Final set of non-linear dynamic equations:

$$[C] := \begin{pmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ \begin{pmatrix} s\varphi s\theta c\psi \\ -c\varphi s\psi \end{pmatrix} & \begin{pmatrix} s\varphi s\theta s\psi \\ +c\varphi c\psi \end{pmatrix} & s\varphi c\theta \\ \begin{pmatrix} c\varphi s\theta c\psi \\ +s\varphi s\psi \end{pmatrix} & \begin{pmatrix} c\varphi s\theta s\psi \\ -s\varphi c\psi \end{pmatrix} & c\varphi c\theta \end{pmatrix}$$

$$\vec{V} = [C]^{-1} \begin{pmatrix} V_{Bx} \\ V_{By} \\ V_{Bz} \end{pmatrix} = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

$$\dot{x} = V_x \; ; \; \dot{y} = V_y \; ; \; \dot{z} = V_z$$

$$\dot{\psi} = \frac{\sin(\varphi)\,\omega_y + \cos(\varphi)\,\omega_z}{\cos\theta}$$

$$\dot{\theta} = \cos(\varphi)\,\omega_y - \sin(\varphi)\,\omega_z$$

$$\dot{\varphi} = \omega_x + \tan(\theta)\sin(\varphi)\,\omega_y + \tan(\theta)\cos(\varphi)\,\omega_z$$

$$\begin{pmatrix} W_{Bx} \\ W_{By} \\ W_{Bz} \end{pmatrix} = \begin{pmatrix} U_{Bx} \\ U_{By} \\ U_{Bz} \end{pmatrix} - \begin{pmatrix} V_{Bx} \\ V_{By} \\ V_{Bz} \end{pmatrix} := \vec{W}_B$$

$$\alpha_y = \operatorname{atan}\left(\frac{W_{By}}{W_{Bx}}\right) \; ; \; \alpha_p = \operatorname{atan}\left(\frac{W_{Bz}}{W_{Bx}}\right)$$

$$\begin{pmatrix} D_{Bx} \\ D_{By} \\ D_{Bz} \end{pmatrix} = \frac{1}{2}\rho\pi r^2 C_D \left(\vec{W}_B \cdot \vec{W}_B\right) \cdot \frac{\vec{W}_B}{\|\vec{W}_B\|}$$

$$\dot{V}_{Bx} = -g c\theta c\psi + \frac{D_{Bx}}{m} - 2\left(\omega_y V_{Bz} - \omega_z V_{By}\right)$$

$$\dot{V}_{By} = \frac{(R_4 - R_2)}{m} - \frac{C_{lz}}{ml}\alpha_y + g\left(c\varphi s\psi - s\varphi s\theta c\psi\right) + \frac{D_{By}}{m} - 2\left(\omega_z V_{Bx} - \omega_x V_{Bz}\right)$$

$$\dot{V}_{Bz} = \frac{(R_1 - R_3)}{m} - \frac{C_{ly}}{ml}\alpha_p - g\left(c\varphi s\theta c\psi + s\varphi s\psi\right) + \frac{D_{Bz}}{m} - 2\left(\omega_x V_{By} - \omega_y V_{Bx}\right)$$

$$\begin{pmatrix} \dot{U}_{Bx} \\ \dot{U}_{By} \\ \dot{U}_{Bz} \end{pmatrix} = [\dot{C}] \begin{pmatrix} U_x \\ U_y \\ U_z \end{pmatrix} + [C] \begin{pmatrix} \dot{U}_x \\ \dot{U}_y \\ \dot{U}_z \end{pmatrix}$$

(see next page!)

$$\begin{pmatrix} \dot{W}_{Bx} \\ \dot{W}_{By} \\ \dot{W}_{Bz} \end{pmatrix} = \begin{pmatrix} \dot{U}_{Bx} - \dot{V}_{Bx} \\ \dot{U}_{By} - \dot{V}_{By} \\ \dot{U}_{Bz} - \dot{V}_{Bz} \end{pmatrix}$$

$$\dot{\alpha}_y = \frac{1}{1 + \left(\frac{W_{By}}{W_{Bx}}\right)^2} \left[\frac{\dot{W}_{By} W_{Bx} - W_{By} \dot{W}_{Bx}}{W_{Bx}^2} \right]$$

$$\dot{\alpha}_p = \frac{1}{1 + \left(\frac{W_{Bz}}{W_{Bx}}\right)^2} \left[\frac{\dot{W}_{Bz} W_{Bx} - W_{Bz} \dot{W}_{Bx}}{W_{Bx}^2} \right]$$

$$\dot{\omega}_x = \frac{1}{I_R} \left(dR_4 - dR_2 \right)$$

$$\dot{\omega}_y = \frac{1}{I_L} \left[(I_L - I_R) \omega_x \omega_z + hR_3 - hR_1 - C_{1y} \alpha_p - C_{2y} \dot{\alpha}_p + l D_{Bz} \right]$$

$$\dot{\omega}_z = \frac{1}{I_L} \left[(I_R - I_L) \omega_y \omega_x + hR_4 - hR_2 + C_{1z} \alpha_y + C_{2z} \dot{\alpha}_y - l D_{By} \right)$$

State vector:

$$\vec{x} = \left[\dot{\psi} \;\; \dot{\theta} \;\; \dot{\varphi} \;\; \dot{x} \;\; \dot{y} \;\; \dot{z} \;\; \dot{\omega}_x \;\; \dot{\omega}_y \;\; \dot{\omega}_z \;\; \dot{V}_{Bx} \;\; \dot{V}_{By} \;\; \dot{V}_{Bz} \;\; \dot{U}_{Bx} \;\; \dot{U}_{By} \;\; \dot{U}_{Bz} \right]^T$$