2 Steady State Characteristics

Contents

Fluid Power Systems
(Hydraulisk Komponentanalyse)

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2.1 Hydraulic Fluids

2.1.1 Introduction

The main purpose of the hydraulic fluid is to transport energy from the pump to the actuators. Secondary purposes involve the lubrication of the moving mechanical parts to reduce wear, noise and frictional losses, protecting the hydraulic components against corrosion and transporting heat away from its sources. The preferred working fluid in most applications is mineral oil, although in certain applications there is a requirement for water-based fluids. Water-based fluids and high water-based fluids provide fire resistance at a lower cost and have the advantage of relative ease of oil storage and fluid disposal. The recommended classification system is as follows:

HFA – dilute emulsions, i.e. oil-in-water emulsions, typically with 95% water content.

HFB – Invert emulsions, i.e. water-in-oil emulsions, typically with 40% water content.

HFC – Aqueous glycols, i.e. solutions of glycol and polyglycol in water, typically with 40% water content.

HFD – Synthetic fluids containing no water, such as silicone and silicote esters.

The selection of the appropriate fluid will require specialist advice from both the component manufacture and the fluid manufacture.

As the most commonly used hydraulic fluid is mineral oil and in the following sections it is the physical properties of commercial mineral oils that is discussed.
The purpose of this chapter is to define certain physical properties which will prove useful and to discuss properties related to the nature of fluids. Because the fluid is the medium of transmission of power in a hydraulic system, knowledge of its characteristics is essential

2.1.2 Fluid density

The mass density, ρ, of a hydraulic fluid is defined as a given mass divided by its volume, see Equation (2.1).

$$\rho = \frac{m}{V} \tag{2.1}$$

where

ρ	mass density $[kg/m^3]$
m	mass of the fluid $[kg]$
V	volume of the fluid $[m^3]$

The mass density is both temperature and pressure dependent. It decreases with increasing temperature but increases with increasing pressure.
A generally accepted empirical expression, the Dow and Fink equation, describes this:

$$\rho(t,p) = \rho_0(t) \cdot \left(1.0 + A_\beta(t) \cdot p - B_\beta(t) \cdot p^2\right) \tag{2.2}$$

where

ρ	mass density $[kg/m^3]$
ρ_0	mass density at atmospheric pressure $[kg/m^3]$
A_β	temperature dependant coefficient $[bar^{-1}]$
B_β	temperature dependant coefficient $[bar^{-2}]$
p	pressure $[bar]$

The density for a hydraulic fluid is normally (DIN 51757) given by the fluid manufacturer as the density at $15\,^{\circ}C$ and atmospheric pressure. This reference density lies between 0.85 and 0.91 g/cm^3 (850-910 kg/m^3) for commercial hydraulic fluids.
The reference mass density in Equation (2.2) may be determined by:

$$\rho_0(t) = \frac{\rho_{15}}{\left(1 + \alpha_t \cdot (t - 15)\right)} \tag{2.3}$$

where

ρ_0	mass density at atmospheric pressure $[kg/m^3]$
ρ_{15}	mass density at atmospheric pressure and $15\,^{\circ}C$ $[kg/m^3]$
α_t	thermal expansion coefficient $[deg^{-1}]$
t	temperature $[^{\circ}C]$

The thermal expansion coefficient is normally regarded as independent of temperature and pressure and lies within the range of 0.0065 to 0.007 deg^{-1}.

The two coefficients in Equation (2.2) are normally referred to as the Dow and Fink coefficients. They have experimentally been found to:

$$A_\beta = \left(-6.72 \cdot 10^{-4} \cdot T^2 + 0.53 \cdot T - 36.02\right) \cdot 10^{-6} \qquad (2.4)$$

$$B_\beta = \left(2.84 \cdot 10^{-4} \cdot T^2 - 0.24 \cdot T + 57.17\right) \cdot 10^{-9} \qquad (2.5)$$

where

A_β	temperature dependant coefficient $[\mathrm{bar}^{-1}]$
B_β	temperature dependant coefficient $[\mathrm{bar}^{-2}]$
T	absolute temperature $[K]$

The variation of the Dow and Fink coefficients with temperature is displayed graphically in Figure 2.1

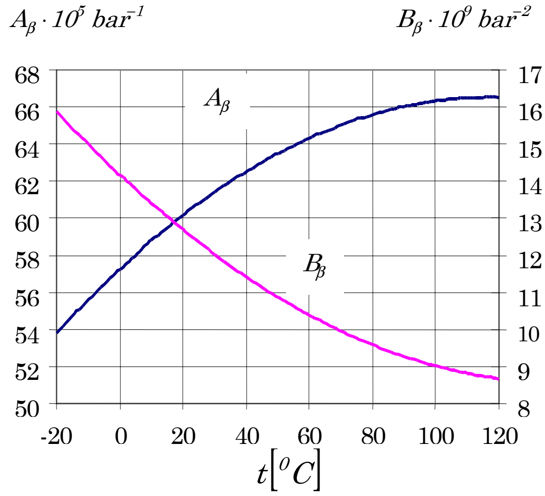

Fig. 2.1 The variation of the Dow and Fink coefficients with temperature

Inserting Equations (2.3)..(2.5) in Equation (2.2) means that the density can be determined by calculations only (no measurements), for any pressure and temperature combination, as long as the reference mass density, ρ_{15}, is known. The variation of the mass density with temperature and pressure is displayed graphically in Figure 2.2.
In Figure 2.2 the mass density is displayed relative to the reference mass density.

2.1.3 Viscosity

The most important of the physical properties of hydraulic fluids is the viscosity. It is a measure of the resistance of the fluid towards laminar (shearing) motion, and is normally specified to lie within a certain interval for hydraulic components in order to obtain the expected performance and lifetime. The definition of viscosities is related to the shearing stress that appear between adjacent layers, when forced to move relative (laminarly) to each other. For a newtonian fluid this shearing stress is defined as:

$$\tau_{xy} = \mu \frac{d\dot{x}}{dy} \qquad (2.6)$$

where

τ_{xy}	shearing stress in the fluid, [N/m^2]
μ	dynamic viscosity, [Ns/m^2]
$\dot{x}$	velocity of the fluid, [m/s]
y	coordinate perpendicular to the fluid velocity, [m]

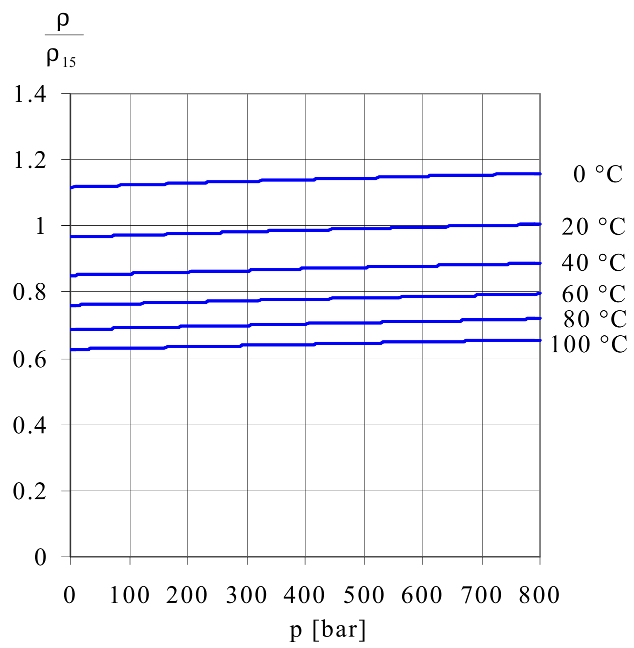

Fig. 2.2 The variation of the mass density with temperature and pressure

In Figure 2.3 the variables associated with the definition of the dynamic viscosity are shown.

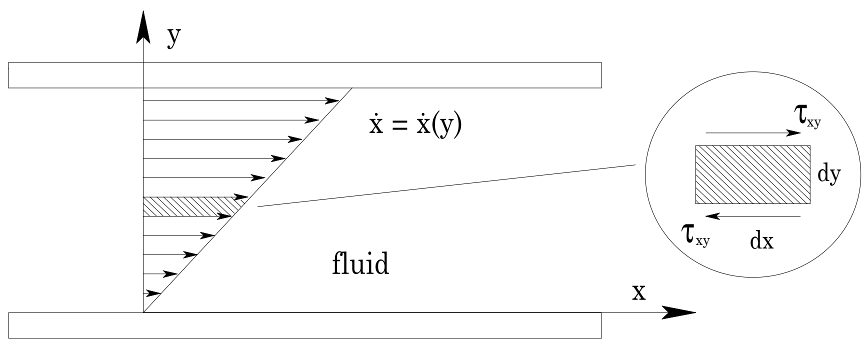

Fig 2.3 Deformation of a fluid element

The usual units for the dynamic viscosity is P for Poise or cP for centipoise. Their relation to the SI-units are as follows: $1\,\text{P} = 100\,\text{cP} = 0.1\,\text{Ns/m}^2$.
For practical purposes, however, the dynamic viscosity is seldom used, as compared to the kinematic viscosity that is defined as follows:

$$\nu = \frac{\mu}{\rho} \tag{2.7}$$

where

ν	kinematic viscosity, [m^2/s]
μ	dynamic viscosity, [Ns/m^2]
ρ	density, [kg/m^3]

The usual unit used for ν is centistoke, cSt, and it relates to the SI units as follows:
$1\,\text{cSt} = 10^{-6}\,\dfrac{\text{m}^2}{\text{s}} = 1\,\dfrac{\text{mm}^2}{\text{s}}$.
A low viscosity corresponds to a "thin" fluid and a high viscosity corresponds to a "thick" fluid. The viscosity depends strongly on temperature and also on pressure. The temperature dependency is complex and is normally, DIN51562 and DIN51563 described by the empirical Uddebuhle-Walther equation:

$$\log_{10}\log_{10}(\nu + 0.8) = C_\nu - m_\nu \cdot \log_{10} t_a \tag{2.8}$$

where

ν	kinematic viscosity, [cSt]
C_ν,m_ν	constants for the specific fluid
t_a	absolute temperature, [K]

This dependency is normally shown in specially designed charts, where the kinematic viscosity shown as function of the temperature becomes a straight line, see Figure 2.4.

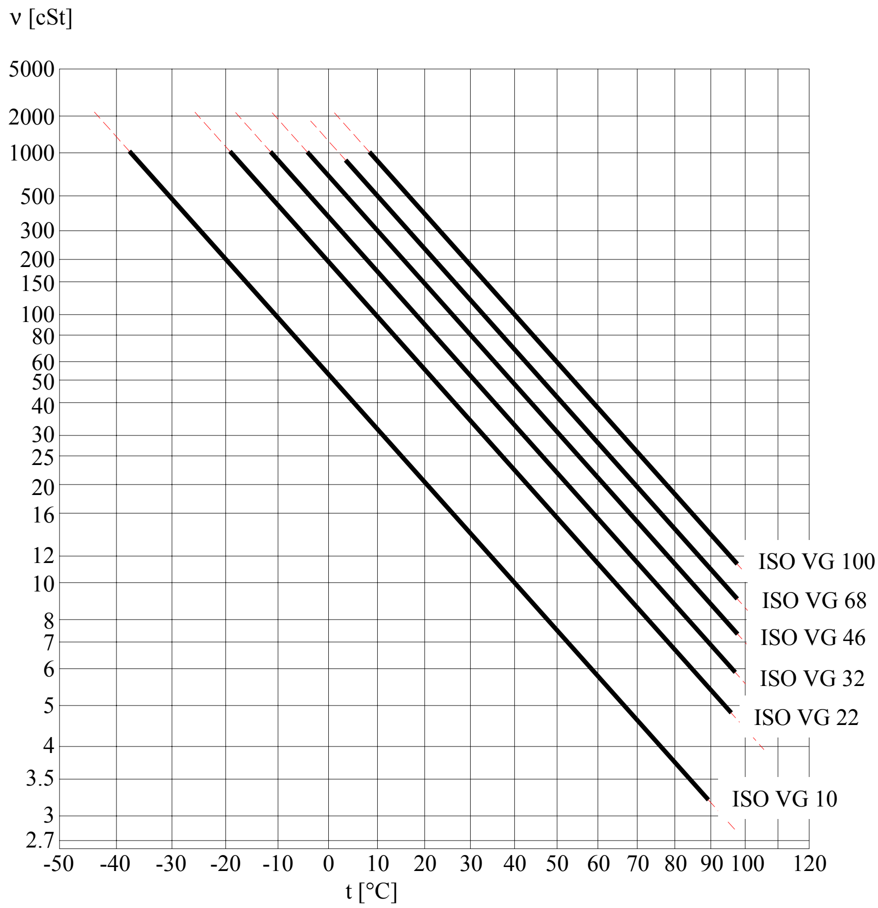

Fig. 2.4 Uddebuhle-chart: The temperature dependency for some of the most commonly used mineral oils. The ISO VG standard refers ν at 40°C

The vertical axis of an Uddebuhle chart is a mapping of log log(ν+0.8), i.e., approximately a double logarithmic axis (especially at higher values of ν). The horizontal axis is a mapping of logT, i.e., a logarithmic axis. A hydraulic fluid is, in general, referred to by its kinematic viscosity at 40°C.
A different way of describing a hydraulic fluid is by means of the viscosity index, where the temperature dependency is related to a temperature sensitive fluid and a temperature insensitive fluid. The hydraulic fluid to be indexed and the 2 reference oils must have the same viscosity at a temperature of 210°F. If that is fulfilled, the viscosity index, V.I., may be determined as:

$$VI = \frac{L - U}{L - H} \cdot 100\% \tag{2.9}$$

where

VI	viscosity index
L	kinematic viscosity at 100°F for the temperature sensitive fluid
U	kinematic viscosity at 100°F for the fluid to be indexed
H	kinematic viscosity at 100°F for the temperature insensitive fluid

Different standards, e.g. DIN ISO 2909, offer a list of reference fluids with different kinematic viscosities at 210°F to pick from. The method dates back to 1929 and the improvement in mineral oil destillation and refining means that many hydraulic fluids come out with an index above 100.
Beside the temperature dependency the viscosity also depends on pressure, especially at higher levels. The general accepted expression is as follows:

$$\mu = \mu_0 \cdot e^{B_\eta p} \tag{2.10}$$

where

μ	dynamic viscosity, [Ns/m^2]
μ_0	dynamic viscosity at atmospheric pressure [Ns/m^2]
B_η	temperature dependant parameter, [bar^{-1}]
p	pressure, [bar]

The parameter B_η may, within temperature ranges from 20°C to 100°C, be determined empirically as:

$$B_\eta = 0.0026 - 10^5 \cdot t \tag{2.11}$$

where

B_η	temperature dependant parameter, [bar^{-1}]
t	temperature, [°C]

The pressure dependency may be rewritten to cover kinematic viscosities:

$$\nu = \frac{\mu_0}{\rho} \cdot e^{B_\eta p} \tag{2.12}$$

where

ν	kinematic viscosity, [m^2/s]

μ_0	dynamic viscosity at atmospheric pressure [Ns/m^2]
ρ	density, [kg/m^3]
B_η	temperature dependant parameter, [bar^{-1}]
p	the pressure, [bar]

In the above it should be remembered that the density increases with pressure, thereby making the kinematic viscosity less sensitive to pressure rise.

2.1.4 Dissolvability

The capability of dissolving air (saturation point) varies strongly for hydraulic fluids with pressure. For pressure levels up to approximately 300 bar, the Henry-Dalton sentence applies:

$$V_a = \alpha_V \cdot V_F \cdot \frac{p_a}{p_{atm}} \tag{2.13}$$

where

V_a	volume of dissolved air in the oil, [m^3]
α_V	Bunsen coefficient, approximately constant at 0.09
V_F	volume of the fluid at atmospheric pressure, [m^3]
p_a	absolute pressure, [bar]
p_{atm}	atmospheric pressure ≈ 1 bar, [bar]

The capability of hydraulic fluids to absorb air is a problem, because the subsequent release of air at lower pressures leads to reduced fluid stiffness.

2.1.5 Stiffness

When pressurized a hydraulic fluid is compressed causing an increase in density. This is described by means of the compressibility which is defined as

$$\kappa_F = \frac{1}{\rho} \cdot \frac{\partial \rho}{\partial p} \tag{2.14}$$

where

κ_F	compressibility of the fluid, [bar^{-1}]
ρ	mass density, [kg/m^3]
p	pressure, [bar]

The reciprocal of κ_F is defined as the stiffness or bulk modulus of the fluid:

$$\beta_F = \frac{1}{\kappa_F} \tag{2.15}$$

where

κ_F	compressibility, [bar^{-1}]
β_F	bulk modulus, [bar]

Based on the above definition it can be shown that for fixed temperature the stiffness is proportional to the pressure rise caused by a compression of the fluid:

$$dp = \frac{\beta_F \cdot dV}{V_0} \tag{2.16}$$

where

dp	increase in pressure, [bar]
β_F	bulk modulus of the fluid, [bar]
dV	the compression, i.e., decrease in volume, [m^3]
V_0	the volume corresponding to the initial pressure, [m^3]

Just like density the bulk modulus and the compressibility are functions of temperature and pressure. Inserting Equation (2.2) in Equation (2.14) and Equation (2.15) leads to:

$$\beta_F(t,p) = \frac{1.0 + A_\beta(t)\cdot p - B_\beta(t)\cdot p^2}{A_\beta(t) - 2\cdot B_\beta(t)\cdot p} \tag{2.17}$$

where

β_F	stiffness of the fluid, [bar]
A_β	temperature dependant coefficient, [bar^{-1}]
p	pressure, [bar]
B_β	a temperature dependant coefficient, [bar^{-2}]

Where the temperature dependant coefficients can be determined from and. It should be noted that Equation (2.17) implies that the fluid stiffness may be calculated for any temperature and pressure combination regardless of the specific type of mineral oil. The variation of the fluid stiffness with temperature and pressure is displayed graphically in Figure 2.5.

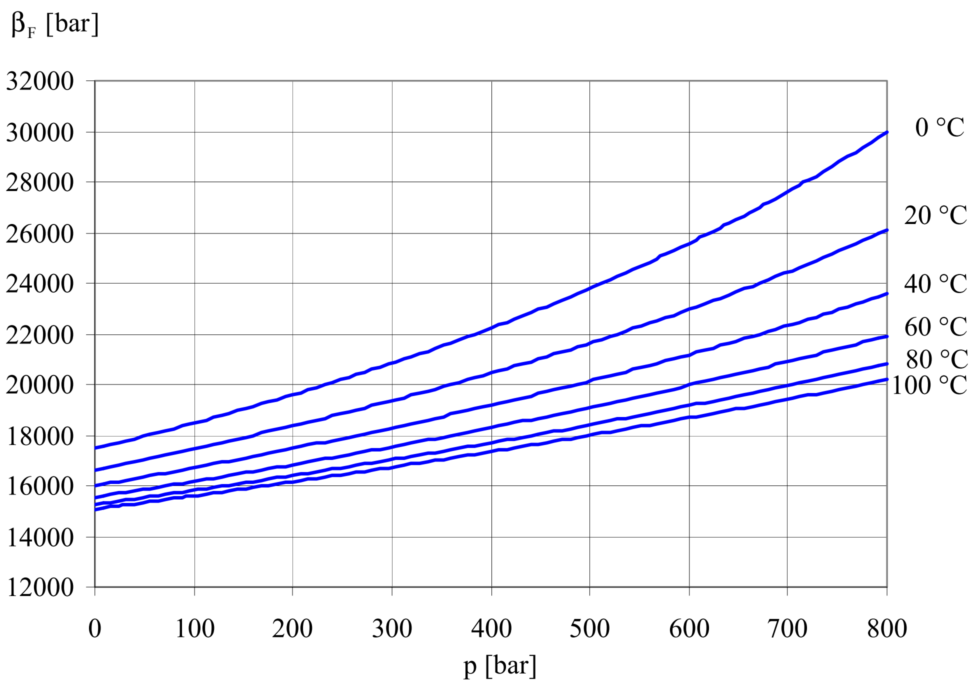

Fig. 2.5 The variation of the fluid stiffness with temperature and pressure

In real systems air will be present in the fluid. The volume percentage at atmospheric pressure will go as high as 20 %. As air is much more compressible than the pure fluid it has, potentially, a strong influence on the effective stiffness of the air containing fluid. If the air, however, is **dissolved** in the fluid there is **no significant effect** on the compressibility. Hence, it is the amount of **free or entrapped** air in the fluid that **markedly reduces** the effective stiffness. Taking the presence of air into account the effective stiffness of the fluid becomes:

$$\beta_{eff}(t,p,\varepsilon_A) = \frac{1}{\frac{1}{\beta_F} + \varepsilon_A\left(\frac{1}{\beta_A} - \frac{1}{\beta_F}\right)} \approx \frac{1}{\frac{1}{\beta_F} + \frac{\varepsilon_A}{\beta_A}} \tag{2.18}$$

where

β_{eff}	effective stiffness of the fluid-air mixture, [bar]
ε_A	the volumetric ratio of free air in the fluid
β_F	stiffness of the pure fluid according to, [bar]
β_A	the air stiffness according to, [bar]
p	pressure, [bar]

The volumetric ratio is defined as:

$$\varepsilon_A = \frac{V_A}{V_F + V_A} \tag{2.19}$$

where

ε_A	volumetric ratio of free air in the fluid
V_A	the volume of air, [m^3]
V_F	volume of the fluid, [m^3]

Assuming adiabatic conditions the volume and stiffness of the air may be determined as:

$$V_A = V_{A0} \cdot \left(\frac{p_{atm}}{p_a}\right)^{\frac{1}{c_{ad}}} \tag{2.20}$$

$$\beta_A = c_{ad} \cdot p_a \tag{2.21}$$

where

V_A	volume of air, [m^3]
V_{A0}	volume of air at atmospheric pressure, [m^3]
p_{atm}	atmospheric pressure ≈ 1 bar, [bar]
p_a	absolute pressure, [bar]
c_{ad}	adiabatic constant for air, 1.4

The volume of the fluid is determined from:

$$V_F(t,p) = V_{F0} \cdot \frac{\rho_0(t_0)}{\rho(t,p)} \tag{2.22}$$

where

V_F	volume of the fluid, [m^3]
V_{F0}	volume of the fluid at atmospheric pressure and a reference temperature, [m^3]
ρ_0	mass density at atmospheric pressure according to Equation (2.3), [kg/m^3]
ρ	the mass density according to Equation (2.2), [kg/m^3]
t_0	reference temperature, [°C]
t	temperature, [°C]
p	pressure, [bar]

From Equation (2.20) and Equation (2.22) it is clear, that the volumetric ratio varies with both temperature and pressure. A reference volumetric ratio at atmospheric pressure is defined:

$$\varepsilon_{A0} = \frac{V_{A0}}{V_{F0} + V_{A0}} \tag{2.23}$$

where

ε_{A0}	the reference volumetric ratio of free air in the fluid at atmospheric pressure
V_{A0}	volume of air at atmospheric pressure, [m^3]
V_{F0}	volume of the fluid at atmospheric pressure and a reference temperature, [m^3]

Knowing this reference, volumetric ratio together with the reference temperature, t_0, may be rearranged to yield an expression for the volumetric ratio directly obtainable from temperature and pressure:

$$\varepsilon_A(t,p) = \frac{1.0}{\left(\frac{1.0 - \varepsilon_{A0}}{\varepsilon_{A0}}\right) \cdot \frac{\rho_0(t_0)}{\rho(t,p)} \cdot \left(\frac{p_{atm}}{p_a}\right)^{\frac{-1}{c_{ad}}} + 1.0} \tag{2.24}$$

In Figure 2.6 the variation of the effective stiffness according to Equation (2.18) is displayed. The variation of the stiffness is dramatic for small pressure levels. The curves in Fig. 2.6 do not take into account the effect of the Henry-Dalton sentence, Equation (2.13), according to which the free air should dissolve at a few bars pressure and subsequently have no effect on the effective stiffness. The Henry-Dalton sentence, however, is for static conditions and in a hydraulic system the pressure variations outside the tank reservoirs are typically so fast, that the hydraulic fluid does not have time to dissolve the free air. Naturally, some air is dissolved, meaning that the curves shown in Fig. 2.6 represents worst case, i.e., instantaneously pressure build up.

Stiffness plays a central role w.r.t. to dynamic performance of hydraulic systems and should be determined/predicted as precisely as possible. This is, however, not an easy task. The sum $V_{F0}+V_{A0}$ in Equation (2.23) is relatively easily determined, whereas V_{F0} or V_{A0} are more elusive.

As a rule of thumb, the stiffness under working conditions used for modelling a system should **not be set above 10000 bar**, unless verified by means of testing.

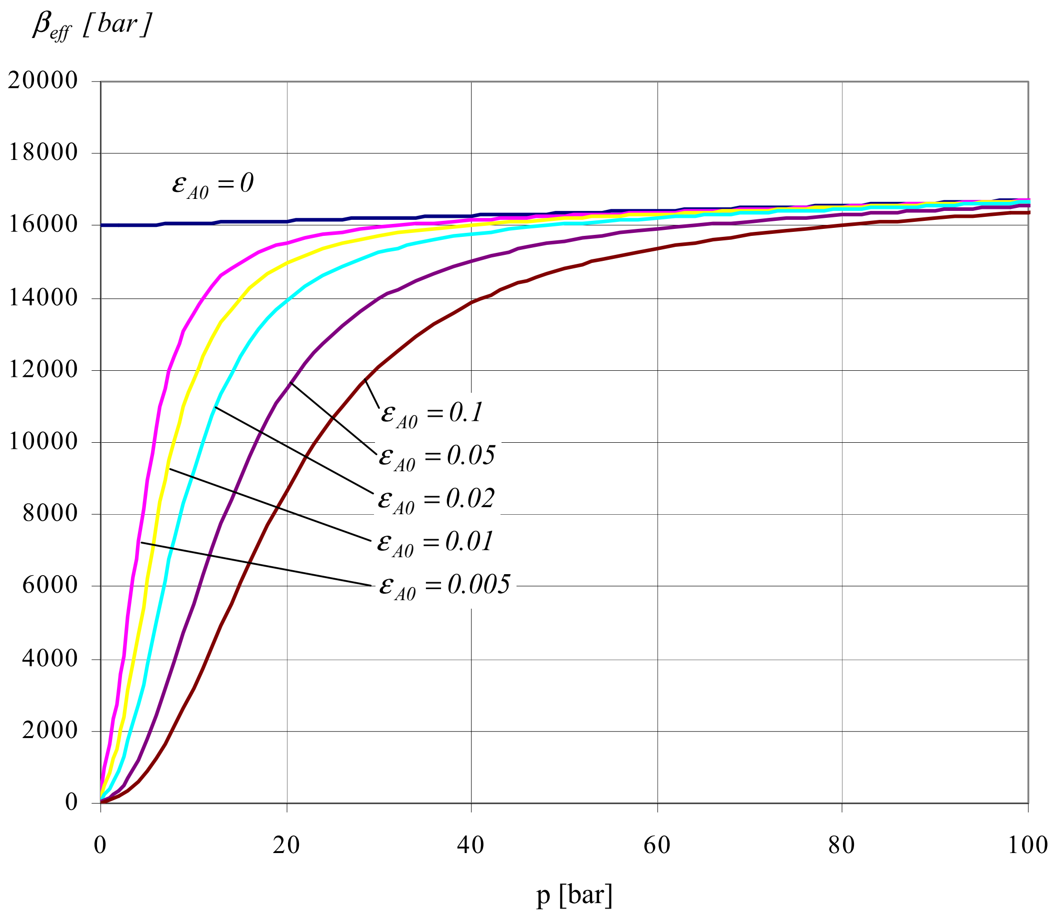

Fig. 2.6 Variation of effective stiffness of fluid-air mixture with respect to pressure and volume ratio of free air at atmospheric pressure. The temperature of the fluid is 40 °C and the compression of the free air is assumed adiabatic

2.2 Flow Characteristics of Spool Valves

2.2.1 Introduction

Hydraulic control valves are devices that use mechanical motion to control a source of fluid power. They vary in arrangement and complexity, depending on their function. Because control valves are the mechanical to fluid interface in hydraulic systems, their performance characteristics are essential. Although emphasis is placed on a principal type of spool valve, the principles apply equal well to other valves, such as different kinds of pressure valves and flow control valves.

The most common used control valve is the spool valve. Two typical spool valve configurations are shown in Figure 2.7. One in the pilot stage and one in the main stage. Spool valves can broadly be classified by the number of "ways" the flow can enter and leave the valve, and the type of centre when the valve spool is in neutral position.

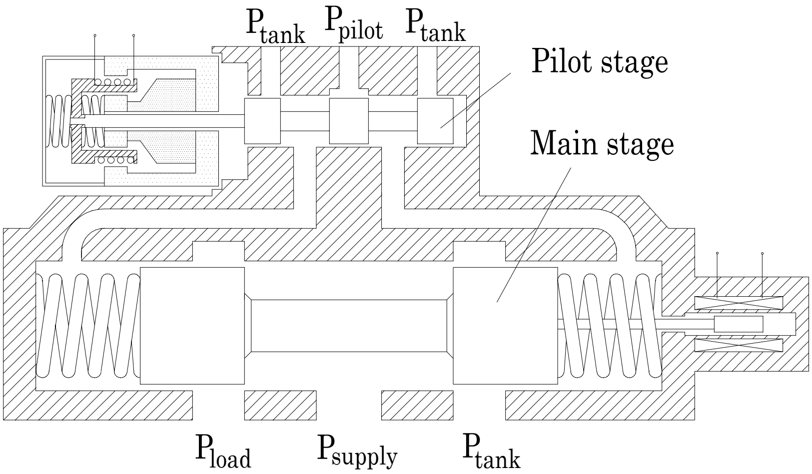

Fig. 2.7 Typical spool valve configurations

Because all valves require a supply, a return, and at least one line to the load, valves are either three-way or four-way (see Figure 2.7). Two-port valves are also available. However, two-way valves cannot provide a reversal in the direction of flow.
If the width of the land is smaller than the port in the valve sleeve, the valve is said to have an *open centre* or to be *underlapped.* A *critical centre* or *zero lapped* valve has a land width identical to the port width and is a condition approached by practical machining. *Closed centre* or *overlapped* valves have a land width greater than the port width when the spool is at neutral.
The above examples serve the purpose of illustrating how flow paths may be created using a variety of restrictions. The actual displacement of the spool which cause the flow restriction is usually of such small value relative to port diameter that the pressure/flow equations obey the Bernoulli equation.

2.2.2 Flow through orifices

The flow restrictions or orifices are a basic means for the control of fluid power. An orifice is a sudden restriction of short length in a flow passage and may have a fixed or variable area. In fluid power is it only inertia and viscous forces that matters. Experience has shown that it is either the inertia forces or the viscous forces that dominate, giving two types of flow regimes. Therefore, it is useful to define a quantity which describes the relative significance of these two forces in a given flow situation. The dimensionless ratio of inertia forces to viscous force is called Reynolds number and defined by

$$\mathrm{Re} = \frac{\rho\,\bar{u}\,d_h}{\mu} \tag{2.25}$$

where ρ is fluid mass density, μ is absolute viscosity, $\bar{u}$ is the average velocity of flow, and d_h is a characteristic length of the flow path.
In our case d_h is taken to be the hydraulic diameter which is defined as:

$$d_h = \frac{4 \times \text{flow area}}{\text{flow perimeter}} \tag{2.26}$$

Flow dominated by viscosity forces is referred to as *laminar* or *viscous* flow. Laminar flow is characterised by an orderly, smooth, parallel line motion of the fluid. Inertia dominated flow is generally *turbulent* and characterised by irregular, eddylike paths of the fluid. In some cases viscosity is only important in the *boundary layer*, while the main flow outside the boundary layer is dominated by inertia and behaves like laminar flow. If the boundary is neglected, the resulting flow is called *potential* flow. Potential flow has no losses while it is frictionless, so Reynolds number is infinite. For potential flow the Navier-Stokes equations reduce to

$$\frac{p}{\rho}+\frac{u^2}{2}=\text{constant} \tag{2.27}$$

Equation (2.27) is Bernoulli's equation with negligible gravity forces.
As an important case where Equation (2.27) is used consider flow through an orifice (see Figure 2.8).

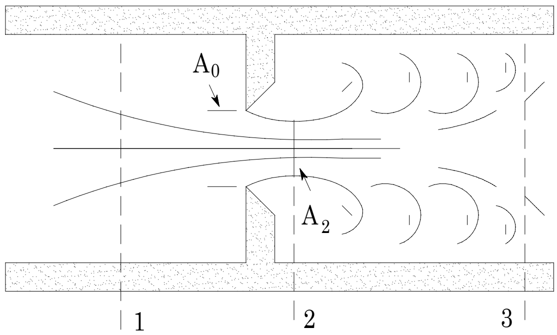

Fig. 2.8 Flow through an orifice; turbulent flow

Since most orifice flow occur at high Reynolds numbers, this region is of great importance. Experience has justified the use of Bernoulli's equation in the region Between point 1 and 2. The point along the jet where the area becomes a minimum is called the *vena contracta.* The ratio between the area at vena contracta A_2 and the orifice area A_0 defines the so called contraction coefficient C_c.

$$C_c = A_2 / A_0 \tag{2.28}$$

After the fluid has passed the vena contracta there is turbulence and mixing of the jet with the fluid in the downstream region. The kinetic energy is converted into heat. Since the internal energy is not recovered the pressures p_2 and p_3 are approximately equal.
Now it is possible to use Bernoulli's equation (2.27) to calculate the relation between the upstream velocity u_1 to the velocity u_0 in vena contracta. Therefore

$$u_2^2 - u_1^2 = \frac{2}{\rho}(p_1 - p_2) \tag{2.29}$$

Applying the continuity equation for incompressible flow yields

$$A_1u_1 = A_2u_2 = A_3u_3 \tag{2.30}$$

Combining Equation (2.29) and Equation (2.30) and solving for u_2 gives

$$u_2 = \left[1-\left(\frac{A_2}{A_1}\right)^2\right]^{-1/2} \cdot \sqrt{\frac{2}{\rho}(p_1 - p_2)} \tag{2.31}$$

In the real world there will always be some viscous friction (and deviation from ideal potential flow), and therefore an empirical factor C_v is introduced to account for this discrepancy. C_v is typically around 0.98. Since $Q = A_2u_2$ the flow rate at vena contracta becomes, by using Equation (2.31).

$$Q = \frac{C_vA_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{\frac{2}{\rho}(p_1 - p_2)} \tag{2.32}$$

Defining the discharge coefficient C_d in Equation (2.32) it is possible to express the orifice flow by the orifice area.

$$C_d = \frac{C_vC_c}{\sqrt{1-C_c^2(A_0/A_1)^2}} \tag{2.33}$$

Now, combining Equation (2.28), (2.32), and (2.33) the orifice equation (in Danish *blændeformlen*) can be written

$$Q = C_dA_0\sqrt{\frac{2}{\rho}(p_1 - p_2)} \tag{2.34}$$

Normally A_0 is much smaller than A_1 and since $C_v \approx 1$, the discharge coefficient is approximately equal to the contraction coefficient. Different theoretical and experimental investigations has shown that a discharge coefficient of $C_d \approx 0.6$ is often assumed for all spool orifices.

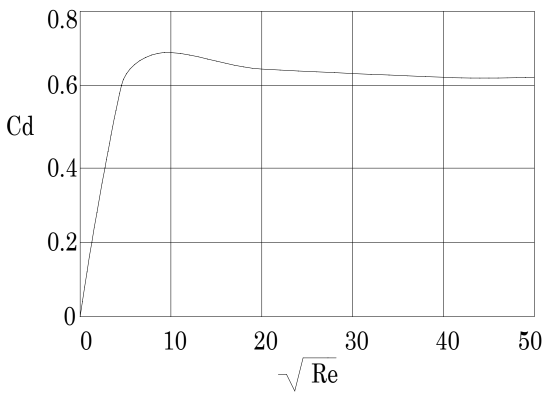

Fig. 2.9 Plot of a discharge coefficient versus Reynolds number for an orifice

At low temperatures, low orifice pressure drop, and/or small orifice openings, the Reynolds number may become sufficiently low to permit laminar flow. Although the analysis leading to Equation (2.34) is not valid at low Reynolds numbers, it is often used anyway by letting the discharge coefficient be a function of Reynolds number. For Re < 10 experimental results shows that the discharge coefficient is directly proportional to the square root of Reynolds number; that is $C_d = \delta\sqrt{Re}$. A typical plot of such a result is shown in Figure 2.9.

2.2.3 General valve analysis

In this section we define some general performance characteristics, such as pressure-flow curves and valve coefficients, which are applicable to all types of valves. Although the analysis is illustrated with a spool type valve, the principles involved are quite general.
Consider the four-way valve shown in Figure 2.10. It is assumed that the valve is connected to a symmetric load, i.e. a rotating motor or a equal area cylinder. The valve geometry is assumed ideal, implying that the orifice edges are perfectly square with no rounding and that there is no radial clearance between the spool and sleeve. It is also assumed that the discharge coefficients for the orifices are equal. The return line pressure p_R is neglected because it is usually much smaller than the other pressures involved.

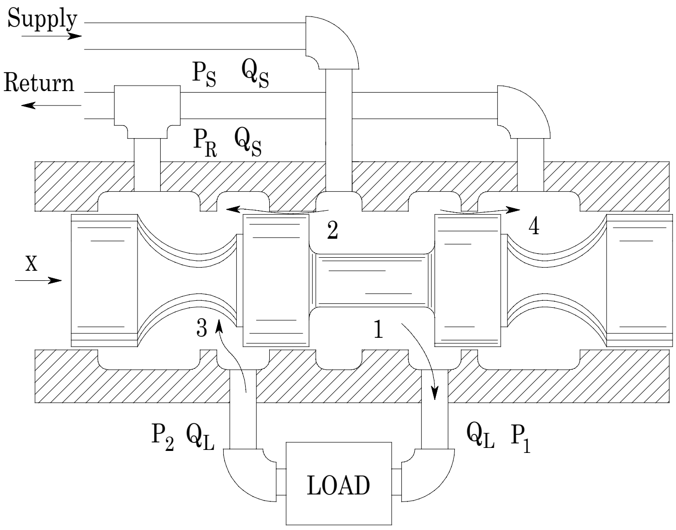

Fig. 2.10 Four-way spool valve

Let the spool be given a positive displacement from the null or neutral position, that is the position x = 0, which is chosen to be the symmetrical position of the spool in its sleeve.
This allows the supply flow Q_S to travel to the load as Q_1, the difference being only leakage flow present, Q_4, across the other land. The flow from the load returns as Q_3, which with the possible addition of the leakage flow Q_2, then forms the return flow. Because we are only interested in the steady-state characteristics, the compressibility flows are zero and the flow continuity equations for the valve chambers are

$$\begin{aligned} Q_L &= Q_1 - Q_4 \\ Q_L &= Q_3 - Q_2 \end{aligned} \tag{2.35}$$

where Q_L is the flow through the load. The load pressure differential is defined as the pressure drop across the load.

$$p_L = p_1 - p_2 \tag{2.36}$$

Flows through the orifices are described by the orifice Equation (2.34). Therefore

$$\begin{aligned} Q_1 &= C_d A_1 \sqrt{\frac{2}{\rho}(p_S - p_1)} \qquad & Q_2 &= C_d A_2 \sqrt{\frac{2}{\rho}(p_S - p_2)} \\ Q_3 &= C_d A_3 \sqrt{\frac{2}{\rho} p_2} \qquad & Q_4 &= C_d A_4 \sqrt{\frac{2}{\rho} p_1} \end{aligned} \tag{2.37}$$

In the vast majority of cases the metering orifices are made so that they are *matched* and *symmetrical.* Matched orifices require that

$$A_1 = A_3 \;; \qquad A_2 = A_4 \tag{2.38}$$

And symmetrical orifices require that (*x* is the spool position)

$$A_1(x) = A_2(-x) \;; \qquad A_3(x) = A_4(-x) \tag{2.39}$$

This means, that in neutral position (x = 0), all four orifice areas are equal ($\approx A_0$). If further the orifice areas varies linear with the stroke, as is usually the case, the areas can be described by only one parameter *w*, defining the width of the slot in the valve sleeve. *w* is the area gradient.

$$A_1 = A_3 = A_0 + x \cdot w \;; \qquad A_2 = A_4 = A_0 - x \cdot w \tag{2.40}$$

The condition that the orifices are matched and symmetrical, gives that

$$Q_1 = Q_3 \;; \qquad Q_2 = Q_4 \tag{2.41}$$

Substituting Equation (2.37) (Q_1 and Q_3), and Equation (2.38) into Equation (2.41) yields

$$p_S = p_1 + p_2 \tag{2.42}$$

Now, Equation (2.36) and Equation (2.42) can be solved simultaneously to obtain

$$p_1 = \frac{p_S + p_L}{2} \;; \qquad p_2 = \frac{p_S - p_L}{2} \tag{2.43}$$

With the relations in Equation (2.41), and Equation (2.43) together with the equations in Equation (2.35) it is possible to find an expression for the load flow as a function of the load pressure.

$$Q_L = C_d A_1 \sqrt{\frac{1}{\rho}(p_S - p_L)} - C_d A_2 \sqrt{\frac{1}{\rho}(p_S + p_L)} \qquad (2.44)$$

Equation (2.44) represent the general steady-state valve equation for a symmetric matched four-way spool valve applied to a symmetric load.

2.3 Valve Coefficients

2.3.1 Introduction

It will be found necessary in a dynamic analysis that the non-linear algebraic equation which describe the pressure-flow curves to be linearised. From Equation (2.44) the load flow can be written as a function of the spool position and the load flow $Q_L = Q_L(x, p_L)$. If x and p_L only changes by a small amount about a operating point (Q_{L0}, p_{L0}, x_0) the general expression for the load flow can be expressed by a Taylor's series. We only consider the first order terms, assuming that the higher order infinitesimals are negligible small. Hence,

$$Q_L = Q_{L0} + \left.\frac{\partial Q_L}{\partial x}\right|_0 \Delta x + \left.\frac{\partial Q_L}{\partial p_L}\right|_0 \Delta p_L + \ldots \qquad (2.45)$$

The partials in Equation (2.45) defines the two most important parameters for a valve. The *flow gain* is defined by

$$k_q \equiv \frac{\partial Q_L}{\partial x} \qquad (2.46)$$

The *flow-pressure coefficient* is defined as

$$k_{qp} \equiv \frac{\partial Q_L}{\partial p_L} \qquad (2.47)$$

Another useful quantity is the *pressure sensitivity* defined by

$$k_p \equiv \frac{\partial p_L}{\partial x} \qquad (2.48)$$

There is the following relation between the quantities

$$\frac{\partial p_L}{\partial x} = \frac{\partial Q_L / \partial x}{\partial Q_L / p_L} \quad \text{or} \quad k_p = \frac{k_q}{k_{qp}} \qquad (2.49)$$

The three quantities k_q, k_{qp}, k_p are called *valve coefficients* and are extremely important in the dynamic analysis of valves in combination with actuators. k_p express the ability to of a valve-actuator combination to breakaway large friction loads. k_{qp} has a direct influence on the damping in the valve-actuator combination. k_q directly affects the open loop gain in a system and therefore has influence on system stability. The valve coefficients evaluated in the neutral position of the valve $(Q_{L0}, p_{L0}, x_0) = (0,0,0)$ are called the *null valve coefficients*. This operating point is the most critical point from a stability viewpoint, while the flow gain is largest, giving high system gain, and the flow-pressure coefficient is smallest, giving a low damping.

2.3.2 A critically lapped valve with linear ports

Many valves are manufactured with a relative linear flow gain near null position, meaning that $A_0 = 0$ in Equation (2.40). Assuming the valve to have ideal geometry the leakage flows are zero. For such a valve the load flow can be expressed by

$$Q_L = C_d w \cdot x \sqrt{\frac{p_S - p_L}{\rho}} \quad ; \quad x > 0 \tag{2.50}$$

while $A_1 = w \cdot x$ and $A_2 = 0$ in Equation (2.44).

$$Q_L = -C_d w \cdot x \sqrt{\frac{p_S + p_L}{\rho}} \quad ; \quad x < 0 \tag{2.51}$$

with $A_2 = -w \cdot x$ and $A_1 = 0$ in Equation (2.44).
Equation (2.50) and (2.51) can be combined into a single equation:

$$Q_L = C_d w \cdot x \sqrt{\frac{1}{\rho}\left(p_S - \frac{x}{|x|} p_L\right)} \tag{2.52}$$

This is the general equation for the pressure-flow curves of an ideal critical centre valve with matched and symmetrical orifices. The Equation (2.52) is plottet in Figure 2.11.

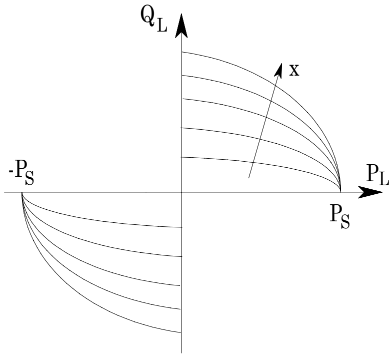

Fig. 2.11 Pressure-flow curves of critical centre four-way valve

The valve coefficients for the important case of an ideal critical centre valve can be obtained by differentiation of Equation (2.44), and are given below in Figure 2.12.

	General valve coefficients	Null valve coefficients
k_q	$C_d w \sqrt{\frac{1}{\rho}(p_S - p_{L0})}$	$C_d w \sqrt{\frac{p_S}{\rho}}$
k_{qp}	$\frac{C_d w \cdot x_0}{2\sqrt{\rho(p_S - p_{L0})}}$	*0*
k_p	$\frac{2(p_S - p_{L0})}{x_0}$	∞

Fig. 2.12 Valve coefficients for a critical centre four-way valve

2.4 Flow Forces on Spool Valves

Consider the steady-state flow through a spool valve as shown in Figure 2.13. When the fluid is flowing through the valve there will be induced some forces acting on the valve.

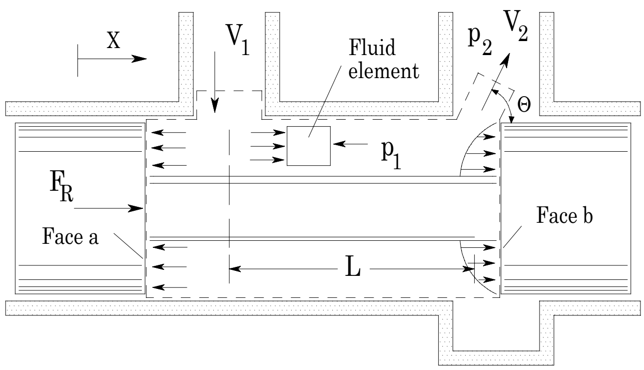

Fig. 2.13 Flow forces on a spool valve due to flow leaving a valve chamber

These forces are normally calculated using a mathematical formulation of Newton's second law suitable for application to a control volume.

$$\vec{F} = \vec{F}_S + \vec{F}_B = \frac{\partial}{\partial t}\int_{CV} \vec{V}\rho \, d\forall + \int_{CS} \vec{V}\rho\vec{V} \cdot d\vec{A} \qquad (2.53)$$

This equation states that the sum of all forces (surface and body forces) acting on a non-accelerating control volume is equal to the sum of the rate of change of momentum inside the control volume (CV) and the net rate of efflux of momentum through the control surface (CS).

Since we are looking for the horizontal force the body force is zero, and the only surface force in horizontal direction is the force F_R, which is the force of the spool on the control volume. Change of momentum inside the control volume occur when the spool position is suddenly changed, say to the right, as shown in Figure 2.13. If the fluid element is being accelerated, the pressure on the left side of the element must be greater than the pressure on the right side. Therefore, the pressure on face a must be greater than the pressure on face b. Thus the transient flow force is due to *acceleration of the fluid in the annular valve chamber*. The direction of this force for the case shown in Figure 2.13 is such that it tends to close the valve – however this is not the general rule. A movement of the spool can also cause the fluid to be decelerated. Applying the momentum equation in the horizontal direction gives

$$F = F_R = \rho L A_n \frac{d(Q/A_n)}{dt} + \rho Q V_2 \cos(\theta) \tag{2.54}$$

where Q is the volumetric flow rate and A_n is the annular area of the spool. The last term in Equation (2.54) can be rewritten as

$$\rho Q V_2 \cos(\theta) = \frac{\rho Q^2}{A_2} = \frac{\rho Q^2}{C_c A_0} \tag{2.55}$$

Where A_0 is the orifice area. The flow Q can be described by the orifice equation as

$$Q = C_d A_0 \sqrt{\frac{2}{\rho}(p_1 - p_2)} = C_c C_v A_0 \sqrt{\frac{2}{\rho}(p_1 - p_2)} \tag{2.56}$$

Obtaining dQ/dt from Equation (2.56), the transient flow force, F_t, becomes

$$F_t = L C_d w \sqrt{2\rho(p_1 - p_2)} \frac{dx}{dt} + \frac{L C_d w x}{\sqrt{(2/\rho)(p_1 - p_2)}} \frac{d(p_1 - p_2)}{dt} \tag{2.57}$$

The last term in Equation (2.57) is normally neglected. The velocity term is more significant because it represents a damping force. The quantity *L* is the axial length between incoming and outgoing flow and is called the *damping length.*
Inserting Equation (2.56) into Equation (2.55), the steady-state axial flow force acting on the valve spool can be obtained as

$$F_s = 2 C_d C_v A_0 (p_1 - p_2) \cos(\theta) \tag{2.58}$$

For a spool with no radial clearance it is well known, and usually assumed, that the jet leaves the control port at an angle of $\theta = 69^o$. The sum of Equation (2.57) and Equation (2.58) give the total flow force, steady-state and transient, opposing the spool motion, while the force from the fluid on the spool is opposite F_R, which is the external force acting *on* the control volume.

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