JOB: Axial & Bending super position

SHEET NO.: 1 OF

CALCULATED BY: DATE:

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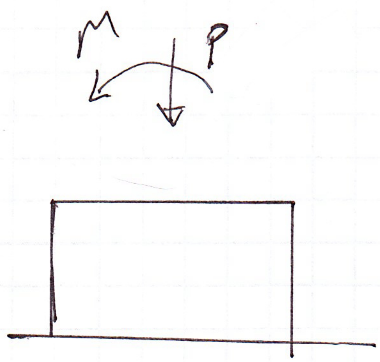
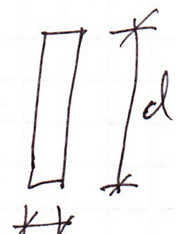
SCALE:





$A = 2\ ft^2$

$I = 0.667\ ft^4$

$S_x = 0.667\ ft^3$

$$\sigma_{Axial} = \frac{2^k}{2ft^2} = 1ksf$$

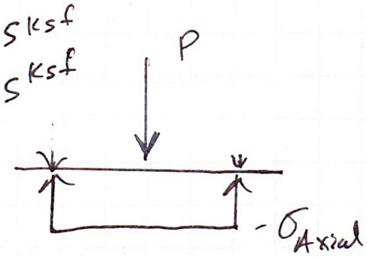
$P = 2^k$

$M = 3^{k-ft}$

$$\sigma_{Bending} = \frac{3^{k-ft}}{0.667\ ft^3} = 4.5^{ksf}$$

$\sigma_c = 1 + 4.5 = 5.5^{ksf}$

$\sigma_t = 1 - 4.5^{ksf} = 3.5^{ksf}$



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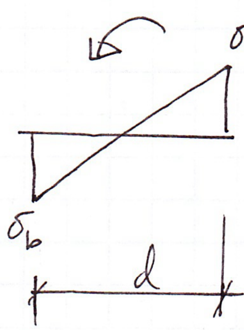

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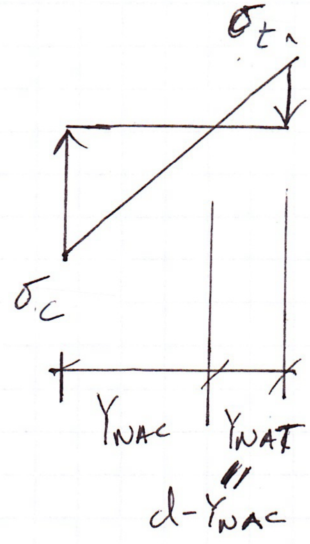

similar Triangles;

$$\frac{\sigma_c}{Y_{NAC}} = \frac{\sigma_T}{d - Y_{NAC}} \Rightarrow \frac{d - Y_{NAC}}{Y_{NAC}} = \frac{\sigma_T}{\sigma_c}$$

$$\Rightarrow \frac{d}{Y_{NAC}} - 1 = \frac{\sigma_T}{\sigma_c} \Rightarrow \frac{d}{\left(\frac{\sigma_T}{\sigma_c} + 1\right)} = Y_{NAC}$$

$$Y_{NAC} = \frac{2'}{\left(\frac{3.5}{5.5}\right) + 1} = 1.22'$$

JOB: Axial & Bending superposition

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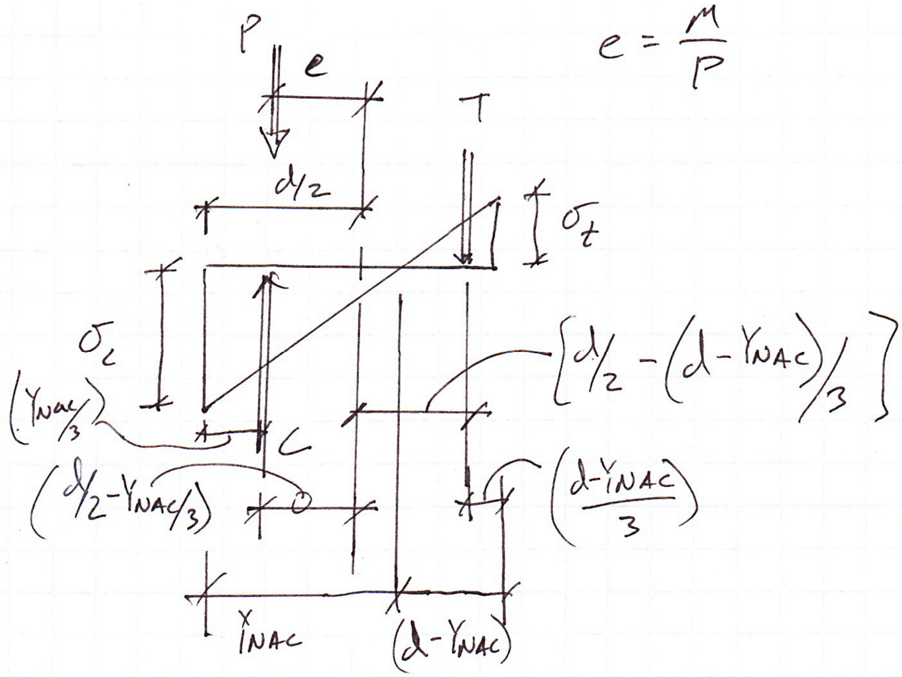
$$e = \frac{M}{P}$$



$$\Sigma F = C - T - P = 0$$

$$\sigma_c \times \frac{b\,Y_{NAC}}{2} - \sigma_t \times \frac{b\,(d - Y_{NAC})}{2} = P$$

$$5.5 \times \frac{1 \times 1.22}{2} - \frac{3.5 \times 1'(2 - 1.22)}{2} = 3.35 - 1.365 = 2^{k} \text{ ok}$$

$$+\Sigma M = C \times \left[\frac{d}{2} - \frac{Y_{NAC}}{3}\right] + T \times \left[\frac{d}{2} - \left(\frac{d - Y_{NAC}}{3}\right)\right] = Pe = M$$

$$3.35 \times [0.59267] + 1.365[0.74''] = 1.99 + 1.01 = 3^{k\text{-}ft} \text{ ok}$$

JOB: Axial & Bending – Equilbrm

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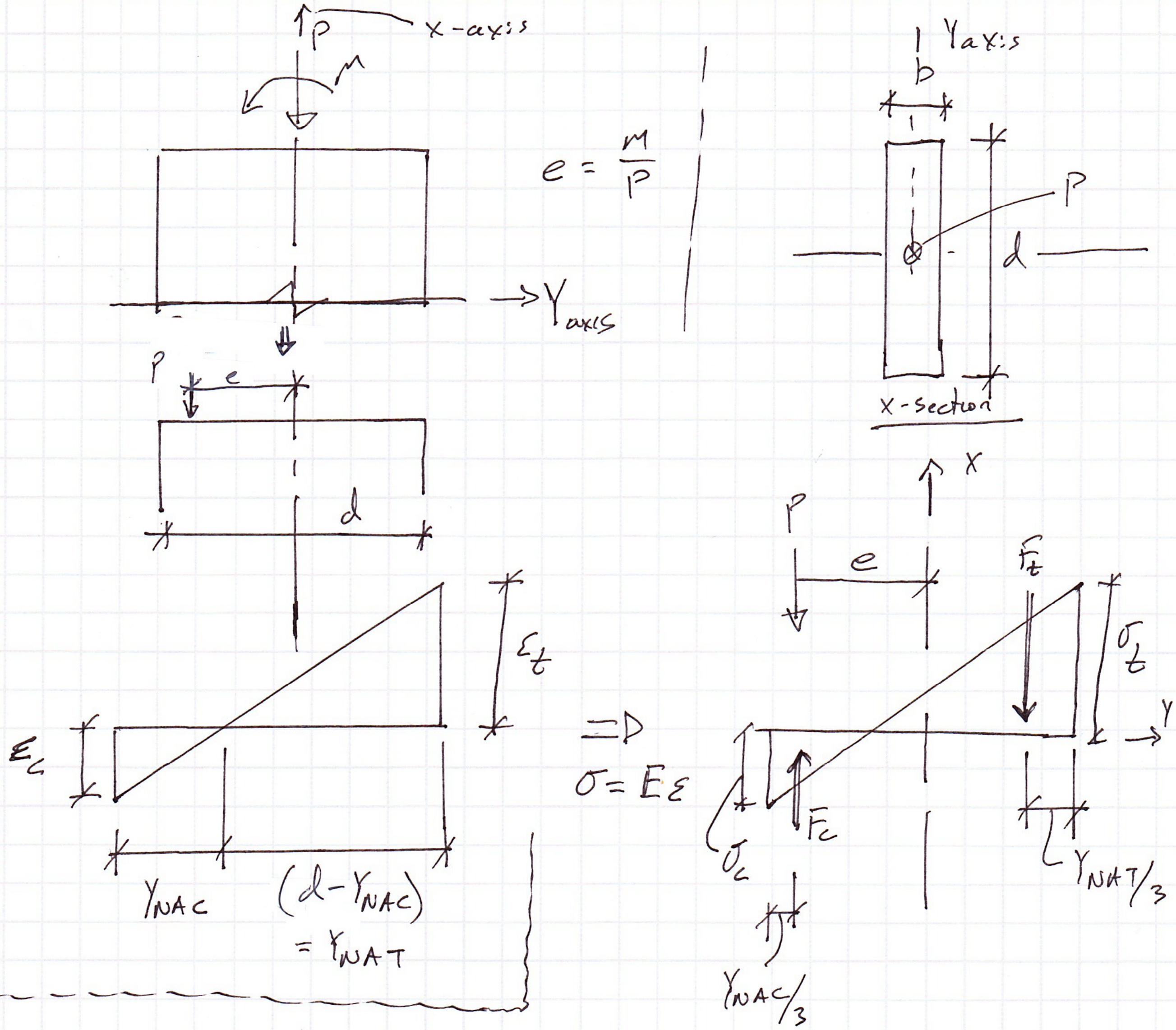
SCALE: Linear Elastic Strain Distribution



(3) unknowns = σ_c, σ_t, Y_{NAC}

(2) Equilibrium Eqns → ΣF_Y ← EQ1 & ΣM ← EQ2

(1) compatibility

$$\frac{\sigma_c}{Y_{NAC}} = \frac{\sigma_t}{(d - Y_{NAC})} \leftarrow \text{EQ3}$$

$$F_c = \frac{b \sigma_c Y_{NAC}}{2}$$

$$F_t = \frac{b \sigma_t Y_{NAT}}{2}, \quad Y_{NAT} = (d - Y_{NAC})$$

JOB: Axial & Bending – Equil. Method

SHEET NO.: 2 OF

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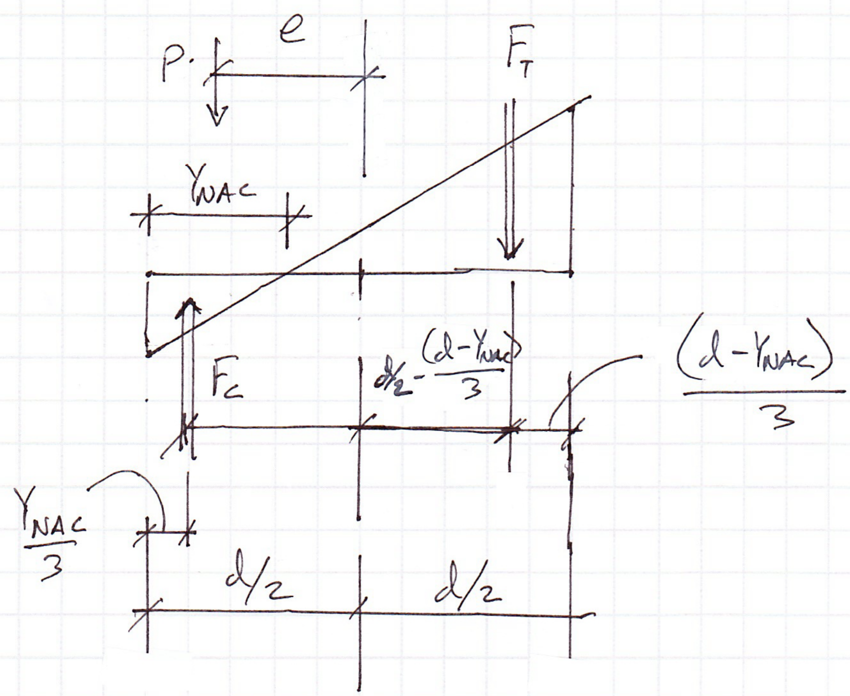
SCALE:



Force Diagram

$$\Sigma F_Y = F_c - P - F_T = 0 \Rightarrow F_c - F_T = P$$

$$\frac{\sigma_c\, b\, Y_{NAC}}{2} - \frac{\sigma_t\, b\,(d - Y_{NAT})}{2} = P \qquad \boxed{EQ\ 1}$$

$$\circlearrowleft^{+}\ \Sigma M = F_c \times \left[\frac{d}{2} - \frac{Y_{NAC}}{3}\right] + F_T \times \left[\frac{d}{2} - \left(\frac{d - Y_{NAC}}{3}\right)\right] - Pe = 0$$

(Neg → −; M → Pe)

$$\sigma_c\left(\frac{b\, Y_{NAC}}{2}\right) \times \left[\frac{d}{2} - \frac{Y_{NAC}}{3}\right] + \sigma_t\left(\frac{b\,(d - Y_{NAT})}{2}\right)\left[\frac{d}{2} - \frac{d - Y_{NAC}}{3}\right] = Pe$$

(M ⇓)

$$\boxed{EQ\ 2}$$

$$\frac{\sigma_c}{Y_{NAC}} - \frac{\sigma_t}{d - Y_{NAC}} = 0 \qquad \boxed{EQ\ 3}$$

JOB: Equil. Method

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SCALE:

We see that it is difficult to solve for the (3) variables b/c it is not a linear system of eqns. Thus, we will solve for σ_c & σ_t & then iterate the value of Y_{NAC} until our (3) Eqns are satisfied.

- use Matrix Algebra

EQ 1 $Ax + By = C$

EQ 2 $Dx + Ey = F$

$$\begin{bmatrix} A & B \\ D & E \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} C \\ F \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} C \\ F \end{bmatrix} \begin{bmatrix} A & B \\ D & E \end{bmatrix}^{-1}$$

(A) → $\frac{bY_{NAC}}{2}$, (B) → $\frac{b(d-Y_{NAT})}{2}$, (C) → $\frac{bY_{NAC}}{2} \times \left(\frac{d}{2} - \frac{Y_{NAC}}{3}\right)$, (D) → $\frac{b(d-Y_{NAT})}{2}\left(\frac{d}{2} - \frac{d-Y_{NAC}}{3}\right)$

$$\begin{bmatrix} \frac{bY_{NAC}}{2} & \frac{b(d-Y_{NAT})}{2} \\ \frac{bY_{NAC}}{2} \times \left(\frac{d}{2} - \frac{Y_{NAC}}{3}\right) & \frac{b(d-Y_{NAT})}{2}\left(\frac{d}{2} - \frac{d-Y_{NAC}}{3}\right) \end{bmatrix} \begin{bmatrix} P \\ M \end{bmatrix} = \begin{bmatrix} \sigma_c \\ \sigma_t \end{bmatrix}$$

(C) → P, M ← (F), σ_c ← (X), σ_t ← (Y)

In excel use
MMult[ ] = For Matrix Mult
MInv.[ ] = For " Inverse
Highlight 2 cells For Result CTL-Shft-Ent

Results Are Same As <u>Superposition</u>