Effective Spring Rate in Sway Bars:

I am sure that many of you have seen the Equation below from Fred Puhn's book *How to Make Your Car Handle.* The following pages of diagrams and equations are my attempt to explain why this equation gives you the effective spring rate at the end of the arm where the end link attaches to the sway bar.

The equation from Puhn's book is only good for spring rate in $\frac{lbf}{in}$, and it assumes generic properties for steel. If that is good enough for you, use his equation; It works. If you would like to know more about why that equation will give you a good number for the effective spring rate of a sway bar, keep reading.

Equation from *How to Make Your Car Handle* Fred Puhn

$$K_{swaybar\_Puhn} \equiv \frac{5000000 \cdot D^4}{0.4244 \cdot A^2 \cdot B + 0.2264 C^3}$$

Sway bar stiffness in $\frac{\text{lbf}}{\text{in}}$

Length of end perpendicular to B (A)

Length of center section (B)

Length of end section (C)

Diameter (D)

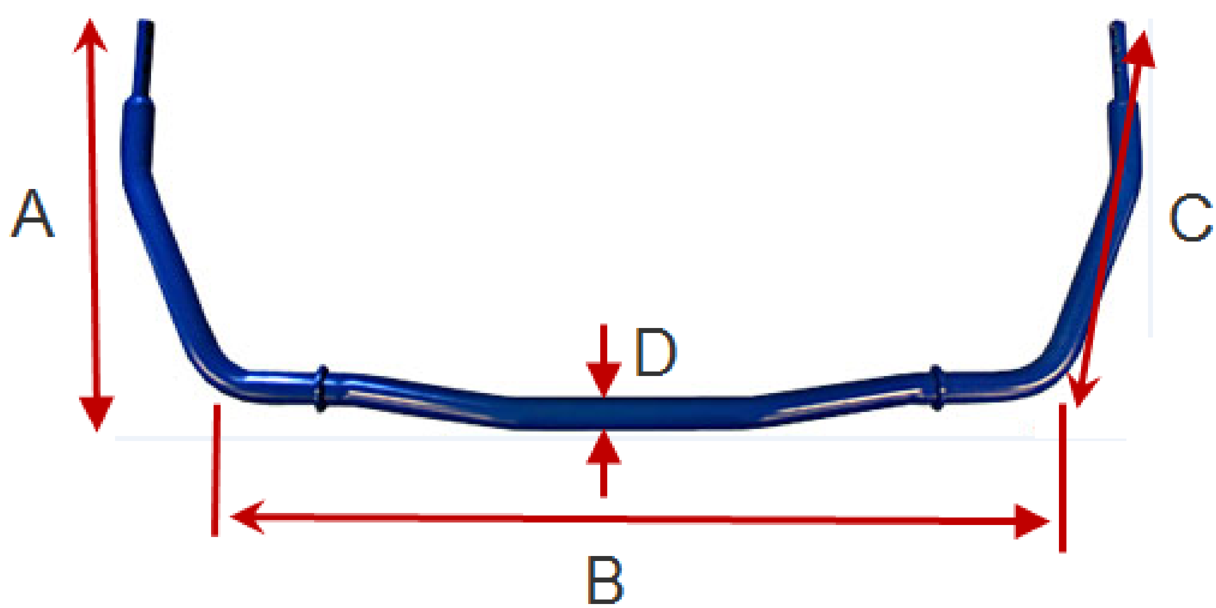


Figure 1: basic sway bar dimensions

Definition of Terms

Length of end perpendicular to B (A)

Length of center section (B)

Length of end section (C)

Outside Diameter (D)

Inside Diameter (d)

Elastic Modulus (E_{steel})

Shear Modulus (G_{steel})

Poisson's Ratio (ν)

Applied Force (P)

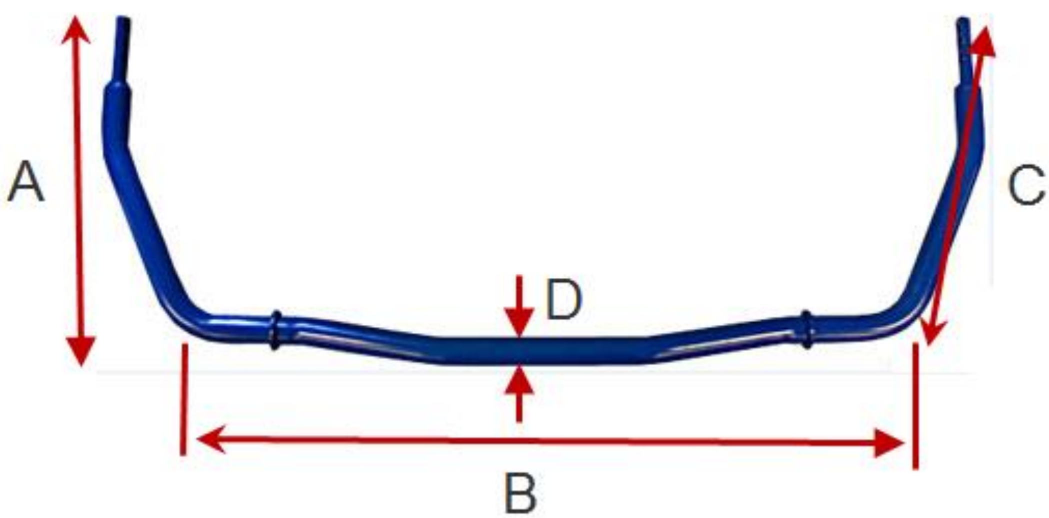
For Solid Round Bars:

Polar Moment of Inertia (J_{bar})

$$J_{bar} \equiv \pi \frac{\left(D^4 - d^4\right)}{32}$$

Moment of Inertia (I_{bar})

$$I_{bar} \equiv \pi \frac{\left(D^4 - d^4\right)}{64}$$



Deflection from Torsion (δ_t)

Deflection from Bending (δ_b)

These deflections are the deflections of the ends (at endlink location)

Angular Deflection of Arm (θ_t) - measured in radians

for small angles $\theta_t \equiv \dfrac{\delta_t}{A}$

the small angle approximation is valid for angles < 15 degrees

Definition for the Moment in a bar under torsional loading:

$$M \equiv \frac{G_{steel} \cdot J_{bar} \cdot \theta_t}{B} \qquad M \rightarrow \frac{\pi \cdot G_{steel} \cdot \delta_t \cdot \left(\frac{D^4}{32} - \frac{d^4}{32}\right)}{A \cdot B}$$

The moment is also the applied force times the perpendicular distance

Therefore: $M \equiv P \cdot A$

Set Both equations for M equal and solve for δ_t

$$\delta_t \equiv \frac{32 \cdot B \cdot P \cdot A^2}{\pi \cdot G_{steel} \cdot \left(D^4 - d^4\right)} \qquad \delta_t \rightarrow \frac{32 \cdot A^2 \cdot B \cdot P}{\pi \cdot G_{steel} \cdot \left(D^4 - d^4\right)}$$

Definition of tip deflection of caltilever beam

$$\delta_b \equiv \frac{P \cdot C^3}{3 \cdot E_{steel} \cdot I_{bar}} \qquad \delta_b \rightarrow \frac{C^3 \cdot P}{3 \cdot \pi \cdot E_{steel} \cdot \left(\frac{D^4}{64} - \frac{d^4}{64}\right)}$$

Solve for spring rate as Force per deflection:

$$k_{bar\_torsion} \equiv \frac{P}{\delta_t} \qquad k_{bar\_torsion} \rightarrow \frac{\pi \cdot G_{steel} \cdot \left(D^4 - d^4\right)}{32 \cdot A^2 \cdot B}$$

$$k_{bar\_bending} \equiv \frac{P}{\delta_b} \qquad k_{bar\_bending} \rightarrow \frac{3 \cdot \pi \cdot E_{steel} \cdot \left(\frac{D^4}{64} - \frac{d^4}{64}\right)}{C^3}$$

We can solve for the effective spring rate by using the individual spring rates for each segment of the bar and combining them in series, or we can add the deflection due to bending of the ends to the deflection at the ends due to torsion in the bar and solve for the effective spring rate.

Combine the deflections for each segment and solve the spring rate:

The total displacement is: $$\delta_{Total} \equiv 2 \cdot \delta_b + \delta_t$$

* We use 2 x's the deflection for bending because the sway bar has 2 segments in bending (each arm)

$$k_{bar} \equiv \frac{P}{\delta_{Total}} \qquad k_{bar} \text{ simplify } \rightarrow \frac{3 \cdot \pi \cdot E_{steel} \cdot G_{steel} \cdot \left(D^4 - d^4\right)}{32 \cdot \left(3 \cdot B \cdot E_{steel} \cdot A^2 + 4 \cdot G_{steel} \cdot C^3\right)}$$

By substituting common values for steel in psi and assuming that the bar is solid (i.e. d=0) we get the following:

$$k_{bar} \left| \begin{array}{l} \text{substitute}, E_{steel} = 30000000 \\ \text{substitute}, G_{steel} = 12000000 \\ \text{substitute}, \pi = 3.14159 \\ \text{substitute}, d = 0 \end{array} \right. \rightarrow \frac{7.0685775e7 \cdot D^4}{60.0 \cdot B \cdot A^2 + 32.0 \cdot C^3}$$

Finally if we divide each term by 141.37154 (equivalent to multiplying by 1… sorry if that is obvious) we get the equation used in Puhn's book. I really have no idea why

$$K_{swaybar\_Puhn} \equiv \frac{5000000 \cdot D^4}{0.4244 \cdot A^2 \cdot B + 0.2264 C^3}$$

Combine the spring rate of each segment in series and calculate the effective spring rate

Effective Spring Rate in force per displacement units

Springs in series:
1 bending - 1 torsion - 1 bending

$$k_{bar} \equiv \left(\frac{1}{k_{bar\_bending}} + \frac{1}{k_{bar\_torsion}} + \frac{1}{k_{bar\_bending}} \right)^{-1}$$

$$k_{bar} \text{ simplify } \rightarrow \frac{3 \cdot \pi \cdot E_{steel} \cdot G_{steel} \cdot \left(D^4 - d^4\right)}{32 \cdot \left(3 \cdot B \cdot E_{steel} \cdot A^2 + 4 \cdot G_{steel} \cdot C^3\right)}$$

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Using Either Method we get the same answer. The general equations that have not assumed material properties can be used for any system of consistent units.

- Typical values for the Elastic Modulus of steel (E_{steel}) can range from 27,000,000 – 30,000,000 psi (190-210 MPa)

- Typical values for the Shear Modulus of steel (G_{steel}) can range from 10,400,000-12,000,000 psi (75000-80000 MPa)

Obviously using different number will change the calculated values slightly.

Many of the aftermarket front sway bars come with multiple end link mounting points. Using the hole which gives the shortest arm length will make the sway bar stiffer, and alternately using the hole which gives the longest arm will make the sway bar softer.

In summary, the torsional stiffness of a sway bar calculated as a function of the applied force and the deflection at the end links comes from the torsional stiffness of the "straight" section and the bending stiffness of each arm.

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