For a saturated soil, Skempton's coefficient $B = 1$, and from Eq. (5.44)

$$\Delta u = \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3) \tag{6.56}$$

Substituting Eq. (6.56) into Eq. (6.55) gives

$$\sigma_3' = K_o(\sigma_1')_o - A(\Delta\sigma_1 - \Delta\sigma_3)$$

Solving for $\Delta\sigma_1 - \Delta\sigma_3$, we obtain

$$\Delta\sigma_1 - \Delta\sigma_3 = \frac{K_o(\sigma_1')_o - \sigma_3'}{A} \tag{6.57}$$

At failure,

$$s_u = \left(\frac{\sigma_1 - \sigma_3}{2}\right)_f = \frac{1}{2}\{[(\sigma_1')_o + \Delta\sigma_1] - [K_o(\sigma_1')_o + \Delta\sigma_3]\}$$
$$= \frac{1}{2}[(\Delta\sigma_1 - \Delta\sigma_3) + (1 - K_o)(\sigma_1')_o] \tag{6.58}$$

Substituting Eq. (6.57) into Eq. (6.58) gives

$$\boxed{s_u = \frac{1}{2}\left[\frac{K_o(\sigma_1')_o - \sigma_3'}{A} + (1 - K_o)(\sigma_1')_o\right]} \tag{6.59}$$

At failure,

$$\frac{\sigma_1'}{\sigma_3'} = \frac{1 + \sin\phi_{cs}'}{1 - \sin\phi_{cs}'}$$

which by substitution into Eq. (6.59) leads to

$$\boxed{\frac{s_u}{\sigma_1'} = \frac{s_u}{\sigma_z'} = \frac{\sin\phi_{cs}'[K_o + A(1 - K_o)]}{1 + (2A - 1)\sin\phi_{cs}'}} \tag{6.60}$$

The essential points are:

1. ***A K_o-consolidated sample of a soil is likely to have a different undrained shear strength than an isotropically consolidated sample of the same soil even if the initial confining pressures before shearing are the same and the slopes of the stress paths are also the same.***
2. ***Failure stresses in soils are dependent on the stress history of the soil.***
3. ***Stress history does not influence the elastic response of soils.***

What's next. . .We have established the main ideas behind the critical state model and used the model to estimate the response of soils to loading. The CSM can also be used with results from simple soil tests (e.g., Atterberg limits) to make estimates of the soil strengths. In the next section, we will employ the CSM to build some relationships among results from simple soil tests, critical state parameters, and soil strengths.

6.10 RELATIONSHIPS BETWEEN SIMPLE SOIL TESTS, CRITICAL STATE PARAMETERS, AND SOIL STRENGTHS

Wood and Wroth (1978) and Wood (1990) used the critical state model to correlate results from Atterberg limit tests with various engineering properties of fine-grained soils. We are going to present some of these correlations. These correlations are very useful when limited test data are available during the preliminary design of geotechnical systems or when you need to evaluate the quality of test results. The correlations utilized water content, which at best is accurate to 0.1%. Most often water content results are reported to the nearest whole number and consequently significant differences can occur between the actual test results and the correlations, especially those involving exponentials. Since we are using CSM and index properties, the relationships only pertain to remolded or disturbed soils.

6.10.1 Undrained Shear Strength of Clays at the Liquid and Plastic Limits

Wood (1990), using test results reported by Youssef et al. (1965) and Dumbleton and West (1970), showed that

$$\frac{(s_u)_{PL}}{(s_u)_{LL}} = R \tag{6.61}$$

where R depends on activity (Chapter 2) and varies between 30 and 100, and the subscripts PL and LL denote plastic limit and liquid limit, respectively. Wood and Wroth (1978) recommend a value of $R = 100$ as reasonable for most soils. The recommended value of $(s_u)_{LL}$, culled from the published data, is 2 kPa (the test data showed variations between 0.9 and 8 kPa) and that for $(s_u)_{PL}$ is 200 kPa. Since most soils are within the plastic range these recommended values place lower (2 kPa) and upper (200 kPa) limits on the undrained shear strength of disturbed or remolded clays.

6.10.2 Vertical Effective Stresses at the Liquid and Plastic Limits

Wood (1990) used results from Skempton (1970) and recommended that

$$(\sigma_z')_{LL} = 8 \text{ kPa} \tag{6.62}$$

The test results showed that $(\sigma_z')_{LL}$ varies from 6 to 58 kPa. Laboratory and field data also showed that the undrained shear strength is proportional to the vertical effective stress. Therefore

$$\boxed{(\sigma_z')_{PL} = R(\sigma_z')_{LL} \approx 800 \text{ kPa}} \tag{6.63}$$

6.10.3 Undrained Shear Strength–Vertical Effective Stress Relationship

Normalizing the undrained shear strength with respect to the vertical effective stress we get a ratio of

$$\boxed{\frac{s_u}{\sigma'_z} = \frac{2}{8} \text{ or } \frac{200}{800} = 0.25} \tag{6.64}$$

Mesri (1975) reported, based on soil test results, that $s_u/\sigma'_{zc} = 0.22$, which is in good agreement with Eq. (6.64) for normally consolidated soils.

6.10.4 Compressibility Indices (λ and C_c) and Plasticity Index

The compressibility index C_c or λ is usually obtained from a consolidation test. In the absence of consolidation test results, we can estimate C_c or λ from the plasticity index. With reference to Fig. 6.16,

$$-(e_{PL} - e_{LL}) = \lambda \ln \frac{(\sigma'_z)_{PL}}{(\sigma'_z)_{LL}} = \lambda \ln R$$

Now, $e_{LL} = w_{LL}G_s$, $e_{PL} = w_{PL}G_s$, and $G_s = 2.7$. Therefore, for $R = 100$,

$$w_{LL} - w_{PL} = \frac{\lambda}{2.7} \ln R \approx 1.7\lambda$$

and

$$\lambda \approx 0.6 I_p \tag{6.65}$$

or

$$\boxed{C_c = 2.3\lambda \approx 1.38 I_p} \tag{6.66}$$

Equation (6.65) indicates that the compression index increases with plasticity index.

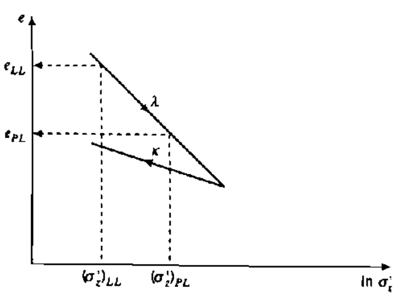

FIGURE 6.16 Illustrative graph of e versus $\ln \Sigma'_z$.

6.10.5 Undrained Shear Strength, Liquidity Index, and Sensitivity

Let us build a relationship between liquidity index and undrained shear strength. The undrained shear strength of a soil at a water content w, with reference to its undrained shear strength at the plastic limit, is obtained from Eq. (6.22) as

$$\boxed{\frac{(s_u)_w}{(s_u)_{PL}} = \exp\left(G_s \frac{(w_{PL} - w)}{\lambda}\right)}$$

Putting $G_s = 2.7$, $\lambda = 0.6 I_p$ in the above equation and recalling that

$$I_L = \frac{w - w_{PL}}{I_p}$$

we get

$$\boxed{(s_u)_w = (s_u)_{PL} \exp(-4.6 I_L) \approx 200 \exp(-4.6 I_L)} \tag{6.67}$$

Clays laid down in saltwater environments and having flocculated structure (Chapter 2) often have in situ (natural) water contents higher than their liquid limit but do not behave like a viscous liquid in their natural state. The flocculated structure becomes unstable when fresh water leaches out the salt. The undistributed or intact undrained shear strengths of these clays are significantly greater than their disturbed or remolded undrained shear strengths. The term sensitivity, S_t, is used to define the ratio of the intact undrained shear strength to the remolded undrained shear strength:

$$S_t = \frac{(s_u)_i}{(s_u)_r} \tag{6.68}$$

where i denotes intact and r denotes remolded. From Eq. (6.67) we can write

$$\boxed{(s_u)_r \approx 200 \exp(-4.6 I_L)} \tag{6.69}$$

For values of $S_t > 8$, the clay is called a quick clay. Quick clay, when disturbed, can flow like a viscous liquid ($I_L > 1$). Bjerrum (1954) reported test data on quick clays in Scandinavia, which yield an empirical relationship between S_t and I_L as

$$\boxed{I_L = 1.2 \log_{10} S_t} \tag{6.70}$$

6.11 SUMMARY

In this chapter, a simple critical state model (CSM) was used to provide some insight into soil behavior. The model replicates the essential features of soil behavior but the quantitative predictions of the model may not match real soil values. The key feature of the critical state model is that every soil fails on a unique surface in (q, p', e) space. According to the CSM, the failure stress state is insufficient to guarantee failure; the soil must also be loose enough (reaches the critical void ratio). Every sample of the same soil will fail on a stress state