A Yield Line Component Method for Bolted Flange Connections

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ABSTRACT

Bolted connections are often used in steel structures to transfer of tension loads into wide flange members. The strength of these connections is determined with a prying action design procedure (outlined in the 13th edition AISC *Steel Construction Manual*) that checks the limit states of bolt tension rupture and bending of the flange. This procedure is valid only for fittings with limited bolt spacing and limited edge distance. This paper discusses a method to determine the local flange bending strength of a wide flange member using the yield line method. The proposed design method includes the effect of prying action on the bolts, and can be applied to many different connection configurations, including connections with large bolt spacing and edge distances and connections with web stiffeners. Comparisons with test data from 10 independent research projects will be used to verify the accuracy of the proposed method.

Keywords: bolted tension connections, hangers, prying action.

Many bolted connections in steel structures rely on the transfer of tension loads into wide flange members as shown in Figure 1. The strength of these connections is determined with the prying action design procedure in the *Steel Construction Manual* (AISC, 2005a), hereafter referred to as the *Manual*, which checks the limit states of bolt tension rupture and bending of the flange. The procedure in the *Manual* is valid only for fittings with limited bolt spacing and edge distance such as clip angles at the end of a beam.

The *Manual* does not provide guidance on how to determine the equivalent length of fittings with large edge distances and bolt spacings. In practice, conservative assumptions are often made. It is commonly assumed that the tributary length per bolt is twice the distance from the center of the bolt to the face of the supporting web. This method is slightly conservative for calculating the elastic stress for wide cantilever beams loaded at the free end (Young, 1989); however, it is extremely conservative for calculating the strength of flanges in bending.

In other cases, unconservative assumptions are sometimes made, where web stiffeners are provided to prevent flange bending, and the stiffened flange is assumed adequate to carry the applied loads with no further calculations. However, tests have shown that flange bending is a common failure mode for connections with web stiffeners (Packer and Morris, 1977; Garrett, 1977; Ghassemieh et al., 1983; Moore and Sims, 1986; Zoetemeijer, 1981).

This paper will discuss a method to determine the local flange bending strength of a wide flange member using the yield line method. The proposed design method includes the effect of prying action on the bolts and can be applied to many different connection configurations, including connections with large spacings and edge distances and connections with web stiffeners. Comparisons with test data from 10 independent research projects will be used to verify the accuracy of the proposed method.

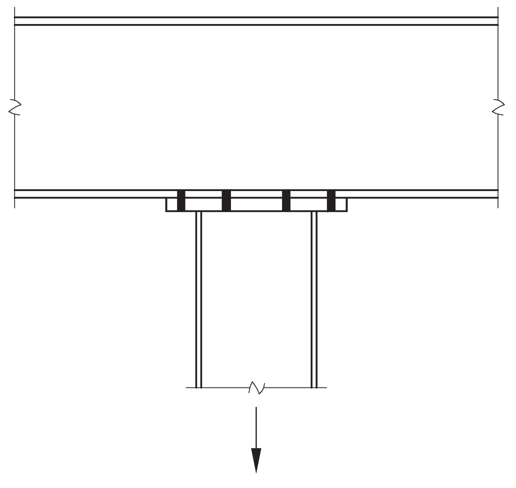

Fig. 1. Bolted hanger connection.

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BACKGROUND

Prying Action

When bolts are loaded in tension, deformation of the connected parts will cause an increase in bolt tension. This additional bolt tension is the prying force, q, shown in Figure 2. Designing for prying action involves checking the limit states of bending of the fitting and tension rupture of the bolts. The two limit states are interdependent—for a given load, an increase in flange thickness leads to a lower prying force on the bolt.

The moment diagram of half of the flange is shown in Figure 2. The moment at the face of the web is always equal to the plastic capacity of the fitting, but the moment at the bolt line can be reduced if required to limit the prying force on the bolt. This behavior is accounted for in the design method in the *Manual*. The background for the design method is provided by Astaneh (1985), Thornton (1985) and Kulak et al. (1987). To calculate the available tensile strength when the connection geometry is known, Equation 1 is applicable:

$$T_a = B\left(\frac{t}{t_c}\right)^2 \left(1+\delta\alpha'\right) \le B \tag{1}$$

where

LRFD	ASD
$t_c = \sqrt{\dfrac{4.44Bb'}{pF_u}}$	$t_c = \sqrt{\dfrac{6.66Bb'}{pF_u}}$

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right] \le 1.00$$

$$\delta = 1 - \frac{d'}{p}$$

$$\rho = \frac{b'}{a'}$$

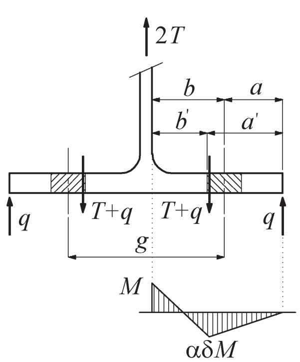


Fig. 2. Model for prying action design method.

$$b' = b - \frac{d_b}{2}$$

$$a' = \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right)$$

B = available tensile strength per bolt, kips
a = distance from the bolt centerline to the edge of the fitting, in.
b = distance from bolt centerline to the face of the web, in.
d_b = bolt diameter, in.
d' = width of the hole along the length of the fitting, in.
F_u = specified minimum tensile strength of connecting element, ksi
p = tributary length of fitting per bolt, in.
t = thickness of the fitting, in.

The Yield Line Method

The yield line method was developed by Hognestad (1953) and Johansen (1962) to determine the ultimate strength of concrete slabs. It is an upper-bound solution based on the principle of virtual work. One form of the upper-bound theorem of limit analysis states that a load calculated based on an assumed mechanism will be greater than or equal to the true limit load.

The yield line method requires the failure pattern to be known prior to calculation of the collapse load. Many patterns may be valid for a particular joint configuration. Because the collapse load is upper bound, the pattern that gives the lowest load will provide results closest to the true failure load. Therefore, selection of the proper yield line pattern is important because an incorrect failure pattern will produce unsafe results.

The collapse load is calculated assuming that a plastic mechanism forms along each line of the chosen failure pattern. To maintain equilibrium, the external work done by the load moving through the virtual displacement, δ, must equal the strain energy due to the plastic moment rotating through virtual rotations, θ_i. The virtual rotations are assumed small, so $\theta_i \approx \tan(\theta_i) \approx \sin(\theta_i)$. The influence of strain hardening and membrane effects are not accounted for in yield line analysis; therefore, there is potentially a large reserve capacity beyond the calculated collapse load.

Some yield line patterns will produce an equation for the load in terms of known geometry, but most cases will require any unknown dimensions to be determined by minimizing the load with respect to the unknown dimension. To do this, the load is differentiated with respect to the unknown dimension and set equal to zero. From this, an equation for the unknown dimension can be determined and substituted into the equation for the load.

The general procedure for deriving an equation based on yield line analysis is as follows:

- Select a valid yield line pattern.
- Determine the equation that describes the external work done by the load moving through the virtual displacement.

$$W_E = P\delta \tag{2}$$

where
P = applied load
δ = virtual displacement

- Determine the equation that describes the internal work done by the rotations along the yield lines,

$$W_I = \sum M_{pi}\theta_i \tag{3}$$

where
M_{pi} = plastic moment capacity of yield line i
= $m_p L_i$
θ_i = virtual rotation of yield line i
m_p = plastic moment capacity per unit length of the fitting
= $F_y t^2/4$
L_i = length of yield line i

- Set the external work equal to the internal work and solve for the load. If required, minimize the load with respect to unknown dimensions.

Traditionally, the prying action equations have been derived using equilibrium methods (Kulak et al., 1987), but the equations can also be derived using energy methods. To show the similarity between the design method for prying action and the yield line equations for flange bending, the *Manual* equation for the required fitting thickness will be derived for the case of an infinitely strong bolt. This exercise will also show the validity of the yield line method for this simple case.

Considering only one side of the connection in Figure 3, the external work is

$$W_E = T\delta \tag{4}$$

The internal work is

$$W_I = \theta m_p(L_1 + L_2) \tag{5}$$

where
L_1 = length of yield line 1
= tributary length per bolt, p
L_2 = length of yield line 2
= net tributary length per bolt, p–d'

Substitute $L_1 = p$ and $L_2 = p - d'$ into Equation 5 to get

$$W_I = \theta m_p(2p - d') \tag{6}$$

For small angles, $\theta = \delta/b'$

$$W_I = \frac{\delta}{b'} m_p \left(2p - d'\right) \tag{7}$$

Substitute $m_p = F_y t^2/4$

$$W_I = \frac{\delta F_y t^2}{4b'}\left(2p - d'\right) \tag{8}$$

Set internal work equal to external work and solve for T_n

$$T_n = \frac{F_y t^2}{4b'}\left(2p - d'\right) \tag{9}$$

The available LRFD strength is

$$\phi T_n = \frac{\phi F_y t^2}{4b'}\left(2p - d'\right) \tag{10}$$

Rearrange Equation 10 and solve for the thickness of the fitting

$$t_{min} = \sqrt{\frac{4Tb'}{\phi F_y \left(2p - d'\right)}} \tag{11}$$

Substitute $\phi = 0.90$ into Equation 11

$$t_{min} = \sqrt{\frac{4.44Tb'}{F_y \left(2p - d'\right)}} \tag{12}$$

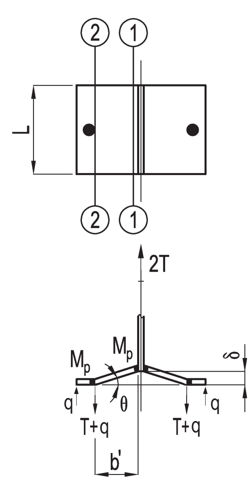


Fig. 3. Yield line model for prying equation.

The LRFD version of the prying equation (on page 9-11 of the *Manual*) is

$$t_{\min} = \sqrt{\frac{4.44Tb'}{pF_u\left(1+\delta\alpha'\right)}} \tag{13}$$

Although the *Manual* procedure uses the ultimate tensile strength, F_u, for prying calculations, which was first suggested by Douty and McGuire (1965) and more recently by Thornton (1992), yield line analysis has traditionally utilized the yield strength, F_y. For comparison with the yield line derivation, F_y will be used here. Replacing F_u with F_y in Equation 13, substituting $\alpha' = 1.0$ for infinitely strong bolts and substituting $\delta = 1 - d'/p$, Equation 12 is obtained.

EXISTING SOLUTIONS

Yield line theory has been presented as a design method for bolted connections in several publications, and many different yield line patterns have been proposed. A component method, similar to the design method proposed in this paper, is currently used in Europe (SCI, 1995; CEN, 2005) to determine the column flange bending strength and the plate bending strength in moment end plate connections.

Zoetemeijer (1974)

The equivalent length concept was first discussed by Zoetemeijer (1974), who used a simplified solution to the yield line pattern in Figure 4 to get an equivalent tributary length per bolt of

$$p_e = 2b + \frac{5a}{8} + \frac{p}{2} \tag{14}$$

where
p = spacing between bolts

Dranger (1977)

The yield line pattern in Figure 5 was solved by Dranger (1977), who determined the strength as a function of the unknown dimension x:

$$P_n = F_y t^2 \left(\frac{x}{b} + \frac{c}{x}\right) \tag{15}$$

Dimension x was then determined by minimizing the load:

$$x = \sqrt{bc} \tag{16}$$

If x from Equation 16 is substituted into Equation 15, the nominal strength is

$$P_n = 2F_y t^2 \sqrt{\frac{c}{b}} \tag{17}$$

where
$c = a + b$

Mann and Morris (1979)

Mann and Morris (1979) presented a yield line pattern with circular corners as shown in Figure 6. The nominal strength is

$$P_n = F_y t^2 \left(\pi + \frac{2a + p - d'}{b}\right) \tag{18}$$

Equation 18 defines the total connection strength, which is twice the strength of each independent yield line pattern forming on both sides of the column web. Mann and Morris also suggested an equation similar to Dranger's (1977) for stiffened connections; however, no guidance was given on how close the stiffener has to be to the bolt for that equation to apply.

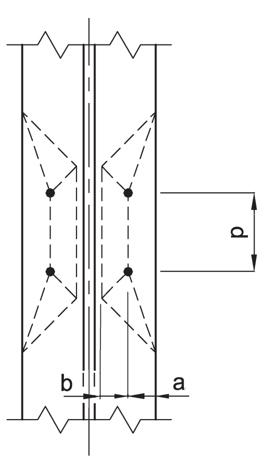


Fig. 4. Yield line pattern from Zoetemeijer (1974).

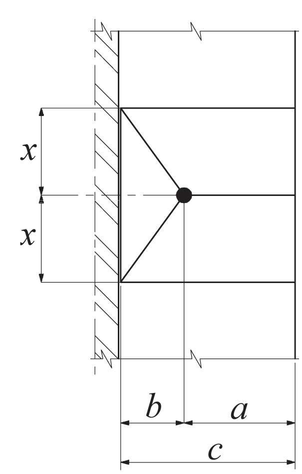


Fig. 5. Yield line pattern from Dranger (1977).

Zoetemeijer (1981)

Zoetemeijer (1981) presented a circular yield line pattern as shown in Figure 7, which he described as a punching failure. For this pull-through mechanism, the prying force is theoretically zero. The yield line solution predicts a nominal strength of

$$P_n = \pi F_y t^2 \tag{19}$$

Thornton and Kane (1999) and Muir and Thornton (2006)

Thornton and Kane (1999) and Muir and Thornton (2006) published the following equation, which provides the average equivalent length per bolt:

$$p_e = \frac{p(n-1)+\pi b+2a}{n} \tag{20}$$

where
n = number of bolt rows

The equation can be derived by dividing the total equivalent length of the bolt group, based on the yield line pattern of Mann and Morris (1979), by the total number of bolts in the joint. The equivalent length is then used with the prying action procedure in the *Manual*. This equation accounts for the prying effect on the bolts; however, the fact that the outermost bolts take significantly more of the load than the inner bolts is neglected.

EXPERIMENTAL RESEARCH

Ten independent research projects were located with experimental results on bolted tension connections. From these tests, four connection types were identified, based on the edge distance and stiffener configuration. These are presented in Table 1. The specimen details are shown in Table A1 of Appendix A.

The generalized experimental load-deflection curve is bilinear with a nonlinear transition point as shown in Figure 8. There are four points of interest on the curve:

1. The proportional limit, where the curve transitions from linear to nonlinear. The load at this deformation may be of interest as a serviceability limit for connections that can allow only very small deformations. The deformation at this point is δ_p, and the load is P_p.
2. The point where the curve transitions from nonlinear to linear at the second linear part of the curve. Loads increased beyond this point are accompanied by large deformations. The deformation at this point is δ_s, and the load is P_s.
3. The point of ¼-in. deformation. This is proposed here as the serviceability limit. The deformation at this point, $\delta_{1/4}$ is ¼ in., and the load is $P_{1/4}$.

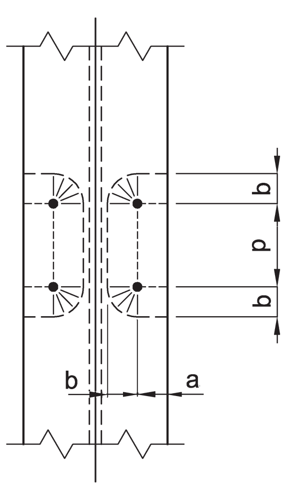


Fig. 6. Yield line pattern from Mann and Morris (1979).

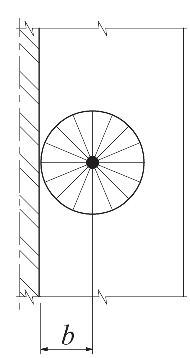


Fig. 7. Circular yield line pattern from Zoetemeijer (1981).

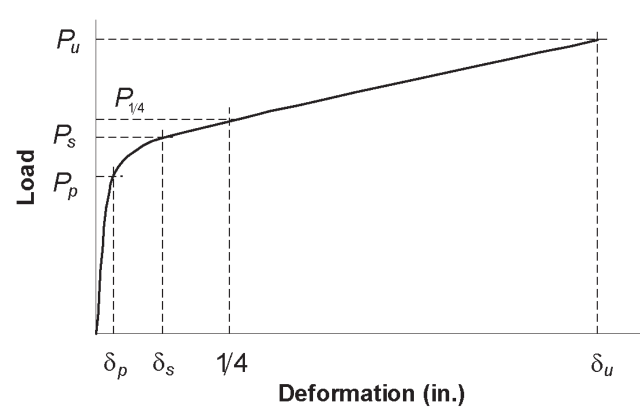
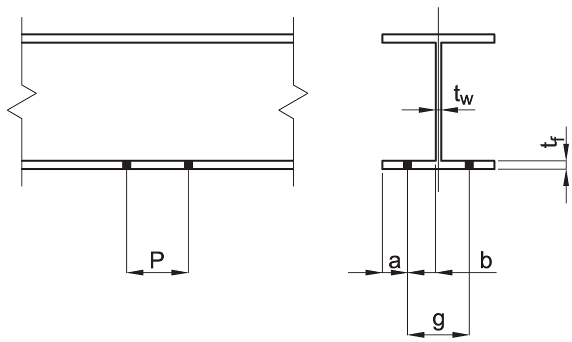
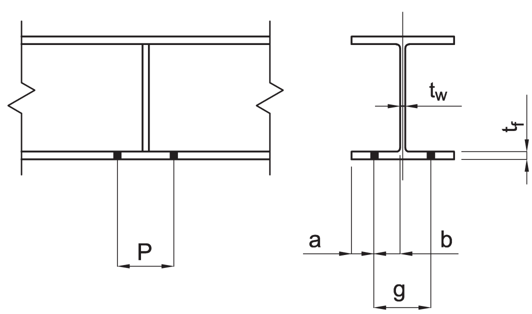
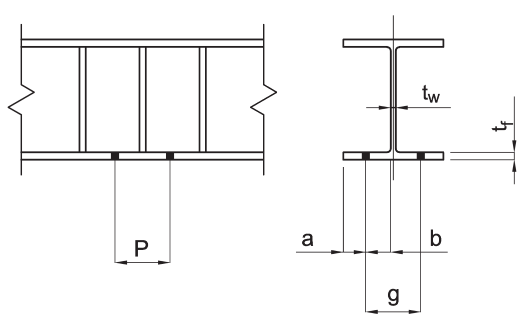
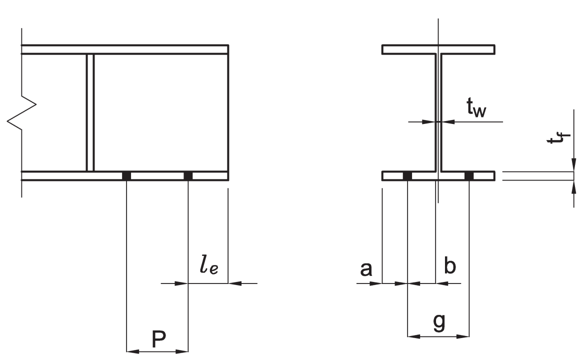


Fig. 8. General load versus deformation curve.

Table 1. Geometry of Experimental Specimens

Specimen Geometry	References
Type 1	
	Garrett (1977) Grogan and Surtees (1999) Hendrick and Murray (1983) Moore and Sims (1986) Packer and Morris (1977) Pynnonen and Granstrom (1986) Tawaga and Gurel (2005) Zoetemeijer (1974)
Type 2	
	Packer and Morris (1977) Garrett (1977) Moore and Sims (1986) Zoetemeijer (1981)
Type 3	
	Garrett (1977)
Type 4	
	Ghassemieh et al. (1983)

4. The ultimate strength. This is a point of interest for structural integrity and ultimate strength calculations. The deformation at this point is δ_u and the load is P_u.

Table A2 in Appendix A contains all of the loads described for each specimen, where they were reported in the referenced document. Many of the listed values are approximate because they were read from graphs of the test data. The experimental failure modes are also listed in Table A2. Most of the specimens with two failure modes listed had the ultimate strength limited by bolt rupture, but only after a large deformation due to flange bending.

For the specimens with thick flanges, the bolts failed before the nonlinear part of the load-deformation curve was reached. For these specimens, the bolt elongation contributed significantly to the total deformation.

For the specimens with thin flanges, the deformation at ultimate strength was as much as 2 in. Under large deformations, the load-transfer mechanism changes from bending to tension, which results in a tension load with a component perpendicular to the axis of the bolt. This component is resisted by the bolts in shear. Many of these tests resulted in bolt fracture due to the applied tension combined with shear, which was caused by large-deformation membrane action of the fitting.

DEVELOPMENT OF PROPOSED DESIGN METHOD

The purpose of this paper is to formulate a simple, accurate and versatile method to design bolted flange connections. To do this, the theoretical and experimental information presented by previous researchers will be analyzed.

The yield line solutions of Zoetemeijer (1974), Dranger (1977), and Mann and Morris (1979) provide accurate results for thin fittings where the limit state of bolt rupture is not applicable. However, where thick flanges dictate that bolt rupture is the controlling limit state, the yield line solutions do not provide a method to calculate the prying force on the bolt.

The method proposed by Thornton and Kane (1999) and Muir and Thornton (2006) explicitly accounts for the prying forces on the bolts; however, an equal amount of axial load is assigned to each bolt. In reality, the outermost bolts will be more highly stressed than the inner bolts, which could lead to an unzipping action.

In this paper, a more refined solution has been developed, where the forces are distributed according to the equivalent length tributary to each bolt and the strength of each bolt is evaluated independently. The equivalent tributary length is calculated using existing yield line solutions.

The Component Method

Many different bolted flange configurations can be analyzed by the yield line method; however, it would be cumbersome for engineers to deal with a separate yield line pattern for each different configuration. To simplify the design process, the component method can be used, where single-bolt (local) yield line patterns are assembled into a larger (global) pattern for the entire bolt group. To do this, the engineer simply selects a local pattern that is identical to each part of the global pattern. The strength of each local pattern is calculated and summed to get the total strength of the global pattern.

In many cases, the local pattern will not be symmetrical about the center of the bolt, and half-patterns can be used. The strength of a half-pattern is simply half of the strength of the whole pattern.

Stiffened Connections

If a flange is not adequate to carry the applied load, stiffeners can be used to reinforce the joint as shown in Figure 9. For stiffeners to be effective, they must be close enough to the bolt to alter the yield line pattern. Using Dranger's (1977) yield line pattern, the stiffeners are effective if

$$x_s < x \tag{21}$$

where

x_s = distance from the center of the bolt to the edge of the stiffener

x = $\sqrt{bc}$

Then, the strength can be determined by substituting x_s for x in Equation 15:

$$P_n = F_y t^2 \left(\frac{x_s}{b} + \frac{c}{x_s} \right) \tag{22}$$

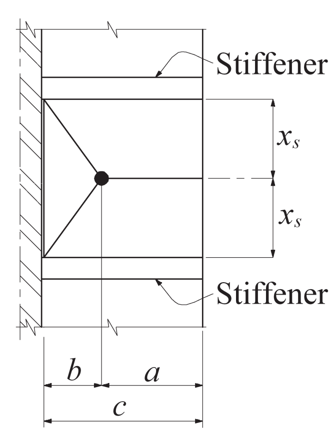


Fig. 9. Yield line pattern for a stiffened flange in bending.

Equivalent Tributary Length Concept

The nominal strength from a given yield line pattern will be equal to that of a straight yield line of length, p. Using Equation 9 with $d' = 0$ and $b' = b$, the nominal strength for a straight yield line is

$$P_n = \frac{F_y t^2 p}{2b} \tag{23}$$

To determine the equivalent tributary length of fitting, the nominal strength of a given yield line solution will be set equal to Equation 23 and solved for p. For the Dranger (1977) pattern in Figure 5, the equivalent length is

$$p_d = 4\sqrt{bc} \tag{24}$$

For single-bolt connections, the equivalent tributary length for the yield line solution of Mann and Morris (1979), shown in Figure 6, is

$$p_m = \pi b + 2a \tag{25}$$

The equivalent tributary length For Zoetemeijer's (1981) circular pattern in Figure 7 is

$$p_c = 2\pi b \tag{26}$$

For the stiffened pattern in Figure 9, the equivalent tributary length is

$$p_s = 2\left(x_s + \frac{cb}{x_s}\right) \tag{27}$$

For single-bolt connections, the yield line solution of Zoetemeijer (1974), given by Equation 14, reduces to

$$p_z = 4b + 1.25a \tag{28}$$

Selection of Proper Yield Line Solution

Because the yield line method is an upper-bound approach, the pattern that gives the lowest load will provide results closest to the true failure load. The normalized equivalent lengths, p_e/c, from the yield line solutions of Zoetemeijer (1974, 1981), Dranger (1977), and Mann and Morris (1979) are plotted against b/a in Figure 10. It can be seen that the Mann and Morris (1979) solution results in the minimum equivalent length for connections with high values of b/a, and the Zoetemeijer (1981) solution produces the minimum equivalent length only for connections with very low values of b/a.

To simplify the design process, it is advantageous to use only one of the available yield line patterns. Analysis of the experimental deformations indicate that the yield line pattern developed by Zoetemeijer (1974), shown in Figure 4, is closest to the actual failure pattern. However, the skewed yield lines are awkward to deal with if stiffeners are present, and for most practical b/a ratios, the difference in strength of the various yield line patterns is small.

The circular yield line pattern presented by Zoetemeijer (1981) will control the design of fittings with large edge distances, a. However, if a limit is placed on the b/a ratio, this yield line pattern will never control the design. The Zoetemeijer solution is equal to the Dranger (1977) solution at $b/a = 0.68$; therefore, if a is limited to $1.47b$ for design purposes, Zoetemeijer's solution will never control. As a slightly conservative (about 5%) limit, the prying action design procedure in the *Manual* (AISC, 2005a) can be used, which limits a to a maximum of $1.25b$.

When comparing the Dranger (1977) pattern to the Mann and Morris (1979) pattern for stiffened flanges, the Dranger pattern more accurately predicts the increase in strength based on the distance from the bolt to the stiffener. This can be verified by reviewing the projects that tested specimens that were identical except for the addition of a stiffener: Packer and Morris's (1977) specimens T6, T7 and T8; Moore and Sims's (1986) specimen T7. For these four specimens, the Mann and Morris model predicted no increase in strength due to the stiffeners; however, the average experimental load increased by 32% compared to identical specimens with no stiffeners. The Dranger model predicted a 37% increase due to the stiffeners.

Due to the simplicity and the more accurate prediction of the strength when stiffeners are present, the Dranger (1977) yield line pattern is proposed here. A plot of p_{min}/p_d versus b/a is shown in Figure 11, where p_d is the tributary length

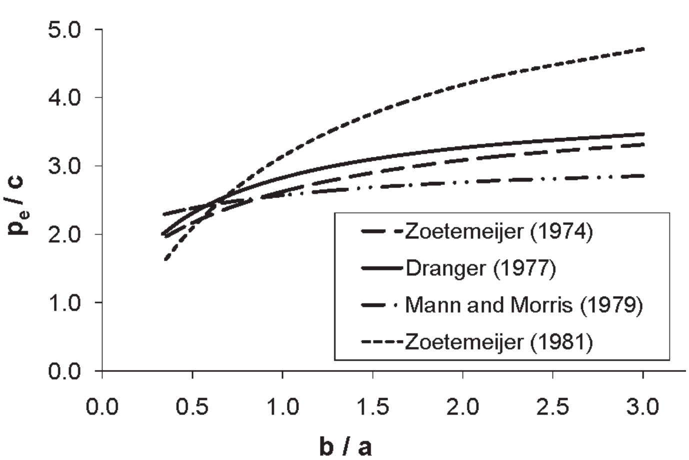


Fig. 10. Comparison of different yield line patterns.

for the Dranger yield line pattern and p_{min} is the minimum tributary length for the yield line patterns of Zoetemeijer (1974), Dranger (1977), and Mann and Morris (1979). It is seen that the Dranger solution is unconservative. However, for most practical b/a ratios, the difference can be neglected because the beneficial effects of strain hardening and membrane action are not accounted for. Figure 11 also shows the curve-fit equation, which can be used as a reduction factor in design if the engineer wants to explicitly account for the difference among the three different solutions. The curve fit for the reduction factor is

$$C_r = 1.0 - 0.11(b/a) + 0.019(b/a)^2 \tag{29}$$

The coefficient of determination, R^2, is 0.99, indicating a very good fit. If the equivalent tributary length has been calculated using Dranger's solution, the minimum of the three solutions can be approximated as

$$p'_{min} = p_d\, C_r \tag{30}$$

where

p'_{min} = approximate minimum equivalent tributary length per bolt

p_d = equivalent tributary length per bolt calculated according to the yield line pattern, developed by Dranger (1977)

Joints with Bolt Rupture as the Controlling Limit State

In joints where the equivalent tributary length at one bolt is larger than the remaining bolts in the joint, the bolt forces will not be distributed equally. When bolt rupture is the controlling limit state, the design procedure must account for this. The component method accounts for the nonequal distribution of bolt forces by assigning the loads in proportion to the tributary length at each bolt.

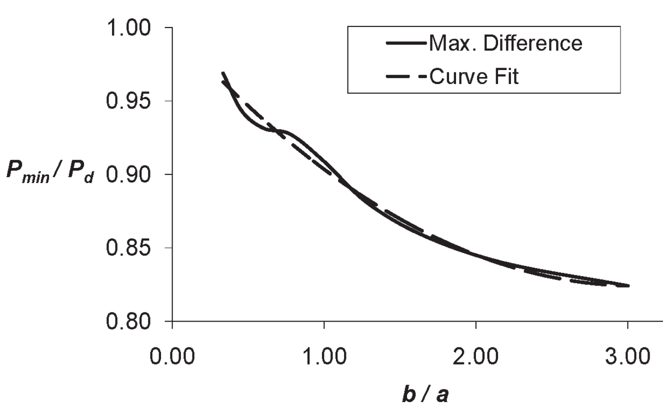


Fig. 11. Comparison of Dranger (1977) yield line solution to the minimum of the Zoetemeijer (1974) and Mann and Morris (1979) solutions.

When bolt rupture controls the design, an additional complication arises because deformation compatibility must be maintained for all bolts in the joint. Under normal conditions, when one of the bolts within the joint ruptures, it is unlikely that the full yield line pattern has formed due to the limited deformation. The local yield line pattern tributary to the adjacent bolt will also be limited to the deformation at bolt rupture. Because the internal energy at the yield lines is proportional to the displacement, deformation compatibility of the adjacent yield lines can be upheld by reducing the strength in proportion to the deformation ratio, δ_r/δ.

$$\frac{\delta_r}{\delta} = \frac{T_a}{T_{\alpha'=1}} \tag{31}$$

where

δ_r = deformation at bolt rupture

δ = deformation at full yield line strength assuming infinitely strong bolts

T_a = strength of the fitting at bolt rupture (calculated using Equation 1)

$T_{\alpha'=1}$ = strength of the fitting assuming a full yield line pattern forms without bolt rupture (calculated using Equation 1 with $\alpha' = 1$)

Using δ_r as the deformation limit for the entire joint, the total strength of the joint as controlled by the critical bolt is

$$P_n = T_{cr}\left[1 + \left(\frac{T_{cr}}{T_{cr(\alpha'=1)}}\right)\left(\frac{\Sigma P_{ei}}{P_{e(max)}} - 1\right)\right] \tag{32}$$

where

T_{cr} = strength of the fitting at the critical bolt (the bolt with the largest equivalent tributary length within the joint) (calculated using Equation 1)

$T_{cr(\alpha'=1)}$ = strength of the fitting at the critical bolt assuming a full yield line pattern forms without bolt rupture (calculated using Equation 1 with $\alpha' = 1$)

ΣP_{ei} = summation of the equivalent tributary lengths for all local yield line patterns within the joint

$P_{e(max)}$ = largest equivalent tributary length for all bolts within the joint

Equation 32 provides a convenient way to deal with the deformation compatibility of the joint; however, when compared to the test results of Ghassemieh et al. (1983), the calculated strengths are very conservative. The conservatism is due to the fact that the equation only accounts for the flexural deformation of the fitting and neglects other deformations within the joint, such as bolt elongation and shear deformation of the fitting. As discussed in the section on experimental research, bolt elongation can be a large portion of the total joint deformation.

Due to the conservatism associated with Equation 32, it is proposed that the strength of each bolt be evaluated independently. Then, the total strength of the joint can be calculated by summing the local capacities for the entire bolt group. To account for the prying force on the bolt, the equivalent tributary length, p_e, is used in the prying action procedure in the *Manual* in lieu of the tributary length, p. This procedure provides nominal strengths that compare well with the experimental loads, as discussed in the Experimental Validation section.

Large Bolt Spacings

If the distance between bolts, p, is greater than the equivalent tributary length from Equation 24, two independent yield lines will form for each bolt as shown in Figure 12a. Figure 12b shows the same bolt pattern with a small bolt spacing, where half-patterns form at each end and a straight pattern forms between the bolts. Figure 13 shows a plot of the equivalent length per bolt versus spacing between bolts. The transition point between the two yield line patterns is at a bolt spacing of $4\sqrt{bc}$.

A similar problem occurs when a bolt is near the end of a member. If the edge distance from the bolt to the end of the member, l_e, is less than $2\sqrt{bc}$, a straight yield line will form between the bolt and the end of the member.

PROPOSED DESIGN METHOD

The proposed design method consists of the following steps:

1. Select a valid yield line pattern local to each bolt in the group. If the yield line pattern is not symmetrical, two half-patterns should be selected.
2. Calculate the strength of each bolt and fitting using the prying action procedure in the *Manual* (AISC, 2005a), replacing p with p_e; F_y should be used in lieu of F_u unless large deformations are acceptable.
3. Repeat for all bolts in the bolt group.
4. Sum the individual strengths to get the total strength of the bolt group.

The equivalent tributary length for the yield line pattern in Figure 5 is

$$p_e = 4\sqrt{bc} \tag{33}$$

Where stiffeners are present, the equivalent length for the pattern in Figure 9 is

$$p_e = 2\left(x_s + \frac{cb}{x_s}\right) \text{ if } x_s < \sqrt{bc} \tag{34a}$$

$$p_e = 4\sqrt{bc} \text{ if } x_s \geq \sqrt{bc} \tag{34b}$$

Straight yield lines will be part of the yield line pattern when the bolt spacing is less than $4\sqrt{bc}$ or the edge distance is less than $2\sqrt{bc}$. The equivalent tributary length per bolt is half the distance between two bolts, $p/2$, or the distance from the bolt to the end of the member, l_e.

In the calculations for the equivalent tributary length, the limit $a \leq 1.25b$ should be used. For connections subjected to combined tension and shear, the bolt tension strength should be reduced to account for the presence of shear.

As shown in the next section, serviceability design of

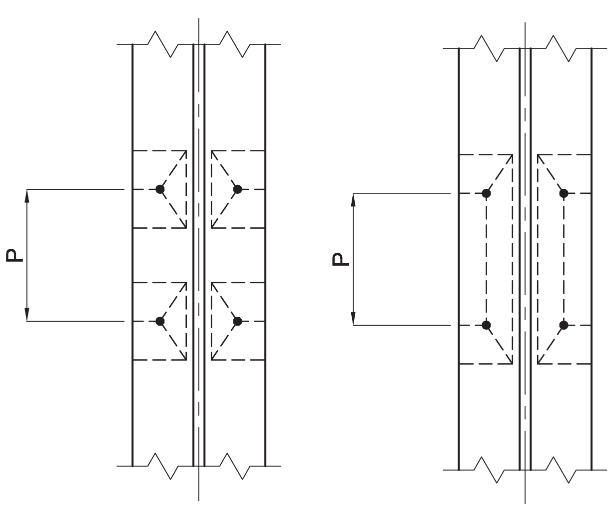


Fig. 12. Effect of bolt spacing on the yield line pattern: (a) large bolt spacing; (b) small bolt spacing.

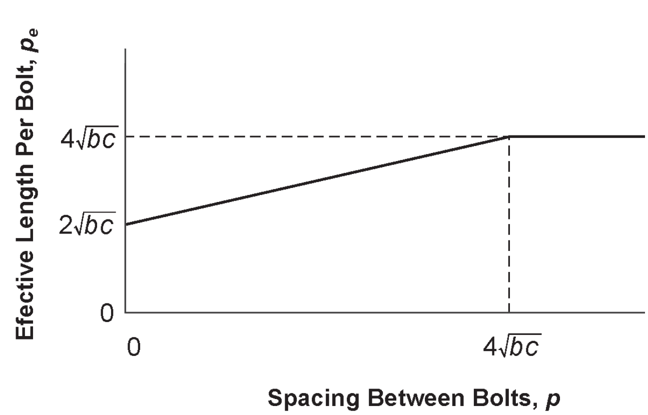


Fig. 13. Equivalent length per bolt versus spacing between bolts.

Table 2. Summary of Calculation Results								
	Using F_y				Using F_u			
[1]	$\frac{P_p}{P_{ny}}$ [2]	$\frac{P_s}{P_{ny}}$ [3]	$\frac{P_{1/4}}{P_{ny}}$ [4]	$\frac{P_u}{P_{ny}}$ [5]	$\frac{P_p}{P_{nu}}$ [6]	$\frac{P_s}{P_{nu}}$ [7]	$\frac{P_{1/4}}{P_{nu}}$ [8]	$\frac{P_u}{P_{nu}}$ [9]
Number of specimens	43	30	23	52	14	11	8	12
Average	0.675	0.932	1.12	1.61	0.490	0.685	0.763	1.13
Standard deviation	0.173	0.184	0.262	0.378	0.157	0.148	0.170	0.269
95% confidence interval (low value)	0.623	0.866	1.01	1.51	0.407	0.597	0.646	0.976
99% confidence interval (low value)	0.607	0.845	0.976	1.48	0.381	0.570	0.609	0.929

connections that can allow only very small deformations should be based on 60% of the nominal load calculated using F_y with the proposed design method. However, for most standard connections, a reduction for stiffness is not required because a ¼-in. deformation allowance is not uncommon in determining the nominal strength of connections for various limit states. For example, in the AISC *Specification for Structural Steel Buildings* (AISC, 2005b), the nominal strength for bearing strength at bolt holes is based on a deformation limit of ¼ in., with an increase in the nominal strength available if more deformation is allowed.

EXPERIMENTAL VALIDATION

The proposed design method was compared to the results of 59 tests from 10 independent research projects. The experimental results are shown in Table A2 of Appendix A. Table A3 in Appendix A shows the calculated nominal strengths and predicted failure modes for all of the specimens. Table A3 also shows the test-to-calculated ratios for each available data point on the experimental curves.

The procedure outlined in the proposed design method was used to calculate the nominal strength of each specimen using the actual yield strengths and the ultimate strengths reported in the referenced documents. Several of the referenced documents reported the yield strength of the tested material, but omitted the ultimate strength; therefore, there were fewer experimental data points to compare with the ultimate strength calculations.

For each specimen, the nominal strength at each local yield line pattern was calculated using Equation 35, with the nominal value of t_c calculated without the resistance factor, as expressed in Equations 36a and 36b for the yield and ultimate strength solutions, respectively.

$$T_e = r_t\left(\frac{t}{t_c}\right)^2\left(1+\delta\alpha'\right) \le r_t \tag{35}$$

$$t_c = \sqrt{\frac{4r_t b'}{p_e F_y}} \tag{36a}$$

$$t_c = \sqrt{\frac{4r_t b'}{p_e F_u}} \tag{36b}$$

where

r_t = strength of tested bolt in tension

Then, the individual strengths of all local yield line patterns within the joint were summed to get the total strength of the joint.

The statistical results are summarized in Table 2, which provides the number of specimens with adequate data to be included in the results, the average, the standard deviation and the low values for the 95% and 99% confidence intervals. Note that P_p, P_s, $P_{1/4}$ and P_u are defined in the section on Experimental Research, and P_{ny} and P_{nu} are the nominal loads calculated with the yield strength and ultimate strength of the fitting, respectively.

The results show that the load at ¼-in. deformation, $P_{1/4}$, can be accurately predicted using F_y with the proposed design method. From column 4 in Table 2, the average test-to-predicted ratio for the 23 specimens is 1.12, and the standard deviation is 0.262. The low values for the 95% and 99% confidence intervals are 1.01 and 0.976, respectively.

The ultimate loads can be accurately predicted using F_u with the proposed design method. However, the deformations at ultimate strength can be very large—Table A2 in Appendix 2 shows experimental deformations greater than 1 in. for several specimens at the maximum test load. From column 9 in Table 2, the average test-to-predicted ratio for the 12 specimens is 1.13, and the standard deviation is 0.269. The low values for the 95% and 99% confidence intervals are 0.976 and 0.929, respectively.

A comparison of columns 4 and 2 of Table 2 indicates that the load at the proportional limit is about 60% of the load at ¼-in. deformation. Based on this, serviceability design of connections that can allow only very small deformations may be based on 60% of the nominal load calculated using F_y with the proposed design method.

EXAMPLES

Example 1

Determine the equivalent tributary length for each bolt in Figure 14.

For bolt 1: The equivalent length is half of the equivalent length from Equation 33 plus half of the distance between bolts 1 and 2.

$$p_e = 2\sqrt{bc} + \frac{p_{12}}{2}$$

For bolt 2: The equivalent length is half of the distance between bolts 1 and 2 plus half of the distance between bolts 2 and 3.

$$p_e = \frac{p_{12}}{2} + \frac{p_{23}}{2}$$

For bolt 3: The equivalent length is half of the equivalent length from Equation 33 plus half of the distance between bolts 2 and 3.

$$p_e = 2\sqrt{bc} + \frac{p_{23}}{2}$$

Example 2

Determine the equivalent tributary length for each bolt in Figure 15.

For bolt 1: Assume $l_e < 2\sqrt{bc}$. The equivalent length is the distance from bolt 1 to the end of the member plus half of the distance between bolts 1 and 2.

$$p_e = l_e + \frac{p}{2}$$

For bolt 2: The equivalent length is half of the equivalent length from Equation 33 plus half of the distance between bolts 1 and 2.

$$p_e = 2\sqrt{bc} + \frac{p}{2}$$

Example 3

Determine the equivalent tributary length for each bolt in Figure 16.

For bolt 1: Assume $l_e < 2\sqrt{bc}$ and $x_s < x$. The equivalent length is the distance from bolt 1 to the end of the member plus half of the equivalent length from Equation 34a.

$$p_e = l_e + x_{s1} + \frac{bc}{x_{s1}}$$

For bolt 2: Assume $x_s < x$. The equivalent length is half of the equivalent length from Equation 34a plus half of the distance between bolts 2 and 3.

$$p_e = \frac{p_{23}}{2} + x_{s2} + \frac{bc}{x_{s2}}$$

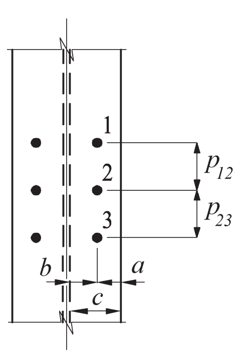


Fig. 14. Example 1.

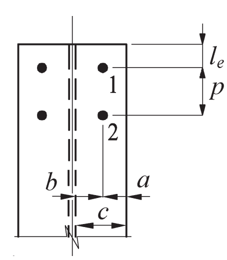


Fig. 15. Example 2.

For bolt 3: The equivalent length is half of the equivalent length from Equation 33 plus half of the distance between bolts 2 and 3.

$$p_e = 2\sqrt{bc} + \frac{p_{23}}{2}$$

Example 4

Determine the equivalent tributary length for each bolt in Figure 17.

For bolts 1 and 2: Assume $x_s < x$. The equivalent length is half of the equivalent length from Equation 33 plus half of the equivalent length from Equation 34a.

$$p_e = 2\sqrt{bc} + x_s + \frac{bc}{x_s}$$

Example 5

Determine the available LRFD strength of the connection in Figure 18 for the limit states of bolt rupture and beam flange bending. The beam is a W21×55 of A992 material. Bolts are ¾-in.-diameter A325 with 13⁄16-in.-diameter holes. The beam gage, g, is 5½ in.

$$B = \phi r_n = 29.8 \text{ kips}$$
$$t_f = 0.522 \text{ in.}$$
$$t_w = 0.375 \text{ in.}$$
$$b_f = 8.22 \text{ in.}$$
$$b = \frac{5.5 \text{ in.} - 0.375 \text{ in.}}{2} = 2.56 \text{ in.}$$
$$a = \frac{8.22 \text{ in.} - 5.5 \text{ in.}}{2} = 1.36 \text{ in.}$$

For design purposes, a must not be greater than 1.25b.

$$a < 1.25\text{b}$$
$$1.36 \text{ in.} < (1.25)(2.56 \text{ in.})$$

Use a = 1.36 in.

$$b' = 2.56 \text{ in.} - \frac{0.75 \text{ in.}}{2} = 2.18 \text{ in.}$$

$$a' = 1.36 \text{ in.} + \frac{0.75 \text{ in.}}{2} = 1.74 \text{ in.}$$

$$\rho = \frac{b'}{a'} = \frac{2.18 \text{ in.}}{1.74 \text{ in.}} = 1.25$$

$$c = b + a = 2.56 \text{ in.} + 1.36 \text{ in.} = 3.92 \text{ in.}$$

For bolts at row 1,

$$P_{eff} = 3.00 \text{ in.}$$

$$t_c = \sqrt{\frac{4.44Bb'}{p_{eff}F_y}} = \sqrt{\frac{(4.44)(29.8 \text{ kips})(2.18 \text{ in.})}{(3 \text{ in.})(50 \text{ ksi})}} = 1.39 \text{ in.}$$

$$\delta = 1 - \frac{d'}{p_{eff}} = 1 - \frac{0.8125 \text{ in.}}{3 \text{ in.}} = 0.729$$

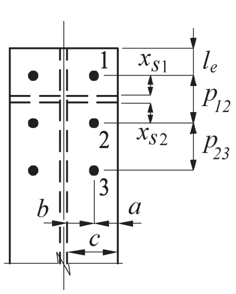


Fig. 16. Example 3.

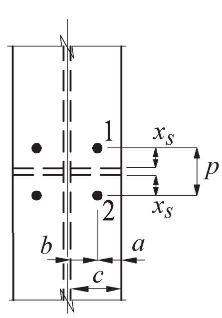


Fig. 17. Example 4.

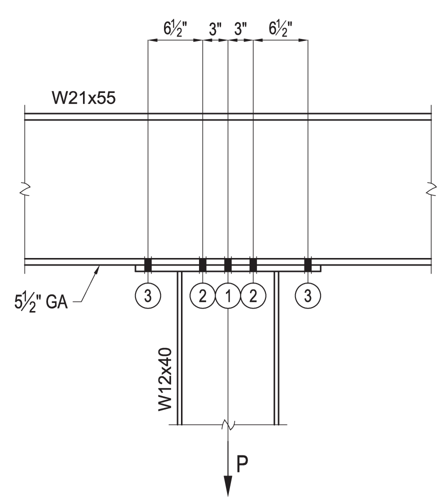


Fig. 18. Example 5—hanger connection without stiffeners.

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right]$$

$$= \frac{1}{(0.729)(1+1.25)}\left[\left(\frac{1.39\text{ in.}}{0.522\text{ in.}}\right)^2 - 1\right]$$

$$= 3.71 > 1.00$$

Use $\alpha' = 1.00$.

$$\phi T_n = B\left(\frac{t}{t_c}\right)^2 (1+\delta\alpha')$$

$$= (29.8\text{ kips})\left(\frac{0.522\text{ in.}}{1.39\text{ in.}}\right)^2 \left[1+(0.729)(1.00)\right]$$

$$= 7.27\text{ kips}$$

For bolts at row 2,

$$p_{eff} = \frac{3\text{ in.}}{2} + \frac{6.5\text{ in.}}{2}$$

$$= 4.75\text{ in.}$$

$$t_c = \sqrt{\frac{4.44Bb'}{p_{eff}F_y}}$$

$$= \sqrt{\frac{(4.44)(29.8\text{ kips})(2.18\text{ in.})}{(4.75\text{ in.})(50\text{ ksi})}}$$

$$= 1.10\text{ in.}$$

$$\delta = 1 - \frac{d'}{p_{eff}}$$

$$= 1 - \frac{0.8125\text{ in.}}{4.75\text{ in.}}$$

$$= 0.829$$

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right]$$

$$= \frac{1}{(0.829)(1+1.25)}\left[\left(\frac{1.10\text{ in.}}{0.522\text{ in.}}\right)^2 - 1\right]$$

$$= 1.84$$

Use $\alpha' = 1.00$.

$$\phi T_n = B\left(\frac{t}{t_c}\right)^2 (1+\delta\alpha')$$

$$= (29.8\text{ kips})\left(\frac{0.522\text{ in.}}{1.10\text{ in.}}\right)^2 \left[1+(0.829)(1.00)\right]$$

$$= 12.3\text{ kips}$$

For bolts at row 3,

$$p_{eff} = 2x + \frac{p}{2}$$

$$= (2)(3.17\text{ in.}) + \frac{6.5\text{ in.}}{2}$$

$$= 9.59\text{ in.}$$

$$t_c = \sqrt{\frac{4.44Bb'}{p_{eff}F_y}}$$

$$= \sqrt{\frac{(4.44)(29.8\text{ kips})(2.18\text{ in.})}{(9.59\text{ in.})(50\text{ ksi})}}$$

$$= 0.776\text{ in.}$$

$$\delta = 1 - \frac{d'}{p_{eff}}$$

$$= 1 - \frac{0.8125\text{ in.}}{9.59\text{ in.}}$$

$$= 0.915$$

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right]$$

$$= \frac{1}{(0.915)(1+1.25)}\left[\left(\frac{0.776\text{ in.}}{0.522\text{ in.}}\right)^2 - 1\right]$$

$$= 0.588$$

$$\phi T_n = B\left(\frac{t}{t_c}\right)^2 (1+\delta\alpha')$$

$$= (29.8\text{ kips})\left(\frac{0.522\text{ in.}}{0.776\text{ in.}}\right)^2 \left[1+(0.915)(0.588)\right]$$

$$= 20.7\text{ kips}$$

The available load for the serviceability limit state is

$$\phi P_n = (2)(7.27\text{ kips}) + (4)(12.3\text{ kips} + 20.7\text{ kips}) = 146\text{ kips}$$

If the ultimate strength, $F_u = 65$ ksi, is used in the design procedure, the available load for the strength limit state is

$$\phi P_n = (2)(9.43\text{ kips}) + (4)(15.9\text{ kips} + 23.0\text{ kips}) = 174\text{ kips}$$

Example 6

If more strength is required for the connection in Example 5, stiffeners can be added as shown in Figure 19. Determine the strength of the bolts and beam flange.

For bolts at row 1,

$$p_{eff} = 3.00 \text{ in.}$$

$$\phi T_n = 7.27 \text{ kips (from Example 5)}$$

For bolts at row 2,

$$x_s = 2.50 \text{ in.}$$

$$\begin{aligned} x &= \sqrt{bc} \\ &= \sqrt{(2.56 \text{ in.})(3.92 \text{ in.})} \\ &= 3.17 \text{ in.} \end{aligned}$$

Thus, $x_s < x$; therefore,

$$\begin{aligned} p_{eff} &= \frac{p}{2} + x_s + \frac{bc}{x_s} \\ &= \frac{3 \text{ in.}}{2} + 2.50 \text{ in.} + \frac{(2.56 \text{ in.})(3.92 \text{ in.})}{2.50 \text{ in.}} \\ &= 8.01 \text{ in.} \end{aligned}$$

$$\begin{aligned} t_c &= \sqrt{\frac{4.44Bb'}{p_{eff}F_y}} \\ &= \sqrt{\frac{(4.44)(29.8 \text{ kips})(2.18 \text{ in.})}{(8.01 \text{ in.})(50 \text{ ksi})}} \\ &= 0.849 \text{ in.} \end{aligned}$$

$$\begin{aligned} \delta &= 1 - \frac{d'}{p_{eff}} \\ &= 1 - \frac{0.8125}{8.01 \text{ in.}} \\ &= 0.899 \end{aligned}$$

$$\begin{aligned} \alpha' &= \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right] \\ &= \frac{1}{(0.899)(1+1.25)}\left[\left(\frac{0.849 \text{ in.}}{0.522 \text{ in.}}\right)^2 - 1\right] \\ &= 0.813 \end{aligned}$$

$$\begin{aligned} \phi T_n &= B\left(\frac{t}{t_c}\right)^2(1+\delta\alpha') \\ &= (29.8 \text{ kips})\left(\frac{0.522 \text{ in.}}{0.849 \text{ in.}}\right)^2\left[1+(0.899)(0.813)\right] \\ &= 19.5 \text{ kips} \end{aligned}$$

For bolts at row 3,

$$x_s = 3.50 \text{ in.}$$

$$x_s = 3.17 \text{ in.}$$

Thus, $x_s > x$; therefore,

$$\begin{aligned} p_{eff} &= 4x \\ &= (4)(3.17 \text{ in.}) \\ &= 12.7 \text{ in.} \end{aligned}$$

$$\begin{aligned} t_c &= \sqrt{\frac{4.44Bb'}{p_{eff}F_y}} \\ &= \sqrt{\frac{(4.44)(29.8 \text{ kips})(2.18 \text{ in.})}{(12.7 \text{ in.})(50 \text{ ksi})}} \\ &= 0.674 \text{ in.} \end{aligned}$$

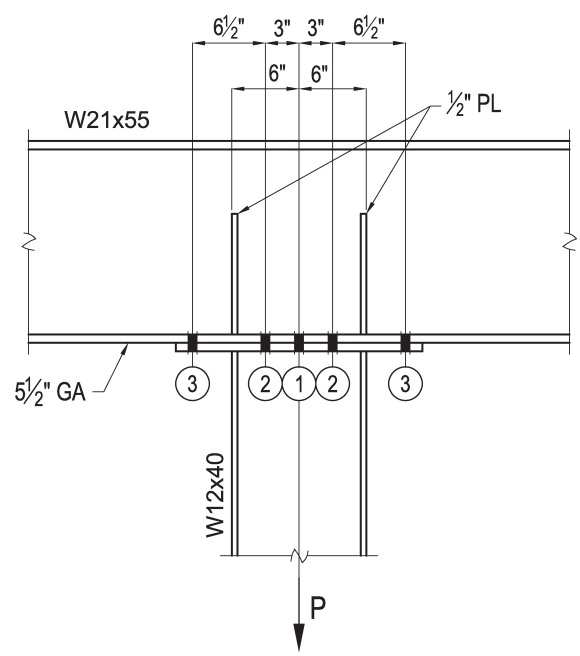


Fig. 19. Example 6—hanger connection with stiffeners.

$$\delta = 1 - \frac{d'}{p_{eff}}$$

$$= 1 - \frac{0.8125 \text{ in.}}{12.7 \text{ in.}}$$

$$= 0.936$$

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right]$$

$$= \frac{1}{(0.936)(1+1.25)}\left[\left(\frac{0.674 \text{ in.}}{0.522 \text{ in.}}\right)^2 - 1\right]$$

$$= 0.317$$

$$\phi T_n = B\left(\frac{t}{t_c}\right)^2 (1+\delta\alpha')$$

$$= (29.8 \text{ kips})\left(\frac{0.522 \text{ in.}}{0.674 \text{ in.}}\right)^2 \left[1+(0.936)(0.317)\right]$$

$$= 23.2 \text{ kips}$$

The available load for the serviceability limit state is

$$\phi P_n = (2)(7.27 \text{ kips}) + (4)(19.5 \text{ kips} + 23.2 \text{ kips}) = 185 \text{ kips}$$

If the ultimate strength, F_u = 65 ksi, is used in the design procedure, the available load for the strength limit state is

$$\phi P_n = (2)(9.43 \text{ kips}) + (4)(21.4 \text{ kips} + 26.2 \text{ kips}) = 209 \text{ kips}$$

CONCLUSIONS

A method has been proposed to calculate the tension strength of bolted flange connections, which includes the effects of prying action. The proposed design procedure, based on yield line theory, is simple, accurate and versatile. It can be used to calculate the strength of many different connection configurations, including stiffened connections, connections with large bolt spacing and connections close to the end of the member.

The bolt forces within a group are distributed according to the equivalent length tributary to each bolt, and the strength of each bolt is evaluated independently. The total strength of the joint is then calculated by summing the nominal strength at each bolt for the entire bolt group.

The calculated strengths were compared to the results of 59 tests from 10 independent research projects, and the proposed design method, which uses the yield strength of the fitting, was shown to be accurate for a deformation limit of approximately ¼ in. The ultimate strength of the fitting can be determined by using the proposed design procedure with the ultimate strength, F_u, of the fitting. However, the deformations at the ultimate strength can be large.

SYMBOLS

B	Available tension per bolt
C_r	Curve fit for the reduction factor is
F_y	Specified minimum yield strength of the fitting
F_u	Specified minimum tensile strength of the fitting
L_i	Length of yield line i
M_{pi}	Plastic moment capacity of yield line i
P	Applied load
P_{ny}	Nominal load calculated with the yield strength of the fitting
P_{nu}	Nominal load calculated with the ultimate strength of the fitting
P_p	Experimental load at the proportional limit
P_s	Experimental load at the nonlinear transition point on the load-deformation curve
P_u	Experimental load at ultimate failure
$P_{1/4}$	Experimental load at ¼-in. deformation
T_a	Available tensile strength of fitting
T_{cr}	Strength of the fitting at the critical bolt (the bolt with the largest equivalent tributary length within the joint) (calculated using Equation 1)
$T_{cr(\alpha'=1)}$	Strength of the fitting at the critical bolt assuming a full yield line pattern forms without bolt rupture (calculated using Equation 1 with α' = 1)
$T_{\alpha'=1}$	Strength of the fitting assuming a full yield line pattern forms without bolt rupture (calculated using Equation 1 with α' = 1)
a	Distance from the bolt centerline to the edge of the fitting, but ≤ 1.25b for calculations using the proposed design method
b	Distance from bolt centerline to the face of the web
d_b	Bolt diameter
d'	Width of the hole along the length of the fitting
l_e	Edge distance from the bolt to the end of the member
m_p	Plastic moment capacity per unit length of the fitting
n	Number of bolt rows
p	Spacing between bolts

p_c Equivalent tributary length per bolt calculated according to the circular yield line pattern developed by Zoetemeijer (1981)

p_d Equivalent tributary length per bolt calculated according to the yield line pattern developed by Dranger (1977)

p_e Equivalent length of fitting tributary to the bolt in question for connection type 4, the equivalent length of fitting tributary to the bolt farthest from the end

p_{el} For connection type 4, the equivalent length of fitting tributary to the bolt closest to the end

p_m Equivalent tributary length per bolt calculated according to the yield line pattern developed by Mann and Morris (1979)

p_{min} Minimum equivalent tributary length per bolt; minimum of p_d, p_z and p_m

p'_{min} Approximate minimum equivalent tributary length per bolt calculated with Equation 30

p_s Equivalent tributary length per bolt calculated according to the stiffened yield line pattern in Figure 9

p_z Equivalent tributary length per bolt calculated according to the yield line pattern developed by Zoetemeijer (1974)

r_t Strength of tested bolt in tension

r_n Nominal strength of bolt in tension

t Thickness of the fitting, in.

x $\sqrt{bc}$

x_s Distance from the center of the bolt to the edge of the stiffener

Δ Virtual displacement

θ_i Virtual rotation of yield line i

δ_p Experimental deformation at the proportional limit

δ_s Experimental deformation at the nonlinear transition point on the load-deformation curve

δ_u Experimental deformation at ultimate failure

APPENDIX A

Table A1. Specimen Properties											
Specimen	F_y (ksi)	F_u (ksi)	t (in.)	b (in.)	a (in.)	p_e (in.)	p_{el} (in.)	d_b (in.)	d' (in.)	r_t (kips)	Notes
Garrett (1977)											
1	41.2	66.5	0.465	1.62	1.51	7.00	—	0.875	0.938	54.1	
2	41.2	66.5	0.465	1.62	1.51	9.00	—	0.875	0.938	54.1	
3	41.2	66.5	0.465	1.62	1.51	9.00	—	0.875	0.938	54.1	
Ghassemieh et al. (1983)											
TH-1	43.5	65.9	0.5	1.5	2.92	5.62	2.08	0.625	0.688	27.1	
TH-2	38.7	70.1	0.5	2.5	2.92	8.23	2.08	0.625	0.688	27.1	
TH-3	45.4	66.5	1	1.5	2.92	5.00	2.08	0.625	0.688	27.1	
TH-4	43.1	73.9	1	2.5	2.92	10.3	2.08	0.625	0.688	27.1	
TH-5	44.9	72.5	0.75	1.5	2.92	5.62	2.08	0.625	0.688	27.1	
TH-6	37.7	65.2	0.75	2.5	2.92	8.23	2.08	0.625	0.688	27.1	
Grogan and Surtees (1999)											
E1	42.8	—	0.559	2.59	2.24	10.0	—	1.18	1.30	64.4	
E15	42.2	—	0.559	2.59	2.24	14.4	—	1.18	1.30	64.4	
Hendrick and Murray (1983)											
1	38.3	—	0.778	2.52	3.26	9.95	—	1.38	1.44	133	
2	34.6	—	0.813	2.50	4.54	10.7	—	1.38	1.44	133	
3	39.7	—	0.718	2.54	3.25	9.57	—	1.13	1.19	89.5	
4	39.7	—	0.718	2.54	3.25	10.14	—	1.50	1.56	159	
Moore and Sims (1986)											
T1	44.8	—	0.268	1.57	1.30	6.22	—	0.630	0.709	25.2	
T8	44.8	—	0.268	1.57	1.30	8.54	—	0.630	0.709	25.2	
Packer and Morris (1977)											
T1	42.9	—	0.268	1.58	1.12	6.04	—	0.630	0.811	36.7	
T2	43.0	—	0.268	1.58	1.12	6.04	—	0.630	0.811	36.7	
T3	42.5	—	0.354	1.56	1.12	6.01	—	0.630	0.811	36.7	
T4	44.7	—	0.268	1.73	0.96	7.26	—	0.630	0.811	36.7	
T5	43.6	—	0.528	1.49	1.12	5.86	—	0.630	0.811	36.7	
T6	43.2	—	0.268	1.58	1.12	8.30	—	0.630	0.811	36.7	
T7	43.9	—	0.268	1.58	1.12	8.30	—	0.630	0.811	36.7	
T8	44.8	—	0.268	1.58	1.12	8.30	—	0.630	0.811	36.7	
Pynnonen and Granstrom (1986)											
1	46.1	68.4	0.366	1.38	1.65	6.07	—	0.630	0.709	31.1	
5	46.1	68.4	0.366	1.74	1.30	6.57	—	0.630	0.709	31.1	
11	46.1	68.4	0.366	1.74	1.30	6.57	—	0.945	1.02	71.6	
15	44.8	64.4	0.551	2.19	3.54	9.07	—	0.945	1.02	71.6	
21	44.8	64.4	0.551	3.42	2.32	10.8	—	0.945	1.02	71.6	

Table A1. Specimen Properties (continued)

Specimen	F_y (ksi)	F_u (ksi)	t (in.)	b (in.)	a (in.)	p_e (in.)	p_{el} (in.)	d_b (in.)	d' (in.)	r_t (kips)	Notes
Tawaga and Gurel (2005)											
T-N	41.8	61.9	0.394	1.44	1.38	6.03	—	0.787	0.866	50.9	
Zoetemeijer (1981)											
1	36.4	—	0.457	2.21	2.36	13.9	—	0.787	0.866	50.9	
2	36.4	—	0.457	3.18	1.38	18.2	—	0.787	0.866	50.9	
4	36.4	—	0.457	2.21	2.36	12.8	—	0.787	0.866	50.9	
5	36.4	—	0.457	3.18	1.38	9.54	—	0.787	0.866	50.9	
6	36.4	—	0.457	3.18	1.38	15.3	—	0.630	0.709	32.9	
7	36.4	—	0.457	3.18	1.38	15.8	—	0.787	0.866	50.9	
8	40.6	—	0.492	4.39	1.36	20.7	—	0.945	1.02	67.5	
9	40.6	—	0.492	2.19	3.56	13.7	—	0.945	1.02	67.5	
10	40.6	—	0.492	4.39	1.36	20.7	—	0.945	1.02	67.5	
11	40.6	—	0.492	2.19	3.56	14.2	—	0.945	1.02	67.5	1
12	40.6	—	0.492	4.39	1.36	27.6	—	0.945	1.02	67.5	
13	40.6	—	0.492	2.19	3.56	13.7	—	0.945	1.02	67.5	
14	40.6	—	0.492	2.19	3.56	17.5	—	0.945	1.02	67.5	1
15	40.6	—	0.492	4.39	1.36	27.6	—	0.945	1.02	67.5	
Zoetemeijer (1974)											
5	37.7	—	0.315	1.68	0.965	5.78	—	0.787	0.866	39.4	
6	37.7	—	0.315	1.28	1.13	5.09	—	0.787	0.866	39.4	
7	38.8	—	0.335	1.77	1.26	6.21	—	0.787	0.866	39.4	
8	41.8	—	0.492	1.77	1.22	6.17	—	0.787	0.866	41.1	
9	41.8	—	0.492	1.38	1.13	5.29	—	0.787	0.866	41.1	
10	39.2	—	0.906	1.61	1.26	5.88	—	0.787	0.866	38.9	
11	43.5	—	0.591	1.87	1.12	6.29	—	0.787	0.866	41.1	
12	43.5	—	0.669	1.69	1.26	6.03	—	0.787	0.866	38.3	
13	37.7	—	0.315	1.68	0.97	5.78	—	0.787	0.866	39.4	
14	37.7	—	0.315	1.68	0.97	5.78	—	0.787	0.866	39.4	
20	30.5	—	0.571	2.48	1.12	7.54	—	0.787	0.866	40.7	
21	30.5	—	0.571	2.48	1.12	7.54	—	0.787	0.866	40.7	
22	30.5	—	0.571	2.48	1.12	7.54	—	0.787	0.866	37.2	
23	30.5	—	0.571	2.48	1.12	7.54	—	0.787	0.866	37.2	

1. Four bolts per bolt row.

Table A2. Experimental Results									
Specimen	P_p (kips)	P_s (kips)	$P_{1/4}$ (kips)	P_u (kips)	δ_p (in.)	δ_n (in.)	δ_u (in.)	**Failure Mode**	**Notes**
Garrett (1977)									
1	56	82	92	164	0.08	0.18	1.39	F	
2	72	102	106	174	0.12	0.23	0.74	F	
3	85	97	117	239	0.13	0.17	1.29	F	
Ghassemieh et al. (1983)									
TH-1	80	—	—	105	0.002	—	0.006	B	
TH-2	40	110	—	140	0.001	0.016	> 0.1	F B	
TH-3	165	—	—	200	0.006	—	—	N	1
TH-4	150	175	—	195	0.007	0.014	0.03	B	
TH-5	130	150	—	170	0.006	0.012	> 0.02	F B	
TH-6	70	—	—	140	0.001	—	0.011	N	2
Grogan and Surtees (1999)									
E1	90	124	—	248	—	—	—	F	
E15	112	169	—	292	—	—	—	F	
Hendrick and Murray (1983)									
1	110	160	—	200	0.003	0.021	0.065	N	1
2	120	160	—	200	0.012	0.033	0.088	N	1
3	60	—	—	200	0.023	—	0.22	N	1
4	110	170	—	200	0.012	0.056	0.14	N	1
Moore and Sims (1986)									
T1	22	27	31	60	0.10	0.18	> 0.7	O	
T8	34	40	43	74	0.16	0.22	> 0.6	O	
Packer and Morris (1977)									
T1	22.5	—	—	63.0	—	—	—	F B	
T2	22.1	—	—	62.8	—	—	—	F B	
T3	36.0	—	—	69.3	—	—	—	F B	
T4	27.0	—	—	45.5	—	—	—	F O	
T5	63.0	—	—	103	—	—	—	O B	
T6	31.5	—	—	73.6	—	—	—	F O	
T7	31.5	—	—	73.6	—	—	—	F O	
T8	29.3	—	—	67.7	—	—	—	F O	
Pynnonen and Granstrom (1986)									
1	51	68	83	97.7	0.03	0.08	> 0.6	F B	
5	22	50	61	90.0	0.05	0.08	> 0.6	F B	
11	30	57	94	151	< 0.01	0.05	> 0.9	F	
15	72	110	130	212	< 0.01	0.03	> 0.9	F B	
21	—	—	—	191	—	—	—	F B	

Table A2. Experimental Results (continued)									
Specimen	**P_p (kips)**	**P_s (kips)**	**$P_{¼}$ (kips)**	**P_u (kips)**	**δ_p (in.)**	**δ_n (in.)**	**δ_u (in.)**	**Failure Mode**	**Notes**
Tawaga and Gurel (2005)									
T-N	61	74	94	97	0.024	0.063	0.32	N	3
Zoetemeijer (1981)									
1	—	—	—	167	—	—	—	F	
2	56	76	79	144	0.08	0.24	1.2	F	
4	—	—	—	161	—	—	—	F	
5	—	—	—	135	—	—	—	F	
6	—	—	—	117	—	—	—	B	
7	—	—	—	133	—	—	—	F	
8	50	80	79	183	0.09	0.18	2.4	B	
9	90	150	140	244	0.04	0.35	2.0	B	
10	40	65	68	183	0.08	0.20	2.2	O	
11	97	150	140	266	0.06	0.35	1.4	B	
12	—	—	—	150	—	—	—	O	
13	130	—	190	221	0.08	—	—	B	
14	250	290	280	300	0.12	0.43	0.59	O	
15	—	—	—	159	—	—	—	O	
Zoetemeijer (1974)									
5	22	32	45	49.5	0.004	0.02	> 0.2	F	4
6	32	45	63	67.4	0.01	0.03	> 0.2	F	4
7	26	40	48	67.4	0.01	0.03	> 0.2	F	4
8	—	—	—	135	—	—	—	F B	
9	—	—	—	135	—	—	—	F B	
10	—	—	—	148	—	—	—	F B	
11	—	—	—	126	—	—	—	F B	
12	—	—	—	153	—	—	—	F B	
13	22	29	36	49.5	0.02	0.04	> 0.2	F	4
14	25	30	36	40.5	0.04	0.05	> 0.2	F	4
20	—	—	—	120	—	—	—	F B	
21	43	63	84	103	0.02	0.04	> 0.2	F B	4
22	—	—	—	111	—	—	—	F B	
23	—	—	—	128	—	—	—	F B	

Failure modes
- N: No failure
- F: Flange bending
- B: Bolt rupture
- O: Other

Notes
1. Maximum test load was 200 kips, which was the machine capacity.
2. Test result for ultimate load was not available; 140-kip load was taken from finite element model.
3. Test was stopped at a load of 97 kips.
4. Loads at ¼-in. deformation were conservatively read from the highest graphed deformations, which were between 0.12 and 0.16 in.

Table A3. Calculation Results													
Specimen	Nominal Strength (kips)		Predicted Failure Mode		$\frac{P_p}{P_{ny}}$	$\frac{P_s}{P_{ny}}$	$\frac{P_{1/4}}{P_{ny}}$	$\frac{P_u}{P_{ny}}$	$\frac{P_p}{P_{nu}}$	$\frac{P_s}{P_{nu}}$	$\frac{P_{1/4}}{P_{nu}}$	$\frac{P_u}{P_{nu}}$	Notes
	Using F_y	Using F_u	Using F_y	Using F_u									
Garrett (1977)													
1	98.6	159	F	F	0.57	0.83	0.93	1.66	0.35	0.52	0.58	1.03	
2	129	176	F	F B	0.56	0.79	0.82	1.35	0.41	0.58	0.60	0.99	2
3	129	176	F	F B	0.66	0.75	0.91	1.85	0.48	0.55	0.66	1.36	2
Ghassemieh et al. (1983)													
TH-1	116	144	F B	F B	0.69	—	—	0.91	0.56	—	—	0.73	1
TH-2	81.8	111	F B	F B	0.49	1.34	—	1.71	0.36	0.99	—	1.26	1
TH-3	204	217	B	B	0.81	—	—	—	0.76	—	—	—	1
TH-4	177	195	B	B	0.85	0.99	—	1.10	0.77	0.90	—	1.00	1
TH-5	182	201	B	B	0.71	0.82	—	0.93	0.65	0.75	—	0.85	1
TH-6	125	167	F B	B	0.56	—	—	—	0.42	—	—	—	1
Grogan and Surtees (1999)													
E1	126	—	F	—	0.71	0.98	—	1.97	—	—	—	—	
E15	181	—	F	—	0.62	0.93	—	1.61	—	—	—	—	
Hendrick and Murray (1983)													
1	233	—	F	—	0.47	0.69	—	—	—	—	—	—	
2	252	—	F	—	0.48	0.63	—	—	—	—	—	—	
3	186	—	F	—	0.32	—	—	—	—	—	—	—	
4	215	—	F	—	0.51	0.79	—	—	—	—	—	—	
Moore and Sims (1986)													
T1	30.0	—	F	—	0.73	0.90	1.03	2.00	—	—	—	—	
T8	41.8	—	F	—	0.81	0.96	1.03	1.77	—	—	—	—	1
Packer and Morris (1977)													
T1	27.4	—	F	—	0.82	—	—	2.30	—	—	—	—	
T2	27.5	—	F	—	0.80	—	—	2.28	—	—	—	—	
T3	47.9	—	F	—	0.75	—	—	1.45	—	—	—	—	
T4	31.1	—	F	—	0.87	—	—	1.46	—	—	—	—	
T5	108	—	F B	—	0.58	—	—	0.95	—	—	—	—	
T6	38.7	—	F	—	0.81	—	—	1.90	—	—	—	—	1
T7	39.3	—	F	—	0.80	—	—	1.87	—	—	—	—	1
T8	40.1	—	F	—	0.73	—	—	1.69	—	—	—	—	1
Pynnonen and Granstrom (1986)													
1	66.1	98.1	F	F	0.77	1.03	1.26	1.48	0.52	0.69	0.85	1.00	
5	54.0	80.1	F	F	0.41	0.93	1.13	1.67	0.27	0.62	0.76	1.12	
11	59.1	87.8	F	F	0.51	0.96	1.59	2.55	0.34	0.65	1.07	1.72	
15	135	194	F	F	0.53	0.81	0.96	1.57	0.37	0.57	0.67	1.09	
21	95.4	137	F	F	—	—	—	2.00	—	—	—	1.39	

Table A3. Calculation Results (continued)													
Specimen	Nominal Strength (kips)		Predicted Failure Mode		$\frac{P_p}{P_{ny}}$	$\frac{P_s}{P_{ny}}$	$\frac{P_{1/4}}{P_{ny}}$	$\frac{P_u}{P_{ny}}$	$\frac{P_p}{P_{nu}}$	$\frac{P_s}{P_{nu}}$	$\frac{P_{1/4}}{P_{nu}}$	$\frac{P_u}{P_{nu}}$	Notes
	Using F_y	Using F_u	Using F_y	Using F_u									
Tawaga and Gurel (2005)													
T-N	69.5	103	F	F	0.88	1.06	1.35	—	0.59	0.72	0.91	—	
Zoetemeijer (1981)													
1	112	—	F	—	—	—	—	1.49	—	—	—	—	1, 3
2	96.8	—	F	—	0.58	0.79	0.82	1.49	—	—	—	—	1
4	103	—	F	—	—	—	—	1.56	—	—	—	—	1
5	49.5	—	F	—	—	—	—	2.73	—	—	—	—	1
6	74.2	—	B	—	—	—	—	1.58	—	—	—	—	
7	83.5	—	F	—	—	—	—	1.59	—	—	—	—	
8	101	—	F	—	0.50	0.79	0.78	1.81	—	—	—	—	1
9	152	—	F	—	0.59	0.99	0.92	1.61	—	—	—	—	3
10	101	—	F	—	0.40	0.64	0.67	1.81	—	—	—	—	1
11	157	—	F	—	0.62	0.96	0.89	1.69	—	—	—	—	
12	133	—	B	—	—	—	—	1.13	—	—	—	—	1, 3
13	152	—	F	—	0.86	—	1.25	1.45	—	—	—	—	1, 3
14	195	—	F	—	1.28	1.49	1.44	1.54	—	—	—	—	1
15	133	—	B	—	—	—	—	1.20	—	—	—	—	1, 3
Zoetemeijer (1974)													
5	31.2	—	F	—	0.71	1.03	1.44	1.59	—	—	—	—	
6	39.2	—	F	—	0.82	1.15	1.61	1.72	—	—	—	—	
7	36.5	—	F	—	0.71	1.10	1.32	1.85	—	—	—	—	
8	84.3	—	F	—	—	—	—	1.60	—	—	—	—	
9	99.8	—	F	—	—	—	—	1.35	—	—	—	—	
10	155	—	B	—	—	—	—	0.95	—	—	—	—	
11	115	—	F B	—	—	—	—	1.10	—	—	—	—	
12	126	—	F B	—	—	—	—	1.21	—	—	—	—	
13	31.2	—	F	—	0.71	0.93	1.15	1.59	—	—	—	—	
14	31.2	—	F	—	0.80	0.96	1.15	1.30	—	—	—	—	
20	67.9	—	F	—	—	—	—	1.77	—	—	—	—	
21	67.9	—	F	—	0.63	0.93	1.24	1.52	—	—	—	—	
22	67.9	—	F	—	—	—	—	1.63	—	—	—	—	
23	67.9	—	F	—	—	—	—	1.89	—	—	—	—	

Notes
1. $x_s < x$
2. $x_s > x$
3. Theory indicates circular yield line controls the design. This was accounted for in the listed values.

REFERENCES

AISC (2005a), *Steel Construction Manual*, 13th ed., American Institute of Steel Construction, Chicago, IL.

AISC (2005b), *Specification for Structural Steel Buildings*, American Institute of Steel Construction, Chicago, IL.

Astaneh, A. (1985), "Procedure for Design and Analysis of Hanger-Type Connections," *Engineering Journal*, AISC, Vol. 22, No. 2, pp. 63–66.

CEN (2005), *Eurocode 3: Design of Steel Structures-Part 1–8: Design of Joints*, European Committee for Standardization.

Douty, R.T. and McGuire W. (1965), "High Strength Bolted Moment Connections," *Journal of the Structural Division*, ASCE, Vol. 91, No. ST2, pp. 101–128.

Dranger, T.S. (1977), "Yield Line Analysis of Bolted Hanging Connections," *Engineering Journal*. AISC, Vol. 14, No. 3.

Garrett, J.L. (1977), "An Investigation of High Strength Bolted Tension Connections," Master's Thesis, Iowa State University.

Ghassemieh, M., Kukreti, A. and Murray, T.M. (1983), "Inelastic Finite Element Analysis of Stiffened End Plate Moment Connections," Report FSEL/AISC 83-02, School of Civil Engineering and Environmental Science, University of Oklahoma, June.

Grogan, W. and Surtees, J.O. (1999), "Experimental Behavior of End Plate Connections Reinforced with Bolted Backing Angles," *Journal of Constructional Steel Research*, Vol. 50, pp. 71–96.

Hendrick, A. and Murray, T.M. (1983), "Column Web and Flange Strength at End-Plate Connections," Report FSEL/AISC 83-01, School of Civil Engineering and Environmental Science, University of Oklahoma, February.

Hognestad, E. (1953), "Yield Line Theory for the Ultimate Flexural Strength of Reinforced Concrete Slabs," *Journal of the American Concrete Institute*, Vol. 24, No. 7.

Johansen, K.W. (1962), *Yield Line Theory*, Cement and Concrete Association, London, England.

Kulak, G.L., Fisher, J.W. and Struik, J.H.A. (1987), *Guide to Design Criteria for Bolted and Rivited Joints*, John Wiley and Sons, New York.

Mann, A.P. and Morris, L.J. (1979), "Limit Design of Extended End-Plate Connections," *Journal of the Structural Division*, ASCE, March, pp. 511–526.

Moore, D.B. and Sims, P.A.C. (1986), "Preliminary Investigations into the Behavior of Extended End-Plate Steel Connections with Backing Plates," *Journal of Constructional Steel Research*, pp. 95–122.

Muir, L.S. and Thornton, W.A. (2006), "Connections," Ch. 3, *Structural Steel Designer's Handbook*, R.L. Brockenbrough and F.S. Merritt, Eds., McGraw-Hill, New York.

Packer, J.A. and Morris, L.J. (1977), "A Limit State Method for the Tension Region of Bolted Beam-Column Connections," *The Structural Engineer*, Vol. 5, No. 10.

Pynnonen, J. and Granstrom, A. (1986), "Beam-to-Column Connections with Backing Plates," Swedish Institute of Steel Construction Report 86:6, April.

SCI (1995), *Joints in Steel Construction-Moment Connections*, The Steel Construction Institute, Ascot, Berkshire, UK.

Tawaga, H. and Gurel, S. (2005), "Strength Evaluation of Bolted Moment Connections Stiffened with Channels," Handout for poster session at the North American Steel Construction Conference.

Thornton, W.A. (1985), "Prying Action—A General Treatment," *Engineering Journal*, AISC, Vol. 22, No. 2, pp. 67–75.

Thornton, W.A. (1992), "Strength and Serviceability of Hanger Connections," *Engineering Journal*, Vol. 29, No. 4, pp. 145–149.

Thornton, W.A. and Kane, T. (1999), "Design of Connections for Axial, Moment and Shear Forces," Ch. 2, *Handbook of Structural Steel Connection Design and Details*, A.R. Tamboli, Ed., McGraw-Hill, New York.

Young, W.C. (1989), *Roark's Formulas for Stress and Strain*, 6th ed., McGraw-Hill, p. 207.

Zoetemeijer, P. (1974), "A Design Method for the Tension Side of Statically Loaded, Bolted Beam-to-Column Connections," *Heron*, Vol. 20, No. 1.

Zoetemeijer, P. (1981), "Semi-Rigid Bolted Beam-to-Beam Column Connections with Stiffened Column Flanges and Flush End Plates," *Joints in Structural Steelwork, Proc. International Conference,* Teesside Polytechnic, April 6–9, John Wiley and Sons.