: SOR51 Figure 8 - Maximum Stiffener Stress
(This contains two extra curves at h/b = 5 and 10 - arbitrary curves from program APA107)

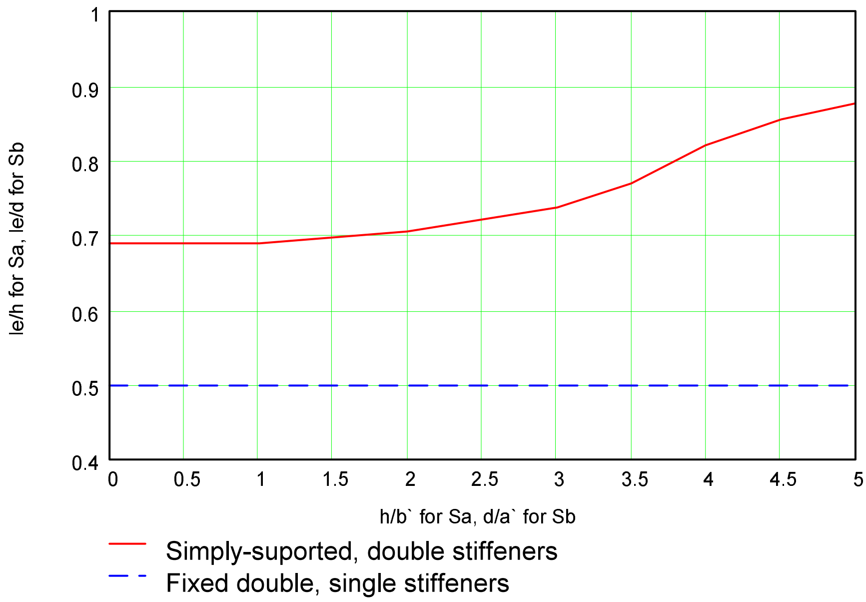


Fig.D12: SOR51 Figure 9 - Effective Strut Length

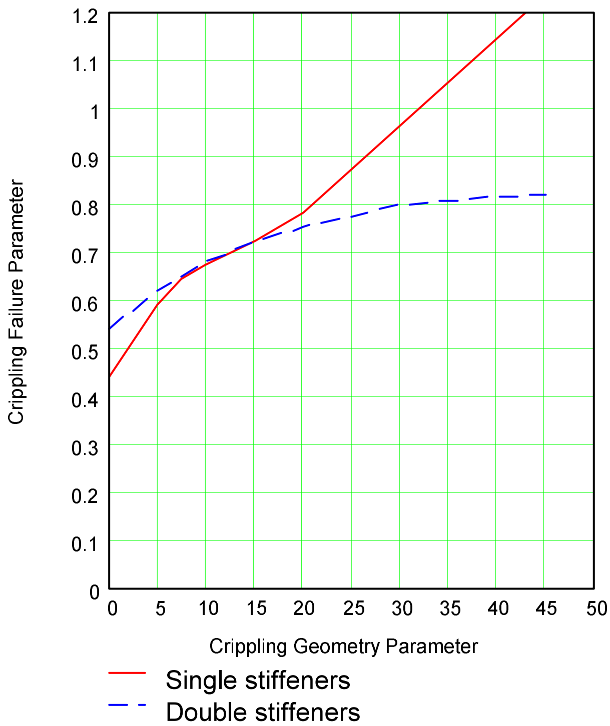


Fig.D13: SOR51 Figure 12 - Forced Crippling Failure of Stiffener

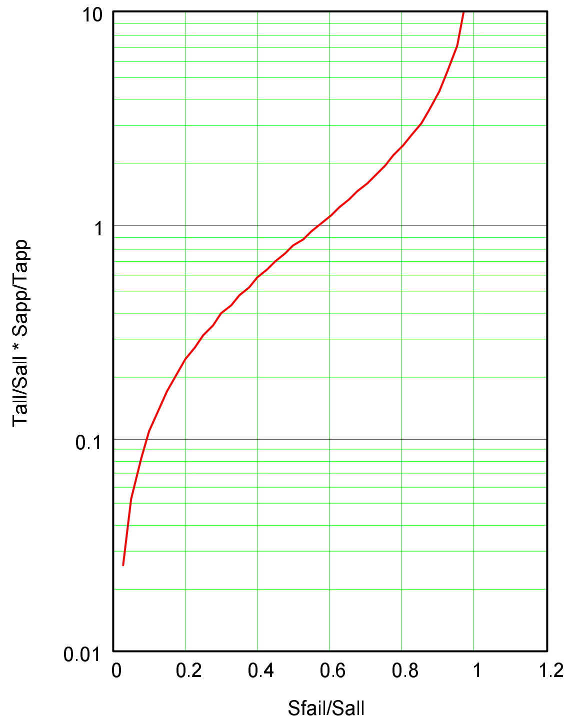


Fig.D14: SOR51 Figure 13 - Combined Shear and Direct Stress on Stiffener

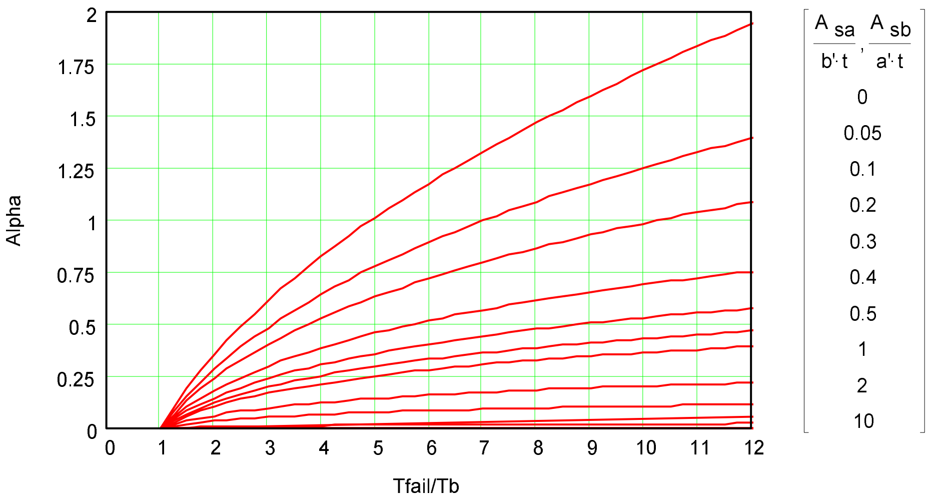


Fig.D15: SOR51 Figure 14a - End Load Coefficient for Edge Member

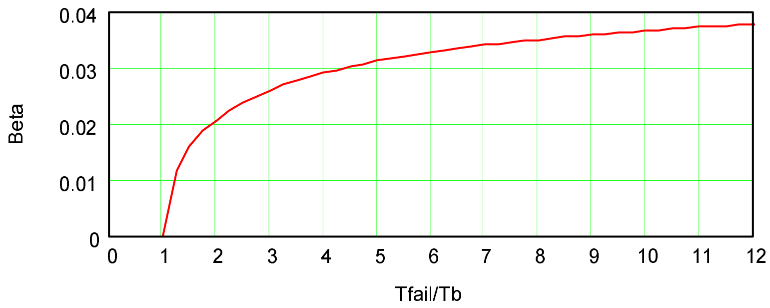


Fig.D16: SOR51 Figure 14b - Bending Moment Coefficient for Edge Member

APA115 Volume 3, Appendix E: APA107 SHEAR ANALYSIS

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E1.0 INTRODUCTION

This appendix gives a description of APA115's implementation of the flat shear panel analysis of Hatfield Report S.O.R. 51. In order to obtain direct comparisons, the implementation presents the output in a similar format to program APA107. However, it is *not* the same program code. Neither does it contain any part of the APA107 code. It has been assembled from the subroutines of a privately-written version of program FH101A (the pre-cursor to APA107) dating back to 1987.

Appendix D gives a full description of the S.O.R. 51 theoretical analysis. It also includes details of some of the modifications that were incorporated into APA107 to allow for some geometric aspects not covered by S.O.R. 51. This appendix basically covers the differences in notation and presentation with a brief description of the meaning of the output.

The S.O.R. 51 analysis implemented by APA115 starts with the main controlling subroutine called SOR51.FOR. This calls three separate subroutines to analyse panels (SOR51P.FOR), stiffeners (SOR51S.FOR) and edge members (SOR51E.FOR). The description of the analysis in this appendix follows the sequence of procedures in these subroutines.

Input to APA107 deviates from the notation of S.O.R. 51 in some instances. In particular, the stiffener area in APA107 does not include any contribution from the skin and the position of the neutral axis is related to the mid-plane of the web whereas APA107 relates it to the interface between stiffener and web.

The main subroutine uses the basic input to APA115 plus some of the generated data related to stiffener section properties - A, A_s x_s, y_s, I_{xs}, I_{ys}, J_s, Γ, I_{es} (using the notation of the main text). In this way, the input and output can be presented in the familiar style of program APA107.

E2.0 GENERATION OF INPUT DATA

This section provides for a comparison between the different notations for the standard input for APA115 and the APA107 input.

The notation used for the input data to the S.O.R. 51 analysis corresponds to that adopted for B.Ae program APA107 and is broadly in line with the S.O.R. 51 report. The notation for APA107 is in upper case but may conflict with that for APA115. Therefore, the notation for APA115 will be converted to italics for this appendix only. In APA115, the dimensions h, d are stored as h_1, h_2 and a, b are stored as a_1, a_2. All parameters referring to stiffener S_a are suffixed -$_1$ and all for stiffener S_b are suffixed -$_2$.

The dimension, H, of stringer S_a corresponds to the panel dimension, A. The dimension, D, of stringer S_b corresponds to the panel dimension, B. For simplicity, only stiffener S_a will be described.

E2.1 PANEL ANALYSIS

E2.1.1 Panel Size, Stiffener Pitch and Flange Codes

This corresponds to the P1 card in APA107. The panel size may be the distance between inside webs or the distance between rivets:

Bonded or blade stiffeners: $A = h_1 - t_{w1}$ $B = h_2 - t_{w2}$

Reversed stiffeners or I-, J-, T-sections:	$A = h_1 - 2 \cdot p_1$	$B = h_2 - 2 \cdot p_2$
C-, L-, Z-sections:	$A = h_1$	$B = h_2$

The stiffener pitches are the same as the APA115 panel dimensions:

All stiffener types	$H = a_1$	$D = a_2$

The flange codes, denoted as KA, KB to avoid confusion with w_{sa}, w_{sb} in S.O.R. 51, are as follows:

Bonded or blade stiffeners	KA = 1	KB = 1
Double stiffeners/attachments, I-, J-, T-sects	KA = 0	KB = 0
All other types	KA = -1	KB = -1

E2.1.2 Panel Thickness and Stiffener Pad

This corresponds to the P2 card in APA107. The panel thickness, T, remains as t. The combined panel thickness plus attached flange thickness is:

All stiffener types	$TSA = t_{a1} + t_{p1}$	$TSA = t_{a1} + t_{p1}$

The web width working directly with the stiffener is:

Bonded stiffeners	$CA = b_{a1}$	$CB = b_{a2}$
Blade stiffeners	$CA = t_{w1}$	$CB = t_{w2}$
I-, J-, T-sections	$CA = 2 \cdot p_1$	$CB = 2 \cdot p_2$
Double stiffeners	$CA = b_{a1}$	$CB = b_{a2}$
C-, L-, Z-sections	$CA = 0$	$CB = 0$

E2.1.3 Basic Stiffener Constants

This corresponds to the P3 card in APA107. The constants correspond to APA115 but the centroid position is referred to the interface of the stiffener and panel:

All stiffener types	$ASA = A_{s1}$	$ASB = A_{s2}$
All stiffener types	$YSA = y_{ss} - t_{ps} + \frac{1}{2} \cdot t$	$YSB = y_{sf} - t_{pf} + \frac{1}{2} \cdot t$
All stiffener types	$ISA = I_{xs1}$	$ISB = I_{xs2}$

E2.1.4 Panel Material Properties

This corresponds to the P4 card in APA107. The material properties are derived using the Ramberg-Osgood formulae:

$$F1P = f_{np} \cdot \left[\frac{m_p \cdot 0.001 \cdot E_p}{f_{np}} \right]^{\frac{1}{m_p}} \qquad F2P = f_{np} \cdot \left[\frac{m_p \cdot 0.002 \cdot E_p}{f_{np}} \right]^{\frac{1}{m_p}}$$

The Young's modulus, EP, remains the same as E_p and the ultimate stress, FULTP, is the same as the allowable stress, f_{allp}, from APA115.

E2.1.5 Applied Loads, Joint Allowables and Proof Factor

This corresponds to the P5 card in APA107. The proof factor, PF, is internally set to 1.5. The applied shear per unit length is based on the side a, or along the length of the S_a stiffener:

$$SPI = \frac{P_Q}{h_1}$$

The strength of the joints are:

For all stiffener types:

$$RPJ = \frac{P_{sult1}}{s_1} \qquad REJ = \frac{P_{sult2}}{s_2}$$

E2.1.6 Edge Member Section Constants

This corresponds to the P6 card in APA107. The panel is assumed to be central under the stiffener. This is conservative for true edge members and really represents a member in the middle of a panel. The panel associated with the edge member is the minimum of 25·t and a/6:

$$ICA = I_{ys1} + \min(25 \cdot t, \frac{a_2}{6})^3 \cdot \frac{t}{12} + b_{p1}{}^3 \cdot \frac{t_{p1} - t}{12}$$

$$ICB = I_{ys2} + \min(25 \cdot t, \frac{a_1}{6})^3 \cdot \frac{t}{12} + b_{p2}{}^3 \cdot \frac{t_{p2} - t}{12}$$

The tension members are given the same values, i.e. ITA = ICA, ITB = ICB.

E2.2 STIFFENER ANALYSIS

E2.2.1 Panel Size, Stiffener Pitch and Flange Codes

This corresponds to the S1 card in APA107. For stiffener S_a, the parameters W, L, WD, WS are the same as D, H, B, KA from the panel analysis.

E2.2.2 Panel Thickness and Stiffener Pad

This corresponds to the S2 card in APA107. For stiffener S_a, the parameters T, TS, C are the same as T, TSA, CA.

E2.2.3 Basic Stiffener Constants

This corresponds to the S3 card in APA107. For stiffener S_a, the parameters AS, YS, IS are the same as ASA, YSA, ISA.

E2.2.4 Panel Material Properties

This corresponds to the S4 card in APA107. The parameters F1, F2, FULT, E are the same as F1P, F2P, FULTP, EP.

E2.2.5 Applied Loads, Joint Allowables and Proof Factor

This corresponds to the S5 card in APA107. SPI is the same as for the panel analysis.

E2.2.6 Stiffener Detail Dimensions

This corresponds to the S6 card in APA107. D1 is the width of flange, from heel to free edge. D2 is the with of stiffener flange lip. D3 is the free width of the stiffener flange, from attachment to free edge. For all sections covered by APA115, D2 = 0.

For I-, J-, T-sections

$$D1 = \frac{1}{2} \cdot (b_a - t_w)$$

$$D3 = \frac{1}{2} \cdot (b_a - p)$$

For C-, L-, Z-sections

$$D1 = b_a$$

$$D3 = b_a - p$$

The code for the stiffener, SF, is 2 for a double stiffener (recognised by the initial 'D' in the stiffener section type) and 1 for a single stiffener. The effective skin width under a closed section stiffener is zero for all APA115 stiffeners.

E2.2.7 Stiffener Material Properties and Applied Stress

This corresponds to the S7 card in APA107. The material properties are derived using the Ramberg-Osgood formulae:

$$F1S = f_{ns} \cdot \left[\frac{m_s \cdot 0.001 \cdot E_s}{f_{ns}} \right]^{\frac{1}{m_s}} \qquad F2S = f_{ns} \cdot \left[\frac{m_s \cdot 0.002 \cdot E_s}{f_{ns}} \right]^{\frac{1}{m_s}}$$

The Young's modulus, ES, remains the same as E_s and the ultimate stress, FULTS, is the same as the allowable stress, f_{alls}, from APA115. The applied stress is simply:

$$SAPPS = \frac{P_s}{A_s}$$

E2.2.8 Stiffener Attachment

This corresponds to the S8 card in APA107. The pitch, PP, is the same as *p*. The attachment types for APA107 are a different notation to APA115. They correspond as follows:

APA115: 'flat head' — APA107: '   bolts'
APA115: 'mush head' — APA107: ' mh.rivet'

APA115: ' csk head' APA107: ' cskrivet'
APA115: ' integral' APA107: ' '

Note, there is no equivalent to the APA107 'spotweld' or 'dimrivet'

E2.2.9 Stiffener Type and Flange Details

This corresponds to the S9 card in APA107. The stiffener types are:

Double stiffeners	STFL = 'ss.dble'
Single stiffeners	STFL = 'single'

The flange width and number of flanges are:

I-, J-, T-sections	$WSP = b_a$	NF = 2
C-, L-, Z-sections	$WSP = b_a + \frac{1}{2} \cdot t_w$	NF = 1

E2.2.10 Stiffener Compound Section Constants

This corresponds to the S10 card in APA107. The constants are calculated directly by APA115:

$$TC = J_s \qquad WC = \Gamma \qquad PMI = I_{es}$$

E2.2.11 Stiffener Attachment Strengths

This corresponds to the S11 card in APA107. These are the same as for the panel analysis, i.e. PSE = RPJ and REJ = RPJ.

E2.3 EDGE MEMBER ANALYSIS

E2.3.1 Panel Size

This corresponds to the E1 card in APA107. For edge member S_b, the parameters L, WP are the same as D, H from the panel analysis.

E2.3.2 Edge Member Dimensions

This corresponds to the E2 card in APA107. For edge member S_b, the parameters T, C, AS are the same as T, CA, ASA from the panel analysis.

E2.3.3 Edge MemberSecion Constants

This corresponds to the E3 card in APA107. For edge member S_b, the parameters IC, IT are the same as ICA, ITA from the panel analysis.

E2.3.4 Applied Load and Buckling Stress

This corresponds to the E4 card in APA107. SPI is the same as for the panel analysis.

E2.3.5 Compression Member Load and Geometry

This is the same as card E5 in APA107. The applied stress is:

$$SAPPE = -\left|\frac{P_s}{A_s}\right|$$

The fibre distances are as follows:

$$YUOC = \min(12.5 \cdot t, -b_a - x_s)$$
$$YUIC = \min(12.5 \cdot t, b_a - x_s)$$
$$YLOC = -x_s$$
$$YLOC = p - x_s$$

E2.3.6 Tension Member Load and Geometry

This is the same as card E6 in APA107. The applied stress is:

$$FAPPE = \left|\frac{P_s}{A_s}\right|$$

The fibre distances are as follows:

$$YUOT = \min(12.5 \cdot t, -b_a - x_s)$$
$$YUIT = \min(12.5 \cdot t, b_a - x_s)$$
$$YLOT = -x_s$$
$$YLOT = p - x_s$$

APA115 Volume 3, Appendix F: COMPARISON OF ANALYSES

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F1.0 INTRODUCTION

This appendix compares the three methods available in the operation of program APA115. Additionally, reference is made to program APA107 - an implementation of report S.O.R. 51 with some modifications and other differences.

F2.0 INITIAL PANEL BUCKLING

The initial panel buckling stress may be obtained from several sources, the type of analysis depending on the particular configuration of the panel and the degree of complexity used for the idealisation of the edge conditions.

The panel for analysis needs to be clearly defined. A panel with no adjacent panels capable of transferring shear at its edges is clearly defined. Its behaviour as a shear panel is limited by its edge members. If these should buckle or fail, the structure is usually inadequate. A nominal panel which is bounded by light stiffeners and part of larger panel could be viewed as an individual panel if its edges are adequately restrained. This usually requires that the stiffeners provide simple-support under the applied load. If this support is not provided, buckling occurs across the stiffeners and the boundaries defining the panel are no longer valid for the purposes of the analysis. The larger panel may be more suitable.

In some structures, engineering judgement may be appropriate to decide that a panel is adequately supported, or isolated from the adjacent structure by its edge members. In this case, the panel is considered as a single panel and an initial assumption of simple-support may be sufficient. However, in some cases this may be unduly pessimistic and the assumption of clamped edges may be more appropriate. Thus, although the panel geometry is well-defined, the restraints could be better idealised.

Sometimes equivalent panel sizes are used. This introduces the concept of an imaginary panel which has the same buckling stress as the physical panel under analysis, but has a buckling coefficient equal to a simply-supported panel. The dimensions of this imaginary panel are the equivalent panel size. A panel with an equivalent size greater than the physical size thus indicates that buckling has occurred across its edges. A panel with an equivalent size less than the physical size indicates restraints greater than simple-support.

The equivalent panel approach was developed to simplify hand calculations (see Ref.6) but it has led to confusion in the past. The equivalent panel is purely imaginary and its boundaries do not physically exist nor do they have any relevance to the surrounding structure. If there has been buckling across the stiffeners, there is no guarantee that the buckles will be contained in the adjacent panels. In such a case, the analysis should be extended to consider a greater panel which includes the adjacent panels. Only when panel edges resisting buckling are reached can the panel be said to be defined in a valid way for the purposes of this analysis.

Inevitably, assumptions need to be made in the analysis. Some of these are concerned with initial panel buckling. The support from the stiffeners and the effective plain panel sizes are related to the type, orientation and dimensions of the stiffeners. These assumptions are incorporated in APA115 and are listed in this section.

In the flat panel analysis, more information is available to determine the effects of buckling across stiffeners. The curved panel analysis lacks this information but some tentative suggestions will be made regarding the use from the flat panel analysis.

The output of APA115 can be brief or extended. The extended output provides more detail about assumptions and calculations performed by the analysis. Several buckling coefficients based on different edge restraints are determined. From these, a coefficient applicable for the structure is selected directly or otherwise derived.

F2.1 FLAT PANELS

The elastic buckling stress is expressed in general form as follows:

$$\tau = K \cdot E \cdot \left(\frac{t}{b}\right)^2$$

where K is the buckling coefficient obtained from theoretical or experimental data. The panel dimension b is the shorter of the two sides. The value of K depends on assumptions about panel geometry and edge restraint. In its simplest form, it may be related purely to aspect ratio. In more detailed analyses, the flexural and torsional properties of stiffeners are included.

F2.1.1 Single Flat Panels

A simple analysis is found in E.S.D.U. 71005 (Ref.7). This provides the shear buckling stress for panels with a combination of either simply-supported or clamped edges, but no intermediate edge restraint between these extremes. As the minimum restraint is a simple-support, it is implicitly assumed that buckling across the stiffeners does not occur. Before using this analysis, some justification of the adequacy of stiffener restraint should be provided.

The buckling coefficient in this analysis is dependent on the panel aspect ratio. Fig.3 shows curves of the buckling coefficient for different edge restraints. This figure is a representation of that in Ref.7, but the curves have been plotted directly from embedded data in the program and act as a verification of the program data.

F2.1.2 Multiple Flat Panels

The results of a more detailed analysis are presented in E.S.D.U. 02.03.02 (Ref.8). This considers long panels with transverse stiffeners in which the stiffeners are considered to act in conjunction with the panel to restrain its overall buckling. The buckling modes may be such that the complete panel buckles or one individual bay buckles or some intermediate mode. The curves in Ref.8 were obtained from Refs.14-16. Although Refs.14-16 go into great depth, the precise computation of the curves has not been determined at this stage. The curves are represented in Figs.4a-4j, obtained from the program in the same manner as those from Ref.7.

The limiting curves for μ_c on Figs.4a, 4b represent the boundary at which buckling occurs across the transverse stiffeners (i.e. the stiffeners of length a and pitch b). At this boundary, the stiffeners are providing minimal restraint - i.e. simple-support.

A value of $\mu = 0$ on Figs.4a, 4b represents a complete loss of restraint on the transverse stiffeners. This effectively implies that these stiffeners are no longer present in the capacity as boundary members. The buckling stress must be based on a long panel, using the shortest dimension, which becomes a instead of b on Figs.4a, 4b.

The buckling coefficients so far considered have been referred to transverse stiffeners with no torsional stiffness. E.S.D.U. provides a range of curves in Ref.8 which make allowance for this. Program APA115 outputs values from all the curves for comparison.

Note the definition of the stiffener inertia used by E.S.D.U. The stiffener inertia about the panel mid-plane is suggested, but it is stated that a combination of half the panel width plus stiffener about the combined neutral axis provides similar results. Program APA115 uses the stiffener inertia alone about the panel mid-plane.

5.1.3 Comparison of Single and Multiple Panels

The consistency between Ref.7 and Ref.8 may be seen where the latter refers to restraints at the extremes of simple-support or clamping. In Figs.3, 4a, 4b, discrete points have been identified where the analyses should concur and at these points, the buckling coefficient is essentially the same. Two of these points will be used as examples.

Taking a/b = 1.5 on Fig.4b at the $\mu = 0$ boundary (point F), a value of K = 6.4 is found. On Fig.3, at b/a = 0.67 on the lowest curve, the same value of K is found. Point F corresponds to a panel simply-supported on all sides.

In the case of a square panel with two sides clamped (the longitudinal sides), point B on Fig.3 at b/a = 0 shows a value of k = 8.15. On Fig.4a, the value of K is the same at a/b = 1. This is the maximum coefficient possible, valid for $a/b \leq 1$.

On Figs.4a, 4b, additional points may be identified at $\mu = 0$ for other aspect ratios. To obtain buckling coefficients corresponding to a particular (a/b), divide the square panel coefficient by $(a/b)^2$ - i.e. $8.15/(a/b)^2$ for Fig.4a and $4.9/(a/b)^2$ for Fig.4b. All the points on the curves at $\mu = 0$ match.

In general, on Fig.4a and Fig.4b, the μ_c curve represents a limited single bay buckle with simply-supported stiffeners and the $\mu = 0$ axis represents a complete panel buckle across totally ineffective transverse stiffeners. Values above the μ_c curve indicate that the stiffeners are providing restraint greater than simple-support. Values in between $\mu = 0$ and $\mu = \mu_c$ indicate a partial buckle across the stiffeners. Some restraint is provided by them, but the design philosophy of the structure may require that even partial buckles are not accepted. For example, integral stiffeners are forced to follow a buckled panel precisely and could experience some localised deformation whereas riveted stiffeners may allow buckles in between attachments and avoid this effect.

Neither Ref.7 or Ref.8 allow for buckling across the longitudinal edges. This action is conceivable, but most real structures provide at least one pair of edges deliberately designed to provide adequate restraint. To cover the case of a panel which is surrounded entirely by other panels, APA115 considers the panel as part of a larger long panel in two orthogonal directions. That is, the panel is considered with stiffener S_a as the transverse stiffener and then with stiffener S_b as the transverse stiffener.

5.1.4 Equivalent Panel Size

An equivalent panel is defined as a simply-supported panel which has the same buckling stress as the physical panel under analysis. The dimensions of this panel are the equivalent panel sizes. Report S.O.R. 51 (Ref.6) refers to equivalent panel sizes and presents a set of curves relating these sizes to aspect ratio and a stiffness parameter for the stiffener. The source of these curves was originally presented by Cook and Rockey (Ref.15) and used by E.S.D.U. (Ref.8), both of which are acknowledged in Ref.6. The B.A.C. design manuals in Ref.17 also quote E.S.D.U. as a reference. All these references should show some consistency, but this has not been found to be the case.

The original reference (Ref.15) contains some complex mathematics resulting in a set of graphs relating buckling coefficient, aspect ratio and a stiffness parameter. The mathematics is beyond the scope of this report and the graphs do not present data in enough detail for the lower stiffness range. The E.S.D.U. data sheet (Ref.8), being a widely used and verified reference, has been used for the program. The remaining two references may have deviated from this during their preparation, which was at a later date.

A comparison was made between the E.S.D.U. data sheet and S.O.R. 51 (Fig.1c). The results deviated by a maximum of 20% for the lower stiffness values but generally showed the same pattern. These curves should be identical and it is suspected that some inaccuracies have been generated in the transformation of the data to the S.O.R. 51 format. It is noted that S.O.R. 51 extends the curves below b/a = 0.2, for which no previous references provide information. The test verification in S.O.R. 51 uses results from the N.A.C.A. analysis (Ref.3) which do not include buckling across stiffeners. Therefore, the extension to the b/a ratio is thought to be arbitrary and possibly unreliable.

APA115 quotes the equivalent flat panel dimensions for reference in the extended output. These may be directly compared with the output which comes from the S.O.R. 51 analysis.

5.1.5 Plasticity Correction

The plasticity correction for shear buckling is η_1, obtained from E.S.D.U. 83044 (Ref.19) using C = 2. Although η_8 is quoted as the coefficient for shear, this gives an identical set of curves to E.S.D.U. 71005 (Ref.7).

5.1.6 Stiffener Inertia Requirement

5.1.7 Other References

S.D.D. data sheets (Ref.17) present a series of curves for the minimum stiffness criterion. Although this makes reference to E.S.D.U. (the original R.Ae data sheets) and comes from the same source as S.O.R. 51, the curves show such a large deviation from the other sources that it is believed they are in error. Also, the upper curve which denotes the critical stiffness parameter is based on clamped longitudinal edges and not consistent with the simply-supported longitudinal edges used in S.O.R. 51.

Boeing (Ref.20) presents a minimum stiffener requirement which appears to be a lot less conservative than the references mentioned above. A comparison is shown in Fig.6. However, this does not necessarily represent a true comparison because the stiffener inertia quoted by Boeing is that of the stiffener alone about its own centroid. This is equal to E.S.D.U. for double stiffeners only.

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