where w is the unit weight of water, T is the top width, and h is the surge height. The kinetic energy of the element is evidently equal to

$$\text{K.E.} = \frac{wV^2yT}{2g} \tag{19-47}$$

where y is the depth of water and V is the velocity of flow. By Eq. (19-23) or Eq. (19-19), as the case may be, the above equation may be reduced to

$$\text{K.E.} = \frac{wh^2gyT}{2c^2} \tag{19-48}$$

Assuming a surge of small height,

$$c = \sqrt{gy}$$

and the above equation becomes

$$\text{K.E.} = \tfrac{1}{2}wh^2T \tag{19-49}$$

The total energy of surge per unit length is, therefore,

$$E = \text{P.E.} + \text{K.E.} = wh^2T \tag{19-50}$$

19-4. Negative Surges. Negative surges are not stable in form, because the upper portions of the wave travel faster than the lower portions (Art. 19-1). If the initial profile of the surge is assumed to have a

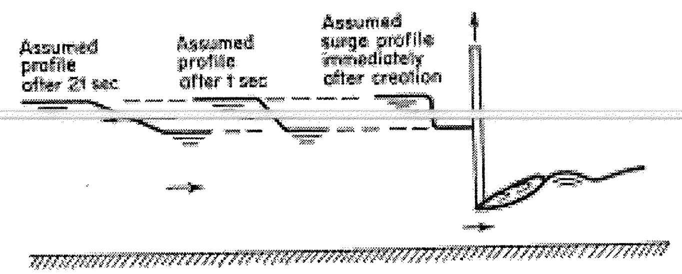

FIG. 19-8. Propagation of a negative surge due to sudden lift of a sluice gate.

steep front, it will soon flatten out as the surge moves through the channel (Fig. 19-8). If the height of the surge is moderate or small compared with the depth of flow, the equations derived for a positive surge can be applied to determine approximately the propagation of the negative surge. If the height of the surge is relatively large, a more elaborate analysis is necessary as follows:

Figure 19-9 shows a type D surge (Fig. 19-2) of relatively large height, retreating in an upstream direction. The surge is caused by the sudden lifting of a sluice gate. The wave velocity of the surge actually varies from point to point. For example, V_w is the wave velocity at a point on the surface of the wave where the depth is y and the velocity of flow through the section is V. During a time interval dt, the change in y is dy. The value of dy is positive for an increase of y and negative for a decrease of y. By the momentum principle, the corresponding change in hydrostatic pressure should be equal to the force required to change the momentum of the vertical element between y and $y + dy$. Considering a unit width of the channel and assuming $\beta_1 = \beta_2 = 1$,

$$\frac{w}{2} y^2 - \frac{w}{2} (y + dy)^2 = \frac{w}{g} (y + \tfrac{1}{2} dy)(V + V_w)\, dV \qquad (19\text{-}51)$$

Simplifying the above equation and neglecting the differential terms of higher order,

$$dy = -\frac{V + V_w}{g} dV \qquad (19\text{-}52)$$

As described previously (Art. 19-1), the whole wavefront can be assumed to be made up of a large number of very small waves placed

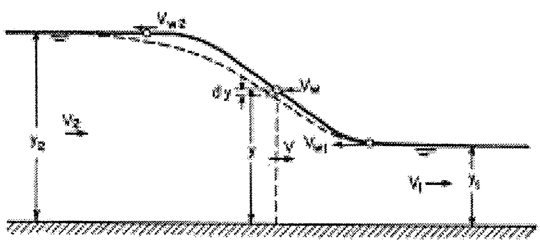

FIG. 19-9. Analysis of a negative surge.

one on top of the other. The velocity of the small wave at the point under consideration may be expressed according to Eq. (19-11) as

$$V_w = \sqrt{gy} - V \qquad (19\text{-}53)$$

Similarly, the velocity at the wave crest is

$$V_{w2} = \sqrt{gy_2} - V_2 \qquad (19\text{-}54)$$

and, at the wave trough,

$$V_{w1} = \sqrt{gy_1} - V_1 \qquad (19\text{-}55)$$

When the surge is not too high, a straight-line relation between V_{w1} and V_{w2} may be assumed. Thus, the mean velocity of the wave may be considered to be

$$V_m = \frac{V_{w1} + V_{w2}}{2} \qquad (19\text{-}56)$$

Now, eliminating V_w between Eqs. (19-52) and (19-53),

$$\frac{dy}{\sqrt{y}} = -\frac{dV}{\sqrt{g}} \qquad (19\text{-}57)$$

Integrating this equation from y_2 to y and from V_2 to V, and solving for V,

$$V = V_2 + 2\sqrt{gy_2} - 2\sqrt{gy} \tag{19-58}$$

From Eq. (19-53),

$$V_w = 3\sqrt{gy} - 2\sqrt{gy_2} - V_2 \tag{19-59}$$

Thus, the wave velocity at the trough of the wave is

$$V_{w1} = 3\sqrt{gy_1} - 2\sqrt{gy_2} - V_2 \tag{19-60}$$

Let t be the time elapsed since the surge was created or, in this case, since the sluice gate was opened. At $t = 0$, the wavelength $\lambda = 0$. After t sec, the wavelength is equal to

$$\lambda = (V_{w2} - V_{w1})t \tag{19-61}$$

The above analysis can be applied similarly to a negative surge of type C.

Example 19-5. Show that the equation of the wave profile, resulting from the failure of a dam is in the form of

$$x = 2t\sqrt{gy_2} - 3t\sqrt{gy} \tag{19-62}$$

where x is the distance measured from the dam site, y is the depth of the wave profile, y_2 is the depth of the impounding water, and t is the time after the dam broke.

Solution. Since the impounding water has zero velocity, or $V_2 = 0$, the wave velocity by Eq. (19-59) is $V_w = 3\sqrt{gy} - 2\sqrt{gy_2}$. Since V_w is in the negative direction of x, $x = -V_w t$, which gives Eq. (19-62).

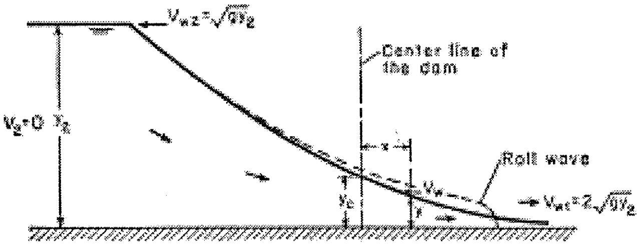

FIG. 19-10. Wave profile due to dam failure.

Equation (19-62) represents a parabola with vertical axis and vertex on the channel bottom, as shown in Fig. 19-10. At the site of the dam, $x = 0$ and the depth $y_c = 4y_2/9$. Owing to the channel friction, the actual profile takes the form indicated by the dashed line. This profile has a rounded front at the downstream end, forming a bore (see Example 18-1). At the upstream end, the theoretical profile thus developed has been checked satisfactorily with experiments by Schoklitsch [12].

19-5. Surge in Power Canals. Engineers are interested in the determination of the maximum stage of water that could be developed as a