

new coord. system - x pointing forwards in the direction of the tyre, z is in the wheel plane pointing downwards, y defined by the two (but is in the matter of the fact wheel axis of revolution, but pointing the 'other way around')

camber - negative of the angle between wheel and car z axis when viewed from the front

toe - negative of the angle between wheel and car x axis when viewed from top

***ass*umption** - ground is parallel to xy plane

$$\alpha_T := 1^\circ \quad \dots \text{toe} \quad \alpha_C := -3^\circ \quad \dots \text{camber}$$

set of equations to describe the tyre coordinate system:

$$\begin{aligned} |\vec{x}| = 1 \quad |\vec{y}| = 1 \quad |\vec{z}| = 1 \quad \vec{y} = \vec{z} \times \vec{x} \\ x_3 := 0 \quad (\text{x parallel to the ground}) \quad \tan(\alpha_T) = -\frac{x_2}{x_1} \\ \tan(\alpha_C) = \frac{z_2}{z_3} \quad \vec{z} \cdot \vec{x} = 0 \quad (\text{to ensure z and x are perpendicular}) \end{aligned}$$

we will calculate x_1 because we know it's orientation (i.e. that it is a positive value):

$$\begin{aligned} x_2 = -x_1 \cdot \tan(\alpha_T) \quad (x_1)^2 + (x_2)^2 + (x_3)^2 = (x_1)^2 + (x_1)^2 \cdot \tan^2(\alpha_T) + 0 = (x_1)^2 \cdot (1 + \tan^2(\alpha_T)) = 1 \\ x_1 := \frac{1}{\sqrt{1 + \tan^2(\alpha_T)}} \quad x_2 := -x_1 \cdot \tan(\alpha_T) \quad x_3 := 0 \end{aligned}$$

we will calculate z_3 because we know it's orientation (i.e. that it is a positive value):

$$\begin{aligned} z_2 = z_3 \cdot \tan(\alpha_C) \\ x_1 \cdot z_1 + x_2 \cdot z_2 + x_3 \cdot z_3 = 0 \quad z_1 = -\frac{x_2 \cdot z_2 + x_3 \cdot z_3}{x_1} = -\frac{x_2 \cdot \tan(\alpha_C) + x_3}{x_1} \cdot z_3 \\ (z_1)^2 + (z_2)^2 + (z_3)^2 = \left[\left(\frac{x_2 \cdot \tan(\alpha_C) + x_3}{x_1} \right)^2 + \tan^2(\alpha_C) + 1 \right] \cdot (z_3)^2 = \dots \\ = \frac{(x_2 \cdot \tan(\alpha_C) + x_3)^2 + (\tan^2(\alpha_C) + 1) \cdot (x_1)^2}{(x_1)^2} \cdot (z_3)^2 = 1 \\ z_3 := \sqrt{\frac{(x_1)^2}{(x_2 \cdot \tan(\alpha_C) + x_3)^2 + (\tan^2(\alpha_C) + 1) \cdot (x_1)^2}} \quad z_2 := z_3 \cdot \tan(\alpha_C) \quad z_1 := -\frac{x_2 \cdot \tan(\alpha_C) + x_3}{x_1} \cdot z_3 \end{aligned}$$

y axis is now pretty straightforward:

$$\vec{y} := \vec{z} \times \vec{x}$$

$$T := \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix} \quad T = \begin{pmatrix} 0.99985 & 0.01743 & -9.13527 \times 10^{-4} \\ -0.01745 & 0.99848 & -0.05234 \\ 0 & 0.05234 & 0.99863 \end{pmatrix}$$