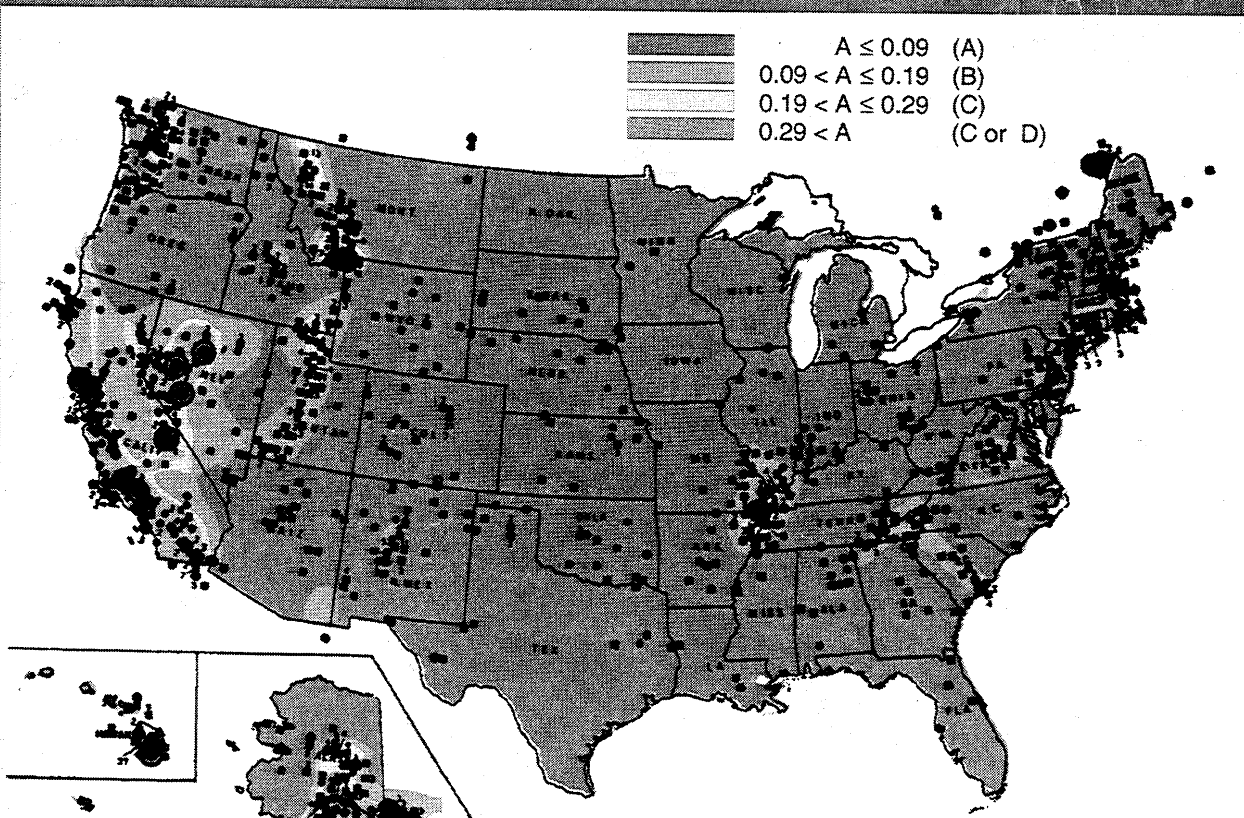
U.S. Department
of Transportation
Federal Highway
Administration

October 1996

Seismic Design of Bridges
Design Example No. 1
Two-Span Continuous
CIP Concrete Box Bridge



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16. Abstract

This document describes one of seven seismic design examples that illustrate "how" to apply AASHTO's seismic analysis and design requirements on actual different bridge types across the United States. Each provides a complete set of "designer's notes" covering the seismic analysis, design, and details for that particular bridge including flow charts, references to applicable AASHTO requirements, and thorough commentary that explains each step. In addition, each example highlights separate issues (skew effects, wall piers, elastomeric bearings, pile foundations, etc.).

The **first example** is a 242' reinforced concrete box girder two span overcrossing with spread footing foundations, SPC-C & A = 0.28g. The **second example** is a 400' 3-span skewed steel plate girder bridge over a river in New England with spread footing foundations, SPC-B & A = 0.15g. The **third example** is a skewed 70' single span prestressed concrete girder bridge with tall-closed seat-type abutments on spread footings, SPC-C & A = 0.36g. The **fourth example** is a 320' reinforced concrete box girder 3-span skewed bridge in the western United States with spread footing foundations, SPC-C & A = 0.30g. The **fifth example** is a 1488' steel plate girder bridge in the inland Pacific Northwest with pile foundations, SPC-B & A = 0.15g. It has nine spans and consists of two units: a four-span tangent (Unit 1) and a five-span with a 1300-foot radius curve (Unit 2). The **sixth example** is a 290' sharply curved (104 degrees) 3-span concrete box girder bridge in the Northwestern United States with pile abutment foundations and drilled shaft pier foundations, SPC-C & A = 0.20g. The **seventh example** is a 717' 10-span prestressed girder bridge with open pile bents, SPC-B & A = 0.10g. The superstructure consists of three continuous span units arranged in a 3-4-3 span series.

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Seismic Design Course

Design Example No. 1

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Prepared by

BERGER/ABAM Engineers Inc.
33301 Ninth Avenue South, Suite 300
Federal Way, Washington 98003-6395

Job No. A95013

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Philip Yen

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Robert Griebenow
James Guarre
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PLEASE NOTE

Data, specifications, suggested practices, and drawings presented herein are based on the best available information, are delineated in accord with recognized professional engineering principles and practices, and are provided for general information only. Procedures, suggested or discussed, should not be used without first securing competent advice respecting their suitability for any given application.

This document was prepared with the help and advice of FHWA, State, academic, and private engineers. The intent of this document is to aid practicing engineers in the application of the AASHTO seismic design specification. BERGER/ABAM and the United States Government assume no liability for its contents or use thereof.

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TABLES

Section I
Introduction

PURPOSE OF DESIGN EXAMPLE

This is the first of a series of seismic design examples developed for the FHWA. A different bridge configuration is used in each example. The bridges are in either Seismic Performance Category B or C sites. Each example will emphasize different features that must be considered in the seismic analysis and design process. The matrix below is a summary of the features purposed for the first seven examples.

DESIGN EXAMPLE NO.	DESIGN EXAMPLE DESCRIPTION	SEISMIC CATEGORY	PLAN GEOMETRY	SUPER-STRUCTURE TYPE	PIER TYPE	ABUTMENT TYPE	FOUNDATION TYPE	CONNECTIONS AND JOINTS
1	Two-Span Continuous	SPC - C	Tangent Square	CIP Concrete Box	Three-Column Integral Bent	Seat Stub Base	Spread Footings	Monolithic Joint at Pier Expansion Bearing at Abutment
2	Three-Span Continuous	SPC - B	Tangent Skewed	Steel Girder	Wall Type Pier	Tall Seat	Spread Footings	Elastomeric Bearing Pads (Piers and Abutments)
3	Single-Span	SPC - C	Tangent Square	AASHTO Precast Concrete Girders	(N/A)	Tall Seat (Closed-In)	Spread footings	Elastomeric Bearing Pads
4	Three-Span Continuous	SPC - C	Tangent Skewed	CIP Concrete	Two-Column Integral Bent	Seat	Spread Footings	Monolithic at Col. Tops Pinned Column at Base Expansion Bearings at Abutments
5	Nine-Span Viaduct with Four-Span and Five-Span Continuous Structs.	SPC - B	Curved Square	Steel Girder	Single-Column (Variable Heights)	Seat	Steel H-Piles	Conventional Steel Pins and PTFE Sliding Bearings
6	Three-Span Continuous	SPC - C	Sharply-Curved Square	CIP Concrete Box	Single Column	Monolithic	Drilled Shaft at Piers, Steel Piles at Abutments	Monolithic Concrete Joints
7	12-Span Viaduct with (3) Four-Span Structures	SPC - B	Tangent Square	AASHTO Precast Concrete Girders	Pile Bents (Battered and Plumb)	Seat	Concrete Piles and Steel Piles	Pinned and Expansion Bearings

REFERENCE AASHTO SPECIFICATIONS

The examples conform to the following specifications.

AASHTO Division I (herein referred to as "Division I")

Standard Specifications for Highway Bridges, American Association of State Highway and Transportation Officials, Inc., 15th Edition, as amended by the Interim Specifications-Bridges-1993 through 1995.

AASHTO Division I-A (herein referred to as "Division I-A" or the "Specification")

Proposed revisions to the *AASHTO Standard Specifications for Highway Bridges, Division I-A; Seismic Design*, NCHRP Project 20-7, Task 45, National Center for Earthquake Engineering Research, Buffalo, New York, July 1994.

ORGANIZATION OF EXAMPLE

Layout of Example

Design Example Number 1 is divided into sections as described below.

Section	Contents
I	Introduction
II	Flowcharts
III	Analysis and Design using Single-Mode Spectral Method with Basic Support Condition
IV	Analysis and Design using Single-Mode Spectral Method with Spring Support Condition
V	Analysis Using Uniform Load Method for Both Basic and Spring Support Conditions
VI	Analysis Using Multimode Spectral Method for Both Basic and Spring Support Conditions
VII	Notations
VIII	References
IX	Input for Computer Analysis

ORGANIZATION OF EXAMPLE (continued)

Summary of Analyses and Supports

As seen in the list of sections, this first example has been worked using three different analysis techniques in order to demonstrate different analysis options the designer can use. The analyses are:

- Uniform Load Method,
- Single-Mode Spectral Method, and
- Multimode Method.

Also, the example has also been worked using two different support conditions to show that two different support conditions produces considerably different column and footing requirements. The conditions are:

1. The Basic Support Condition uses no soil springs, considers the full Igross of the column, and allows the superstructure to slide longitudinally at the abutments.

2. The Spring Support Condition, considers soil springs under supports, uses one-half Igross of the column, and restrains the superstructure longitudinally using the soil at the abutments.

The reader should be aware that the assumptions made regarding the absence or presence of the soil springs and their effectiveness can have considerable effects on the design and performance of the structure. The choices of spring constants, spring strengths, and appropriate load-displacement relations for the soil are an evolving science. Therefore, the reader should expect that changes regarding the modeling of soil effects will occur in the future. The assumptions for modeling soil springs are often prescribed by local departments of transportation; therefore, the approaches taken in this example may not conform to those used by some agencies.

A summary of these support conditions is given below.

	Basic Support Condition	**Spring Support Condition**
Support Stiffness	Rigid	Springs
Column Stiffness	Igross	0.5 * Igross
Abutment Type	Seat Type	Stub Wall
Restraint of Superstructure	Unrestrained Longitudinally	Restrained Longitudinally

ORGANIZATION OF EXAMPLE
(continued)

Emphasis of Design Example No. 1

In the main body of the document (Section III), the example is worked using the Single-Mode Spectral Analysis Method for the Basic Support Condition. Following this in Section IV of the document, the same analysis method is used in the design and analysis of the structure with the Spring Support Condition.

The Uniform Load Method and Multimode Method analyses are included in Sections V and VI to illustrate their application. These methods are used with both the basic and spring supported conditions. After all the analyses are explained, results are compared to highlight similarities and differences. No design calculations are presented in Sections V or VI.

Finally, the following four additional issues have been treated in depth due to their importance in seismic design.

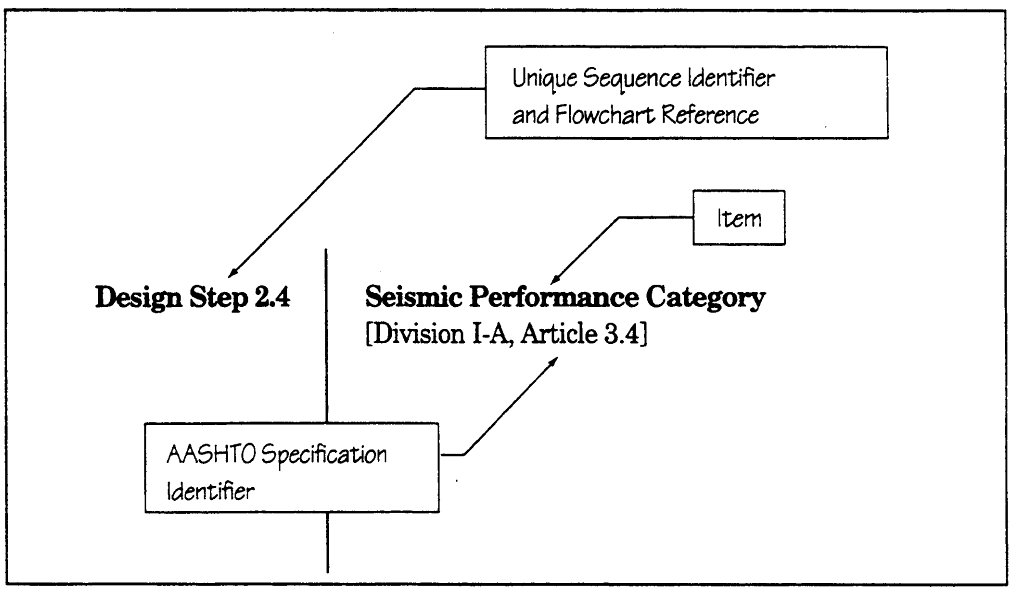
- Development of plastic hinging forces in the transverse direction of a multiple-column bent (See Design Step 7.4.2).
- Selection of controlling forces for various components (See Design Step 8).
- Design of column transverse reinforcement (See Design Step 10.1).
- Design size of footing under columns (See Design Step 11.1).

FLOWCHARTS AND DESIGN STEPS

This first example follows the outline given in detailed flowcharts presented in Section II, Flowcharts. The flowcharts consist of a main flowchart, which generally follows the one currently used in AASHTO Division I-A, and several subcharts that detail the operations that occur for each Design Step.

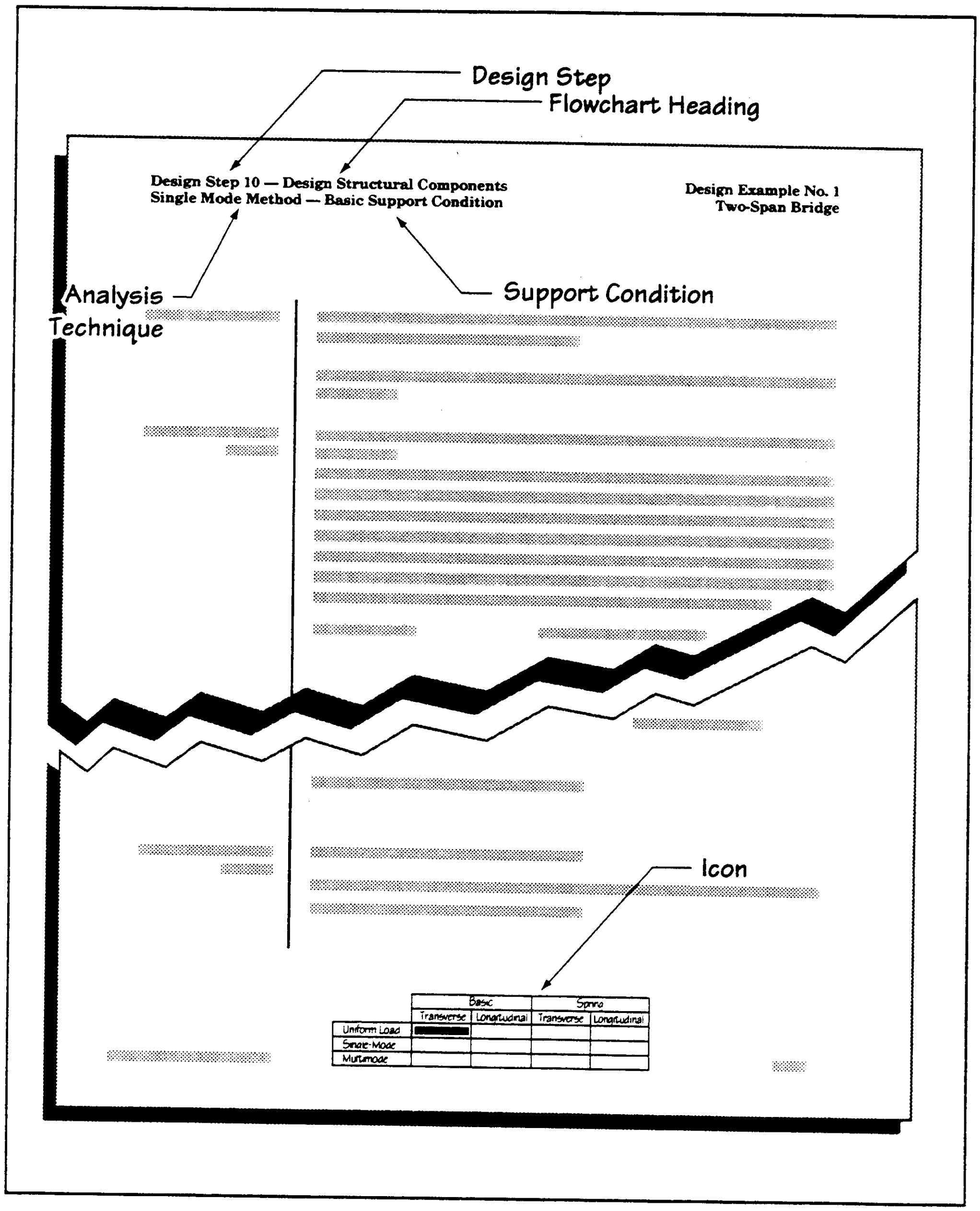
The purpose for having Design Steps is to present the example in a logical and sequential manner. This allows for easier referencing within the example itself. Each Design Step has a unique number in the left margin of the calculation document. The title is located to the right of the Design Step number. Where appropriate, there is a reference to either Division I or Division I-A of the AASHTO Specification beneath the title.

An example is shown below.



HEADINGS AND ICON

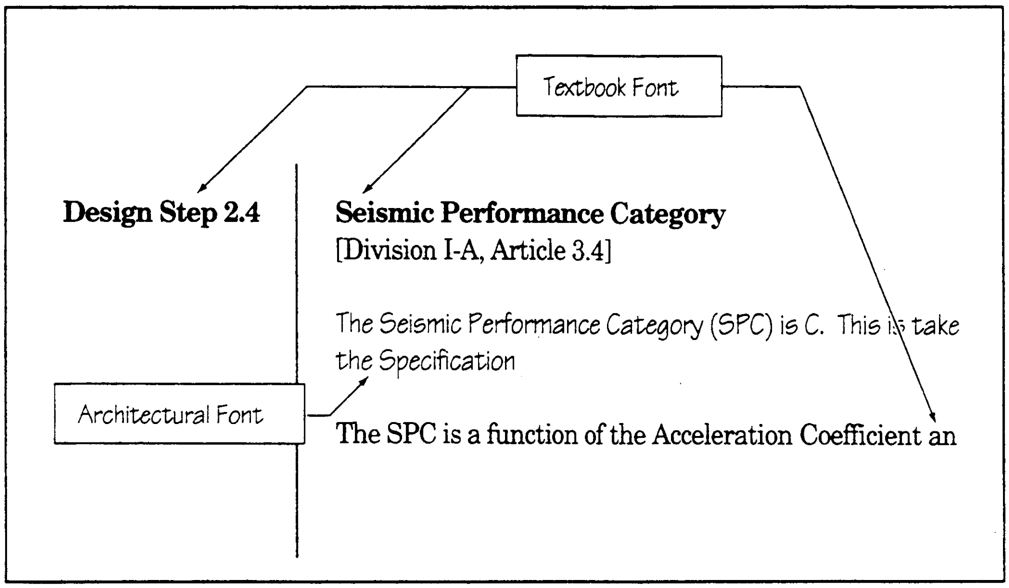
The header at the top of each page provides (in abbreviated notation) a description of the design example section. To help the user keep track of the analysis and support types as they are discussed, an icon is included at the bottom of each page in analysis Design Steps 6.3 and 6.4. The analysis method, the support condition, and the direction of loading are indicated by the icon, as shown below.



USE OF DIFFERENT FONT TYPES

In the example, two primary type fonts have been used. One type is similar to the type used for textbooks, and it is used for all section headings and for commentary. The other is an architectural font that appears handwritten, and it is used for all primary calculations. The material in the architectural font is the essential calculation material and essential results.

An example of the use of the fonts is shown below.

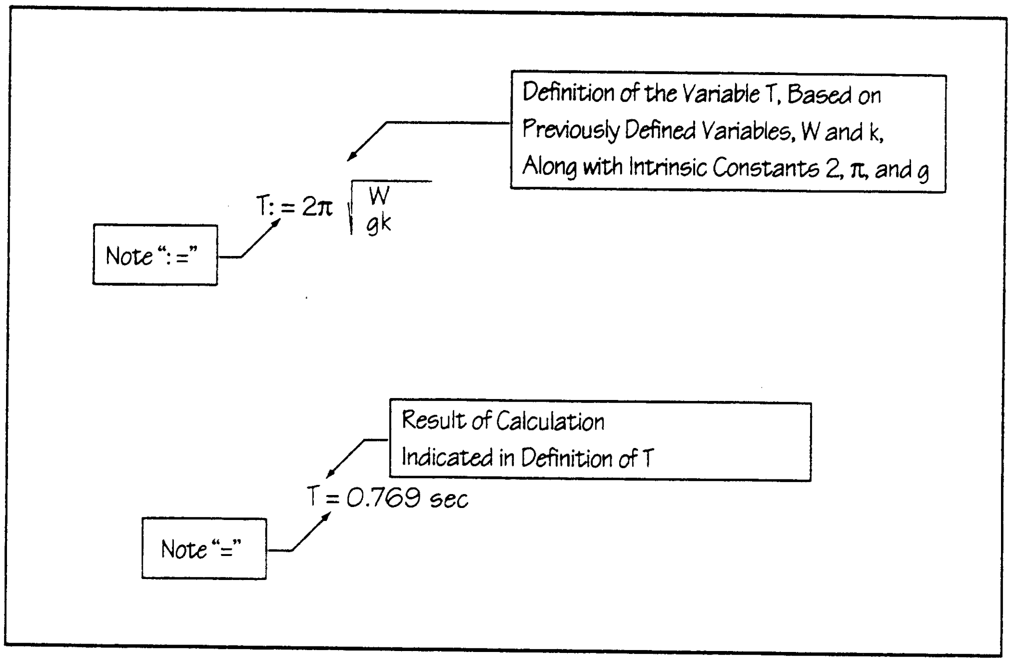


USE OF MATHCAD®

The calculations have been performed using the program Mathcad® to provide consistent results and quality control.

The variables used in equations calculated by the program are defined before the equation, and the **definition** of either a variable or an equation is distinguished by a ':= ' symbol. The **echo** of a variable or the result of a calculation is distinguished by a '=' symbol, i.e., no colon is used.

An example is shown below.

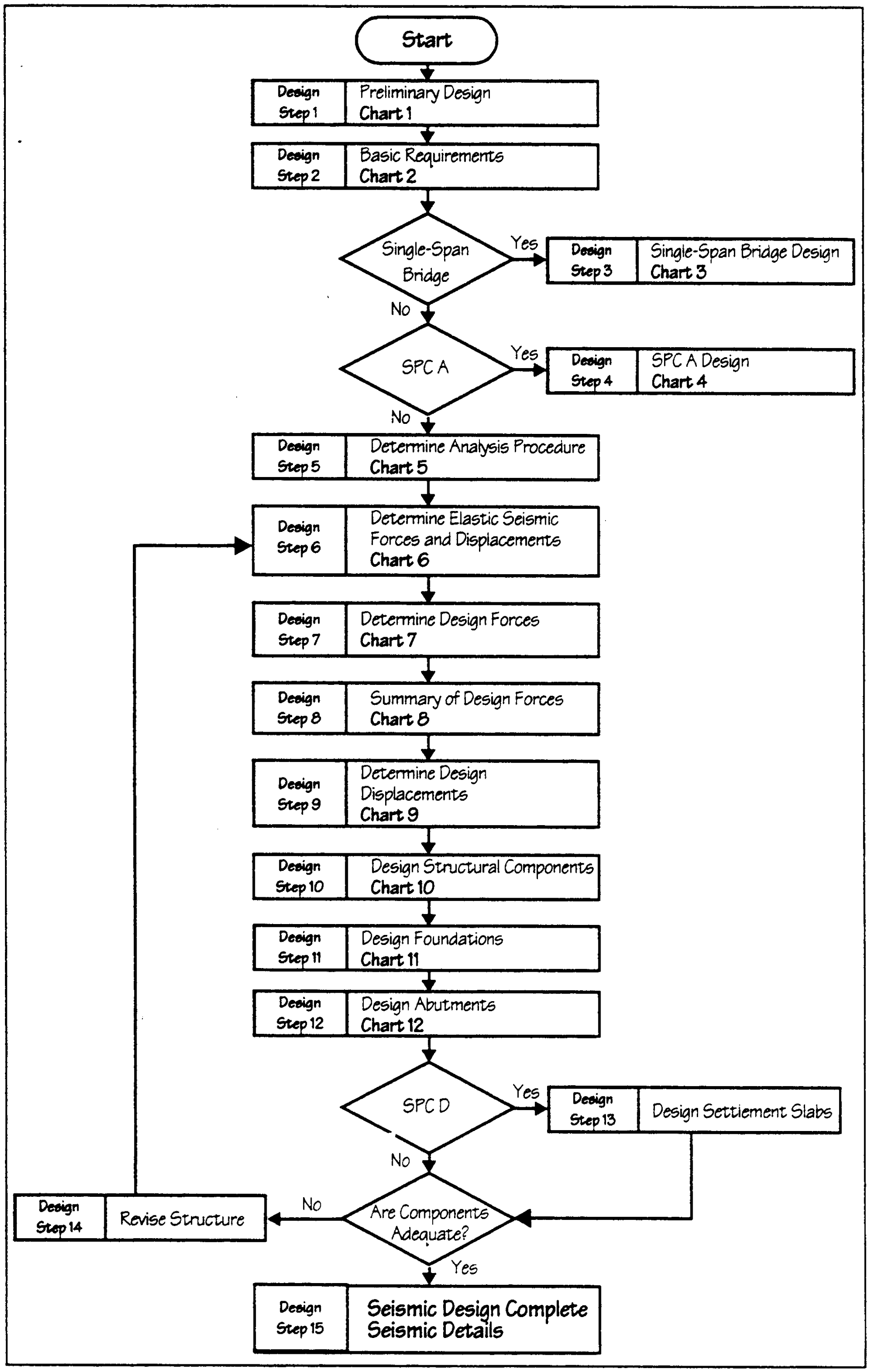


Note that Mathcad® carries the full precision of the variables throughout the calculations, even though the listed result of a calculation is rounded off. Thus, hand-calculated checks made using intermediate rounded results may not give the same number as that being checked.

Also, Mathcad® does not allow the superscript " ′ " to be used in a variable name. Therefore, the specified compressive strength of concrete is defined as f_c in this example (not f'_c).

Section II
Flowcharts

FLOWCHARTS



Main Flowchart — Seismic Design AASHTO Division I-A

FLOWCHARTS
(continued)

To Be Developed

Chart 1 — Preliminary Design

FLOWCHARTS
(continued)

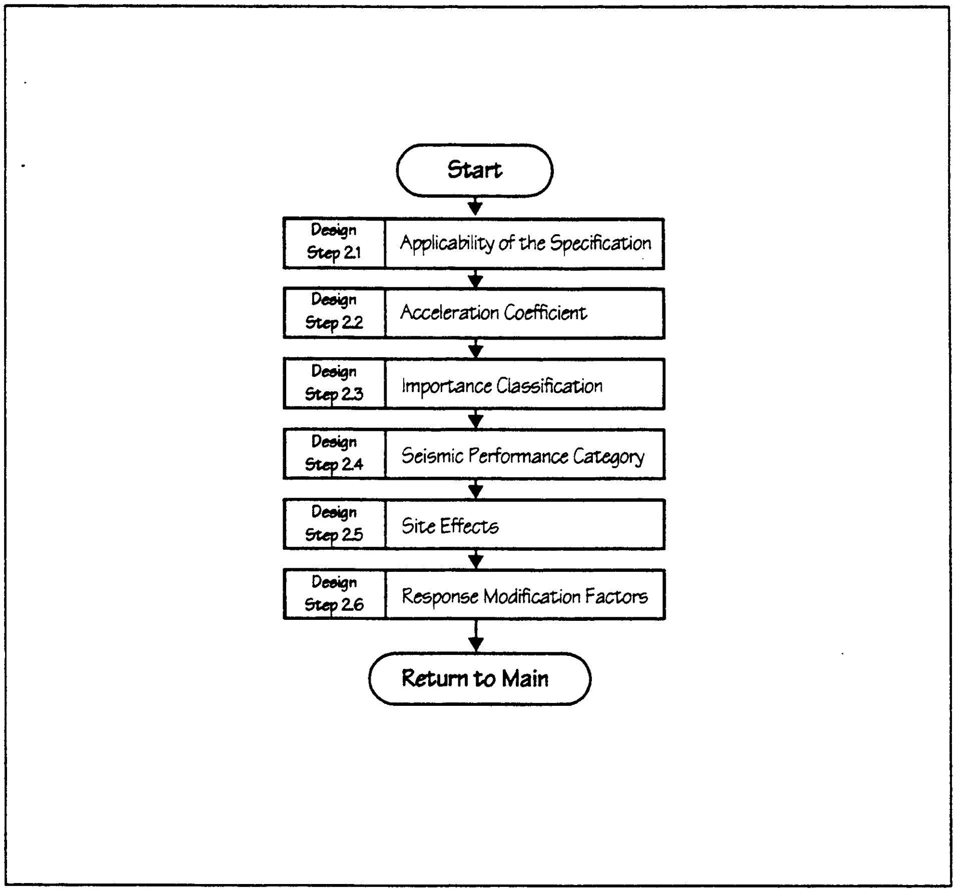


Chart 2 — Basic Requirements

FLOWCHARTS
(continued)

To Be Developed

Chart 3 — Single-Span Bridge Design

FLOWCHARTS
(continued)

To Be Developed

Chart 4 — SPC A Design

FLOWCHARTS
(continued)

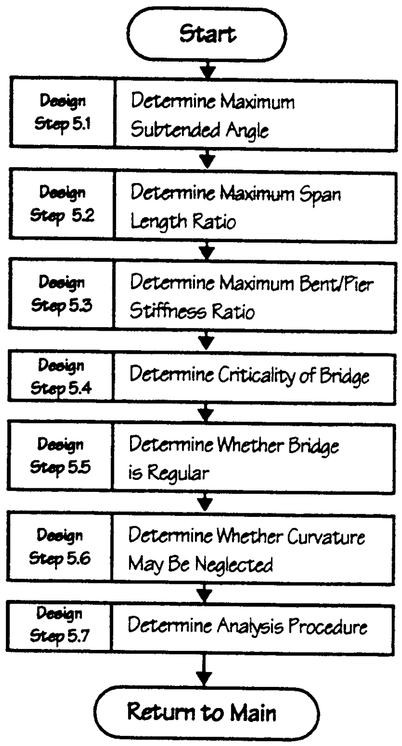


Chart 5 — Determine Analysis Procedure

FLOWCHARTS
(continued)

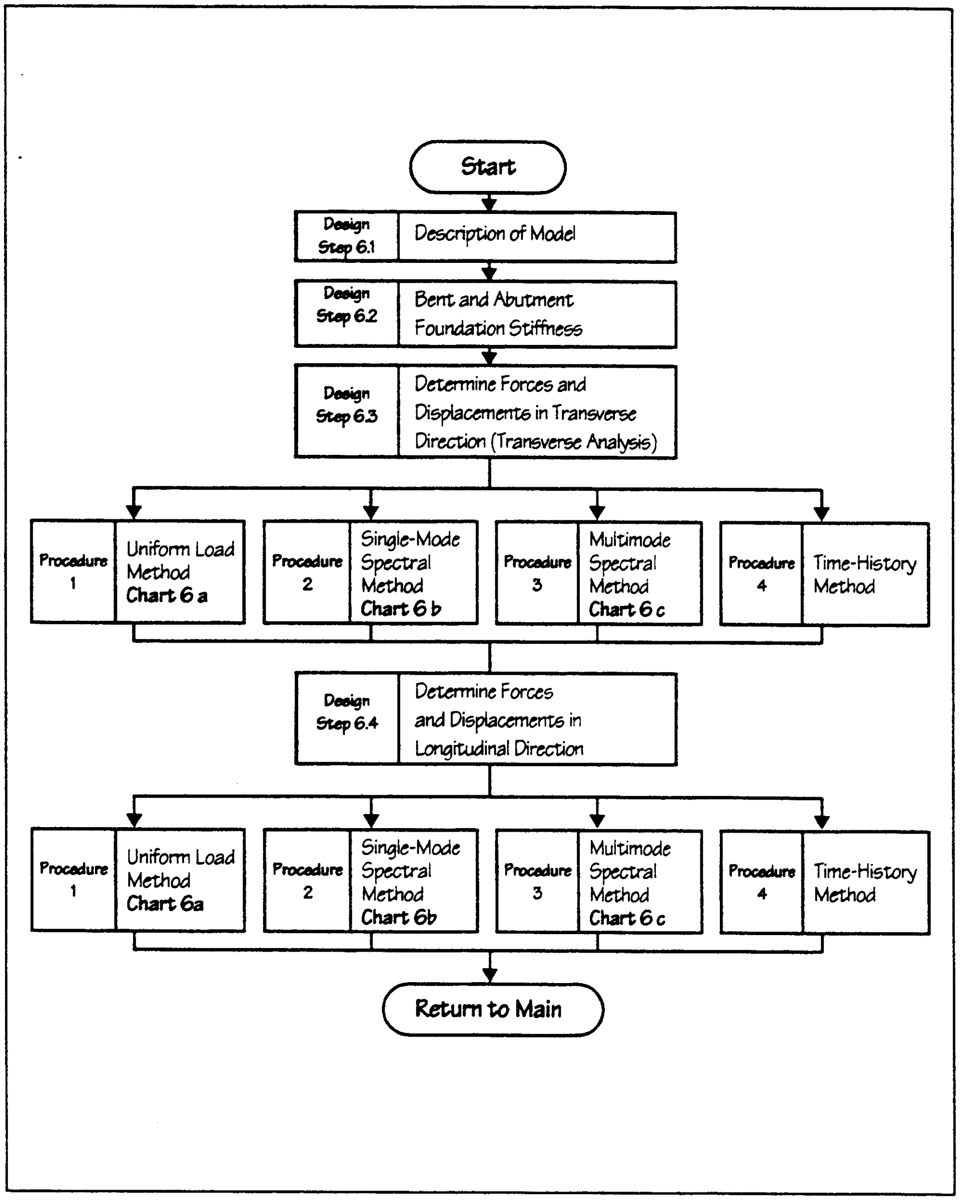


Chart 6 — Determine Elastic Seismic Forces and Displacements

FLOWCHARTS
(continued)

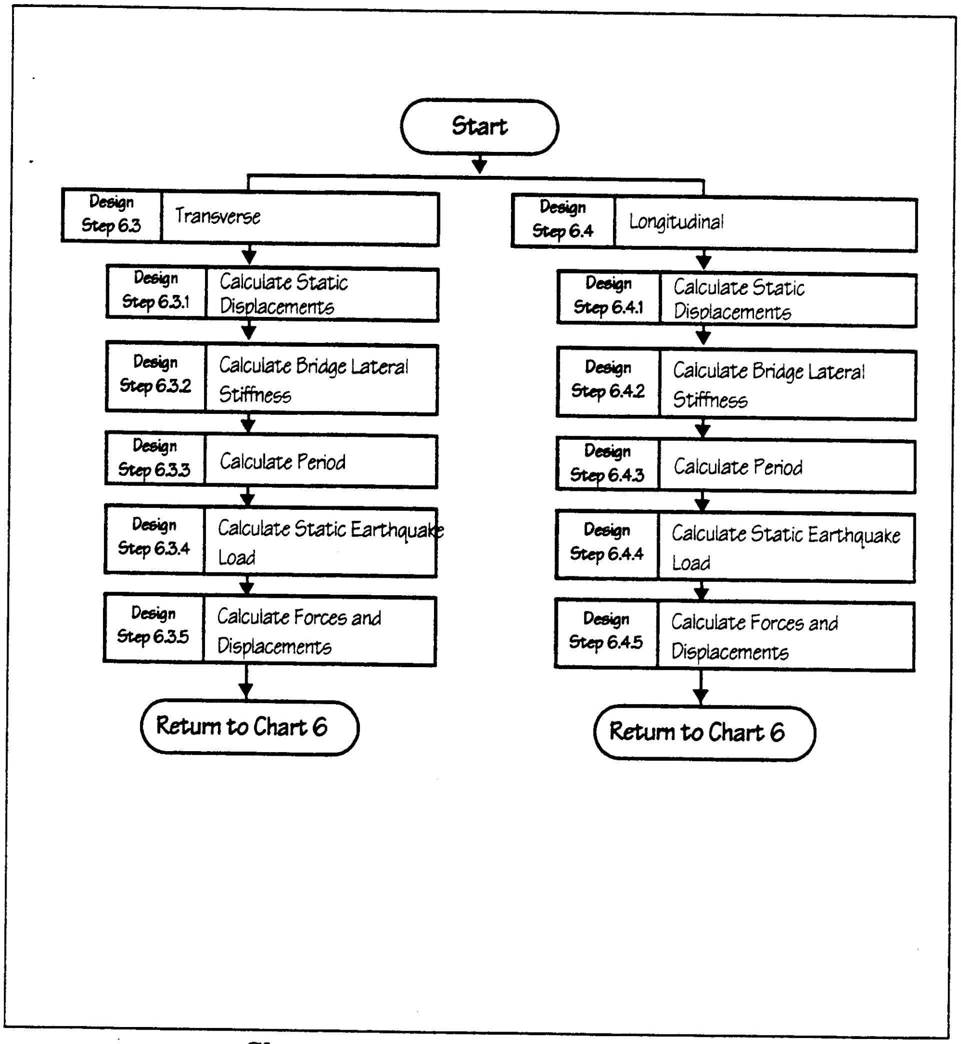


Chart 6a — Uniform Load Method

FLOWCHARTS
(continued)

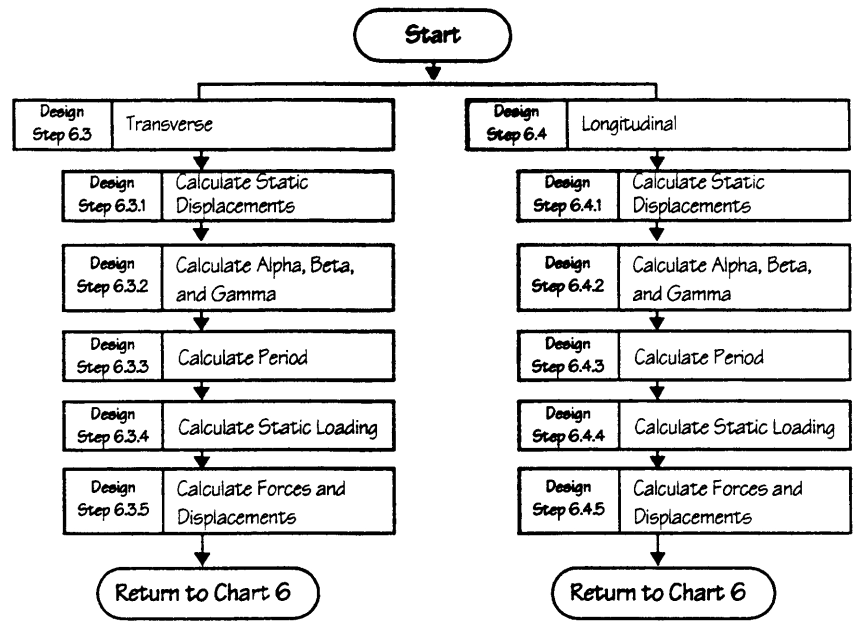


Chart 6b — Single-Mode Spectral Method

FLOWCHARTS
(continued)

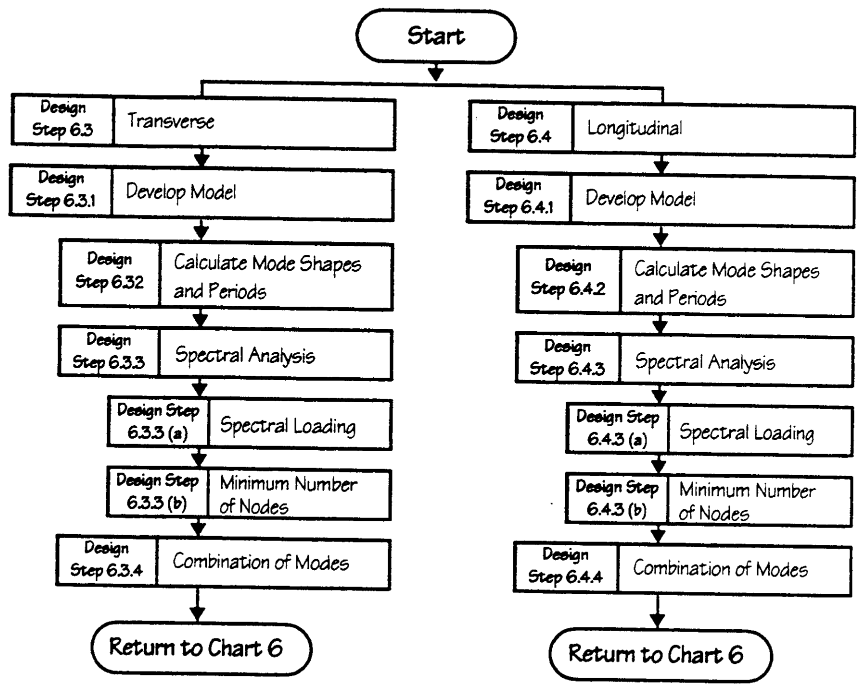


Chart 6c — Multimode Spectral Method

FLOWCHARTS
(continued)

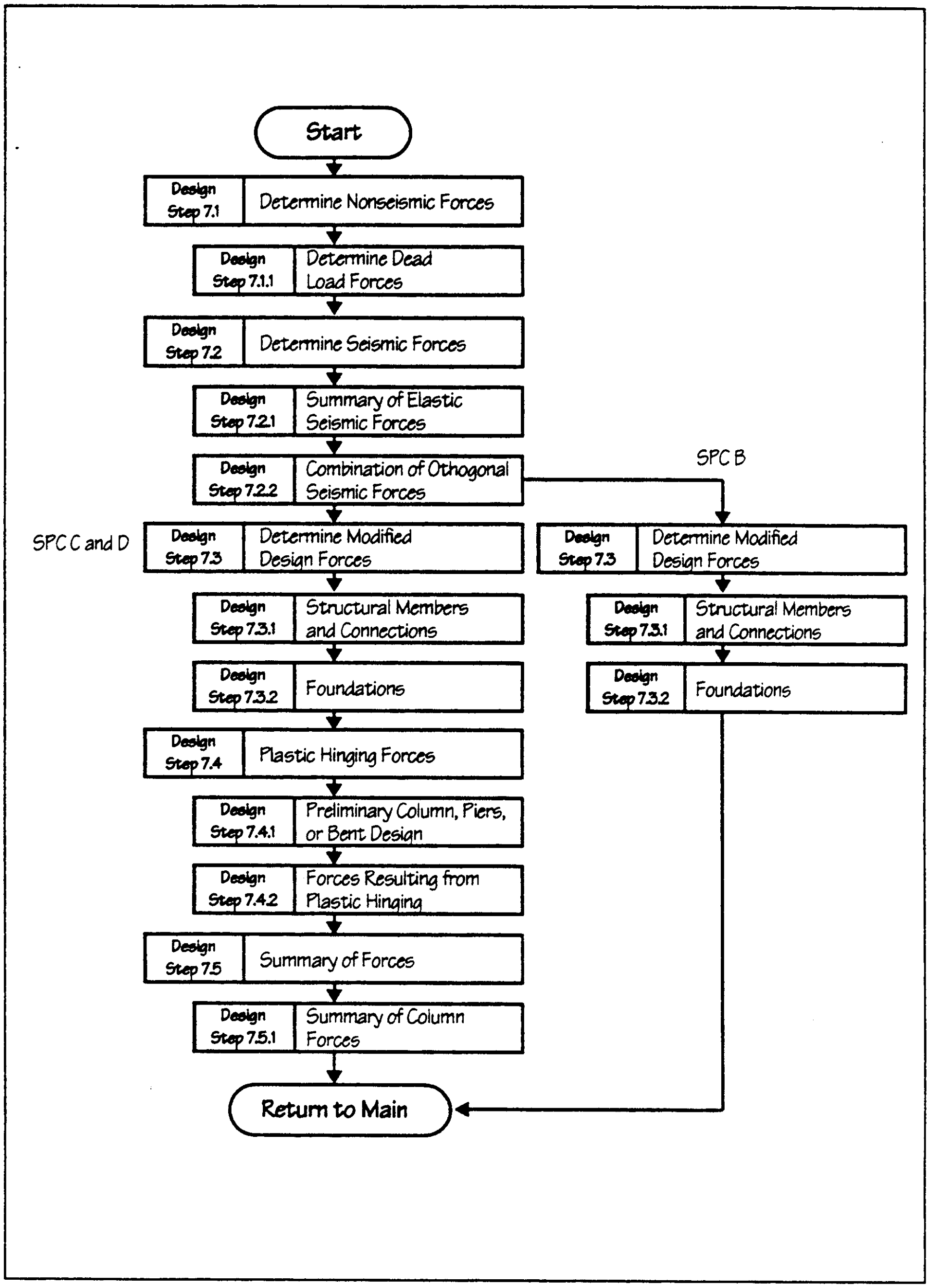


Chart 7 — Determine Design Forces

FLOWCHARTS
(continued)

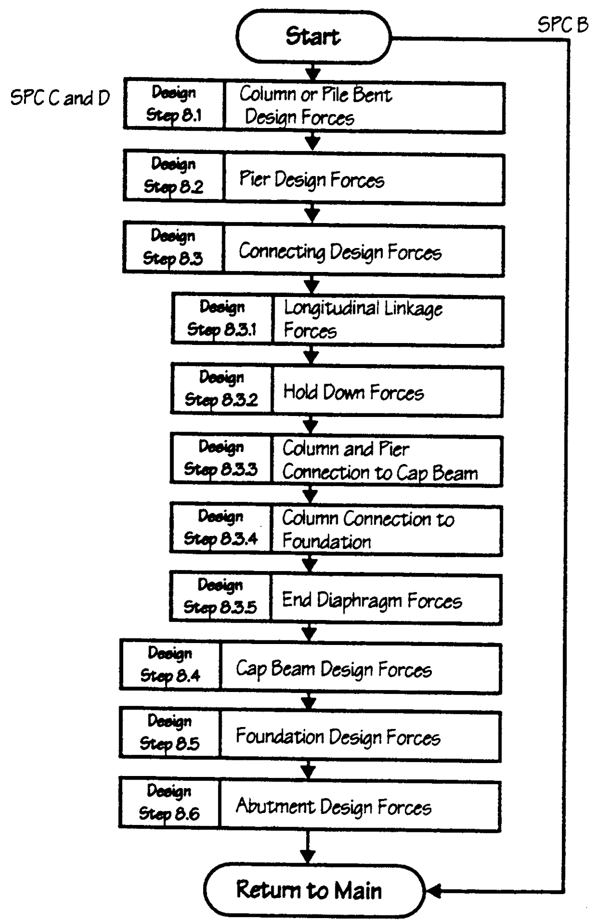


Chart 8 — Summary of Design Forces

FLOWCHARTS
(continued)

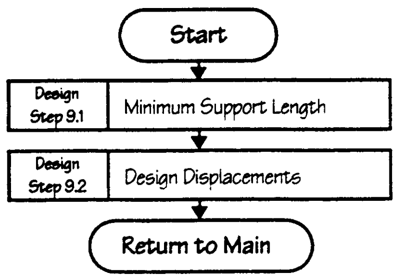


Chart 9 — Determine Design Displacements

FLOWCHARTS
(continued)

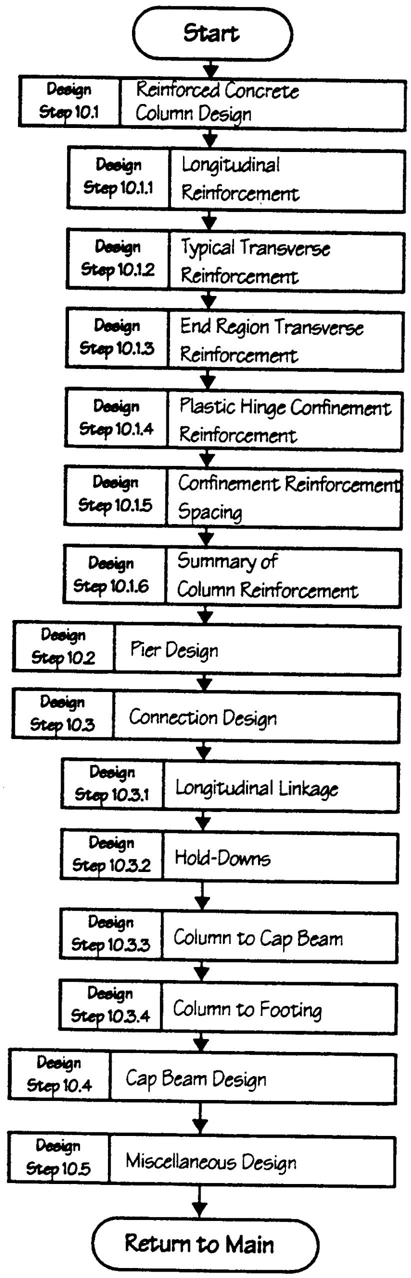


Chart 10 — Design Structural Components

FLOWCHARTS
(continued)

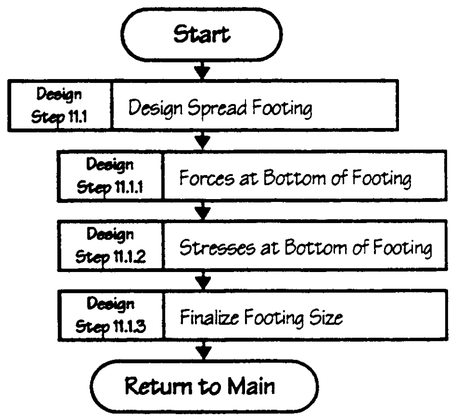


Chart 11 — Design Foundations

FLOWCHARTS
(continued)

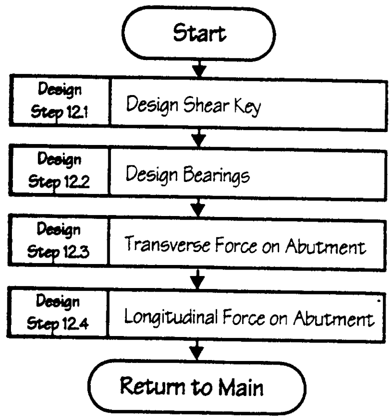


Chart 12 — Design Abutments

Section III
Analysis and Design Using Single-Mode Spectral Method with Basic Supports

SECTION III — ANALYSIS AND DESIGN USING SINGLE-MODE SPECTRAL METHOD WITH BASIC SUPPORTS

DATA

The bridge is to be built near Glacier National Park, Montana. The soil is a 250-foot-deep glacial deposit of sand and gravel.

REQUIRED

Design the bridge for seismic loading using the Proposed Revisions to the *AASHTO Standard Specifications for Highway Bridges*, Division I-A: Seismic Design, 1995.

SOLUTION

DESIGN STEP 1 — PRELIMINARY DESIGN

The preliminary seismic design of the bridge has been completed and the results are shown in this section.

The initial iterative design process, required to size the columns of the intermediate bent is not shown here because it requires the knowledge of techniques that will be discussed within the example itself. After these techniques are understood, it is a simple matter to perform various quick hand analyses to obtain preliminary sizes of the substructure elements.

In this example, 5-, 4-, and 3-foot-diameter columns were checked as potential candidates for the bent. The 4-foot-diameter column was selected because it resulted in a reasonable footing size and amount of longitudinal column reinforcement. The initial configuration and geometry of the bridge are shown in Figures 1a, 1b, 1c, and 1d. The intended seismic resisting mechanisms are as follows. In the transverse direction, both the superstructure and the relatively flexible bent act to resist transverse forces. The superstructure essentially acts as a simply supported horizontal beam that spans between pinned supports at the abutments. This behavior is illustrated in Figure 2. In the longitudinal direction, the intermediate bent columns are assumed to resist the entire longitudinal seismic force. This behavior is illustrated in Figure 3.

The abutments are seat-type abutments with space behind the end diaphragm that allows free longitudinal movement of the superstructure. Due to the lack of longitudinal restraint at the abutment, the bent acts alone to resist the longitudinal forces. The issue of plastic hinging forces in the column is, therefore, addressed.

DESIGN STEP 1
(continued)

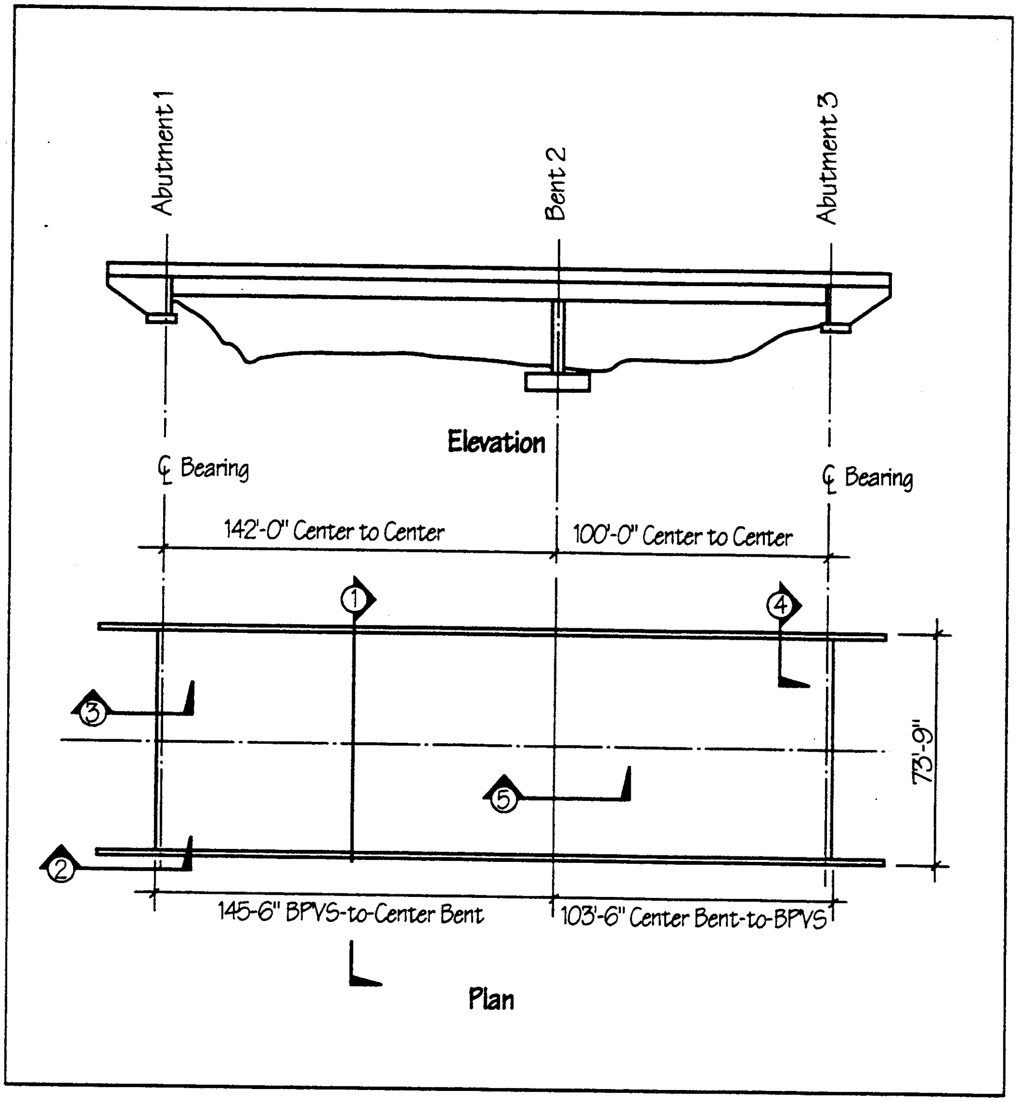


Figure 1a — Bridge Layout with Seat-Type Abutment

DESIGN STEP 1
(continued)

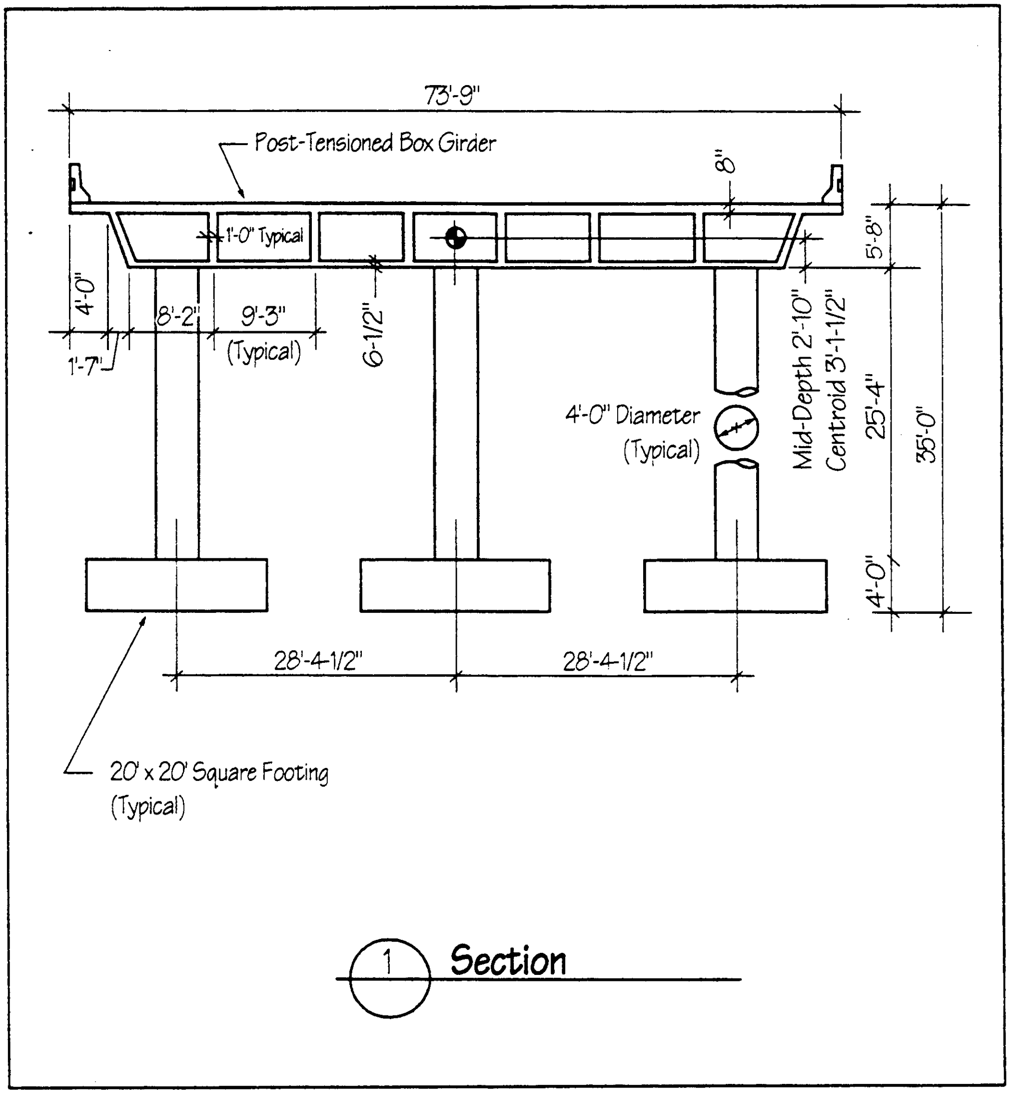


Figure 1b — Bridge Layout with Seat-Type Abutment

DESIGN STEP 1
(continued)

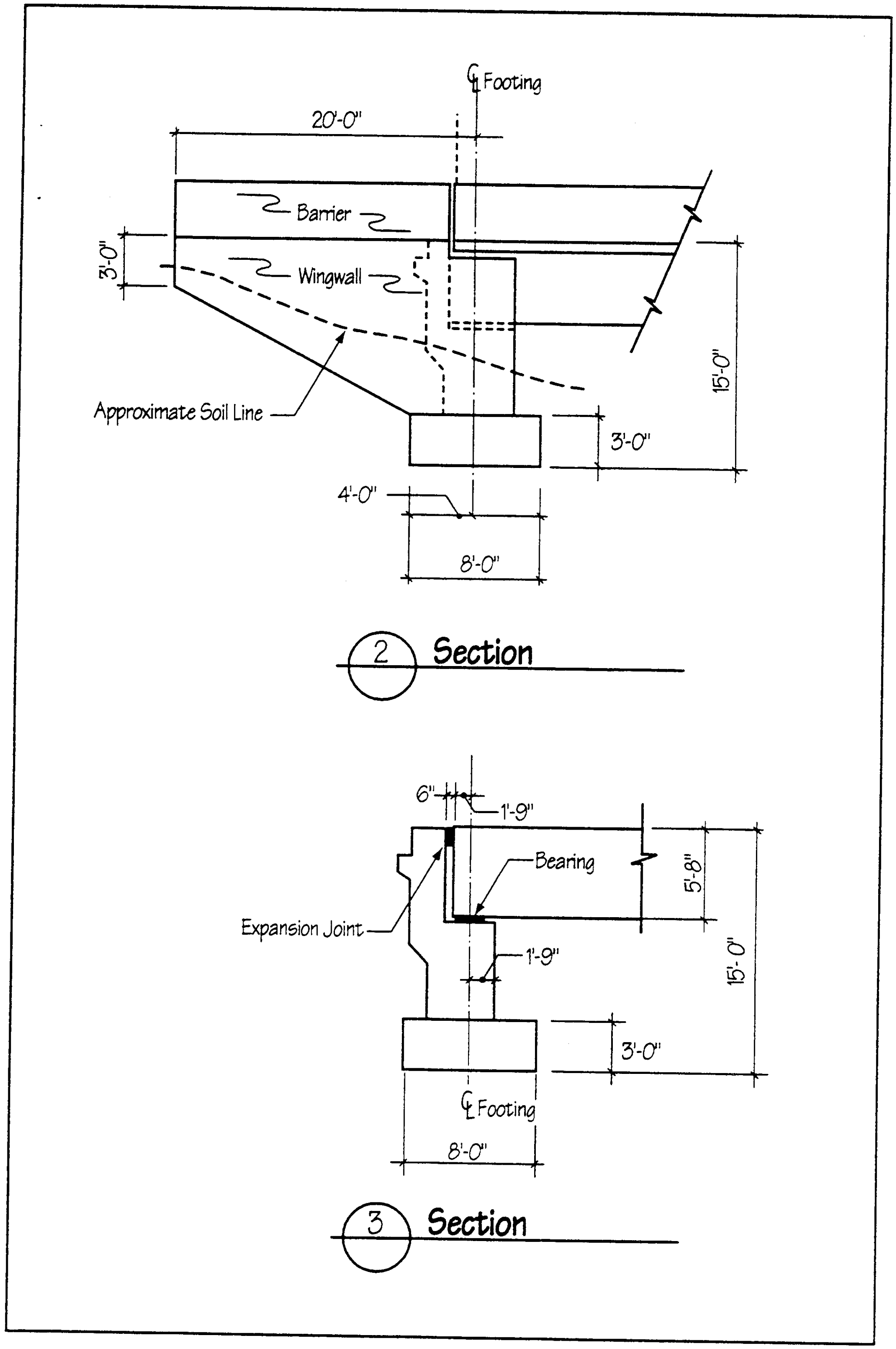


Figure 1c — Bridge Layout with Seat-Type Abutment

DESIGN STEP 1
(continued)

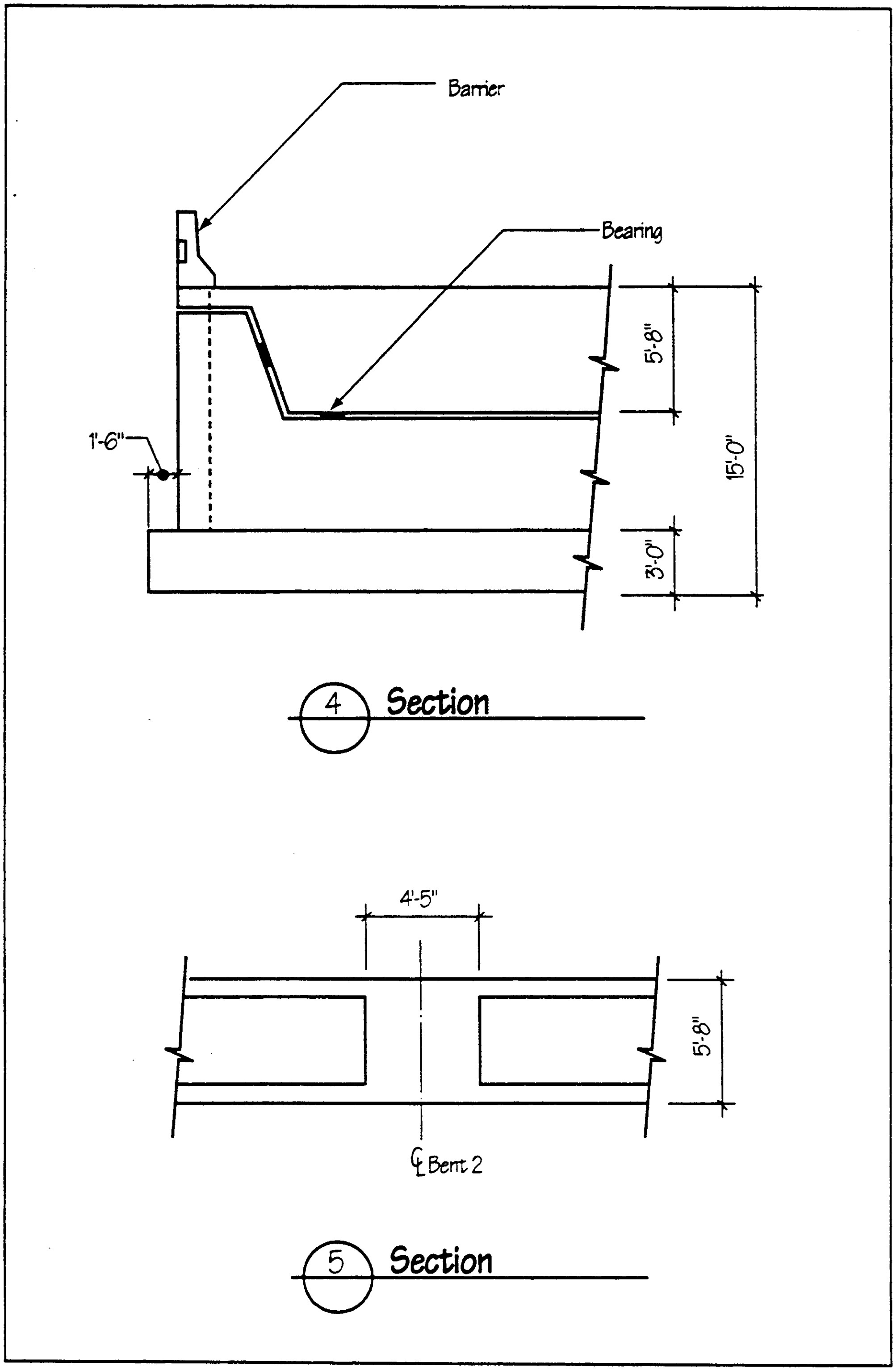


Figure 1d — Bridge Layout with Seat-Type Abutment

DESIGN STEP 1
(continued)

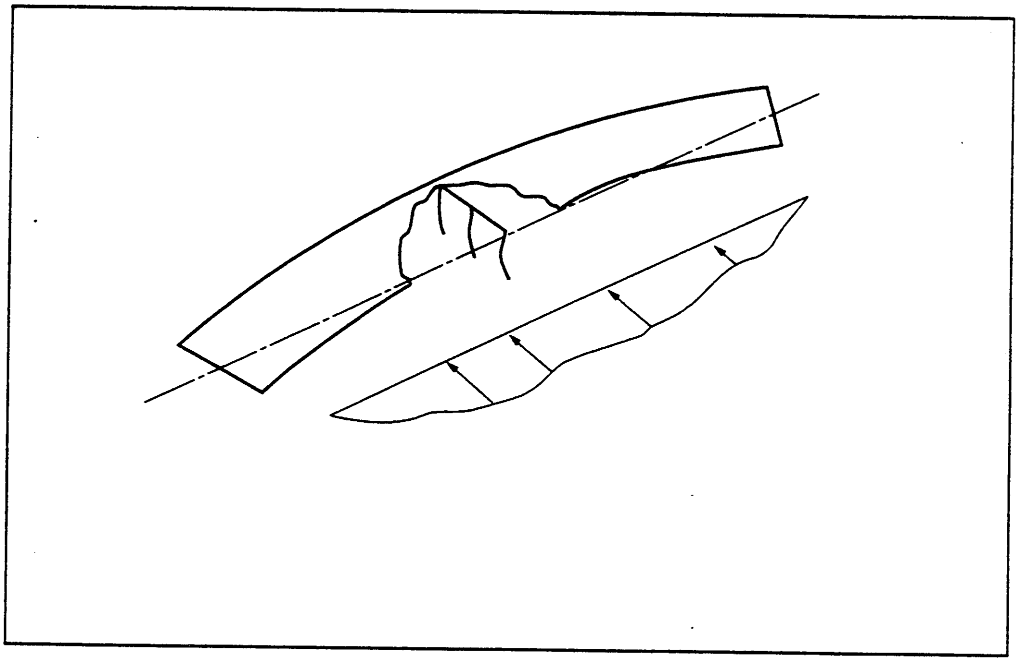


Figure 2 — Transverse Seismic Behavior

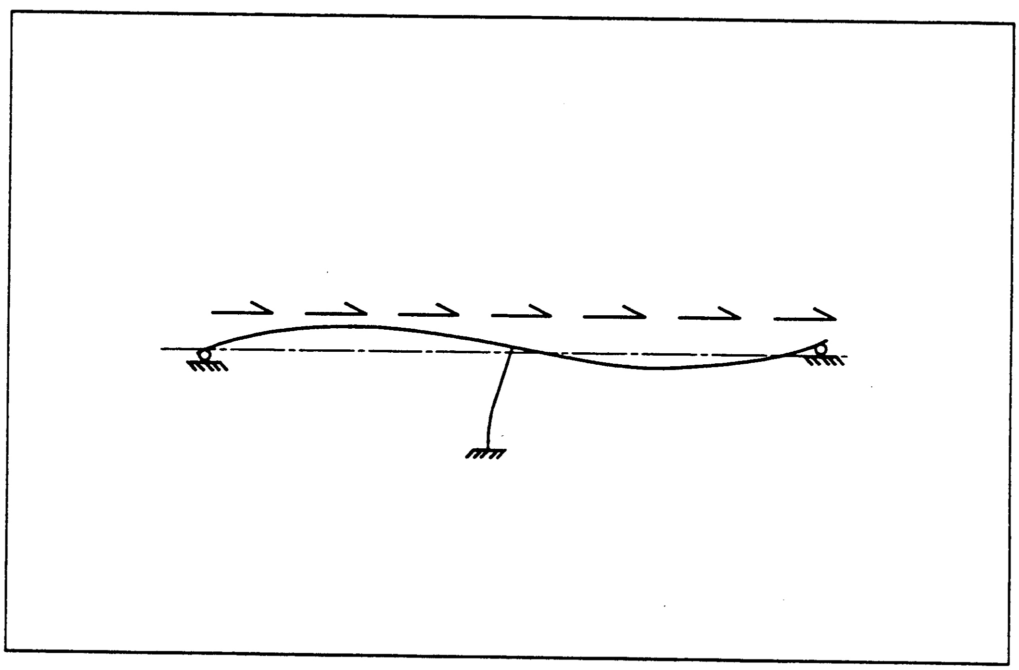


Figure 3 — Longitudinal Seismic Behavior

DESIGN STEP 1
(continued)

The abutments are assumed to provide restraint in the transverse direction. They also are assumed to allow free rotation about a vertical axis.

In both the transverse and longitudinal directions, the bases of the columns are considered fixed against rotation. Because the flexural stiffness of the superstructure is large relative to the columns, the bent columns tend to be nearly fixed at the top under both longitudinal and transverse loads. The moment of inertia of the column was assumed to be that of the full cross section, "Igross."

Several types of bearings could be used to accommodate the expected displacements. Elastomeric bearings with provision for sliding between the bearing and the diaphragm under large displacements would work, as would PTFE bearings against a sliding surface. The transverse restraint would be provided by the abutment and its integral wingwalls acting against the soil.

Superstructure

Properties of the structure and its elements are as follows.

$L := 242 \cdot ft$ — Overall length of bridge

$A_d := 120 \cdot ft^2$ — Cross-sectional area of superstructure and deck

$A_{cb} := 25 \cdot ft^2$ — Cross-sectional area of cap beam

$I_{yd} := 51000 \cdot ft^4$ — Moment of inertia of superstructure in horizontal plane

$I_{zd} := 575 \cdot ft^4$ — Moment of inertia of superstructure in vertical plane

$f_c := 4000 \cdot \frac{lbf}{in^2}$ — Compressive strength of concrete

DESIGN STEP 1
(continued)

$E_c := 3600 \cdot \frac{kip}{in^2}$ Young's modulus of concrete (based on Division I, Article 8.7.1)

$E_c = 5.184 \cdot 10^5 \cdot \frac{kip}{ft^2}$ Equivalent Young's modulus in the units used for the analyses

Substructure

The 4-foot-diameter circular columns each have moments of inertia and cross-sectional areas as given below.

The columns are founded on spread footings that have been preliminarily sized at 20 feet square and 4 feet thick.

$I_c := \frac{\pi \cdot 4^4}{64} \cdot ft^4$ $I_c = 12.57 \cdot ft^4$ Moment of inertia of one column

$A_c := \frac{\pi \cdot 4^2}{4} \cdot ft^2$ $A_c = 12.57 \cdot ft^2$ Cross-sectional area of one column

DESIGN STEP 2 — BASIC REQUIREMENTS

Design Step 2.1 — Applicability of Specification
[Division I-A, Article 3.1]

The bridge has two spans that total 242 feet, and the bridge is made of reinforced concrete. Thus, the Specification applies.

Design Step 2.2 — Acceleration Coefficient
[Division I-A, Article 3.2]

From Figure 3 of the Specification, the Acceleration Coefficient A is 0.28.

A site investigation by a qualified geotechnical engineer or seismic hazard assessment specialist may be used to develop more accurate acceleration data. Such an investigation is required if the structure is near an active fault, if long-duration earthquakes are expected, or if design for a long return period is required due to great importance of the structure. In addition, some agencies may require acceleration coefficients that are different than those given in the AASHTO Specification.

Design Step 2.3 — Importance Classification
[Division I-A, Article 3.3]

The Importance Classification (IC) of this bridge is taken to be II. It is assumed not to be essential for use following an earthquake.

Design Step 2.4 — Seismic Performance Category
[Division I-A, Article 3.4]

The Seismic Performance Category (SPC) is C . This is taken from Table 1 of the Specification.

The SPC is a function of the Acceleration Coefficient and the Importance Classification.

Design Step 2.5 — Site Effects
[Division I-A, Article 3.5]

The site conditions affect the design through a coefficient based on the soil profile. In this case, SOIL PROFILE TYPE II corresponds to 250-foot-deep cohesionless soil (sand and gravel).

Design Step 2.5 (continued)

The Site Coefficient S for this type soil is 1.2 per Table 2 of the Specification.

A geotechnical investigation may be made by qualified professionals to establish site-specific seismic response information (e.g., site-specific response spectra). This is typically done on a site-by-site basis. In some cases, State Departments of Transportation (DOTs) have developed representative spectra for soil types and seismic hazards in their jurisdiction. These are then used in lieu of the information in Article 3.5. Lacking such specific information, the structural engineer should decide whether to have site-specific information generated or use the approach given in this section.

Design Step 2.6

Response Modification Factors
[Division I-A, Article 3.7]

Since this bridge is classified as SPC C, appropriate Response Modification Factors (R Factors) must be selected for use later in establishing appropriate design force levels.

In this case, Table 3 of the Specification gives the following R Factors.

$R = 5$ For the substructure since multiple-column bents are used

$R = 0.8$ For the superstructure to abutment connection (bearings and shear keys)

These factors will be used to ensure that inelastic effects are restricted to elements that can be designed to provide reliable, ductile response that can be inspected after an earthquake to assess damage and that can be repaired relatively easily. The foundations do not fit this constraint and thus will be designed not to experience inelastic effects. For instance, the bent foundations and superstructure elements will be designed to have moment strengths greater than the maximum (plastic) moment that the columns can develop.

DESIGN STEP 3 SINGLE-SPAN BRIDGE DESIGN

Not applicable.

DESIGN STEP 4 SEISMIC PERFORMANCE CATEGORY A DESIGN

Not applicable.

DESIGN STEP 5 — DETERMINE ANALYSIS PROCEDURE

Design Step 5.1 — Determine Maximum Subtended Angle
[Division I-A, Article 4.2]

The bridge is not curved in the horizontal plane.

Design Step 5.2 — Determine Maximum Span Length Ratio
[Division I-A, Article 4.2]

The maximum span length ratio is 1.42 = 142 ft/100 ft.

Design Step 5.3 — Determine Maximum Bent/Pier Stiffness Ratio
[Division I-A, Article 4.2]

There are only two spans; thus this step does not apply.

Design Step 5.4 — Critical Bridge
[Division I-A, Article 4.2.3]

Assume that the bridge is not critical.

If the bridge is large, expensive, required to be functional immediately following the design earthquake, or complex geometrically, then the Specification recommends that time-history analyses (Division I-A Procedure 4) be used to analyze the structure.

Design Step 5.5 — Regular Bridge
[Division I-A, Article 4.2]

Table 5 of the Specification gives the requirements for determining whether a bridge is regular. The requirements are based on limiting values of the parameters determined in the steps above.

The bridge is regular since there is no curve, and the span length ratio is less than 3.

Design Step 5.6 — Curvature
[Division I-A, Article 4.2.2]

Not applicable; no curvature.

Design Step 5.7

Analysis Procedure
[Division I-A, Article 4.2]

Since this bridge is not a single-span bridge and since it is not a SPC A bridge, the analysis requirements of Article 4 must be satisfied. Table 4 of the Specification is used to select the minimum analysis requirements.

From Table 4 of the Specification, either the Uniform Load Method (Procedure 1) or the Single-Mode Spectral Method (Procedure 2), may be used to analyze this structure since it has less than six spans.

These are the <u>minimum</u> methods that can be used; the Multimode Spectral Method (Procedure 3) or the Time-History Method (Procedure 4) could be used in lieu of Procedures 1 and 2.

For this example, Procedure 2 is used for the analysis.

DESIGN STEP 6

DETERMINE ELASTIC SEISMIC FORCES AND DISPLACEMENTS

Design Step 6.1

Description of Model

The structural analysis program SAP90 (CSI, 1992) was used for the analyses. The model used is shown in Figure 4 and includes a single line of frame elements for the superstructure and individual elements for the columns and cap beam.

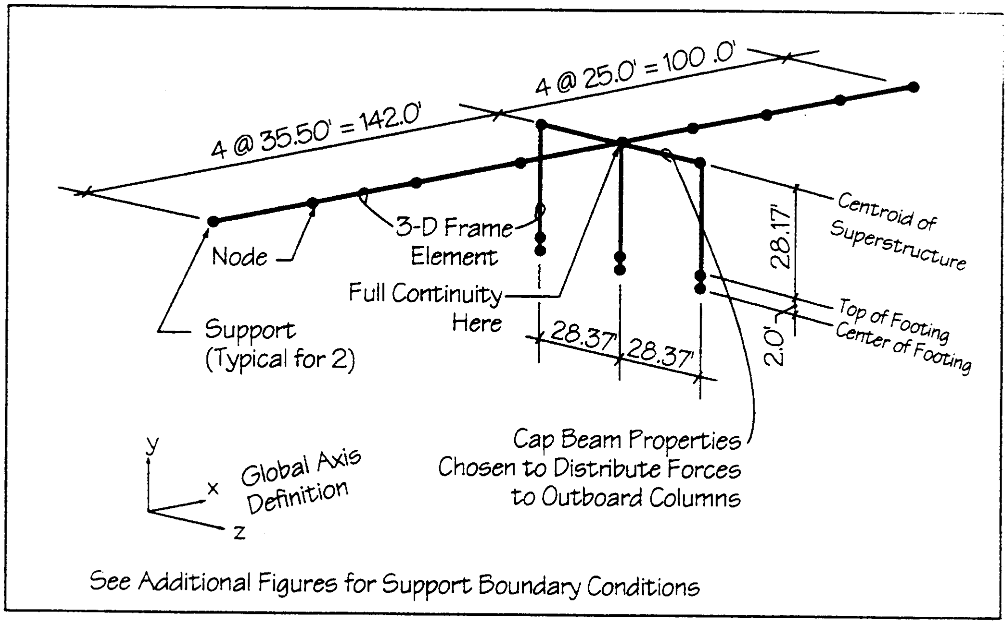


Figure 4 — Structural Model of Bridge

Superstructure

As shown in Figure 4, the superstructure has been collapsed into a single line of 3-D frame elements. This is a reasonable approach for most bridges that have regular geometry. The model is used solely for the determination of seismic forces, so the fact that such a "stick" model does not give the correct forces for other loadings, for instance dead loads, is not a concern. Many designers use such an approach for the seismic model, and further discussion of setting up the seismic model is given by FHWA (1987) and Caltrans (1994).

Enough nodes must be used along the length of the superstructure to accurately characterize the response and forces. For a uniform cross section such as this one, nodes at the quarter points are sufficient. If a tapered box girder had been used, additional nodes may have been

Design Step 6.1 (continued)

required. Because the superstructure is integral with the bent, full continuity is used at the seismic model's superstructure-bent intersection. Moments of inertia and torsional stiffness of the superstructure are based on uncracked cross-sectional properties.

Bent

The bent is modeled with 3-D frame elements that represent the cap beam and individual columns. In the actual structure, internal forces are transferred between the superstructure and the bent in a nearly continuous fashion along the length of the cap beam. In the seismic model, the superstructure forces are transferred at the single point where the superstructure and bent intersect. Due to this difference, the forces in the cap beam from the seismic model do not well represent "actual" forces, and the stiffnesses of the cap beam must be adjusted to better represent "actual" distribution of forces to the columns. In the model, both the cap beam moments of inertia and its torsional stiffness were increased by several orders of magnitude. There is no precise rule for adjusting the properties, although the columns should typically attract approximately equal forces. Judgment must be used in assessing the values used.

Figure 5 shows the relation between the actual column and the "stick" model of 3-D frame elements.

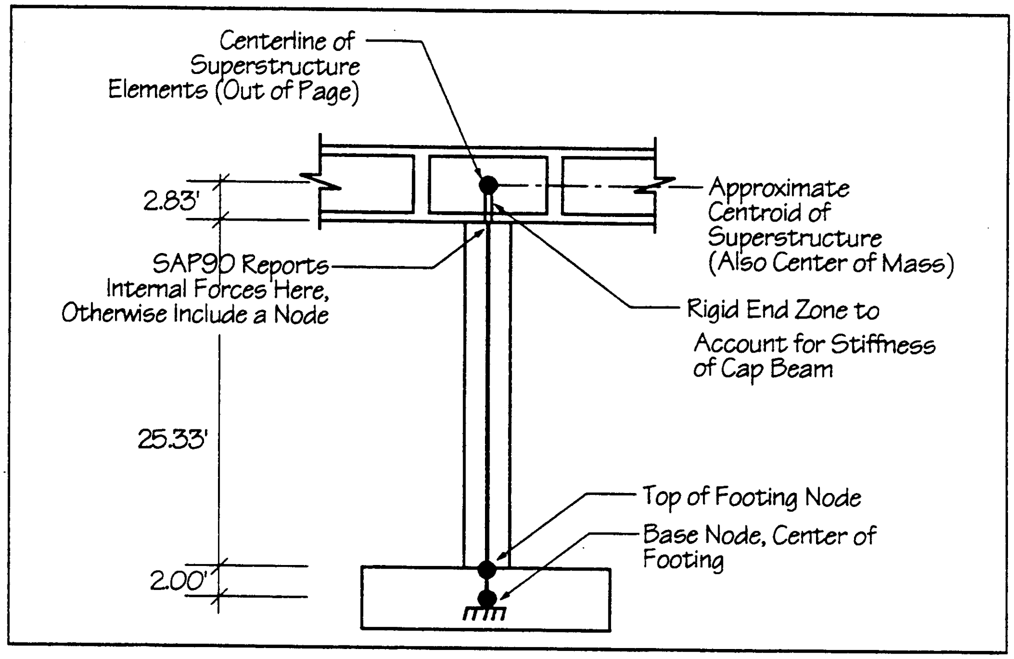


Figure 5 — Details of Column Elements

Design Step 6.1 (continued)

The upper ends of the column elements have a rigid end zone to account for the stiffness of the cap beam. A separate element was not used here since SAP90 reports internal member forces at the end of the rigid zone. In this case, the rigid zone extended from the superstructure work line to the soffit of the superstructure. Some designers prefer to use a shorter end zone to account for flexibility of the joint region.

The work line of the superstructure was taken at the geometric center of the box girder. This was the location assumed for the centroidal axis early in the design. As the cross section becomes better defined, the location of the work line should be moved to the actual centroid. In this case, it was not. However, the difference this introduces is small since the actual centroid is but 3 inches higher than the center of the box.

Only one element was used for the column between the top of footing and the superstructure. The length of the column in this case did not warrant additional elements. If the clear height of the column had been much longer or if the columns had a tapered or flared cross section, additional elements would have been used. For this model, the moments of inertia and torsional properties of the columns are based on an uncracked section. There are data to suggest that the actual column flexural stiffnesses, particularly at actual seismic load levels, are somewhat less than those corresponding to the uncracked section. This is discussed in more detail in Section IV.

Design Step 6.2

Bent and Abutment Foundation Stiffnesses

Bent Foundations

The footings were considered fixed against both translation and rotation.

Such an approach is often used for the footings, particularly for preliminary analyses and for "bounding" the response. It is rare that spread footings and supporting soil are stiff enough to produce fixed conditions. The actual stiffness of the soil can be included by estimating spring constants for the soil. This is considered in Section IV. As an approximation to the stiffness of the footing and soil for this model, the elastic properties of the column were used for an element that extended from the top of the footing to the mid-depth of the footing. There is no unique rule for handling this.

Design Step 6.2 (continued)

Additionally, the location of the node representing the bottom of the "stick" model varies from designer to designer. For the case where the base is considered fixed, this is less of a consideration than when springs are included. If springs are included it is important to coordinate with the geotechnical engineer in developing spring constants. This includes coordinating the position of the footing node. For instance, the node location may be set at the top of the footing, and the geotechnical engineer then develops constants based on everything below the top of the footing.

Abutments

The abutments were not directly modeled, nor was the soil flexibility accounted for. Restraints were applied to the model as shown in Figure 6.

The restraints act either normal to or colinearly with the superstructure work line. They are also located at the centerline of the bearings in the longitudinal direction. For a regular bridge with no skew, this is a reasonable approach.

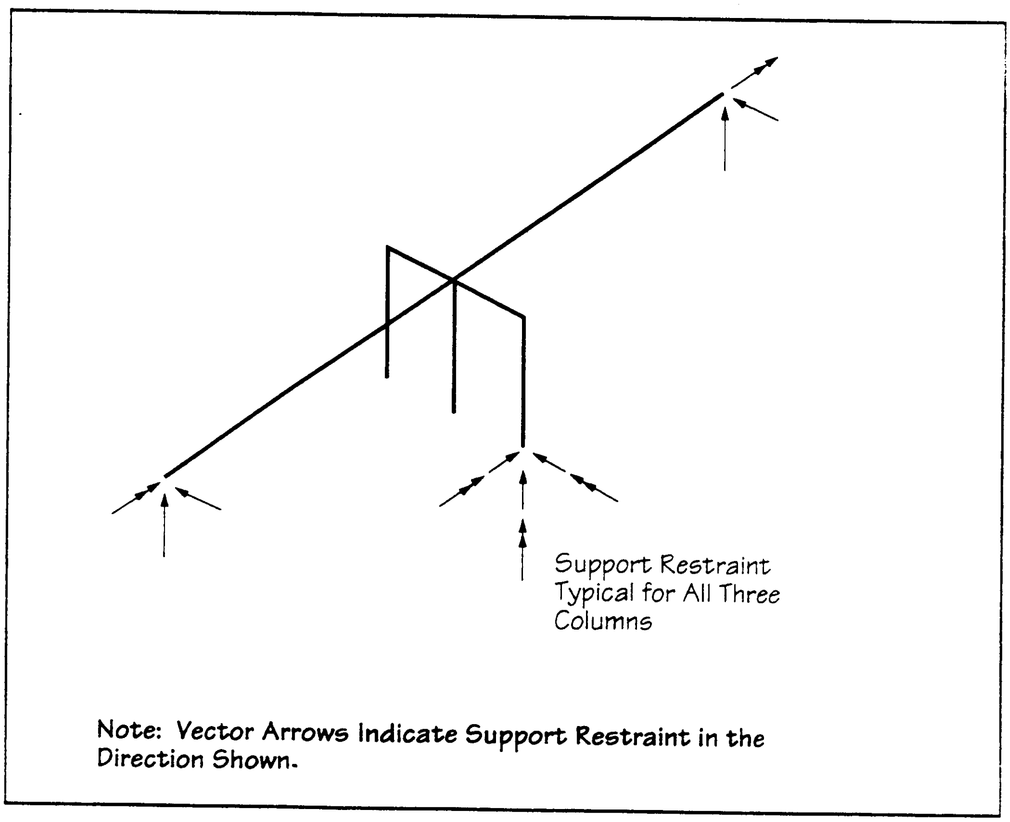


Figure 6 — Details of Supports for Basic Foundation Model

Design Step 6.2 (continued)

The model allows longitudinal response that is unrestrained at the abutment. A gap between the end of the superstructure and the abutment backwall that is larger than the expected seismic displacement must be included if no longitudinal force is to be developed. Depending on the site acceleration coefficient, soil conditions, and bridge configuration, this gap may be a reasonable size to accommodate with available joint configurations, or it could be too large.

In such a case, the longitudinal movement would be unrestrained until the superstructure came into contact with the abutment backwall. Then a longitudinal force would develop. This effect can be modeled and is described in the *Seismic Design and Retrofit Manual for Highway Bridges*, FHWA (1987).

The abutments are considered fixed against translation in the transverse direction. The forces required to produce such fixity would be developed by the soil acting against both the abutment wingwalls and acting against the backwall of the abutment. These forces would be passed through the abutment and into the superstructure via the abutment shear keys. The actual flexibility of the soil-abutment system can be modeled, and this is discussed in Section IV.

Torsional response of the superstructure is restrained in the model by the abutments. Such fixity is assumed to occur as the result of the gravity contact forces existing between the superstructure and the bearings.

Design Step 6.3

Transverse Analysis, Single-Mode Spectral Method

[Division I-A, Article 4.4]

The Single-Mode Spectral Method is used for the analysis of the bridge.

For regular bridges, a reasonable estimate of the seismically induced inertial forces can be made by considering the response of the structure in a single mode of vibration, hence the name Single-Mode Spectral Method. Typically, regular configuration structures predominantly respond to earthquake ground motion in their fundamental modes of vibration. The fundamental modes are those with the longest period in each primary direction. The shape of these modes can usually be approximated by the deformed shape of the structure when its self-weight is applied in the direction under consideration.

Because the weight of most bridges is nearly uniform along their lengths, a uniform load is often applied to calculate the deformed shape. The Single-Mode Spectral Method is part of a category of dynamic analysis techniques called "generalized single-degree-of-freedom systems." Thorough discussion of such techniques can be found in Clough and Penzien (1993) or other standard structural dynamics texts.

This method could be applied using only hand calculations. However, SAP90 will be used so that all the member internal forces and all the joint displacements are obtained at one time. This is considerably easier than performing all the calculations by hand. However, certain elements of the analysis will be done by hand, and simple checks of the intermediate answers will be performed by hand. Such checks help the designer gain confidence in the results.

Design Step 6.3.1

Static Displacements

[Division I-A, Article 4.4, *(Step 1)*]

The displacements of the bridge under a uniform transverse load as shown in Figure 7 are calculated. The displacements resulting from a 100-kip/foot loading are given in Table 1.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

Design Step
6.3.1
(continued)

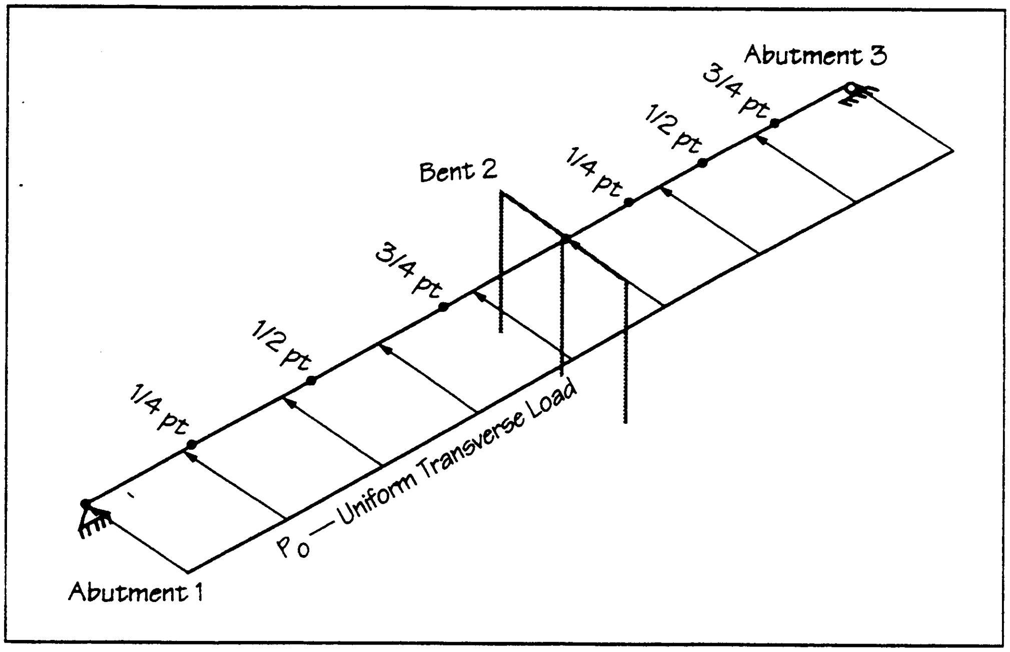


Figure 7 — Uniform Load

Table 1
Transverse Displacements for 100-kip/foot Uniform Load

Displacements (feet)								
Abut 1	1/4 Span	1/2 Span	3/4 Span	Bent 2	1/4 Span	1/2 Span	3/4 Span	Abut 2
0.0000	0.0688	0.1218	0.1488	0.1454	0.1254	0.0921	0.0489	0.0000

The load intensity may be set at 1.0 force per length. However, the loading is often set to an arbitrary higher value to provide larger forces and displacements. This avoids the use of many small numbers, and the results may be scaled down later.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode	▬			
Multimode				

Design Step 6.3.2

Calculate α, β, and γ Factors
[Division I-A, Article 4.4, *(Step 2)*]

The factors, α, β, and γ are calculated using the displacements determined in the previous step and Division I-A, Equations (4-5)~~,~~ (4-6), and (4-7), respectively, as shown below.

$$\alpha := \int_0^L v_s(x)\, dx$$

Division I-A Eqn (4-5)

$$\beta := \int_0^L w(x)\cdot v_s(x)\, dx$$

Division I-A Eqn (4-6)

$$\gamma := \int_0^L w(x)\cdot v_s(x)^2\, dx$$

Division I-A Eqn (4-7)

$v_s(x)$ is the displacement along the length of the bridge.
$w(x)$ is the weight of the bridge per unit length.
L is the total length of the bridge.

The equations as given in AASHTO are written in integral form, whereas the displacements are known only at discrete locations. Typically, numerical integration is used to calculate the three factors. This has been done using the trapezoidal rule, where the average value of the variable to be summed for each incremental length is calculated and multiplied by the appropriate length. This has been set up using a spreadsheet in Table 2. The intermediate results are given for the quarter points along the bridge, and the final summations (integrations) are given at the bottom of Columns 5, 6, and 7 of the table.

Design Step
6.3.2
(continued)

The results are $\alpha = 23.104$ ft^2
$\beta = 464.4$ k-ft
$\gamma = 55.96$ k-ft^2

Table 2
AASHTO Single-Mode Spectral Analysis Method
for Calculation of Seismic Load

Assumptions:					
p_o =	100.0	k/ft	A =	0.28	
g =	32.2	ft/sec^2	2.5*A =	0.70	
w(x) =	20.1	k/ft	S =	1.2	

1	2	3	4	5	6	7	8
Location	Node Distance x (ft)	Tributary Length dx (ft)	Displ Due to Uniform Loading $v_s(x)$ (ft)	α (x) (ft^2)	β (x) (k-ft)	γ (x) (k-ft^2)	Equiv. Static EQ Loading $p_e(x)$ (k-ft)
Abut 1	0.0	0.0	0.0000	0.00	0.00	0.00	0.00
1/4 pt	35.5	35.5	0.0688	1.22	24.55	1.69	8.03
1/2 pt	71.0	35.5	0.1218	3.38	68.02	6.98	14.23
3/4 pt	106.5	35.5	0.1488	4.80	96.54	13.19	17.37
Bent 2	142.0	35.5	0.1454	5.22	104.94	15.43	16.97
1/4 pt	167.0	25.0	0.1252	3.38	67.99	9.25	14.62
1/2 pt	192.0	25.0	0.0921	2.72	54.61	6.07	10.76
3/4 pt	217.0	25.0	0.0489	1.76	35.44	2.73	5.71
Abut 3	242.0	25.0	0.0000	0.61	12.29	0.60	0.00
Sum =		242.0		23.10	464.38	55.96	

(continued on next page)

Design Step
6.3.2
(continued)

Table 2 (continued)
AASHTO Single-Mode Spectral Analysis Method for Calculation of Seismic Load

Summary of Coefficients

$\alpha =$	23.10 ft^2	α = sum of [$v_s(x)$ *dx]
$\beta =$	464.38 k-ft	β = sum of [$w(x)$ * $v_s(x)$ * dx]
$\gamma =$	55.96 k-ft^2	γ = sum of [$w(x)$ * $v_s(x)^2$ * dx]

Calculate Period T, C_s and $p_e(x)$

T =	0.172 sec	$T = 2\pi * (\gamma / p_o * g * \alpha)^{1/2}$
$C_s =$	1.30	$C_s = 1.2*A*S / T^{2/3}$ or
C_s (min) =	0.70	$C_s = C_s$ or 2.5*A (whichever is less)
$p_e(x) =$	116.8 *$v_s(x)$	$p_e(x) = \beta * C_s / \gamma * w(x) * v_s(x)$

The numerical integration used in the table for α, β, and γ is described below. As an example, the calculation of each of these terms is given for the 1/2 point of Span 1. It should be noted that the values of α, β, and γ at the 1/2 point actually represent the values of these variables between the 1/4 and 1/2 points of Span 1.

$$\alpha_{half\_pt} := \frac{0.1218 + 0.0688}{2} \cdot 35.5 \qquad \alpha_{half\_pt} = 3.38$$

$$\beta_{half\_pt} := \frac{0.1218 + 0.0688}{2} \cdot 20.1 \cdot 35.5 \qquad \beta_{half\_pt} = 68.00$$

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode	▬			
Multimode				

$$\gamma_{half\_pt} := \frac{0.1218^2 + 0.0688^2}{2} \cdot 20.1 \cdot 35.5 \qquad \gamma_{half\_pt} = 6.98$$

Design Step 6.3.3

Fundamental Period
[Division I-A, Article 4.4, *(Step 3)*]

The period is calculated using Equation (4-8) of the Specification. This has also been calculated in the table and the result is given below.

T = 0.172 sec

Design Step 6.3.4

Equivalent Transverse Load
[Division I-A, Article 4.4, *(Step 4)*]

The equivalent lateral load is defined by Equation (4-9) of the Specification. When only discrete displacements are known along the length of the bridge, the result of Equation (4-9) will be a unique intensity of loading for each displacement point.

The intensities of loading for the various points are given in Column 8 of Table 2.

The constant C_s in Equation (4-9) relates the period of the structure to the amount of lateral force that the earthquake induces. The Specification defines C_s as

$$C_s := \frac{1.2 \cdot A \cdot S}{T^{\frac{2}{3}}}$$

Division I-A
Eqn (3-1)

Where A is the acceleration coefficient.
S is the site coefficient.
T is the period.

The value given by Equation (3-1) need not exceed 2.5 times A, which is equal to 0.70. As a result, the value of C_s is capped at this maximum value for low periods. The form of C_s for this bridge is given in Figure 8.

Design Step 6.3.4 (continued)

If a site-specific response spectra had been generated by a geotechnical engineer, that spectra would have been used instead of Equation (3-1). This substitution is allowed per Division I-A, Article 3-6.

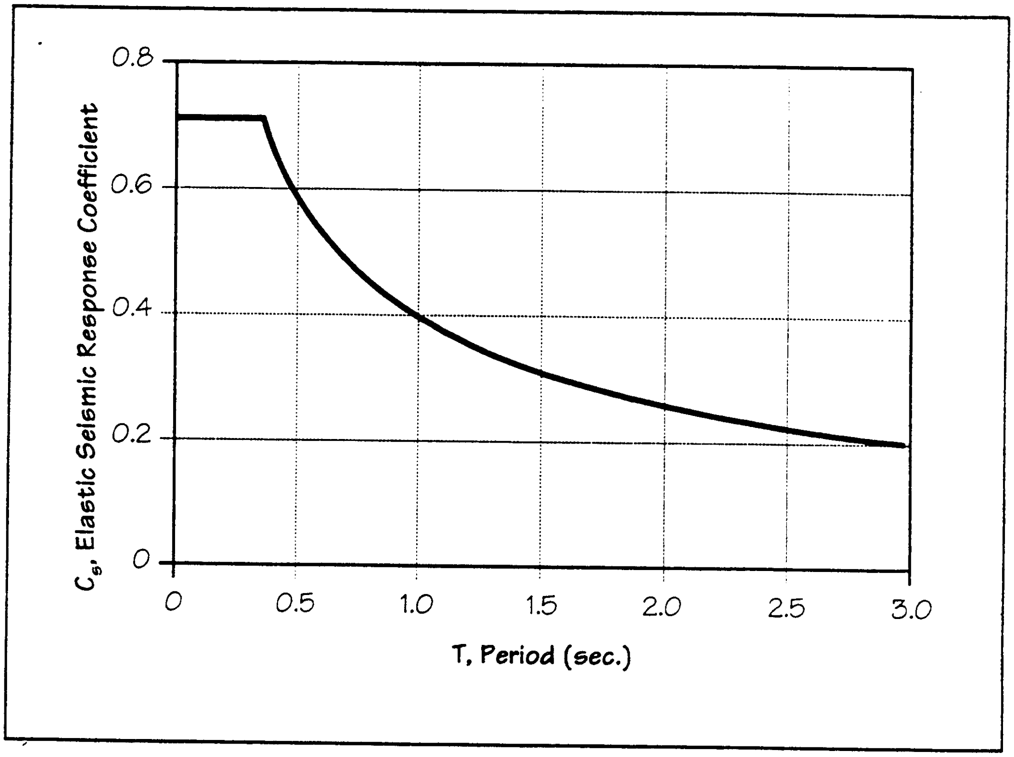


Figure 8 — Relation Between Elastic Seismic Response Coefficient and Period

Design Step 6.3.5

Displacements and Member Forces
[Division I-A, Article 4.4, *(Step 5)*]

The loads determined in the previous step are then applied to the bridge and the response determined. In this case, the loads are applied as shown in Figure 9. The loading is assumed to vary linearly between the quarter points.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

Design Step
6.3.5
(continued)

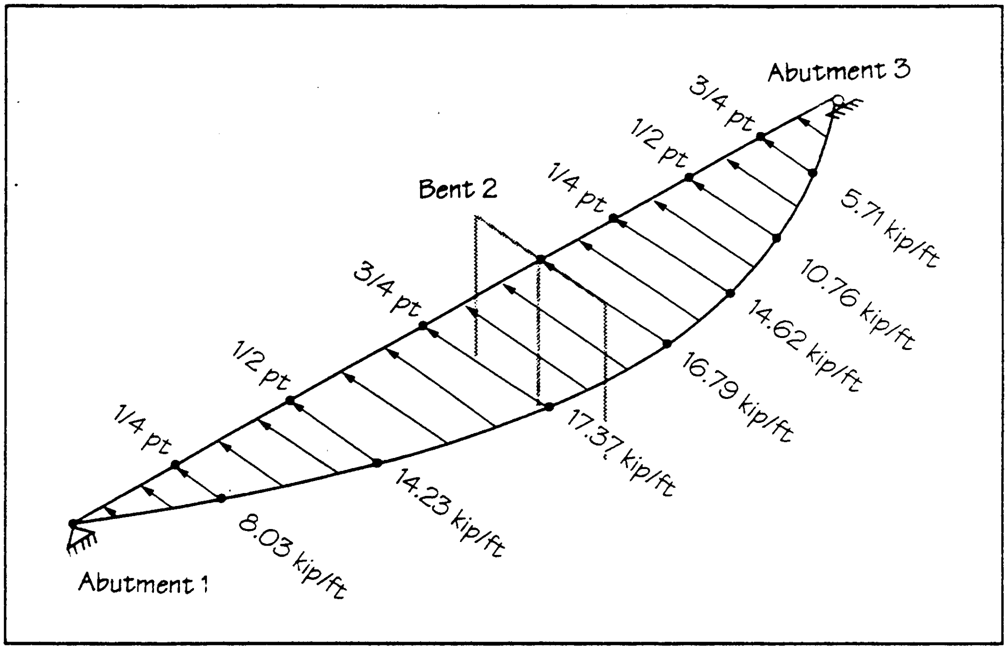


Figure 9 — Single-Mode Spectral Loads for Transverse Direction

The response values of the structure, both internal forces and displacements, are given in Table 3.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

Design Step
6.3.5
(continued)

Table 3
Response for Single-Mode Method, Transverse Direction, and Basic Foundation

			Forces and Moments				
			Longitudinal		Transverse		
			Shear (kips)	Moment (kip-ft)	Shear (kips)	Moment (kip-ft)	Axial (kips)
Abutment 1			0	0	1277	583	0
Bent 2	Center	Top	0	0	77.8	1062	0
		Bottom	0	0	77.8	910	0
	Outboard	Top	8.1	110	77.2	1053	42.5
		Bottom	8.1	94.7	77.2	902	42.5
Abutment 3			0	0	1241	828	0

Displacements (ft)								
Abut 1	1/4 pt	1/2 pt	3/4 pt	Bent 2	1/4 pt	1/2 pt	3/4 pt	Abut 3
0	0.0095	0.017	0.0208	0.0204	0.0175	0.0128	0.0068	0

Figure 10 illustrates the nomenclature used with Table 3. The same nomenclature will be used with all the subsequent tables that have the same form.

The SAP90 input file for this analysis is F1TSM4C.

It is seen in the table that longitudinal forces are developed in the columns of the bent even though loading is only applied in the transverse direction. This is due to the unequal spans. Since the bent is not in the center of the bridge there is some rotation about a vertical axis that is induced in the bent. This then causes longitudinal forces to develop in the columns.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

Design Step
6.3.5
(continued)

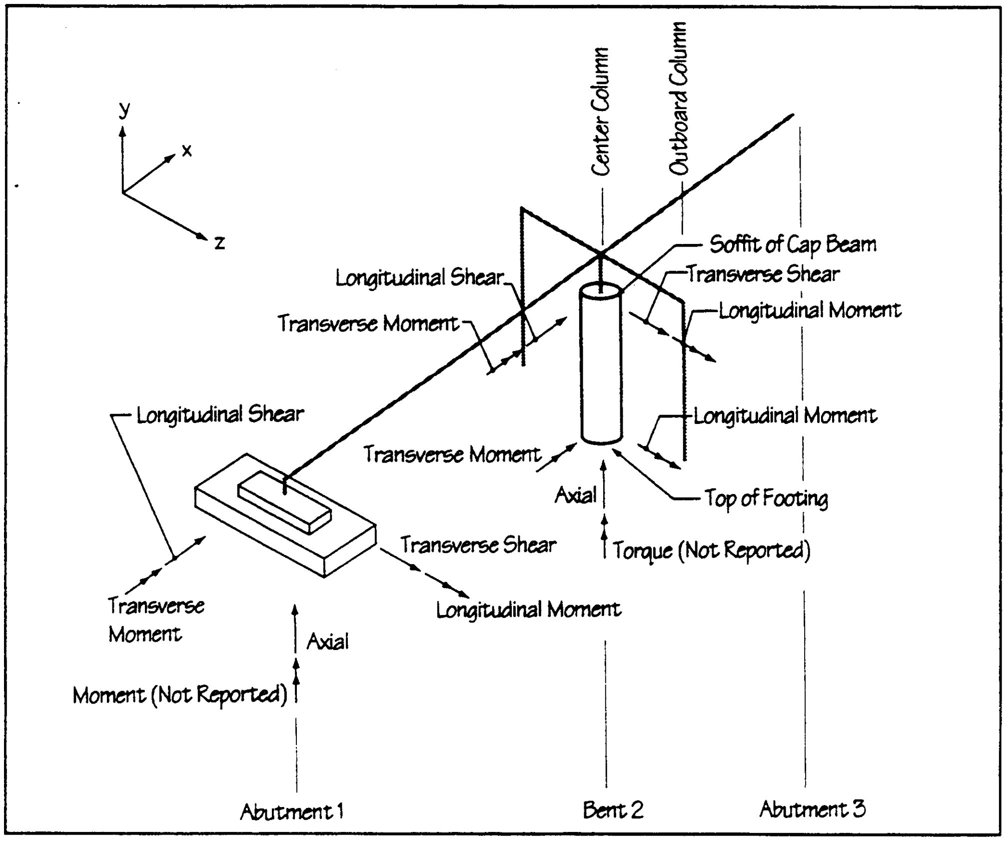


Figure 10 — Nomenclature for Results

✓ 6.3 Check No. 1 Transverse Direction

This is a hand check of the transverse fundamental period of the bridge as calculated by Single-Mode Method of the AASHTO Specification. Since the force applied to the bridge is directly a function of the period, this check is helpful in assessing whether a reasonable force will result.

Assume that half the mass of the bridge is resisted by the bent alone. Then use the bent stiffness to determine the period.

Recall the following values for the bridge.

$L := 242 \cdot ft$ Overall length of bridge

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode	▬			
Multimode				

Design Step 6.3.5 (continued)

$A_d := 120 \cdot ft^2$ Cross-sectional area of deck

$A_{cb} := 25 \cdot ft^2$ Cross-sectional area of capbeam

$E_c := 518400 \cdot \frac{kip}{ft^2}$ Young's modulus for concrete

$H := 27.33 \cdot ft$ Approximate height of columns (clear height plus extension into footing)

$I_c := 12.57 \cdot ft^4$ Moment of inertia of columns

$A_c := 12.57 \cdot ft^2$ Area of columns

Calculate the weight of the structure, including all the superstructure, the cap beam, two traffic barriers, and half of the columns. Half of the columns are assumed tributary to the superstructure and half to the foundations.

$\gamma_{conc} := 0.150 \cdot \frac{kip}{ft^3}$ Density of reinforced concrete

The cap beam is approximately 60 feet long, and the two barriers weigh 0.9 kip/foot.

$$W := \left(A_d \cdot L + A_{cb} \cdot 60 \cdot ft + 3 \cdot A_c \cdot \frac{H}{2}\right) \cdot \gamma_{conc} + 0.9 \cdot \frac{kip}{ft} \cdot L$$

Total weight of bridge

$W = 4876 \cdot kip$ Weight of bridge

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode	▬			
Multimode				

Design Step 6.3.5 (continued)

Stiffness of bent

$$k_b := 3 \cdot \left(\frac{12 \cdot E_c \cdot I_c}{H^3} \right) \qquad k_b = 11492 \cdot \frac{kip}{ft}$$

Period based on half of bridge weight considered tributary to bent

$$T := 2 \cdot \pi \cdot \sqrt{\frac{W}{2 \cdot g \cdot k_b}} \qquad T = 0.510 \cdot sec$$

The period determined from the Single-Mode Method was 0.172 second. The hand calculation is 378 percent larger than this number. This is clearly a poor estimate of the period. The reason is that the bridge behaves more like a simply supported beam in the transverse direction.

✓ 6.3 Check No. 2 Transverse Direction

Calculate the transverse fundamental period considering the superstructure as a simple beam.

The stiffest load path for transverse load is through the superstructure to the abutments rather than through the columns to the spread footings. Thus, the fundamental transverse period may be estimated using the standard expression for the fundamental period of vibration of a simply supported beam with uniform distribution of mass and stiffness. Because the bent stiffness is much less than the superstructure, the bent may be neglected.

The expression for the period may be found in any structural dynamics text. In this case, Clough and Penzien (1993) was used. The vibration shape is that of the sine function evaluated between 0 and $\pi/2$ radians, as shown in Figure 11.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode	▬			
Multimode				

Design Step
6.3.5
(continued)

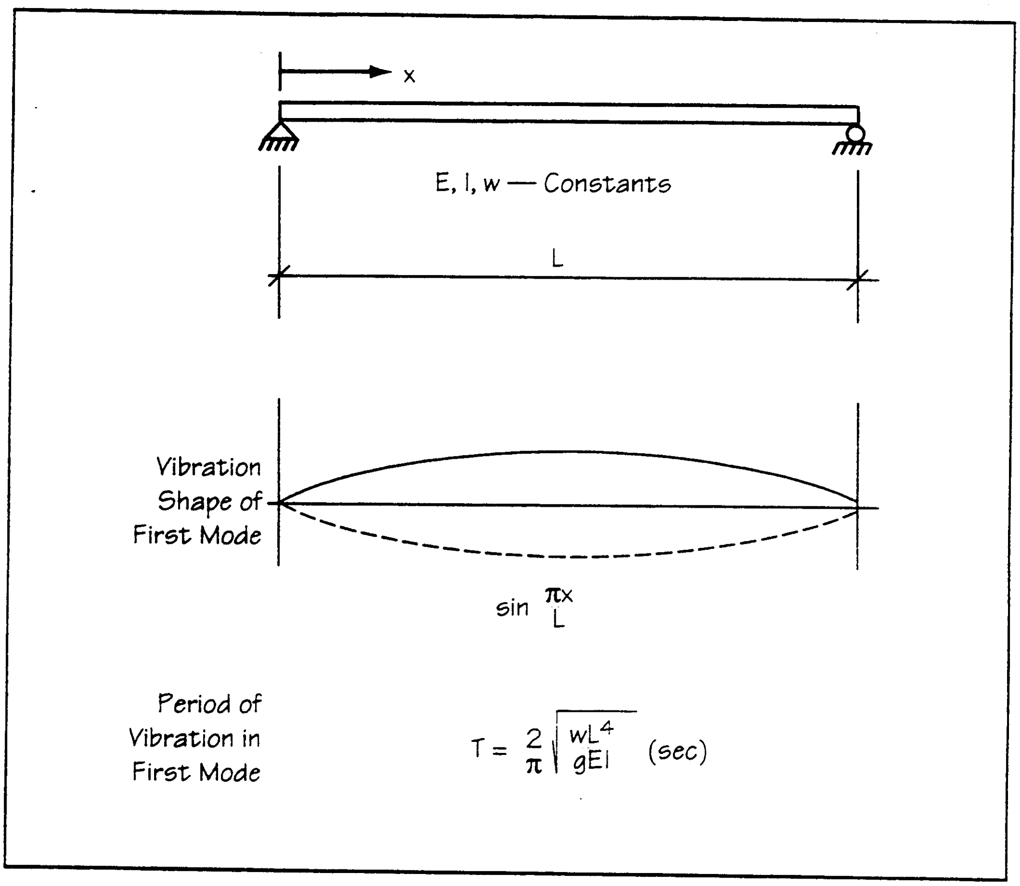


Figure 11 — Vibration of Simply Supported Beam with Uniform Stiffness and Weight

Recall the following values for the bridge.

$W := 4876 \cdot kip$	Weight of bridge, including capbeam and columns
$E_c := 518400 \cdot \dfrac{kip}{ft^2}$	Young's modulus of concrete
$L := 242 \cdot ft$	Length of bridge
$I_{yd} := 51000 \cdot ft^4$	Moment of inertia of superstructure about the vertical axis

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

Design Step
6.3.5
(continued)

The mass of the bridge per unit length is given by

$$m := \frac{W}{g \cdot L} \qquad m = 0.626 \cdot kip \cdot \frac{sec^2}{ft^2}$$

The period then is given by

$$T := \frac{2}{\pi} \cdot \sqrt{\frac{m \cdot L^4}{E_c \cdot I_{yd}}} \qquad T = 0.181 \cdot sec$$

The period determined from the Single-Mode Method was 0.172 second, which is within 6 percent of the value determined by the check. This is a much better check than the first method because a better approximation of the response was assumed. The importance of understanding the dynamic response is emphasized by the results of the checks. If a poor approximation is used in the check, the results of the actual analysis may be questioned. Thus, several quick checks should be performed where possible.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode	▬			
Multimode				

Design Step 6.4

Longitudinal Analysis, Single-Mode Spectral Method
[Division I-A, Article 4.4]

Perform a longitudinal analysis using the basic foundation condition.

Design Step 6.4.1

Static Displacements
[Division I-A, Article 4.4, *(Step 1)*]

The displacements of the bridge due to a uniform longitudinal loading are calculated. These are given in Table 4.

Table 4
Longitudinal Displacements for 100-kip/foot Uniform Load

Displacements (feet)								
Abut 1	1/4 Span	1/2 Span	3/4 Span	Bent 2	1/4 Span	1/2 Span	3/4 Span	Abut 2
2.551	2.550	2.547	2.542	2.535	2.539	2.541	2.543	2.543

Inspection of the displacements indicates that for longitudinal loading they are nearly the same for all points along the bridge. The method shown in Table 2 could be used to determine the equivalent loading or it could be approximated using the average displacement. The average displacement method is described below and the tabular method is used for longitudinal loading with spring supports.

Recall the following values.

$w := 20.1 \cdot \frac{kip}{ft}$ — Weight of bridge per unit length (including capbeam and columns)

$v_s := 2.54 \cdot ft$ — Approximate longitudinal displacement under 100 kip/ft load (from SAP90 run F1LSM4C)

$L := 242 \cdot ft$ — Length of bridge

$p_o := 100 \cdot \dfrac{kip}{ft}$ Uniform load applied in the longitudinal direction

Design Step 6.4.2

Calculate α, β, and γ Factors
[Division I-A, Article 4.4, *(Step 2)*]

Calculate α

Because the displacement does not vary along the length, Equation (4-5) of the Specification gives

$\alpha := v_s \cdot L$ $\qquad \alpha = 614.7 \cdot ft^2$

Calculate β

Because neither the displacement or the weight varies along the length, Equation (4-6) of the Specification gives

$\beta := w \cdot v_s \cdot L$ $\qquad \beta = 12355 \cdot kip \cdot ft$

Calculate γ

Because the displacement and weight are constant along the length, Equation (4-7) of the Specification gives

$\gamma := w \cdot {v_s}^2 \cdot L$ $\qquad \gamma = 31382 \cdot kip \cdot ft^2$

Design Step 6.4.3

Fundamental Period
[Division I-A, Article 4.4, *(Step 3)*]

The period of the bridge in the longitudinal direction is given by

$$T := 2 \cdot \pi \cdot \sqrt{\frac{\gamma}{p_o \cdot g \cdot \alpha}} \qquad T = 0.791 \cdot sec$$

Division I-A
Eqn (4-8)

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

Design Step 6.4.4

Equivalent Longitudinal Load
[Division I-A, Article 4.4, *(Step 4)*]

The elastic seismic response coefficient C_s is given by the following equation. Additionally, recall that A and S are

$A := 0.28$ $\qquad$ $S := 1.2$

$$C_s := \frac{1.2 \cdot sec^{\frac{2}{3}} \cdot A \cdot S}{T^{\frac{2}{3}}} \qquad C_s = 0.471$$

Division I-A
Eqn (3-1)

Note that the constant 1.2 in Equation (3-1) carries units of seconds raised to the 2/3 power. This is the result of A and S being nondimensional and the requirement that Cs also be nondimensional.

Cs need not exceed 2.5*A, which is equal to 0.70. This limit does not control in the longitudinal direction.

Thus, the equivalent lateral load is given by

$$p_e := \frac{\beta \cdot C_s}{\gamma} \cdot w \cdot v_s \qquad p_e = 9.47 \cdot \frac{kip}{ft}$$

Division I-A
Eqn (4-9)

Design Step 6.4.5

Displacements and Member Forces
[Division I-A, Article 4.4, *(Step 5)*]

The response values of the structure, both internal forces and displacements, are given in Table 5. The SAP90 input file for this analysis is F1LSMPE.

At this point, the assumption that the abutments offer no restraint to the longitudinal movement may be assessed.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode		▬		
Multimode				

Design Step 6.4.5 (continued)

The displacements at the abutments are 0.242 foot at either end. This is 2.9 inches, which is easily accommodated with expansion joints. Thus, the assumption made originally is reasonable.

If the displacement had been too large, the abutment soil resistance and provided gap would be combined to produce a secant stiffness, and the analysis would be repeated. FHWA (1987) provides discussion of this procedure.

Table 5
Response for Single-Mode Method, Longitudinal Direction, and Basic Foundation

			Forces and Moments				
			Longitudinal		Transverse		
			Shear (kips)	Moment (kip-ft)	Shear (kips)	Moment (kip-ft)	Axial (kips)
Abutment 1			0	0	0	0	105
Bent 2	Center	Top	771	9978	0	0	35.5
		Bottom	771	9566	0	0	35.5
	Outboard	Top	760	9790	0	0	35.5
		Bottom	760	9481	0	0	35.5
Abutment 3			0	0	0	0	211

Displacements (ft)								
Abut 1	1/4 pt	1/2 pt	3/4 pt	Bent 2	1/4 pt	1/2 pt	3/4 pt	Abut 3
0.2417	0.2416	0.2413	0.2408	0.2401	0.2404	0.2407	0.2408	0.2409

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode		▬		
Multimode				

Design Step 6.4.5 (continued)

✓ 6.4 Check No. 1 Longitudinal Direction

Calculate the longitudinal stiffness and period of the bridge if the superstructure is considered rigid.

Recall the following values.

$I_c := 12.57 \cdot ft^4$ Moment of inertia of one column

$H := 27.33\, ft$ Height of columns (clear height plus extension into footing)

$E_c := 518400 \cdot \frac{kip}{ft^2}$ E of concrete

$W := 4876 \cdot kip$ Weight of bridge

Because the superstructure is considered rigid, the lateral stiffness of the bent will be that of three columns fixed against rotation at both ends, but free to translate at the top.

$$K := \left(\frac{12 \cdot E_c \cdot I_c}{H^3}\right) \cdot 3 \qquad K = 11492 \cdot \frac{kip}{ft}$$

The period is then given by the standard expression.

$$T := 2 \cdot \pi \cdot \sqrt{\frac{W}{g \cdot K}} \qquad T = 0.722 \cdot sec$$

Division I-A
Eqn (4-3)

This value is within 9 percent of the previously calculated longitudinal fundamental period. The deck is not rigid. Thus, the assumption that it is rigid artificially increases the structure stiffness, thereby reducing the period.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

Design Step 6.4.5 (continued)

This check was based upon the assumption that the superstructure was rigid. An upper bound to the longitudinal period could be obtained by considering the superstructure to have zero flexural stiffness. The columns would then, effectively, act as cantilevers fixed at their base. The reader can perform this calculation simply by substituting "3" for the constant "12" in the equation for stiffness.

✓ 6.4 Check No. 2 Longitudinal Direction

This check accounts for the stiffness of the superstructure by determining the longitudinal stiffness using the moment distribution method.

Calculate the stiffness of the bridge subject to a concentrated longitudinal load applied at the top of the columns. Then use this to calculate the fundamental period of vibration in the longitudinal direction.

Recall the following.

$I_d := 575 \cdot ft^4$ Moment of inertia of deck and superstructure

$I_b := 3 \cdot 12.57 \cdot ft^4$ Moment of inertia of three columns

$L_l := 142 \cdot ft$ Length of left span

$L_r := 100 \cdot ft$ Length of right span

$H := 27.34 \cdot ft$ Height of columns (clear height plus extension into footing)

$E_c := 518400 \cdot \frac{kip}{ft^2}$ Young's modulus of concrete

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode		▬		
Multimode				

Design Step
6.4.5
(continued)

$$k_{ld} := \frac{3 \cdot I_d}{4 \cdot L_l} \qquad k_{ld} = 3.04 \cdot ft^3$$

Relative stiffness of left span (use "modified" stiffness to simplify the distribution of moments)

$$k_{rd} := \frac{3 \cdot I_d}{4 \cdot L_r} \qquad k_{rd} = 4.31 \cdot ft^3$$

Relative stiffness of right span

$$k_c := \frac{I_b}{H} \qquad k_c = 1.38 \cdot ft^3$$

Relative stiffness of bent columns

$$DF_l := \frac{k_{ld}}{k_{ld} + k_{rd} + k_c} \qquad DF_l = 0.348$$

Distribution factor for left span

$$DF_r := \frac{k_{rd}}{k_{ld} + k_{rd} + k_c} \qquad DF_r = 0.494$$

Distribution factor for right span

$$DF_c := \frac{k_c}{k_{ld} + k_{rd} + k_c} \qquad DF_c = 0.158$$

Distribution factor for bent

For longitudinal loading of the bridge, the longitudinal sway case is the only moment distribution that must be carried out. For this sway case, restrain the column to superstructure joint from rotating and apply a sway displacement. The actual displacement is arbitrary. Thus, for convenience, let the displacement correspond to that which would produce a 1000 kip-ft moment in the column. Then distribute the moments. See Figure 12.

The shear carried by the columns is

$$V := \frac{(842 + 921) \cdot kip \cdot ft}{H} \qquad V = 64.5 \cdot kip$$

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

Design Step
6.4.5
(continued)

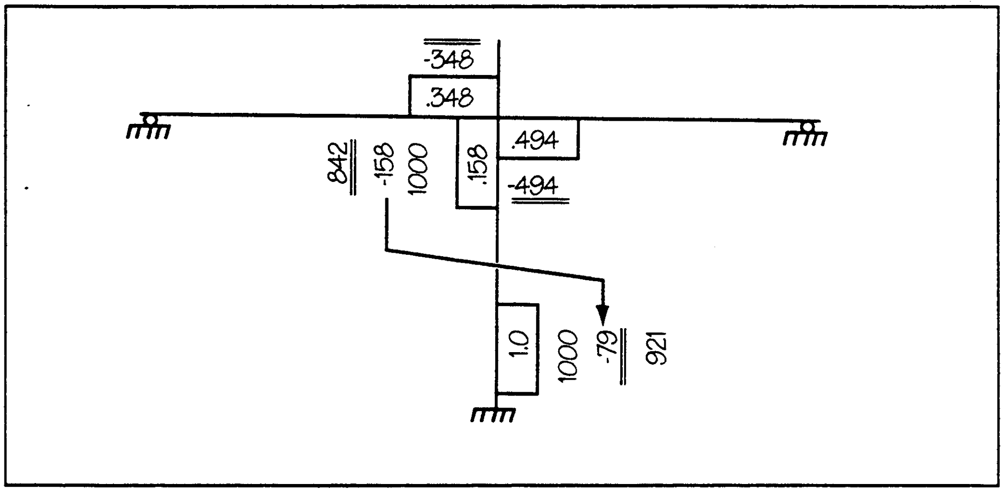


Figure 12 — Distribution of Sway Moments for Longitudinal Loading

Using the conjugate beam method (or moment-area method), the deflection at the top of the columns may be calculated. Because the bases of the columns are considered fixed, the deflection of the columns is simply the moment of the curvature diagram taken about the top of the columns. See Figure 13.

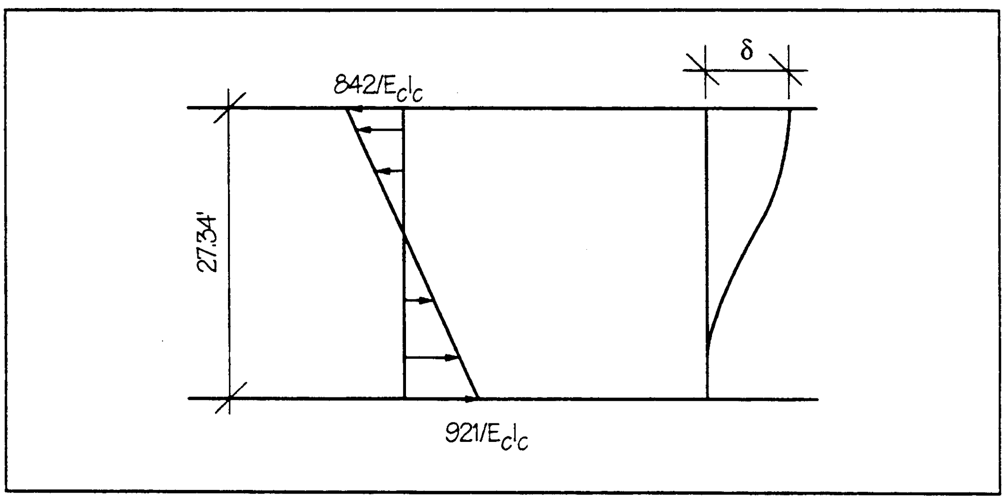


Figure 13 — Curvature Diagram and Deflected Shape of Columns

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

Design Step 6.4.5 (continued)

$M_t := 842 \cdot kip \cdot ft$ $\quad M_b := 921 \cdot kip \cdot ft$ $\quad$ Moments at top and bottom of columns, respectively

The deflection at the top is then

$$\delta := \frac{1}{E_c \cdot I_b} \cdot \left[\left(M_t + M_b \right) \cdot H^2 \cdot \frac{1}{2} \cdot \frac{2}{3} - M_t \cdot H^2 \cdot \frac{1}{2} \right]$$

$$\delta = 0.00637 \cdot ft$$

The stiffness of the bridge in the longitudinal direction is then

$$K := \frac{V}{\delta} \qquad K = 10119 \cdot \frac{kip}{ft}$$

Then the period of vibration can be calculated if the weight of the bridge is known.

$W := 4876 \cdot kip$ $\quad$ Weight of bridge

$$T := 2 \cdot \pi \cdot \sqrt{\frac{W}{g \cdot K}} \qquad T = 0.769 \cdot sec$$

Division I-A
Eqn (4-3)

The Single-Mode Method longitudinal fundamental period is 0.791 second, which is about 3 percent higher than that calculated here. The reason is that the actual column has a rigid end zone of 2.83 feet where it frames into the cap beam. It has been neglected here to simplify calculations.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

DESIGN STEP 7 — DETERMINE DESIGN FORCES FOR BASIC SUPPORT CONDITION

INTRODUCTION

As previously discussed, for the "basic support condition" under seismic loading, the bridge behaves much differently in the longitudinal direction than it does in the transverse direction. Under longitudinal seismic loads, the bridge is free to slide at the abutments. All the longitudinal seismic load is therefore taken by the intermediate bent columns.

Under transverse seismic loads, the bridge is wide relative to its length, and the superstructure acts as a horizontal diaphragm spanning between the abutments. Because the bent columns are flexible in the transverse direction relative to this diaphragm, the columns take a small percentage of the transverse seismic load.

Therefore, for this bridge, inelastic response is expected in the longitudinal direction, but not in the transverse direction. The reason is that the plastic hinging moment capacity of the columns, resulting from the column reinforcement design using the longitudinal modified design moments, are larger than the elastic moments in the transverse direction. As a result, it is permissible according to the Specification to omit the consideration of plastic hinging in the transverse direction.

However, for the purposes of demonstrating the approach, plastic hinging will be considered in the transverse direction. The results will also be used throughout the remainder of the example. This is conservative and therefore leads to slightly higher design shear forces for the columns and slightly higher plastic hinging forces transferred from the column to the cap beam and footing. The conservatism, in this case, is not unduly large.

It should be noted that conservatism with respect to shear design is not a bad practice. The penalty for having columns with shear strengths less than that corresponding to the maximum expected forces may well be brittle failure that leads to collapse. Such failures have occurred in past earthquakes.

Design Step 7.1

Determine Nonseismic Forces

Design Step 7.1.1

Determine Dead Load Forces

The dead load forces are summarized in Table 6 below. Because this is a continuous post-tensioned bridge, the secondary effects due to post-tensioning should also be included, but are left out of this example.

Table 6
Dead Load Forces with Basic Supports

Location	Load Case	Longitudinal Force V_L (kips)	Longitudinal Force M_L (k-ft)	Transverse Force V_T (kips)	Transverse Force M_T (k-ft)	Vertical P (kips)
Abut 1	Dead Load	0	0	0	0	1075
Bent 2, Top Exterior Column	Dead Load	0	366	0	0	1098
Abut 3	Dead Load	0	0	0	0	577

Design Step 7.2

Determine Seismic Forces

Design Step 7.2.1

Summary of Elastic Seismic Forces

As was discussed previously, the Single-Mode Spectral Method results are used to determine the modified design forces.

A summary of the full elastic seismic forces for an earthquake along each of the principal axis (both longitudinal and transverse) is shown in Table 7, which is a condensed version of results from Tables 3 and 5.

The column forces listed are those for the exterior columns. These forces will be used to determine the design forces so that the combination of nonzero longitudinal and transverse earthquake forces can be shown. The forces on the center column are slightly larger, although the difference does not affect the number of bars selected.

Design Step
7.2.1
(continued)

Table 7
Full Elastic Seismic Forces with Basic Support

Location	Load Case	Longitudinal Force V_L (kips)	Longitudinal Force M_L (k-ft)	Transverse Force V_T (kips)	Transverse Force M_T (k-ft)	Vertical P (+/- kips)
Abutment 1	Longitudinal Analysis	0	0	0	0	105
	Transverse Analysis	0	0	1277	583	0
Bent 2, Top	Longitudinal Analysis	760	9790	0	0	36
Exterior Column	Transverse Analysis	8	110	77	1053	43
Abutment 3	Longitudinal Analysis	0	0	0	0	211
	Transverse Analysis	0	0	1241	828	0

Design Step
7.2.2

Combination of Orthogonal Seismic Forces
[Division I-A, Article 3.9]

Before the seismic forces are combined with the dead load to create the modified design forces, the seismic forces along the two principal axes must be combined in LC1 and LC2 (without dead load). See Table 8 for a summary of these forces.

The definition of LC1 and LC2 is as follows.

LC1 = 100 percent of the Longitudinal Analysis Results + 30 percent of the Transverse Analysis Results

LC2 = 30 percent of the Longitudinal Analysis Results + 100 percent of the Transverse Analysis Results

Note that all the forces in LC1 and LC2 are the full elastic seismic forces.

These forces are combinations using the full elastic seismic results and have not been modified by the R Factor yet. At this stage, the designer

Design Step
7.2.2
(continued)

could elect to check for these forces combined with dead load, if other load cases such as stream flow control the size of the substructure.

For example, in Bent No. 2, the longitudinal column moment for LC1 is derived as follows.

$M = (1.0 * M_L) + (0.3 * M_T)$
$M = (1.0 * 9790) + (0.3 * 110) = 9823$ k-ft

All other forces in the table are calculated similarly.

Table 8
Orthogonal Seismic Force Combinations
LC1 and LC2 with Basic Supports

Location	Load Case	Longitudinal Force V_L (kips)	Longitudinal Force M_L (k-ft)	Transverse Force V_T (kips)	Transverse Force M_T (k-ft)	Vertical P (+/- kips)
Abutment 1	Load Case 1	0	0	383	175	105
	Load Case 2	0	0	1277	583	32
Bent 2, Top Exterior Column	Load Case 1	762	9823	23	316	48
	Load Case 2	236	3047	77	1053	53
Abutment 3	Load Case 1	0	0	372	248	211
	Load Case 2	0	0	1241	828	63

Design Step 7.3

Determine Modified Design Forces
[Division I-A, Article 7.2.1(A)]

For design of members and foundations, the modified design forces replace the Group VII load combination found in Table 3.22.1A of Division I. These modified forces, along with the forces associated with plastic hinging in the columns, are used in the seismic design of the various components of the bridge.

Design Step 7.3 (continued)

The modified design forces use the R Factor in modifying the elastic seismic forces. Looking at the entire bridge as a system, the intent of the Specification is to force the plastic hinging to occur in the columns. Therefore, inelastic action is prevented from occurring in the cap beam or foundation.

There is a distinction between modified design forces for a) structural members and connections, and b) foundations.

Design Step 7.3.1

Modified Design Forces for Structural Members and Connections

The Specification makes a distinction between the modified forces for members and connections verses the modified forces for foundations calculated in Design Step 7.3.2. Use Equation (7-1) in Division I-A to calculate the maximum forces in each member.

Group Load = 1.0 (D + B + SF + E + EQM) — Division I-A Eqn (7-1)

For this example, forces B, SF, and E are assumed zero, only D and EQM forces are combined. The equation reduces to

Group Load = 1.0 (D + EQM)

Where EQM = (LC1 or LC2 forces) divided by R

a) Response Modification Reduction Factor, R
[Division I-A, Article 3.7, Table 3]

The R Factor is used to modify EQM, and applies to specific forces for specific members. The decision of which R value to apply to each member is a critical one. R is never directly applied to the axial load nor to the shear force in a column. Although when the plastic hinging forces in the column are calculated in Design Step 7.4, it will be shown how both axial and shear forces are affected indirectly.

Design Step
7.3.1
(continued)

In this example, R reduces the seismic column moments, but increases the seismic lateral shear force on the connection of the superstructure to the abutment. Recall that R was determined in Design Step 2.6, and a summary of the R values used to modify EQM is presented below.

R = 5.0	For moments in multiple column bents
R = 0.8	For shear and axial connection force of superstructure to abutment
R = 1.0	For connection of column to superstructure or foundation

b) Calculate the Modified Design Forces with EQM

Once the R values have been established, the value of EQM can be calculated.

Table 9 summarizes the modified design forces. The R value used for each force is given in the table.

For example, in Bent No. 2, the longitudinal column moment using LC1 is derived as follows.

T8

M = (D + EQ/R)
M = (366 + 9823/5) = 2331 k-ft

T6

All other forces in the table are calculated similarly.

Design Step
7.3.1
(continued)

Table 9
Modified Design Forces for Structural Members and Connections with Basic Supports

Location	Load Case	Longitudinal Force V_L (kips)	Longitudinal Force M_L (k-ft)	Transverse Force V_T (kips)	Transverse Force M_T (k-ft)	Vertical Force P_{max} (kips)	Vertical Force P_{min} (kips)
Abutment 1	R Factor Used in EQM	1	-	0.8	Not Used	0.8	0.8
	Load Case 1	0	-	479	-	1206	944
	Load Case 2	0	-	1596	-	1114	1036
Bent 2, Top Exterior Column	R Factor Used in EQM	1	5.0	1	5.0	1	1
	Load Case 1	762	2331	23	63	1147	1049
	Load Case 2	236	975	77	211	1152	1045
Abutment 3	R Factor Used in EQM	1	-	0.8	Not Used	0.8	0.8
	Load Case 1	0	-	465	-	841	313
	Load Case 2	0	-	1551	-	656	498

✓ from T8

Design Step
7.3.2

Modified Design Forces for Foundations
[Division I-A, Article 7.2.1(B)]

Use Equation (7-2) in Division I-A to calculate the maximum forces in the bent column foundations.

Group Load = 1.0 (D + B + SF + E + EQF)

Division I-A
Eqn (7-2)

For this example, forces B, SF, and E are assumed zero, only D and EQF forces are combined. The equation reduces to

Group Load = 1.0 (D + EQF)

Where EQF = (LC1 or LC2 forces) divided by R

Design Step
7.3.2
(continued)

a) Recall the Response Modification Reduction Factor, R
[Division I-A, Article 7.2.1(B)]

R = 1 for calculating the modified design forces in the foundation.

b) Calculate the Modified Design Forces with EQF

Table 10 summarizes the values of EQF modified design forces. The forces at the bases of the columns have been calculated in the same manner as those previously calculated for the tops of the columns.

For example, in Bent No. 2, the longitudinal column moment using LC1 is derived as follows.

Table 5

$M = (D + EQ/R)$

$M = (366 + [9481 \times 1.0 + 95 \times 0.30]/1.0) = 9876$ k-ft

Table 6

All other forces in the table are calculated similarly.

Table 10
Modified Design Forces for Foundations with Basic Supports

Location	Load Case	Longitudinal Force V_L (kips)	Longitudinal Force M_L (k-ft)	Transverse Force V_T (kips)	Transverse Force M_T (k-ft)	Vertical Force P_{max} (kips)	Vertical Force P_{min} (kips)
Abutment 1	R Factor Used in EQF	1	-	1	Not Used	1	1
	Load Case 1	0	-	383	-	1180	970
	Load Case 2	0	-	1277	-	1107	1044
Bent 2 Exterior Column	R Factor Used in EQF	1	1	1	1	1	1
	Load Case 1	762	9876	23	271	1147	1049
	Load Case 2	236	3305	77	902	1152	1045
Abutment 3	R Factor Used in EQF	1	-	1	Not Used	1	1
	Load Case 1	0	-	372	-	788	366
	Load Case 2	0	-	1241	-	640	514

Design Step 7.4

Plastic Hinging Forces

Before the forces due to plastic hinging can be calculated, the preliminary longitudinal column reinforcement must be determined.

Design Step 7.4.1

Preliminary Column, Piers, or Bent Design
[Division I-A, Article 7.6.2 (A and B)]

The previous design step derived the Seismic Group Loads to be used in the seismic design of the bridge. This design step focuses on the preliminary design of the Bent No. 2 columns. Both the longitudinal and transverse reinforcement in the column will be designed for the seismic load case.

Depending on the Seismic Performance Category, the column may be controlled by dead load combined with seismic loads, or other loads such as live loads or stream flow loads. This example deals only with the seismic load combinations.

Division I-A, Article 7.6.2(b) mentions moment magnification in the columns. Currently, the magnitude and method of computing magnified moments for seismic loadings are under review by AASHTO. Some refer to the Division I, Article 8.16.5; others feel that moment magnification during seismic loadings should not be included for concrete columns.

For concrete columns, a good approximation to account for a magnified moment is to multiply the maximum axial load in the column by the full elastic deflection at the column top. This additional moment is then added to the primary seismic moment before designing the reinforcement. Because the columns in this example are stiff, and the effect small, magnification has been ignored.

Basic substructure data for column design

$f_c := 4000 \cdot psi$ Concrete strength

Circular column section properties

$b_w := 4.0 \cdot ft$

Design Step 7.4.1 (continued)

$$A_g := \frac{\pi \cdot b_w^{\,2}}{4} \qquad A_g = 12.57 \cdot ft^2$$

a) Summary of Modified Design Forces for the Preliminary Column Design
[Division I-A, Article 7.2.3]

Below is a summary of the controlling Modified Design Forces for the preliminary column design, taken from Table 9. By inspection, LC1 controls.

$Pmax_u := 1146 \cdot kip$ — Maximum axial load

$Pmin_u := 1050 \cdot kip$ — Minimum axial load

$M_L := 2331 \cdot kip \cdot ft$ — Longitudinal moment

$M_T := 63 \cdot kip \cdot ft$ — Transverse moment

For a circular column, the modified biaxial bending moment can be converted to a moment about a single axis by

$$M_u := \sqrt{M_L^{\,2} + M_T^{\,2}} \qquad M_u = 2332 \cdot kip \cdot ft$$

The above forces will be used in the design of the longitudinal reinforcement in the column.

b) Calculate the ϕ Factor for Both the Maximum and Minimum Axial Load for Use with M_u

Division I-A, 7.6.2(B) specifies a ϕ factor that varies from 0.9 to 0.5 depending on the axial load. Because the lower bound of ϕ equals 0.5 when the axial stress exceeds $0.2*f'_c$, column charts that use a lower bound of ϕ equal to 0.7 are not applicable.

For this example problem, the nominal capacity of the column is plotted as P_n, M_n without a ϕ factor. Therefore, the factored loads must be divided by ϕ before being plotted on the capacity chart. See Figure 14 for the general curves for design capacity, nominal capacity, and plastic capacity.

Design Step
7.4.1
(continued)

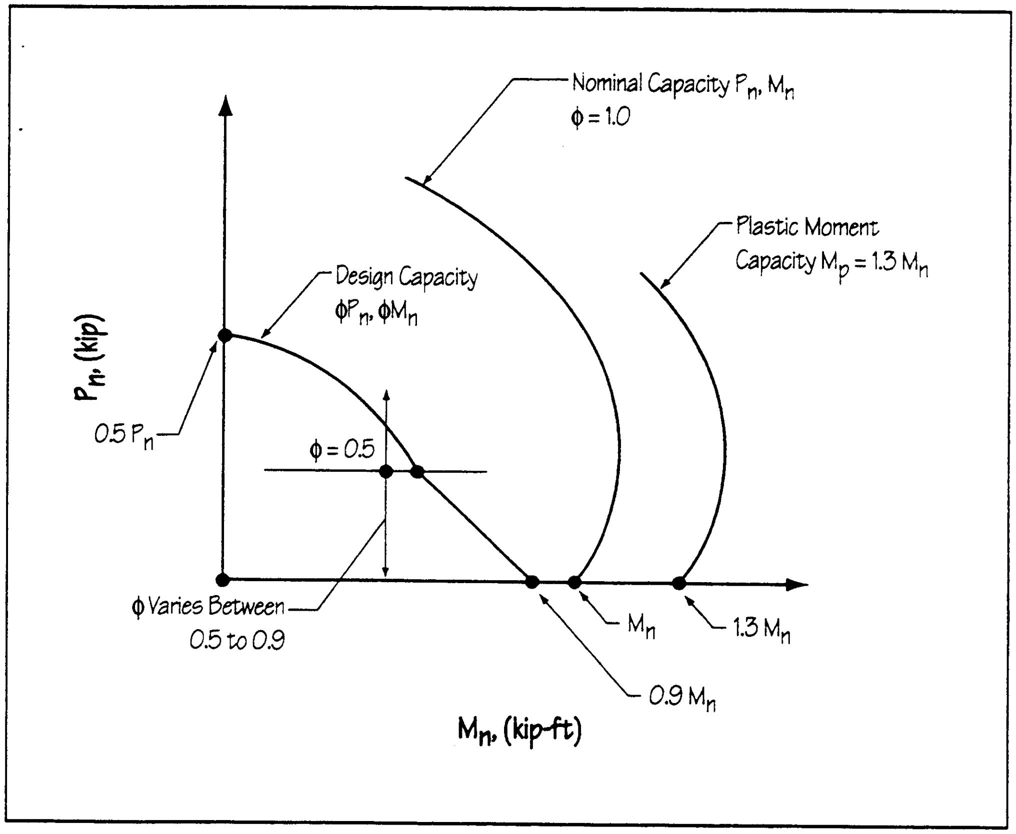


Figure 14 — Column Interaction Curves, General

Compute the maximum and minimum column axial stress, σ_{max} and σ_{min}.

$Pmax_u = 1146 \cdot kip$ $\qquad$ $Pmin_u = 1050 \cdot kip$

$A_g = 12.57 \cdot ft^2$ $\qquad$ Gross area of the column cross section

$$\sigma_{max} := \frac{Pmax_u}{A_g} \qquad \sigma_{max} = 633 \cdot psi$$

$$\sigma_{min} := \frac{Pmin_u}{A_g} \qquad \sigma_{min} = 580 \cdot psi$$

Design Step 7.4.1 (continued)

Per Division I-A, Article 7.6.2(B), $\phi = 0.5$ when the maximum axial stress exceeds $0.2*f'_c$, and varies linearly from 0.5 to 0.9 for stresses less than $0.2*f'_c$. See Figure 15.

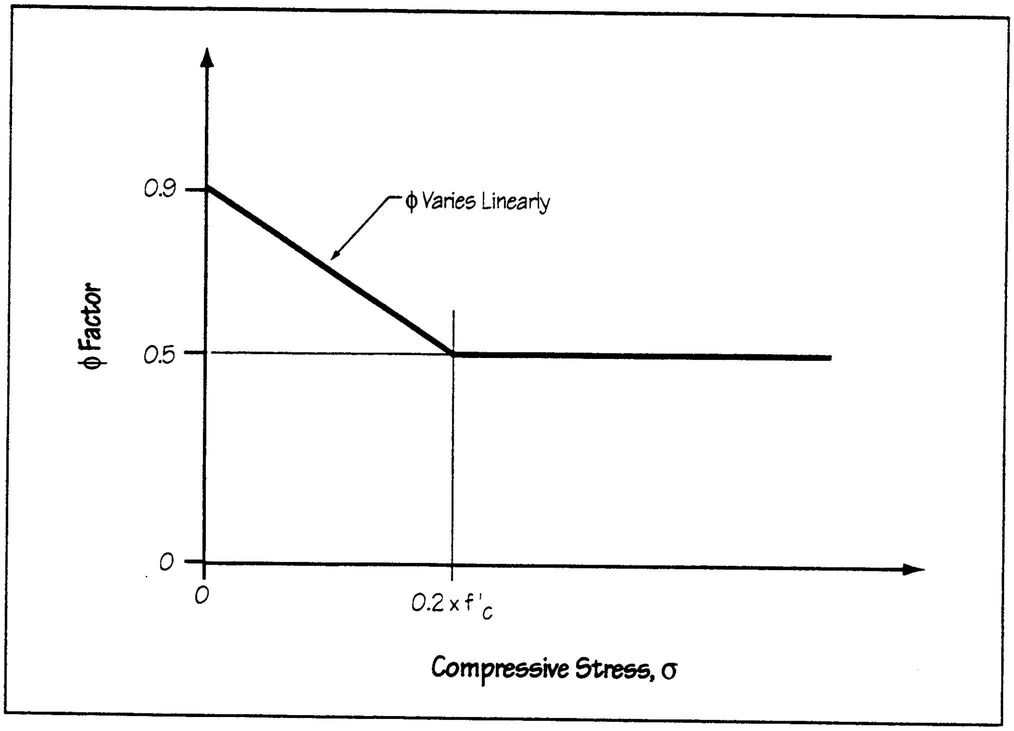


Figure 15 — ϕ Factor versus Compressive Stress

$f_c = 4000$ psi $\qquad 0.2\ f_c = 800$ psi

Because the column axial stresses σ_{max} and σ_{min} are less than $0.2*f'_c$, both ϕ_{max} and ϕ_{min} are between 0.9 and 0.5.

ϕ_{max} is associated with the maximum axial load

$$\phi_{max} := 0.9 - \frac{\sigma_{max}}{0.2 \cdot f_c} \cdot (0.9 - 0.5) \qquad \phi_{max} = 0.58$$

Design Step
7.4.1
(continued)

ϕ_{min} is associated with the minimum axial load.

$$\phi_{min} := 0.9 - \frac{\sigma_{min}}{0.2 \cdot f_c} \cdot (0.9 - 0.5) \qquad \phi_{min} = 0.61$$

c) Calculate the P_u / ϕ and M_u / ϕ Forces and Plot on the P_n, M_n Column Capacity Curve

Controlling forces associated with maximum axial load

$$Pmax_u = 1146 \cdot kip \qquad M_u = 2332 \cdot kip \cdot ft$$

$$\frac{Pmax_u}{\phi_{max}} = 1965 \cdot kip \qquad \frac{M_u}{\phi_{max}} = 3997 \cdot kip \cdot ft$$

Controlling forces associated with minimum axial load

$$Pmin_u = 1050 \cdot kip$$

$$\frac{Pmin_u}{\phi_{min}} = 1722 \cdot kip \qquad \frac{M_u}{\phi_{min}} = 3823 \cdot kip \cdot ft$$

Try a 48-inch-diameter column with (22) #11 bars (1.90 percent reinforcement). The above values are plotted on Figure 16.

The column capacity curve in Figure 16 graphs the nominal strength of P_n verses M_n for a 48-inch-diameter column with a reinforcement ratio of 1.90 percent. Note that the ϕ factor has not been accounted for in this curve.

Now plot the forces for the two load cases calculated above on the curve.

Design Step
7.4.1
(continued)

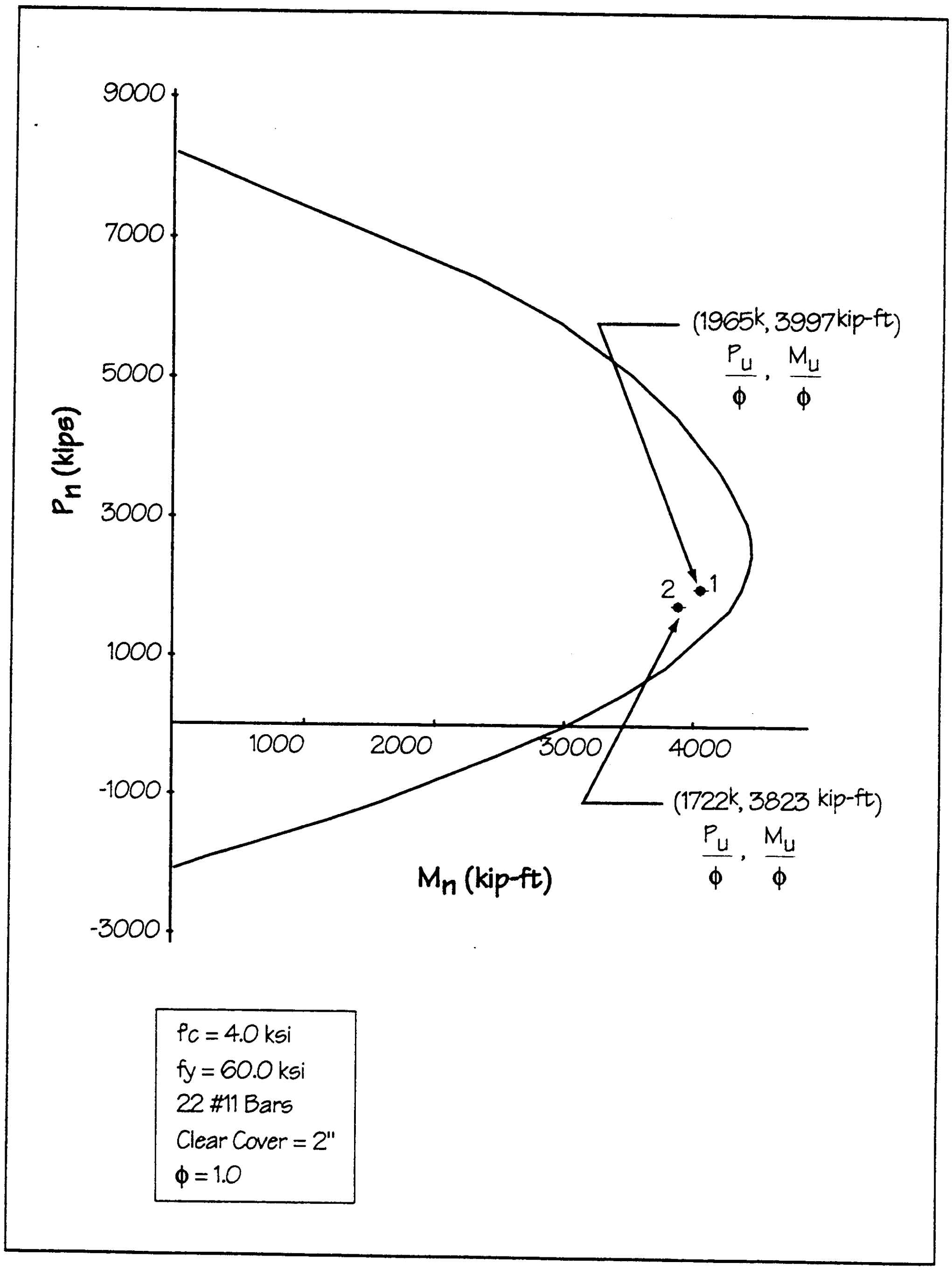


**Figure 16 — Column Interaction Diagram
Plot Factored Loads**

Design Step 7.4.1 (continued)

d) Select the Preliminary Longitudinal Reinforcement

Because the forces for both load cases plot inside the capacity curve for a column with 1.9 percent steel, this reinforcement is sufficient. Also, the 1.9 percent reinforcement provided is between 1 and 6 percent allowed per Division I-A, Article 7.6.2(A).

Therefore, the 48-inch-diameter column with (22) #11 bars is selected.

Design Step 7.4.2

Forces Resulting from Plastic Hinging
[Division I-A, Article 7.2.2(B)]

Before proceeding with this design step, refer again to the introduction of Design Step 7.

After the column cross-sectional dimensions and longitudinal reinforcement have been determined, the forces resulting from plastic hinging in the columns must be computed before the spiral transverse reinforcement can be established. The plastic hinging forces at the top and bottom of the column are also necessary to establish the connection forces transferred to the cap beam and foundation. See Figure 17.

Plastic hinging forces are calculated for bridges in Seismic Categories C and D. The importance of using these forces versus having to use the full elastic forces in design cannot be overemphasized. When the columns are allowed to form a plastic hinge at the top and bottom of the column during a design level earthquake, the hinge acts as a "fuse" to limit the forces transferred to some of the bridge components. This fuse limits the forces transferred into both the footing at the bottom of the column and the cap beam at the top of the column. It also limits the shear (and sometimes the axial force) that the column has to be designed for. The Specification does allow the above components to be designed without calculating the plastic hinging forces. Typically, plastic hinging forces are less in Seismic Categories C and D.

Article 7.2.2(A) of Division I-A covers plastic hinging perpendicular to the plane of the bent. The shear associated with plastic hinging is the sum of the plastic moments at the top and bottom of the column, divided by the column height. In this example

V_p = 2(5200 kip-ft)/25.33 ft = 411 kips

Design Step
7.4.2
(continued)

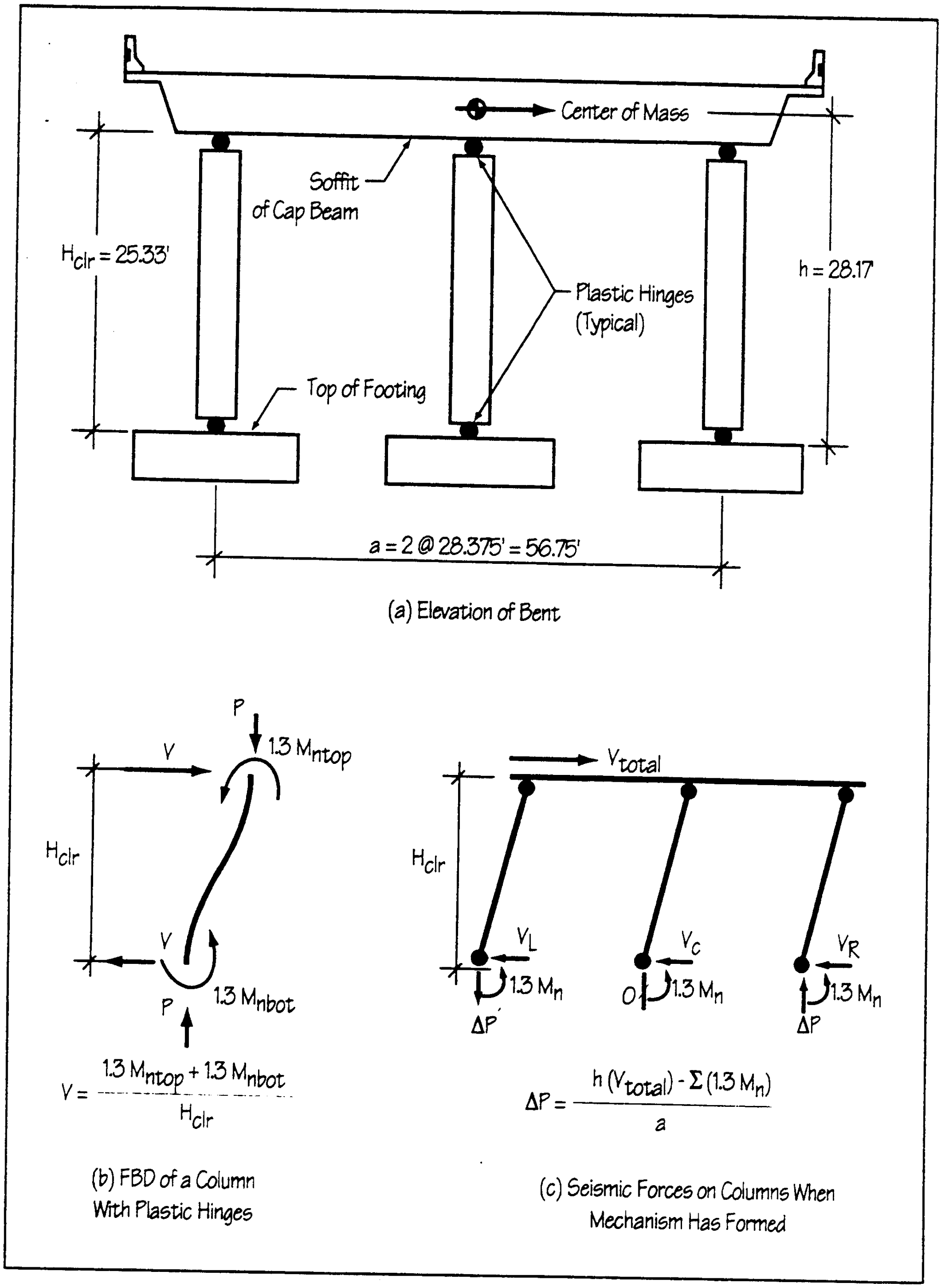


Figure 17 — Plastic Hinging Mechanism in Bent

Design Step
7.4.2
(continued)

Article 7.2.2(B) of Division I-A gives four steps to calculate the plastic hinging forces for bents with two or more columns in the plane of the bent. In this design example, these steps are broken down into 11 intermediate steps (1a through 4f) in order to add further clarification. Table 11 is a tool to tabulate the results of each step. Each step has a brief narrative and an example force.

Given: The preliminary design of the column established the longitudinal reinforcement (22 #11 bars in a 48-inch-diameter column). The column capacity curve of P_n verses M_n will again be used in this section. The axial dead load on each column is 1098 kips. - Table 6

Step 1a — For an axial load corresponding to the dead load, determine the moment capacity, M_n, of each column in the bent using Figure 18.

M_n = 4000 k-ft

Step 1b — Calculate the column overstrength plastic moment capacity 1.3*M_n for each column.

1.3*M_n = 5200 k-ft

Step 2 — In this example, the columns are assumed to be fixed both top and bottom. Therefore, use Equation (1) in Table 11 to calculate the corresponding column shear force, V, in each column.

V = 411 kips

Step 3a — Compute the overturning axial force, ΔP, in the exterior columns using Equation (2) in Table 11. Note that this is a +/- load because the earthquake is bidirectional.

ΔP = 337 kips

Step 3b — Calculate the adjusted axial force in the exterior columns using Equation (3) in Table 11, which assumes a simple rigid frame. Note that for this symmetrical three-column bent, the center column load does not change.

Example: For the right exterior column, the adjusted $Pmax_p$ = 1435 kips

Table 11
Plastic Hinging Forces in Columns

Assumption:

h =	28.17 ft	$V = 2 * 1.3M_n / H$	Eq. (1)
H =	25.33 ft	$\Delta P = [\Sigma(V)*h - \Sigma(1.3M_n)] / a$	Eq. (2)
a =	56.75 ft	Adjusted $P = P +/- \Delta P$	Eq. (3)

Step	Column Moments (kip-ft)					Column Shears V (kips)				Δ P	Column Axial Forces P (kips)		
	M Level	Left	Center	Right	Total	Left	Center	Right	Total	(kips)	Left	Center	Right
Given											1098	1098	1098
1a	M_n	4000	4000	4000									
1b	$1.3*M_n$	5200	5200	5200	15600								
2						411	411	411	1232				
3a										337			
3b											761	1098	1435
4a	M_n	3750	4000	4150									
4b	$1.3*M_n$	4875	5200	5395	15470								
4c						385	411	426	1221				
4d										334			
4e											764	1098	1432

4f

Difference in Column Plastic Shear Capacities (%) = $\frac{1232 - 1221}{1232} * 100 = 0.8$ %
(from Steps 2 and 4c)

Since % difference in total shear between Steps 2 and 4b is less than 10%, stop.

The forces in the individual columns in the plane of a bent corresponding to "column hinging" are

Forces Associated with	a) Axial Forces	$P\ min_p$ =	764 kips
<u>Minimum Axial Load</u>	b) Moment	$Mmin_p = 1.3*Mmin_n$ =	4875 kip-ft
	c) Shear Force	$V\ min_p$ =	385 kips
Forces Associated with	a) Axial Forces	$P\ max_p$ =	1432 kips
<u>Maximum Axial Load</u>	b) Moment	$Mmax_p = 1.3*Mmax_n$ =	5395 kip-ft
	c) Shear Force	$V\ max_p$ =	426 kips

Design Step
7.4.2
(continued)

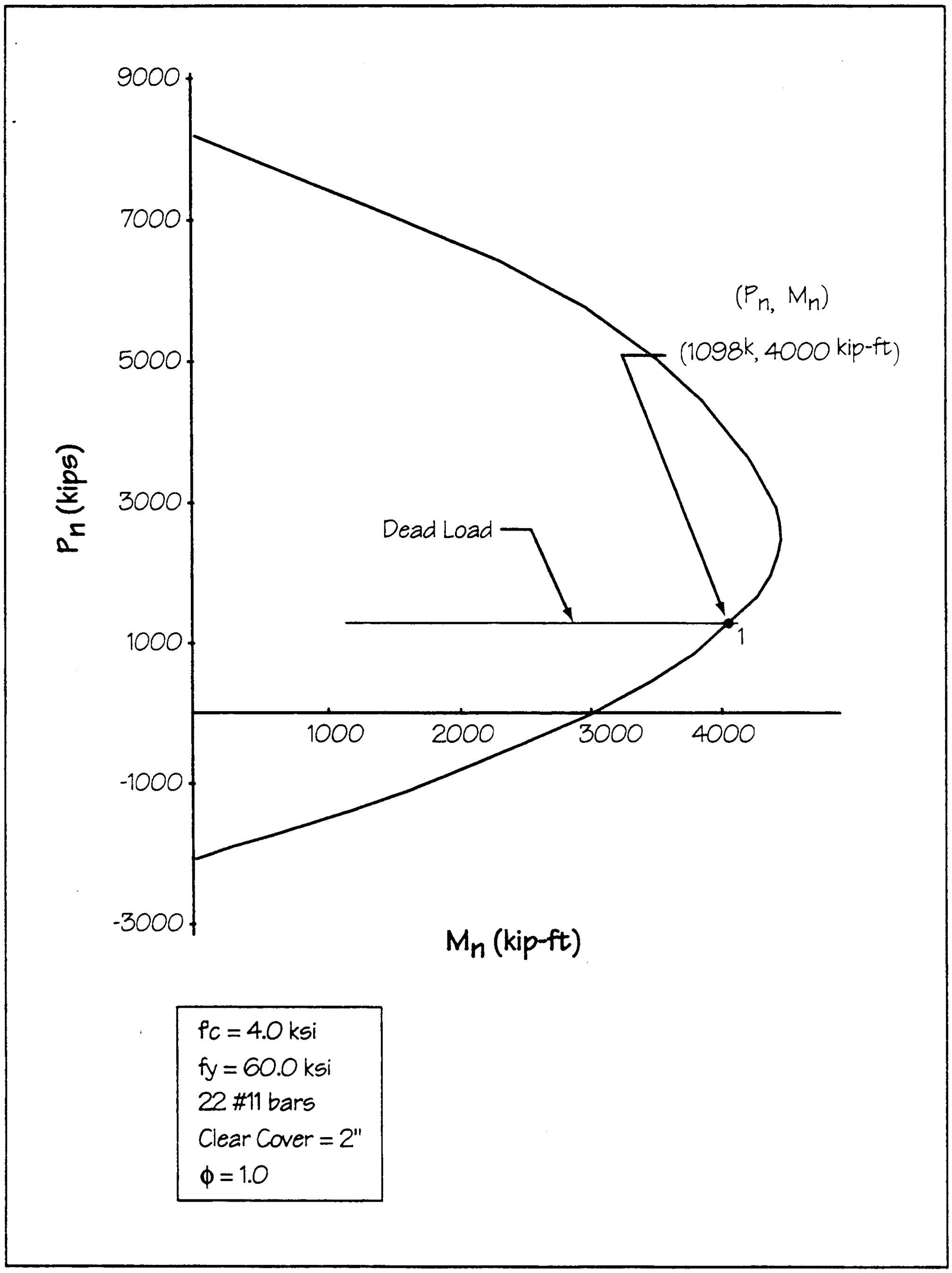


Figure 18 — Column Interaction Diagram
***Step 1a* Nominal Moment Capacity**

Design Step 7.4.2 (continued)

Step 4a — Determine the revised moment capacity, M_n, for each column using the new axial loads in *Step 3b*. See Figure 19 for the plot.

Example: For the right exterior column, $Mmax_n$ = 4150 k-ft

Step 4b — Calculate the revised column overstrength plastic moment capacity, 1.3*M_n for each column.

Example: For the right exterior column, $Mmax_n$ = 1.3*$Mmax_n$ = 5395 k-ft

Step 4c — Find the new column shears, V, using the revised values of 1.3*M_n in *Step 4b*. Note that these are the final shear forces associated with the overstrength plastic moment capacity of the column.

Example: For the right exterior column, $Vmax_p$ = 426 kips

Step 4d — Calculate the revised overturning forces in the exterior columns using the revised column shears in *Step 4c*.

Example: For the right exterior column, ΔP = 334 kips

Step 4e — Calculate the revised axial forces in the exterior columns using the revised overturning forces in *Step 4d*.

Example: For the right exterior column, the adjusted $Pmax_p$ = 1432 kips

Step 4f — Since the difference between the total shear forces in *Steps 2* and *4c* are within 10 percent of each other, it is not necessary to return to *Step 4a* for another iteration. Note that these are the final axial forces associated with the overstrength plastic moment capacity of the column.

A summary of plastic hinging forces is given below.

Forces associated with minimum axial load	$Pmin_p$ = 764 kip $Mmin_p$ = 4875 kip-ft $Vmin_p$ = 385 kip
Forces associated with maximum axial load	$Pmax_p$ = 1432 kip $Mmax_p$ = 5395 kip-ft $Vmax_p$ = 426 kip

Design Step
7.4.2
(continued)

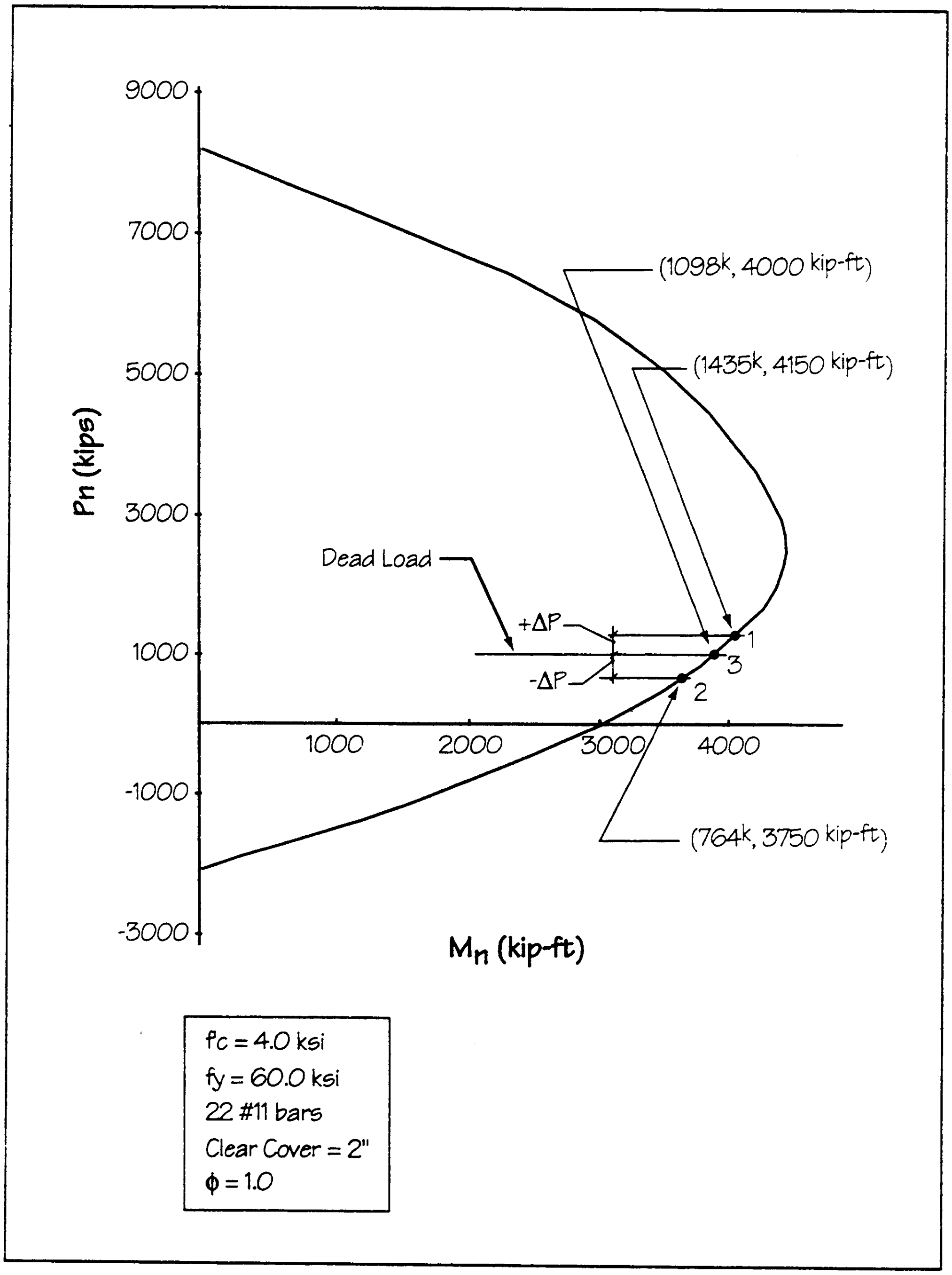


Figure 19 — Column Interaction Diagram
***Step 4a* Nominal Moment Capacities**

Design Step 7.5

Summary of Forces

Design Step 7.5.1

Summary of Column Forces

Table 12 summarizes the forces at the top of the outboard columns of Bent 2. These forces were derived in Design Steps 7.1 to 7.4.

Table 12
Summary of Forces on Bent 2
Outboard Columns with Basic Supports

	Description of Force Type	Load Case	Longitudinal Force V_L (kips)	Longitudinal Force M_L (k-ft)	Transverse Force V_T (kips)	Transverse Force M_T (k-ft)	Vertical Force P_{max} or P (kips)	Vertical Force P_{min} (kips)
Table 6	Dead Load	Dead Load	0	366	0	0	1098	-
table 7	Elastic Seismic Forces	Longitudinal Analysis	760	9790	0	0	36	-
		Transverse Analysis	8	110	77	1053	43	-
Table 8	Orthogonal Seismic Forces	Load Case 1	762	9823	23	316	48	-
		Load Case 2	236	3047	77	1053	53	-
Table 9	Modified Forces Member/Conn	Load Case 1	762	2331	23	63	1146	1050
		Load Case 2	236	975	77	211	1152	1045
V from table 10	Modified Forces Foundations	Load Case 1	762	10189	23	316	1146	1050
		Load Case 2	236	3413	77	1053	1152	1045
	Plastic Hinging Forces	Longit Direction w/P	411	5200	-	-	1098	-
		Transv Direction w/P_{max}	-	-	426	5395	1432	-
		Transv Direction w/P_{min}	-	-	385	4875	-	764

DESIGN STEP 8 — SUMMARY OF DESIGN FORCES

Review the Introduction of Design Step 7 before proceeding with this design step.

Design Step 8.1 — Column or Pile Bent Design Forces

[Division I-A, Article 7.2.3]

The selection of design forces, used to design the column reinforcement for seismic loads, is outlined in Division I-A, Article 7.2.3. The shear force associated with plastic hinging in the column is generally smaller than the full elastic force in seismic Categories C and D. For this example, column forces in the longitudinal direction are used for column design.

a) Axial Forces per Division I-A, Article 7.2.3(a)

Use either the modified forces calculated in Design Step 7.3.1 (same as elastic forces because R = 1) or the plastic hinging forces calculated in Design Step 7.4.2. Both modified forces and plastic hinging values are summarized in Table 12.

Elastic (Modified) Forces for LC1

T6 T8

$P = 1098 +/- 48$ kips

$Pmin_u = 1050$ kips $Pmax_u = 1146$ kips   ← Use

Hinging Forces

$P = 1098$ kips

In this design example, the modified axial forces are selected for column design.

b) Moments per Division I-A, Article 7.2.3(b)

Use the modified moments calculated in Design Step 7.3.1 and summarized in Table 12. The biaxial moment is

$M_u = 2331$ kip-ft   ← Use

Design Step 8.1 (continued)

c) Shear Forces per Division I-A, Article 7.2.3(c)

Use either the modified forces calculated in Design Step 7.3.1 (same as elastic forces because R = 1), or the plastic hinging forces calculated in Design Step 7.4.2. Both modified forces and plastic hinging values are summarized in Table 12.

Elastic (Modified) Forces $V_u = 762$ kips

Hinging Forces $Vmax_p = 426$ kips ← Use

As is normally the case, the shear forces associated with plastic hinging are smaller than the elastic forces.

Therefore, use the force associated with hinging.

d) Summary of Column Forces

See Figure 20.

$Pmin_u = 1050$ kips
$M_u = 2331$ kip-ft
$Vmax_p = 426$ kips

$Pmax_u = 1146$ kips

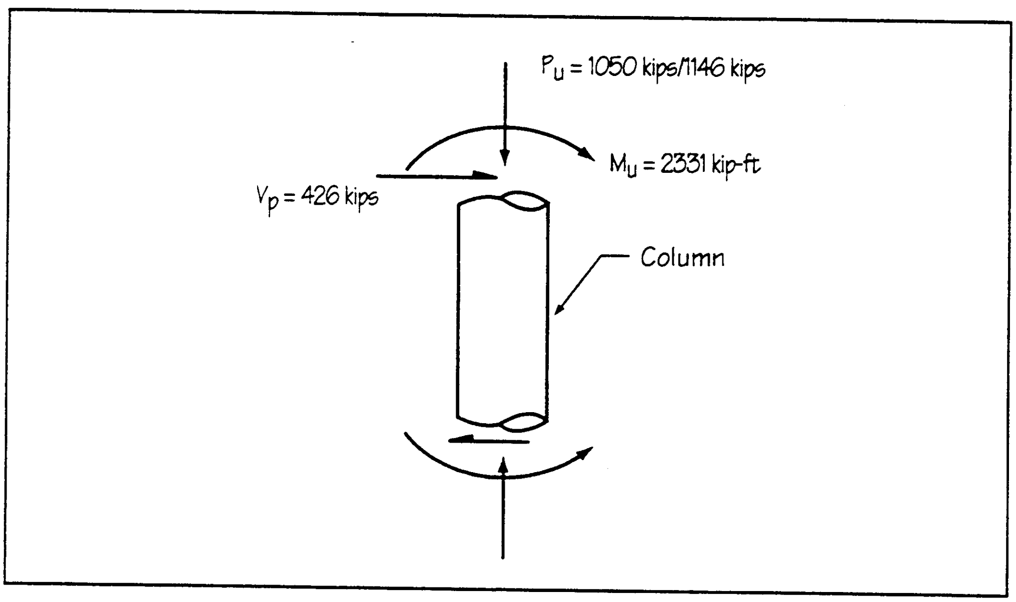


Figure 20 — Summary of Column Forces

Design Step 8.2

Pier Design Forces

Not applicable.

Design Step 8.3

Connection Design Forces
[Division I-A, Article 7.2.5]

Design Step 8.3.1

Longitudinal Linkage Connections
[Division I-A, Article 7.2.5(A)]

There are no linkage connections; therefore, provisions are not applicable.

Design Step 8.3.2

Hold-Down Forces
[Division I-A, Article 7.2.5(B)]

Hold-down devices must be supplied if the vertical force induced at the supports by the earthquake loading exceeds 50 percent of the dead load reaction. The effects due to torsion in the superstructure are negligible.

The maximum vertical seismic force at the abutment is 211 kip. – T8

The dead load reaction is 577 kip.

The seismic load is about 36 percent of the dead load case. Thus, no hold-down devices are required.

Design Step 8.3.3

Column and Pier Connection to Cap Beam
[Division I-A, Article 7.2.5(C)]

For seismic loads, the recommended connection design forces between the column and superstructure cap beam are the forces developed at the top of the column due to column hinging as determined in Design Step 7.4.2. The plastic hinging values are summarized in Table 12. The code allows the designer to use the modified forces calculated in Design Step 7.3.1 if they are smaller, but it is not recommended.

A summary of the maximum forces due to column hinging is given below.

Maximum axial forces due to hinging

$Pmax_p = 1432$ kips

Design Step 8.3.3 (continued)

Maximum moment due to hinging

$Mmaxp = 1.3*Mmax_n = 5395$ kip-ft

Maximum shear force due to hinging

$Vmax_p = 426$ kips

Design Step 8.3.4

Column Connection to Foundation
[Division I-A, Article 7.2.5(C)]

The recommended connection design force between the column and foundation are the forces developed at the bottom of the column due to column hinging. The column dead load has not been added in this design example. These forces are the same as in Design Step 8.3.3 and are summarized below.

$Pmax_p = 1432$ kips
$Mmax_p = 5395$ kip-ft
$Vmax_p = 426$ kips

Design Step 8.4

Cap Beam Design Forces

The cap beam must be designed for the load combinations in Division I, Table 3.22.1A, except for Group VII seismic loads, Division I-A is to be used instead. This design example does not present the dead or live load forces on the cap beam, only the seismic forces.

It is recommended that the forces due to plastic hinging at the top of the column be transferred into the cap beam. These forces are summarized in Design Step 8.3.3 above. Figure 21 identifies how these forces are applied to the cap beam. To restrict the formation of plastic hinges to top of column sections, cap beam sections must remain elastic at internal force levels that form plastic hinges in sections at the top of columns.

Design Step 8.4 (continued)

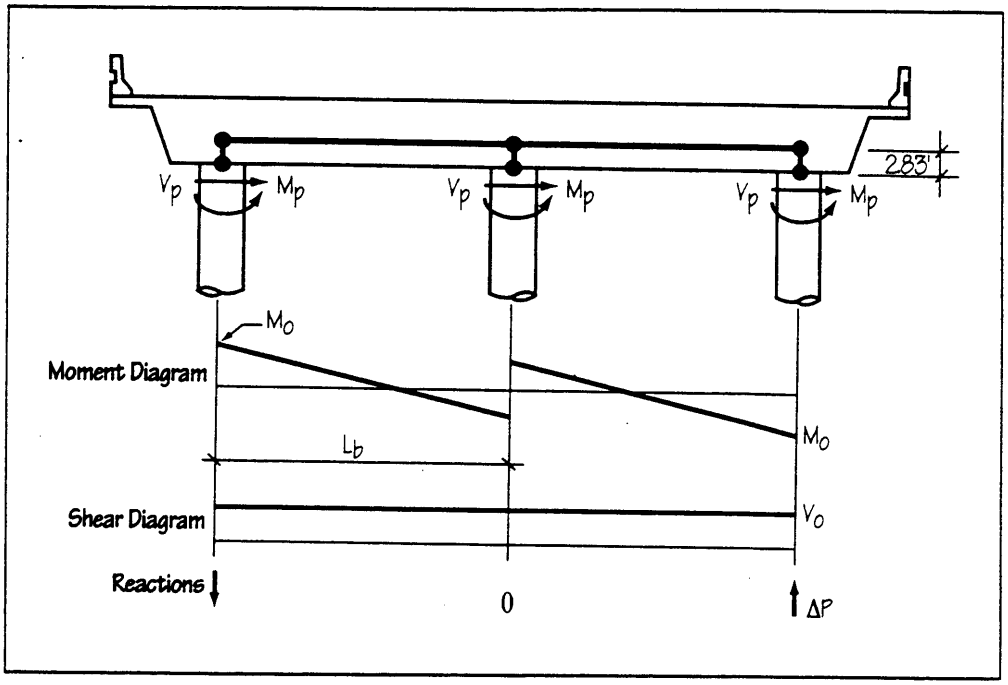


Figure 21 — Plastic Hinging Forces on Cap Beam

Because of symmetry, the maximum peak moment in the cap beam occurs at the outboard columns. The plastic moment transferred from the column is M_p.

$Mmax_p$ = 5395 kip-ft

The column shear associated with the plastic moment V_p acts at a distance 2.83 feet below the c.g. of the cap beam.

$Vmax_p$ = 426 kips

Because of symmetry, the maximum seismic moment in the cap beam due to these column forces occurs over the outside columns.

$M_o = M_p + (2.83' * V_p)$

= 5395 + (2.83*426) = 6601 kip-ft

Therefore, due to these column plastic hinging forces, the seismic shear in the cap beam is constant in between the columns.

$V_o = (M_o + M_{o/2}) / 28.38 \text{ ft} = 349 \text{ kips}$

Design Step 8.5

Miscellaneous Design Forces

Nothing is included in this design step for the basic support condition.

Design Step 8.6

Foundation Design Forces

[Division I-A, Article 7.2.6]

The design forces for the spread footing under the Bent No. 2 columns may be either a) or b) below.

a) Modified Design Forces for Foundations
[Division I-A, Article 7.2.1(B)]

These forces were calculated in Design Step 7.3.2 and are summarized in Table 12, LC1 being the controlling load case. The small transverse forces are ignored. Note that R = 1.0 represents the full elastic seismic forces.

$Pmin_u = 1050$ kip $\quad Pmax_u = 1146$ kips
$M_u = 10{,}189$ kip-ft
$V_u = 762$ kips

b) Forces from Column Plastic Hinging
[Division I-A, Article 7.2.2]

Because of the one-half uplift criteria, the forces associated with both the maximum and minimum axial load need to be considered. The plastic hinging forces were calculated in Design Step 7.4. Refer again to the introduction of Design Step 7 before proceeding.

Plastic hinging forces in longitudinal direction

$P_p = 1098$ kips
$M_p = 5200$ kips-ft
$V_p = 411$ kips

Design Step 8.6 (continued)

Plastic hinging forces in transverse direction (minimum axial load)

$Pmin_p$ = 764 kips
$Mmin_p$ = 4875 kip-ft
$Vmin_p$ = 385 kips

Plastic hinging forces in transverse direction (maximum axial load)

$Pmax_p$ = 1432 kips
$Mmax_p$ = 5395 kip-ft
$Vmax_p$ = 426 kips

Conclusion: Use the forces associated with plastic hinging in the transverse direction. The transverse plastic hinging moment is much smaller than the modified moment (5,395 versus 10,189 kip-ft), and only slightly more conservative than the longitudinal plastic hinging moment (5,395 versus 5,200 kip-ft). See Figure 22.

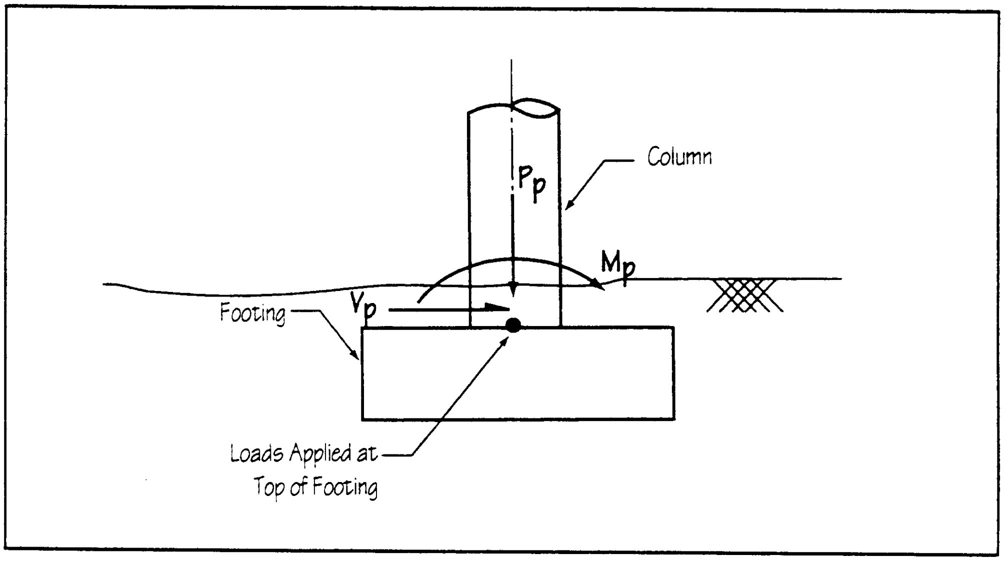


Figure 22 — Summary of Forces on Footing

Design Step 8.7

Abutment Design Forces
[Division I-A, Article 7.2.7]

Only vertical and transverse forces on the abutment are considered in this design example. The superstructure transfers the seismic force to the abutment through the shear key. The abutment then has to transfer the transverse seismic force into the soil.

a) Forces Transferred from Superstructure to Abutment Shear Key
[Division I-A, Article 7.4.2(B)]

Division I-A, Article 7.4.2(B) refers back to Division I-A, Article 7.2.6 for design forces for the abutment. Since there are no hinging effects at the abutment, the controlling forces are the Modified Design Forces for Connections calculated in Design Step 7.3.1. Table 9 lists the seismic forces on the abutment at the level of the bearings. Note that $R = 0.8$ for connections. A summary of the forces at Abutment No. 1 are

Vertical Reaction

$P := 1206 \text{ kip}$ with $R = 0.8$

Transverse Shear

The transverse shear is transferred from the superstructure through the shear key to the abutment. For the shear key design, use the EQM force with $R = 0.8$.

$V_T := 1596 \text{ kip}$ with $R = 0.8$

b) Transverse Forces Transferred from the Abutment into the Soil

Refer to the Modified Design Forces for foundations, summarized in Table 10 of Design step 7.3.2. For this condition, $R = 1.0$ for all cases where the force is transferred into the soil.

Vertical Reaction

$P = 1180 \text{ kips}$ with $R = 1.0$

Design Step 8.7 (continued)

Transverse Shear

V_T = 1277 kips with R = 1.0

Longitudinal Shear

None.

DESIGN STEP 9

DETERMINE DESIGN DISPLACEMENTS
[Division I-A, Article 7.3]

Design Step 9.1

Minimum Support Length
[Division I-A, Article 7.3.1]

The bearing seats supporting the expansion ends of the bridge must provide a minimum support length at least N inches wide. See Figure 23.

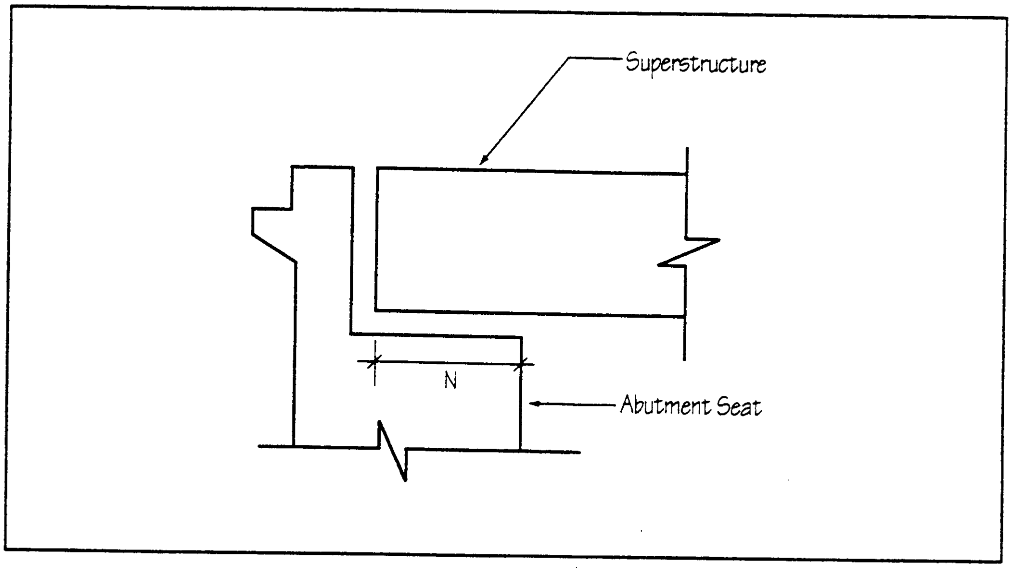


Figure 23 — Minimum Support Length at Abutment

$L := 242 \cdot ft$ Length between abutments

$H := 27.34 \cdot ft$ Average height of columns between expansion joints

$S := 0$ Skew

From Division I-A, Equation (7-3A)

$$N := \left(12 \cdot in + 0.03 \cdot L \cdot \frac{in}{ft} + 0.12 \cdot H \cdot \frac{in}{ft}\right) \cdot \left(1 + 0.000125 \cdot S^2\right)$$

$N = 1.88 \cdot ft$

The width of the abutment stemwall shown in Figure 1 is 3 feet 6 inches; thus, the width provided on top of the stemwall is more than "N."

Design Step 9.2

Design Displacements

The displacement from the Single-Mode Spectral Method in the longitudinal direction with the basic spring case is 0.24 foot.

DESIGN STEP 10

DESIGN STRUCTURAL COMPONENTS

[Division I-A, Article 7.6.2(C)]

This section concentrates on the critical components that resist the seismic forces.

Design Step 10.1

Column Design

Basic column data, see Figure 24 for details.

$f_c := 4000 \cdot psi$ Concrete strength

$f_{yh} := 60 \cdot \frac{kip}{in^2}$ Yield stress of spiral reinforcing

$H_{clr} := 25.33 \cdot ft$ Column clear height

$b_w := 48 \cdot in$ Outside diameter of column

Diameter of concrete core, measured to the outside of the transverse spiral reinforcement. Assume a 2-inch clear cover.

$d_{core} := b_w - 2 \cdot (2 \cdot in)$ $d_{core} = 44 \cdot in$

Summary of Controlling Column Design Forces from Design Step 8.1
[Division I-A, Article 7.2.3]

$Pmin_u := 1050\ kip$ $Pmax_u := 1146\ kip$
$M_u = 2331\ kip\text{-}ft$
$Vmax_p := 426\ kip$

Design Step 10.1.1

Determine Longitudinal Reinforcement

Use the preliminary longitudinal column reinforcement computed in Design Step 7.4.1. Use 22 #11 bars longitudinally.

Design Step
10.1.1
(continued)

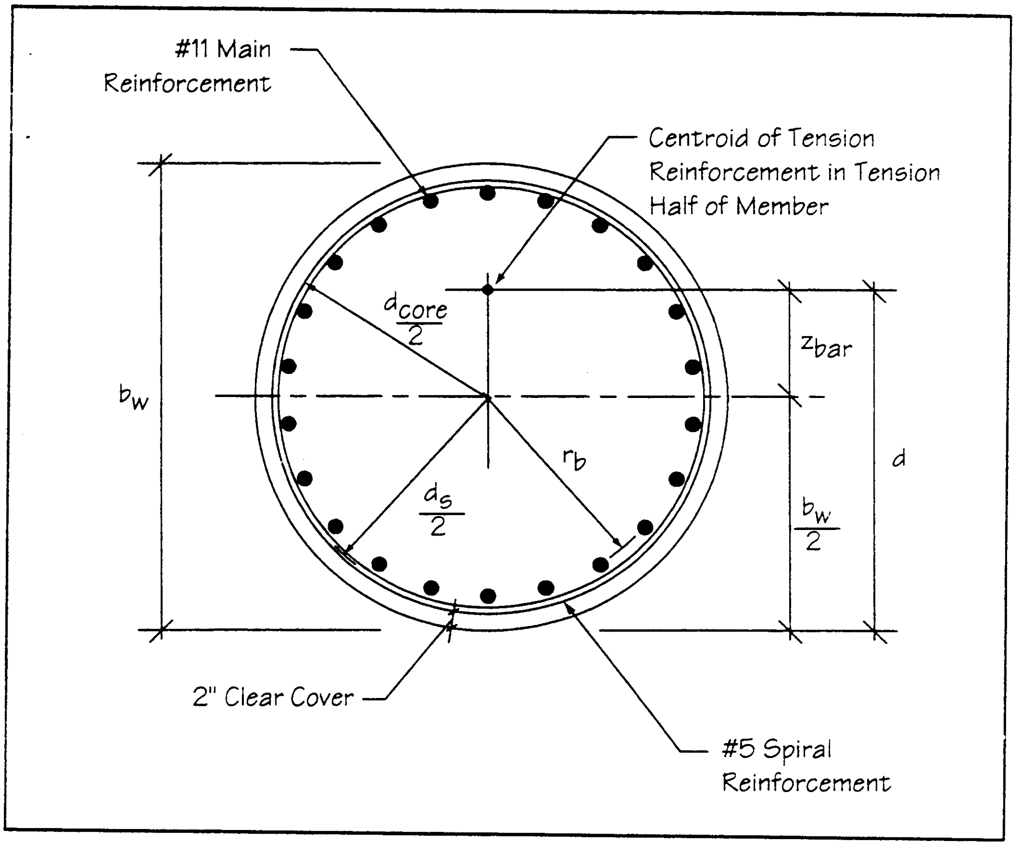


Figure 24 — Cross Section of Column

Design Step
10.1.2

Determine Typical Transverse Reinforcement
[Division I-A, Article 7.6.2(C)]

a) The required shear strength of the column section must be at least that calculated per Division I, Article 8.16.6.1 using the ϕ value from Division I, Article 8.16.1.2 for shear.

$\phi := 0.85$ $\qquad$ $f_c = 4000 \cdot psi$

$b_w := 48 \cdot in$ $\qquad$ Diameter of circular section

$Vmax_p = 426 \cdot kip$ $\qquad$ Factored shear force on section

Design Step
10.1.2
(continued)

The require shear strength of the section, V_n, must be at least

$$V_n := \frac{Vmax_p}{\phi}$$

Division I
Eqn (8-46)

$$V_n = 501 \cdot kip$$

b) The concrete shear strength of the column, calculated per Division I, Article 8.16.6.2, applies only to the middle portion between the "end regions" of the column. Refer to Design Step 10.1.3 for the reinforcement requirements in the end regions.

For computing shear strength of circular sections d need not be less than the distance from extreme compression fiber to centroid of the tension reinforcement in opposite half of member. Refer to Figure 24 for the variables.

$d_{core} = 44 \cdot in$ $\quad\quad$ $b_w = 48 \cdot in$

The location of the #11 longitudinal bars from the center of the column is

$$r_b := \frac{d_{core}}{2} - 0.625 \cdot in - \frac{1.375}{2} \cdot in$$

$$r_b = 20.7 \cdot in$$

$$z_{bar} := \frac{2}{\pi} \cdot r_b$$

Centroid of tension side reinforcement (Popov, 1976)

$$d := \frac{b_w}{2} + z_{bar}$$

$d = 37.2 \cdot in$

$$V_c := 2 \cdot \sqrt{f_c} \cdot b_w \cdot d$$

Division I
Eqn (8-51)

$$V_c := 226 \cdot kip$$

Design Step
10.1.2
(continued)

c) The required shear strength of the reinforcement calculated per Division I, Article 8.16.6.3.

$V_s := V_n - V_c$ Division I Eqn (8-47)

$V_s = 275 \cdot kip$

The minimum spiral spacing, based on a maximum of 3-inch clearance between spirals per Division I, Article 8.18.2.2.3.

$s := 3.5 \cdot in$

The resulting shear reinforcement is then

$A_v := \frac{V_s}{f_{yh}} \cdot \frac{s}{d}$ Division I Eqn (8-53)

$A_v = 0.43 \cdot in^2$ For two legs of the spiral reinforcement

Because the cross-sectional area of #5 spiral reinforcing is 0.31 (in^2/leg) or 2*0.31 = 0.62 (in^2/2 legs), the provided A_v is greater than the required A_v (0.62 > 0.43); therefore, use #5 bars at 3.5-inch spacing.

Design Step
10.1.3

Determine Minimum Transverse Reinforcement in Column "End Region" [Division I-A, Article 7.6.2(C)]

The end regions must meet the following two criteria.

Criteria 1 — The shear strength of the concrete V_c shall be in accordance with Division I, Article 8.16.6.2 when the axial load associated with the shear produces an average compression stress in excess of 0.1f'$_c$ over the core concrete area of the support members. As the average compression stress increases from 0 to 0.1f'$_c$, the strength V_c increases linearly from 0 to the value given in Division I, Article 8.16.6.2.

Design Step 10.1.3 (continued)

Column core area = A_{core}

$d_{core} = 44 \cdot in$

$$A_{core} := \frac{\pi \cdot d_{core}^2}{4} \qquad A_{core} = 10.6 \cdot ft^2$$

Check for the minimum column core axial stress, σ_a, even though the shear force is associated with the maximum axial force. This is conservative.

$Pmin_u = 1050 \cdot kip$

$$\sigma_a := \frac{Pmin_u}{A_{core}} \qquad \sigma_a = 0.69 \cdot \frac{kip}{in^2}$$

In order to use the full concrete shear capacity, the minimum axial stress in the column is

$$0.1 \cdot f_c = 0.4 \cdot \frac{kip}{in^2}$$

Because the minimum axial stress in the column is more than 0.1*f'$_c$ (0.69 ksi > 0.40 ksi), the full value of V_c can be used. In some columns, the minimum axial load may be less than 0.1*f'$_c$, and would therefore require a reduction in V_c.

$V_c = 226 \cdot kip$

Therefore, because the concrete shear capacity is the same as in Design Step 10.1.2, the required transverse reinforcement in the end regions is the same as in the typical region.

Use #5 spiral at 3.5 inches on center.

Design Step 10.1.3 (continued)

Criteria 2 — Extent of column "End Region" above footing and below top of column is the maximum of the following.

a. Diameter of column = 48 inches
b. $H_{clr}/6$ = 25.33 ft / 6 = 4.2 ft = 51 inches <------- Controls
c. 18 inches minimum

Design Step 10.1.4

Determine the Transverse Reinforcement for Confinement at Plastic Hinges [Division I-A, Article 7.6.2(D)]

The core of the column must be confined by spiral in plastic hinge regions by the largest of 1) the criteria in this section, and 2) the criteria in Division I-A, Article 7.6.2(C) in Design Step 10.1.2.

The volumetric ratio of spiral reinforcement is the greater of that required by Equation (7-4) or (7-5) in Division I-A for spiral reinforcement where

Gross area of column A_g

$b_w = 48 \cdot in$

$A_g := \dfrac{\pi \cdot b_w^2}{4}$ $\qquad A_g = 12.6 \cdot ft^2$

$A_{core} = 10.6 \cdot ft^2$ Column core area

$f_c = 4 \cdot \dfrac{kip}{in^2}$ Concrete strength

$f_{yh} := 60 \cdot \dfrac{kip}{in^2}$ Yield strength of spiral reinforcement

Design Step 10.1.4 (continued)

Therefore, the volumetric ratio per Equation (7-4) is

$$\rho_s := 0.45 \cdot \left(\frac{A_g}{A_{core}} - 1 \right) \cdot \frac{f_c}{f_{yh}}$$

Division I-A Eqn (7-4)

$\rho_s = 0.0057$

The volumetric ratio per Equation (7-5) is

$$\rho_s := 0.12 \cdot \frac{f_c}{f_{yh}}$$

Division I-A Eqn (7-5)

$\rho_s = 0.008$ <---------- Controls

The pitch of the spiral is 3.5 inches.

$s = 3.5 \cdot in$

The diameter of the spiral, measured to the middle of the spiral, d_s

$d_{core} = 44 \cdot in$

$d_s := d_{core} - 0.625 \cdot in$ $\qquad d_s = 43.4 \cdot in$

Therefore, the area of one leg of the spiral is Asp.

$$A_{sp} := \frac{\rho_s \cdot s \cdot d_{core}^{2}}{4 \cdot d_s}$$

(Wang and Salmon, 1992)

$A_{sp} = 0.31 \cdot in^2$

This is the required area of one leg of the spiral. Because a #5 bar has a cross-sectional area of 0.31 inch^2, the #5 spiral at 3.5 inches on center will work. This is also the area of spiral required in the end region.

The #5 spiral at 3.5 inches on center still controls.

Design Step 10.1.5

Calculate the Spacing of Transverse Reinforcement for Confinement [Division I-A, Article 7.6.2(E)]

Criteria 1 — Spiral must be provided in the column the length determined in Design Step 10.1.3, per Division I-A, Article 7.6.2(C)2.

Minimum length = 51 inches top and bottom

The spiral must be extended into the cap and footing the distance determined in Division I-A, Article 7.6.4. See Design Step 15 for more details of column spiral.

A minimum of 15 inches or one-half the column diameter
= 48/2 = 24 inches <--- Controls

Criteria 2 — Not applicable.

Criteria 3 — Maximum spacing.

The maximum spacing of 3.5 inches is okay.

Criteria 4 — The spiral may not be lapped in the plastic hinge zones without full strength lap welds. This criteria should be placed on the drawings.

Lap splices in the center region of the column must follow the criteria in Division I-A, Article 7.6.2(F).

Design Step 10.1.6

Summary of Column Reinforcement

Use 22 #11 bars longitudinally. Use #5 spiral at 3.5 inches on center. See Figure 25 for final column design details.

Design Step
10.1.6
(continued)

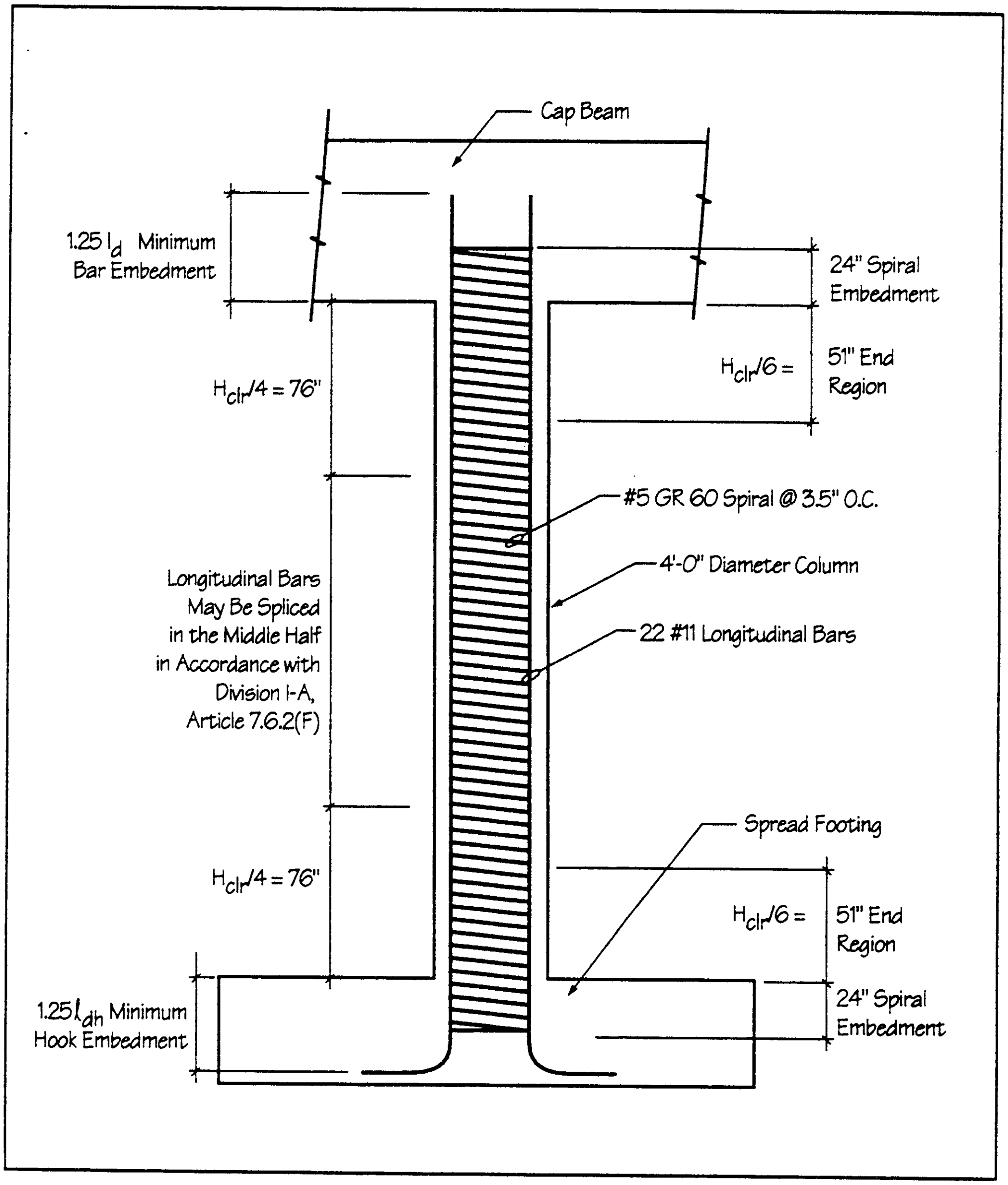


Figure 25 — Column Reinforcement Details

Design Step
10.2

Pier Design

Not applicable.

Deign Step 10.3 — **Connection Design**

Design Step 10.3.1 — Longitudinal Linkage

Not applicable.

Design Step 10.3.2 — Hold Downs

Not applicable.

Design Step 10.3.3 — Connection of Column to Cap Beam

The connections of the column to both the cap beam and footing must be designed. These designs should include the development of the longitudinal column steel into both elements and the assurance that the joint shear stresses are lower than the limiting amount set by the Specification. Both of these conditions require the forces corresponding to plastic hinging as the design forces.

a) Development Length
[Division I, Article 8.25 and Division I-A, Article 7.6.4]

The straight development length of a #11 bar per Division I, Article 8.25, without the additional requirements of Division I-A, is as follows.

Basic data

$f_c := 4000$ — Concrete compressive strength (psi)

$A_b := 1.56$ — Area of #11 bar (in^2)

$d_b := 1.410$ — Diameter of #11 bar (in)

$f_y := 60000$ — Yield stress (psi)

Design Step 10.3.3 (continued)

Basic development length equation

$$L_{db} := 0.04 \cdot A_b \cdot \frac{f_y}{\sqrt{f_c}} \cdot in \qquad L_{db} = 59.2 \cdot in$$

But not less than

$$0.0004 \cdot d_b \cdot f_y \cdot in = 33.8 \cdot in$$

Does not control

Modify the basic length for bar spacing greater than 6 inches on center. The actual spacing is 6.3 inches. [Division I, Article 8.25.3.1]

$$L_d := 0.8 \cdot L_{db}$$

Modify length for probable steel yield stress, 1.25 fy
[Division I-A, Article 7.6.4]

$$L_d := 1.25 \cdot L_{db}$$

Therefore, the final development length of a straight bar must take into account both of the above effects.

$$L_d := 0.8 \cdot 1.25 \cdot L_{db} \qquad L_d = 59.2 \cdot in$$

The length available for developing a straight #11 bar is 68 inches less cover, say 64 inches. Thus, there is sufficient length for development of the bar into the cap beam. Refer back to Figure 25 in Design Step 10.1.6.

b) Joint Shear Stress
[Division I-A, Article 7.6.4]

The average maximum joint shear stress is to be limited. This is an average stress because it may be calculated based on the cross-sectional area of the joint. It should also correspond to the loading condition that produces the largest stress. Typically, this corresponds to plastic hinging of the columns. The stress limit applies to both the horizontal and vertical directions.

Design Step 10.3.3 (continued)

Because the cap beam is designed such that plastic hinging will not occur in it, the vertical shear stress corresponding to column plastic hinging is easier to calculate. The vertical shear stress will be that required to transfer the forces passed into the joint by the column reinforcement. The actual force level is difficult to assess because the steel is distributed around the perimeter. Therefore, the shear force is approximated.

The joint shear stress calculations are not covered in the design example.

Design Step 10.3.4

Connection of Column to Footing
[Division I, Article 8.29 and Division I-A, Article 7.6.4]

a) Development Length

The development length of a hooked #11 bar per Division I, Article 8.29, without the additional requirements of Division I-A, is as follows.

Basic data

$f_c := 4000$ Concrete compressive strength (psi)

$A_b := 1.56$ Area of #11 bar (in^2)

$d_b := 1.410$ Diameter of #11 bar (in)

$f_y := 60000$ Yield stress (psi)

Basic development length for standard hooks

$$L_{hb} := 1200 \cdot \frac{d_b}{\sqrt{f_c}} \cdot in \qquad L_{hb} = 26.8 \cdot in$$

Modify basic length for bar cover greater than 2.5 inches
(Division I, Article 8.29.3.2)

$$L_{dh} := 0.7 \cdot L_{hb}$$

Design Step 10.3.4 (continued)

Modify length for probable steel yield stress, 1.25 fy
[Division I-A, Article 7.6.4]

$$L_{dh} := 1.25 \cdot L_{hb}$$

Therefore, the final development length of a hooked bar must take into account both of the above effects.

$$L_{dh} := 0.7 \cdot 1.25 \cdot L_{hb} \qquad L_{dh} = 23.4 \cdot in$$

Length available for development of the hooks is 60 inches less cover and less the thickness of the mat of steel in the bottom of the footing. The total of these thicknesses is about 6 inches. Thus, there is sufficient length for development. To ensure proper force transfer ability and to simplify the construction, the hooks should be extended to the bottom mat of steel. Refer back to Figure 25 in Design Step 10.1.6.

b) Joint Shear Stress
[Division I-A, Article 7.6.4]

Not calculated in this design example.

Design Step 10.4

Cap Beam Design

The cap beam seismic design forces were determined in Design Step 8.3.2. Figure 21 shows the resulting moment and shear diagram due to seismic loads only. For the final Group VII loads, the dead load moment and shear must be added. The cap beam can then be designed using Division I of AASHTO with the typical detailing requirements for a reinforced concrete beam.

Note that the forces due to column hinging must also be applied to the cap beam in the longitudinal direction. These forces are then transferred into the superstructure box beam. Because of the built-in negative moment in the box girder when the shoring is removed, the positive moment from the hinging will probably not control. Had the superstructure been built with precast units, this positive moment may have been critical.

DESIGN STEP 11 — DESIGN FOUNDATIONS

[Division I-A, Article 7.4.2]

In this design example, only spread footings will be addressed.

Design Step 11.1 — Design Spread Footings (under Bent No. 2 Columns)

[Division I-A, Article 7.4.2]

The footings under Bent 2 are individual spread footings located under each of the three columns. The following soil properties are assumed to apply to this bridge site. The actual properties would be specified in the geotechnical soil reports.

$q_{eq} := 24 \cdot \frac{kip}{ft^2}$ Ultimate soil pressure permitted under seismic loads

$\gamma_{soil} := 0.110 \cdot \frac{kip}{ft^3}$ Unit weight of soil

$\gamma_{conc} := 0.150 \cdot \frac{kip}{ft^3}$ Unit weight of concrete in footing

$\mu = 0.5$ Sliding coefficient

Design Step 11.1.1 — Find Forces at Bottom of Footing

The forces at the top of the footing are taken from Design Step 8.6, and are associated with the plastic hinging forces at the bottom of the column. The forces associated with both maximum and minimum axial loads will be considered in the design. Note that column dead load has been ignored in this design example.

Forces w/minimum axial force	Forces w/maximum axial force
$Pmin_p := 764 \cdot kip$	$Pmax_p := 1432 \cdot kip$
$Mmin_p := 4875 \cdot kip \cdot ft$	$Mmax_p := 5395 \cdot kip \cdot ft$
$Vmin_p := 385 \cdot kip$	$Vmax_p := 426 \cdot kip$

Design Step
11.1.1
(continued)

The moment at the bottom of the footing M_f must include the moment due to V_p times the footing depth d_f. Therefore, $M_f = M_p + V_p * d_f$. See Figure 26.

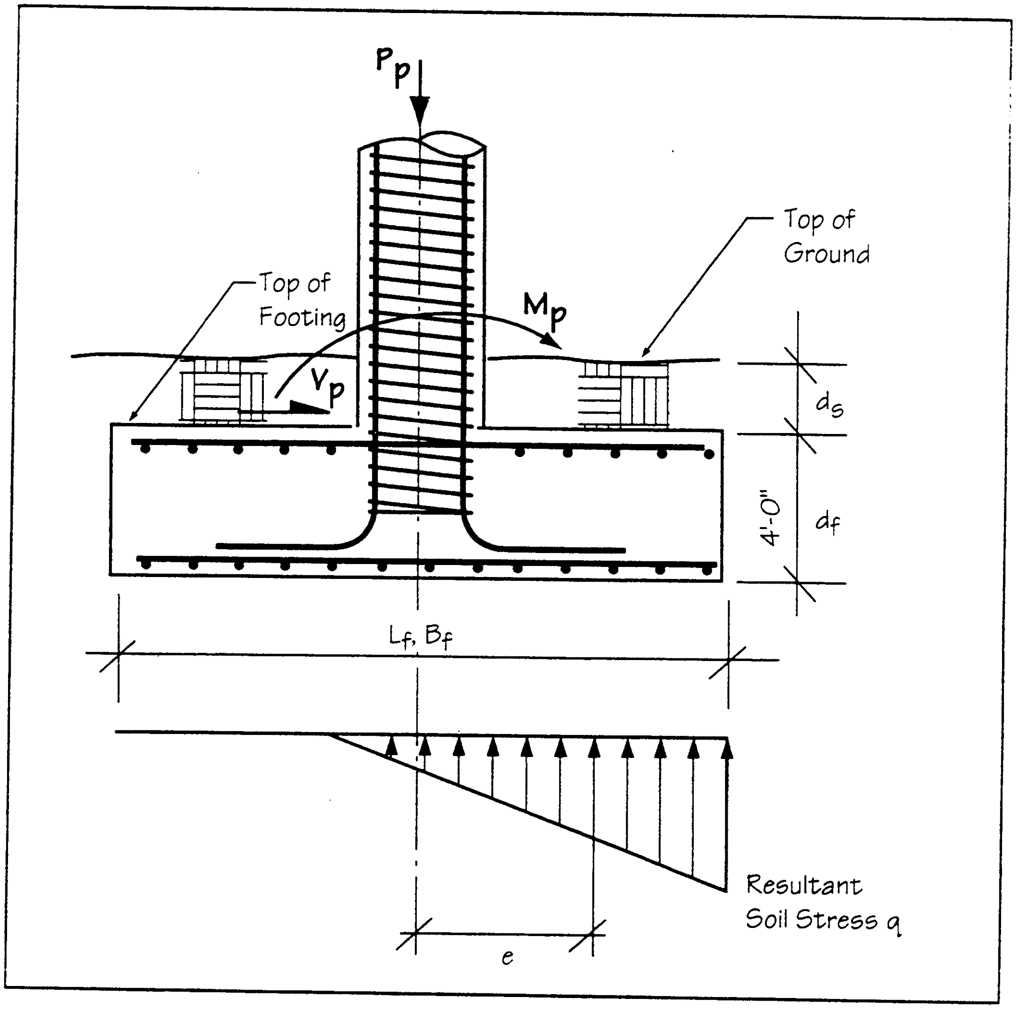


Figure 26 — Column Footing Details and Soil Pressure Distribution

An initial footing depth must be assumed in order to calculate M_f.

$d_f := 4 \cdot ft$ Assumed footing depth

Design Step 11.1.1 (continued)

Moment associated with $Pmin_n$

$Mmin_p = 4875 \cdot ft \cdot kip$

$Vmin_p = 385 \cdot kip$

$Mmin_f := Mmin_p + Vmin_p \cdot d_f$ $\quad Mmin_f = 6415 \cdot kip \cdot ft$

Moment associated with $Pmax_p$

$Mmax_p = 5395 \cdot ft \cdot kip$

$Vmax_p = 426 \cdot kip$

$Mmax_f := Mmax_p + Vmax_p \cdot d_f$ $\quad Mmax_f = 7099 \cdot kip \cdot ft$

The axial load at the bottom of the footing must include the dead weight of the soil and the self-weight of the footing. Assume a 2-foot minimum soil cover on top of the footing. Assume a 20-foot-square footing size for weight purposes. Refer to Figure 26.

$L_f := 20 \cdot ft$ — Length of footing

$B_f := 20 \cdot ft$ — Width of footing

$d_s := 2 \cdot ft$ — Depth of soil above footing

$d_f = 4 \cdot ft$ — Thickness of footing

Therefore, the axial dead load due to the self-weight of the footing, and the weight of the soil above the footing, is P_d.

$$\gamma_{soil} = 0.11 \cdot \frac{kip}{ft^3}$$

Design Step 11.1.1 (continued)

$$\gamma_{conc} = 0.15 \cdot \frac{kip}{ft^3}$$

$$P_d := \gamma_{soil} \cdot (L_f \cdot B_f \cdot d_s) + \gamma_{conc} \cdot (L_f \cdot B_f \cdot d_f) \qquad P_d = 328 \cdot kip$$

Therefore, the total axial load at the bottom of the footing, P_f

Minimum axial load

$$Pmin_p = 764 \cdot kip$$

$$Pmin_f := Pmin_p + P_d \qquad Pmin_f = 1092 \cdot kip$$

Maximum axial load

$$Pmax_p = 1432 \cdot kip$$

$$Pmax_f := Pmax_p + P_d \qquad Pmax_f = 1760 \cdot kip$$

Design Step 11.1.2

Find Stresses at Bottom of Footing

In addition to checking for maximum soil stresses, footing uplift must be considered. Per Division I-A, Article 7.4.2(B), the footing can have a separation of the soil up to one-half of the contact area of the foundation under seismic loading. This is only allowed under foundations not susceptible to loss of strength under cyclic loading. Half uplift will probably be controlled by the load case with the minimum axial load.

The stresses under the footing can then be checked for forces associated with both maximum and minimum axial loads. In cases where the soil material has a large ultimate capacity, as with this bridge, the soil stress limit will usually not control the design. Half uplift usually controls the size of the footing. See Figure 26 for the general soil stress.

Design Step 11.1.2 (continued)

a) Check maximum soil pressure at toe of footing under minimum axial load.

The effective eccentricity of the axial load is

$Mmin_f = 6415 \cdot kip \cdot ft$ $\quad$ $Pmin_f = 1092 \cdot kip$

$$e_{min} := \frac{Mmin_f}{Pmin_f} \qquad e_{min} = 5.9 \cdot ft$$

For there to be any uplift, the eccentricity must be greater than $L_f / 6$, which it is by inspection. To ensure that there is no more than one-half uplift on the footing, the eccentricity e must be less than the $L_f / 3$.

$L_f = 20 \cdot ft$

$$\frac{L_f}{3} = 6.7 \cdot ft$$

Because $L_f/3$ is greater than e_{min} (6.7 > 5.9 ft)a 20-ft-sq footing is large enough to prevent half uplift

The maximum soil pressure at the toe of the footing, allowing for uplift of the footing, is q (because $e_{min} > L_f / 6$).

$$q := \frac{2 \cdot Pmin_f}{3 \cdot B_f \cdot \left(\frac{L_f}{2} - e_{min}\right)}$$

(Bowles, 1988)

$$q = 8.8 \cdot \frac{kip}{ft^2}$$

Since q is less than q_{eq} (8.8 < 24 ksf) , okay

b) Check maximum soil pressure at toe of footing under maximum axial load.

The effective eccentricity of the axial load is

$Mmax_f = 7099 \cdot kip \cdot ft$ $\quad$ $Pmax_f = 1760 \cdot kip$

Design Step
11.1.2

$$e_{max} := \frac{Mmax_f}{Pmax_f} \qquad e_{max} = 4 \cdot ft$$

To ensure that there is no more than one-half uplift on the footing, the eccentricity e must be less than the $L_f / 3$.

$$L_f = 20 \cdot ft$$

$$\frac{L_f}{3} = 6.7 \cdot ft$$

Because $L_f/3$ is greater than e_{max}, (6.7 > 4.0 ft), a 20-ft-sq footing is large enough to prevent half uplift

The maximum soil pressure at the toe of the footing, allowing for uplift of the footing, is q (because $e_{max} > L_f / 6$).

$$q := \frac{2 \cdot Pmax_f}{3 \cdot B_f \cdot \left(\frac{L_f}{2} - e_{max} \right)}$$

(Bowles, 1988)

$$q = 9.8 \cdot \frac{kip}{ft^2}$$

Because q is less than q_{eq}, (9.8 < 24 ksf) is okay

c) Check for sliding beneath the footing.

$Vmin_p = 385$ kips — Plastic hinging shear force

$\mu = 0.5$ — Sliding coefficient

$Pmin_p = 1092$ kips — Minimum axial load

The capacity against sliding under the footing, $\mu * Pmin_p$, must be greater than $Vmin_p$.

$\mu * Pmin_p = 546$ kips — Greater than $Vmin_p$, sliding is okay

Design Step 11.1.3

Finalize Footing Size

Because the maximum soil pressure under the seismic load combination $\dot{q}$ = 9.8 ksf is less than the allowable limit of 24 ksf, use the 20-foot-square footing. Note that half-uplift condition is reached before the soil stress limit is reached.

Using the triangular-shaped soil pressure computed above, the designer can now design the footing for flexure and shear using Division I of AASHTO. Note that top reinforcement should be included in the footing in order to support the weight of the soil above the footing due to the uplift condition. Some agencies also require a minimum amount of shear reinforcement in the footing.

DESIGN STEP 12 — DESIGN ABUTMENTS

This example problem concentrates on the development of seismic forces for key elements of the abutment. Unlike the special seismic design and detailing requirements for the columns in Division I-A, the design and detailing of the abutment reinforcement uses the typical requirements in Division I.

Design Step 12.1 — Design the Abutment Shear Key

Figure 27 shows the transverse shear force from the superstructure into the top of the abutment seat through the shear keys. In the longitudinal direction, the superstructure is free to slide. In the transverse direction, the concrete shear key does not allow the bridge to move transversely. Because this critical connection is very stiff and does not allow any dissipation of energy through yielding, the shear key is designed for a force greater than the full elastic seismic shear computed in the analysis.

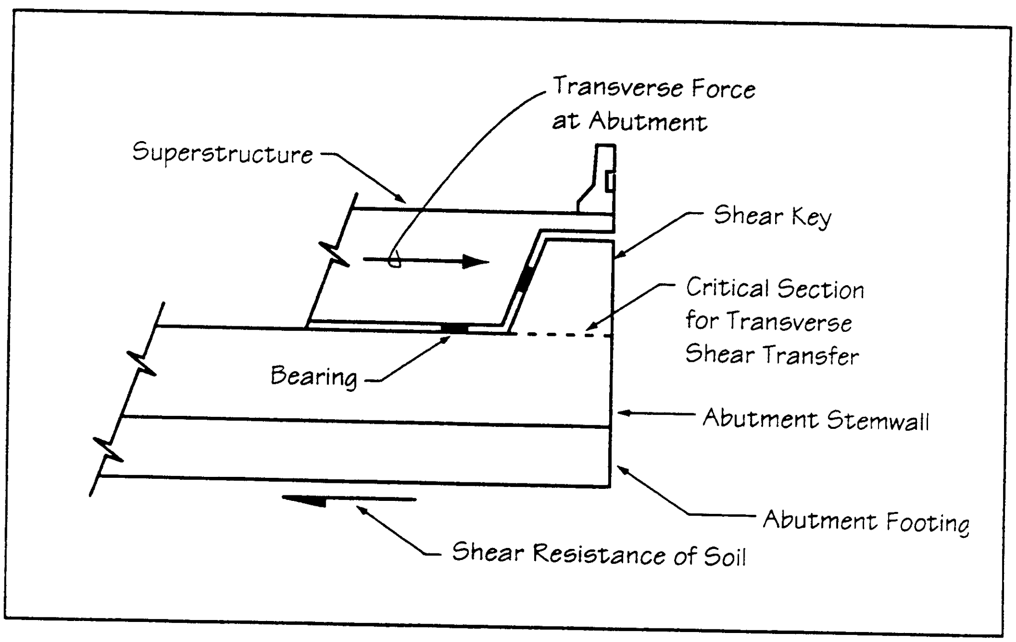


Figure 27 — Abutment Shear Key

Design Step 12.1 (continued)

The shear friction provisions of Division I, Article 8.16.6.4 will be used to calculate the transfer of this horizontal force.

Given

$f'_c := 4000 \cdot psi$

$f_y := 60000 \cdot psi$

$\phi := 0.85$ — Division I Article 8.16.1.2.2

The transverse shear force on the shear key was found in Design Step 8.7, Part (a). This transfer assumed an R Factor of 0.8.

$V_u := 1596 \cdot kip$ — Seismic transverse shear

Required shear capacity

$$V_n := \frac{V_u}{\phi}$$ Division I Eq (8-46)

$V_n = 1878 \cdot kip$

Check Division I, Article 8.16.6.4.5 for the minimum area of concrete to resist the shear transfer force (in plan view).

a) $A_{cv} := \dfrac{V_n}{0.2 \cdot f'_c}$ $\qquad A_{cv} = 16.3 \cdot ft^2$

b) $A_{cv} := \dfrac{V_n}{800 \cdot psi}$ $\qquad A_{cv} = 16.3 \cdot ft^2$

Design Step 12.1 (continued)

Therefore, assuming that there is one shear key on each side of the superstructure, each key must have this area. Based on a 3-foot 6-inch-thick abutment wall, each shear key must have a length of

$$L_s := \frac{A_{cv}}{3.5 \cdot ft} \qquad L_s = 4.7 \cdot ft$$

Use a shear key length of $\quad L_s := 5 \cdot ft$

It is not advisable to place an additional shear key at the centerline of the roadway section due to the unpredictability of how much force each shear key will resist. The stemwall thickness can either be increased locally to reduce this length, or the stemwall can be extended out beyond the edge of the bridge to provide for the required shear key area.

The shear friction reinforcement required for each shear key is based on Division I, Article 8.16.6.4.4.

$\mu := 1.0 \qquad$ For an intentionally roughened surface

$$A_{vf} := \frac{V_n}{f_y \cdot \mu} \qquad A_{vf} = 31.3 \cdot in^2$$

Because the shear area length is 5 feet long for each direction, the required shear reinforcement per foot is

$$A_{vf} := \frac{A_{vf}}{L_s} \qquad A_{vf} = 6.3 \cdot \frac{in^2}{ft}$$

Design Step 12.2

Design Bearings

The bearings used at the abutment for the basic support condition must carry the vertical loads, and slide horizontally under the longitudinal seismic load. The full elastic seismic deflection in the longitudinal direction is approximately 3 inches. A PTFE sliding surface could be used.

Design Step 12.3

Transverse Force on Abutment

As determined in Design Step 8.7 (Part b), the maximum transverse shear force transferred from the superstructure to the abutment and then into the soil is 1277 kips. This force is resisted by two primary load paths.

1) Friction between the bottom of the abutment footing and the soil. Use a sliding coefficient of friction, μ=0.5 times the dead load.

2) Passive pressure developed against the wingwalls.

   The wingwall on the leading edge is not considered fully effective because it pushes against soil that is on a slope. The wingwall on the backside is fully effective as the abutment tries to pull the wingwall into the confined soil behind the abutment.

Design Step 12.4

Longitudinal Force on Abutment

Not applicable for this support condition.

DESIGN STEP 13

DESIGN SETTLEMENT SLABS

Not applicable.

DESIGN STEP 14

REVISE STRUCTURE

Not required.

DESIGN STEP 15

SEISMIC DETAILS

A number of details emphasizing the seismic issues discussed in this example are included within this section. These details, extracted from actual bridge plans, relate to the code requirements of Division I, Division I-A, and local agency requirements.

Footing Detail (Figure 28)

Both top and bottom reinforcement is required. Top reinforcement is required to support the weight of the soil above the footing during uplift conditions. Bottom reinforcement is required to resist forces due to soil bearing pressure. Shear reinforcement permits a reduction in footing thickness and is required by some agencies.

Footing-to-Column Joint Details (Figures 29 and 30)

Critical to the footing-to-column connection is the transverse and spiral reinforcement. End region requirements are specified in Division I-A, Article 7.6.2. Spiral embedment in the footing is specified in Division I-A, Article 7.6.4. Both a continuous spiral option and a discontinuous spiral option are detailed. The discontinuous spiral option is not addressed in AASHTO, but is recommended by some agencies as an alternative that is easier to construct.

Spiral Details (Figures 31 and 32)

Spiral splice details include lap splice and welded splice options as shown. Welded splices are permitted in all column regions. Lap splices are permitted only within the center half of the column height. Refer to Division I-A, Article 7.6.2.

DESIGN STEP 15
(continued)

Column-to-Bent Joint Details (Figures 33 and 34)

Critical to the column-to-bent connection is the transverse and spiral reinforcement. End region requirements are specified in Division I-A, Article 7.6.2. Spiral embedment in the bent is specified in Division I-A, Article 7.6.4. Both a continuous spiral option and a discontinuous spiral option at the construction joint is detailed. The discontinuous spiral option is not addressed in AASHTO, but is recommended by some agencies as an alternative that is easier to construct.

Shear Key Detail (Figure 35)

Typically, the shear key is designed to resist transverse seismic forces, yet it must allow longitudinal movement due to shrinkage and contraction/expansion. The shear key detail illustrates the concept of providing Ultra High Molecular Weight (UHMW) polyethylene plus a neoprene filler to provide a low-friction contact surface. Note that the steel plates and angles armor the shear key to minimize damage during a seismic event, and note that the same detail can be used with sloping shear keys.

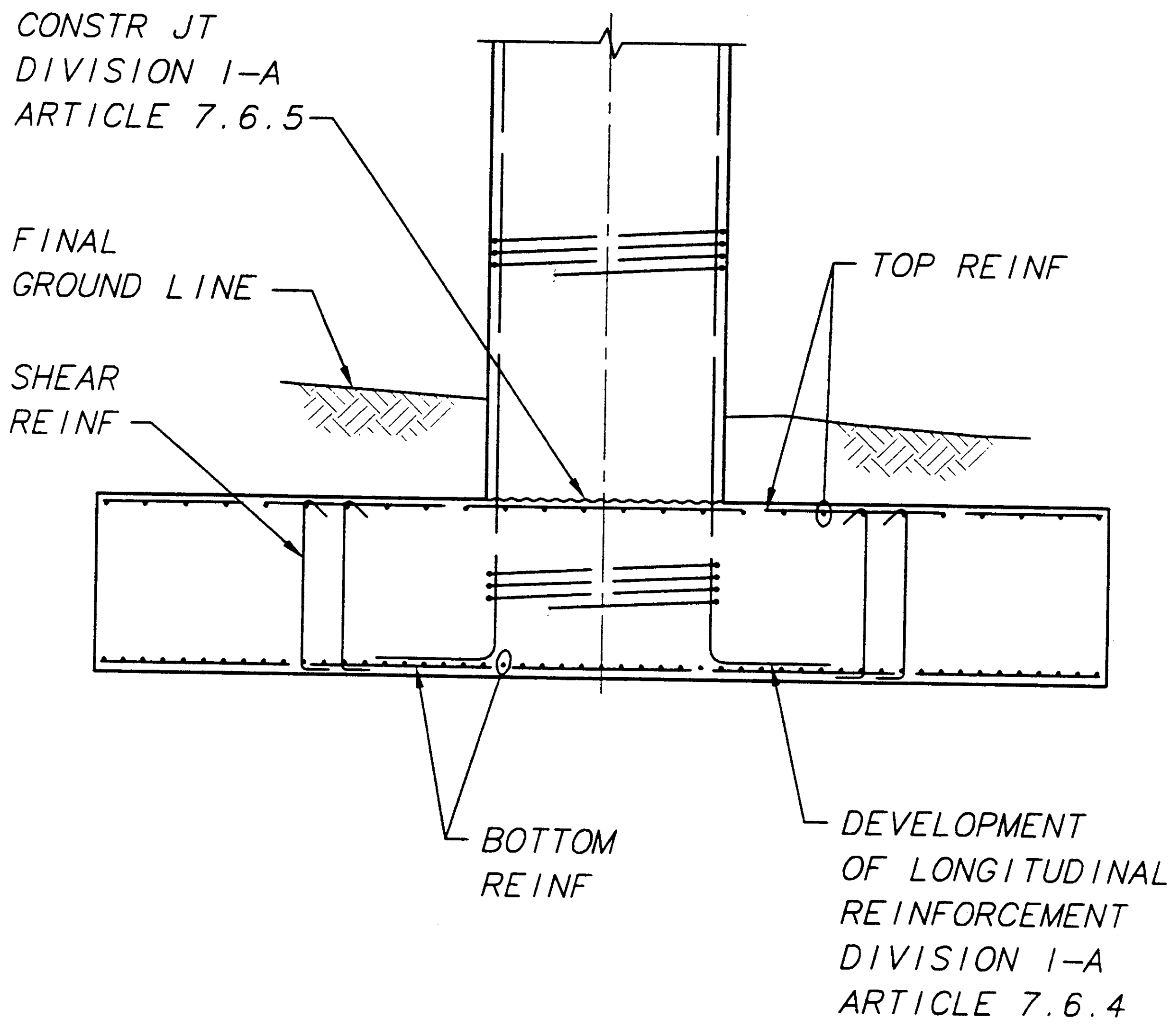


Figure 28 — Footing Detail

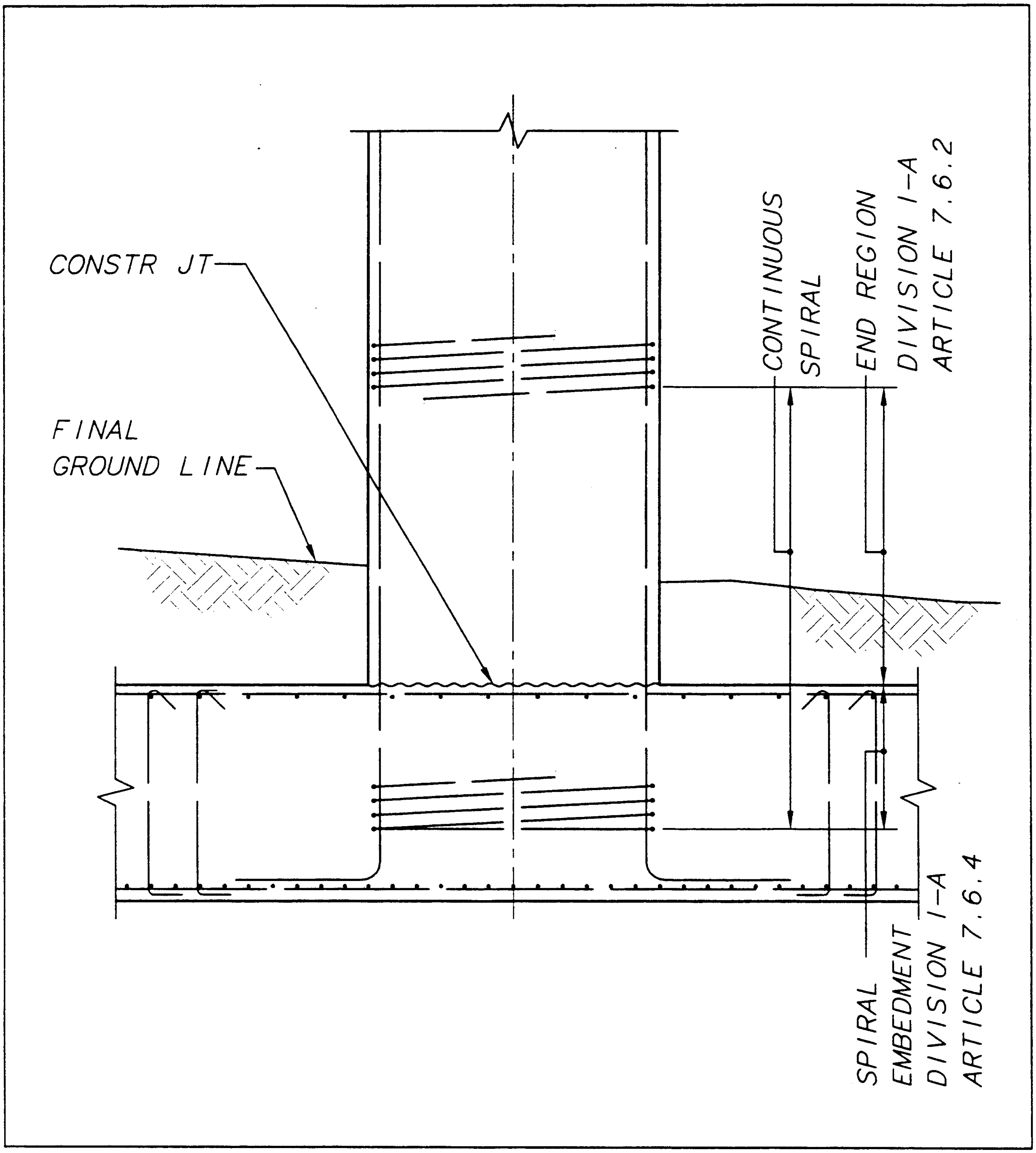


Figure 29 — Footing-to-Column Joint Detail (Continuous Spiral Option)

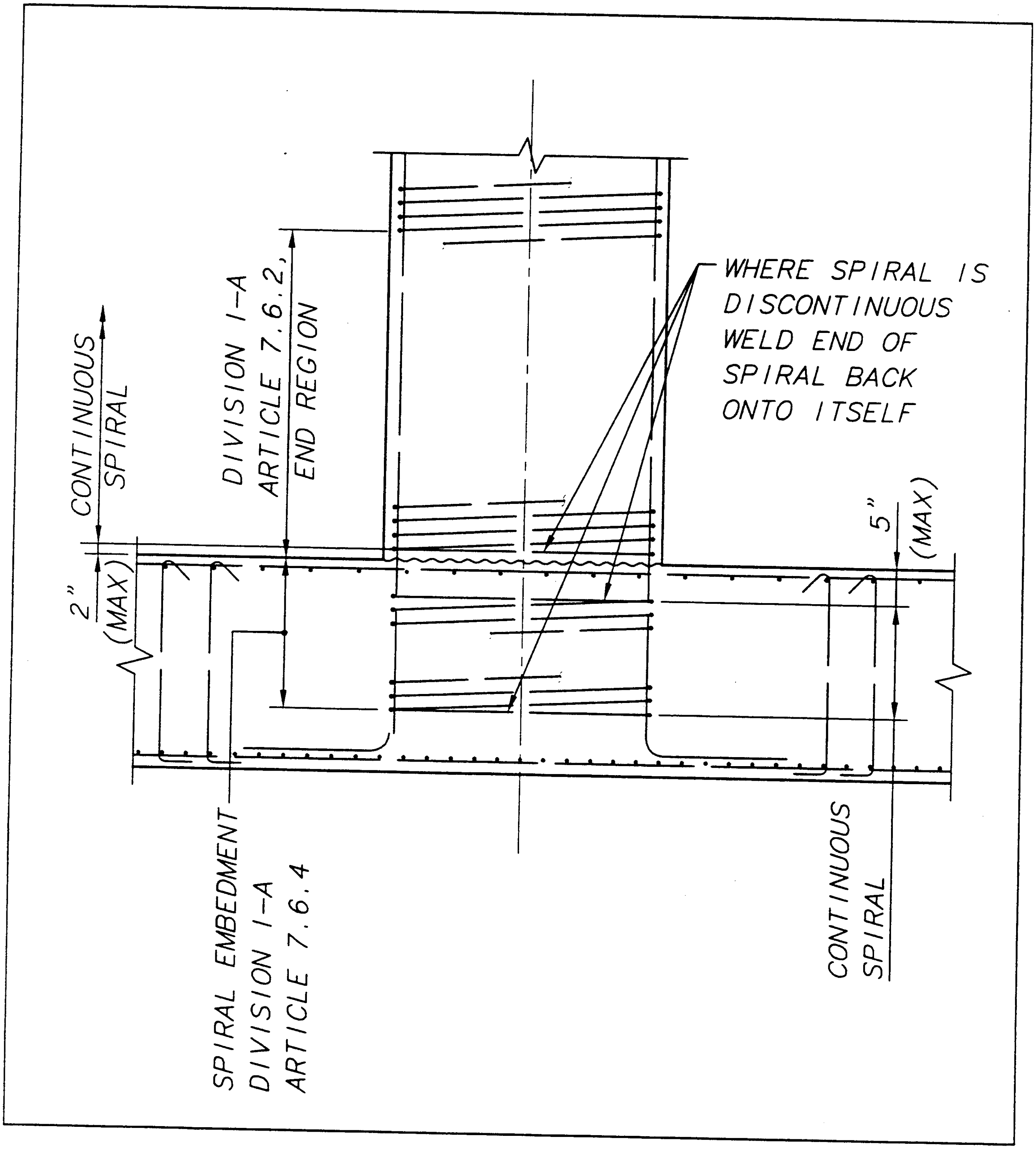


Figure 30 — Footing-to-Column Joint Detail (Discontinuous Spiral Option)

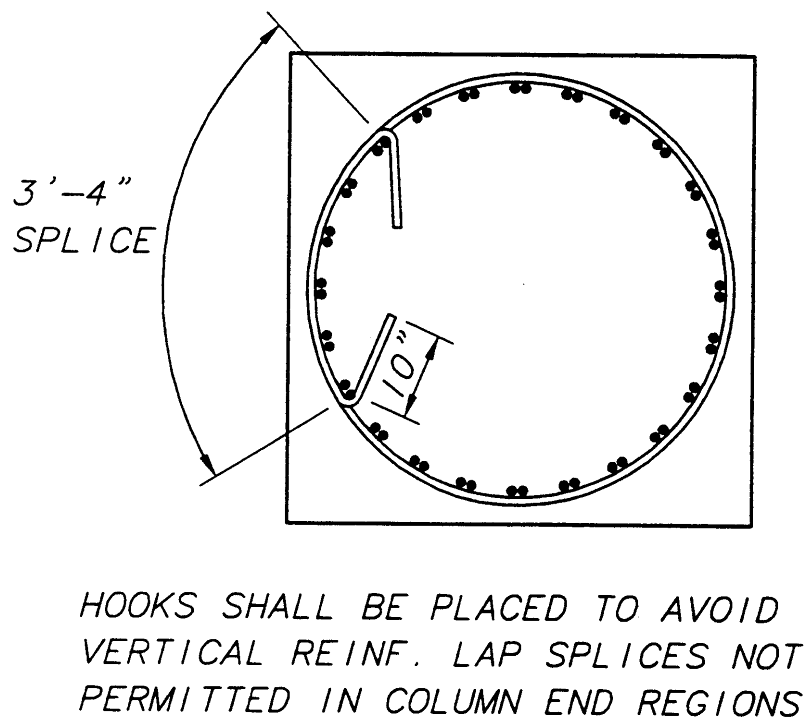


Figure 31 — Lap Splice Spiral Detail

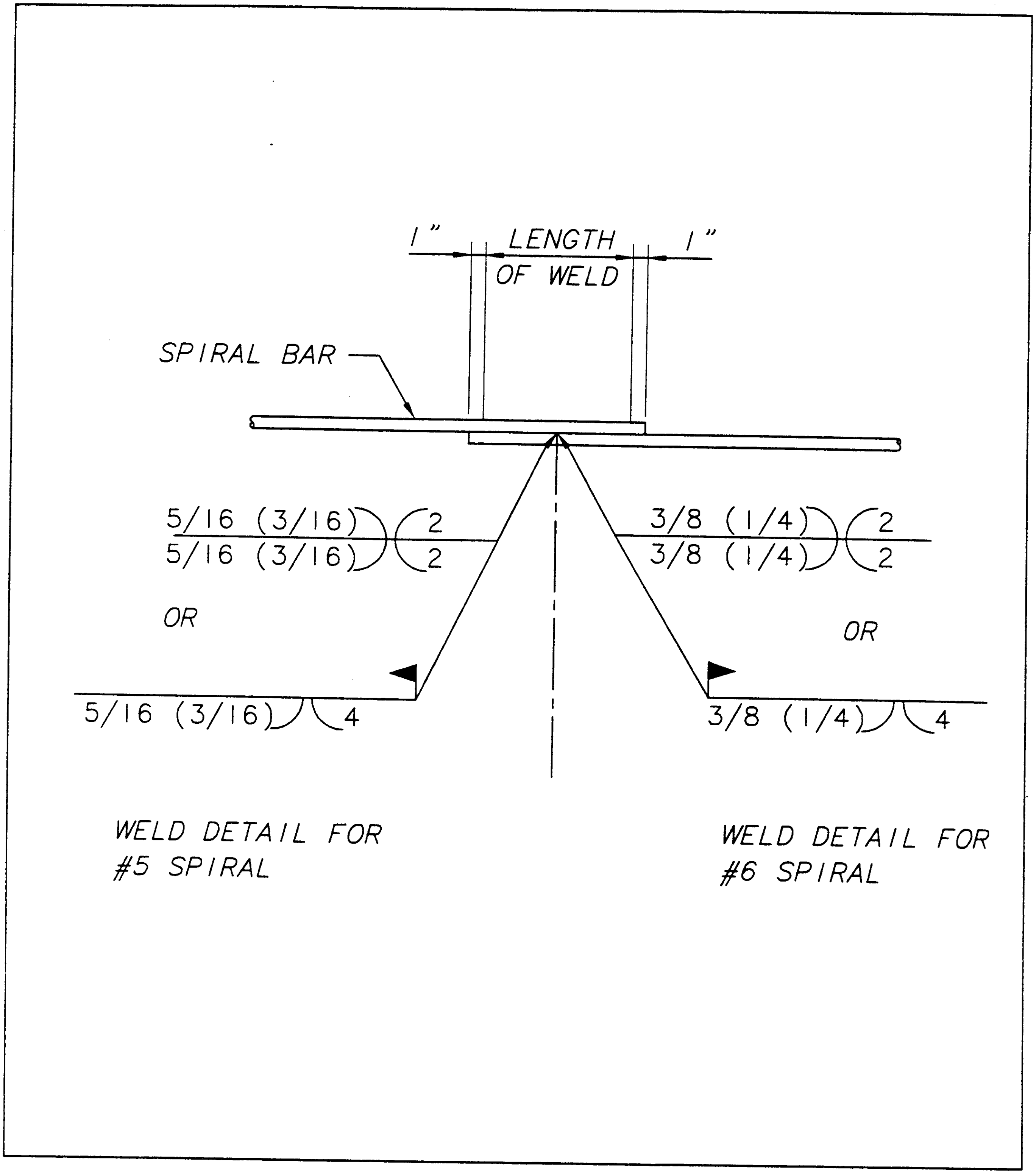


Figure 32 — Welded Splice Spiral Detail

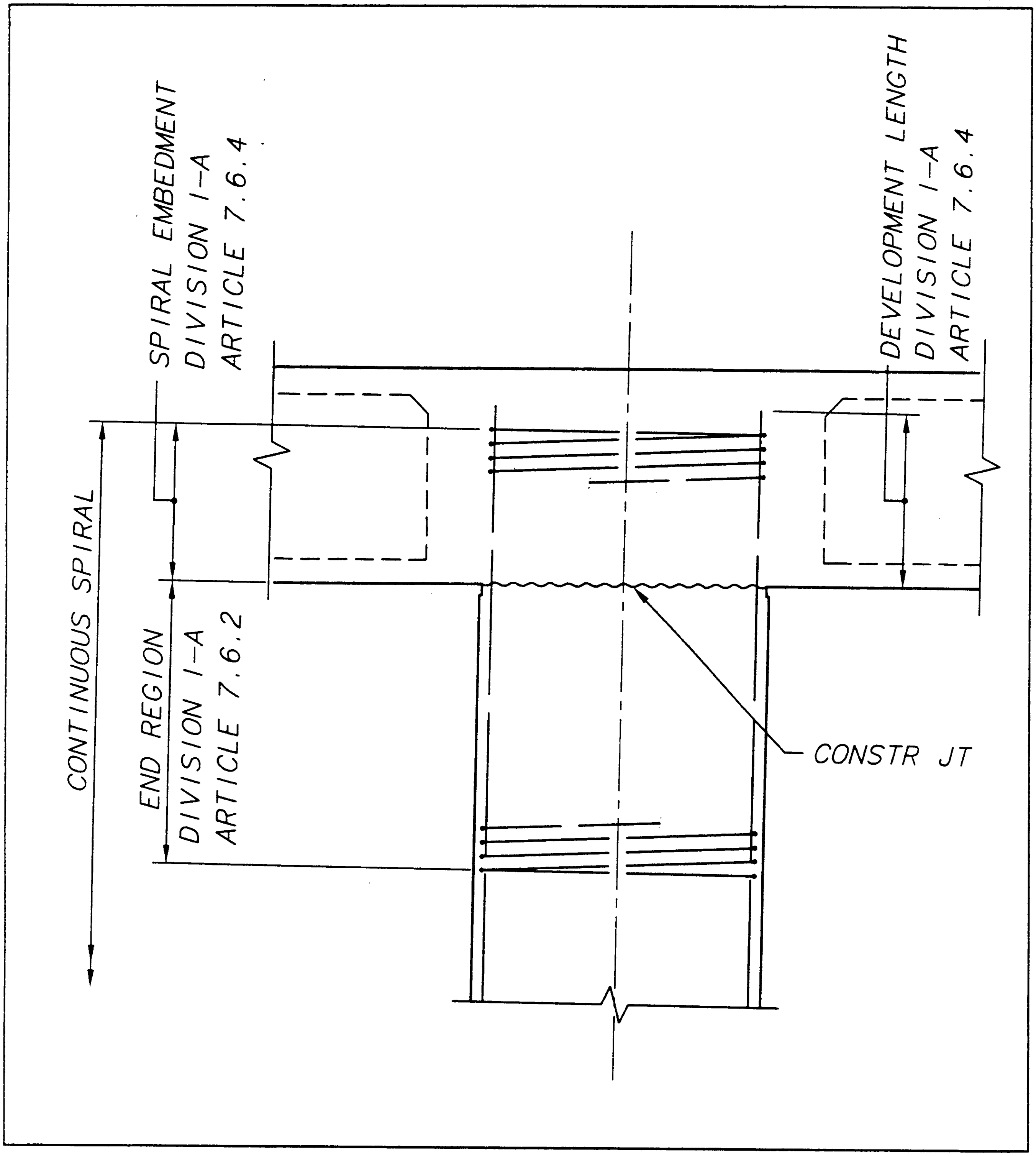


Figure 33 — Column-to-Bent Joint Detail
(Continuous Spiral Option)

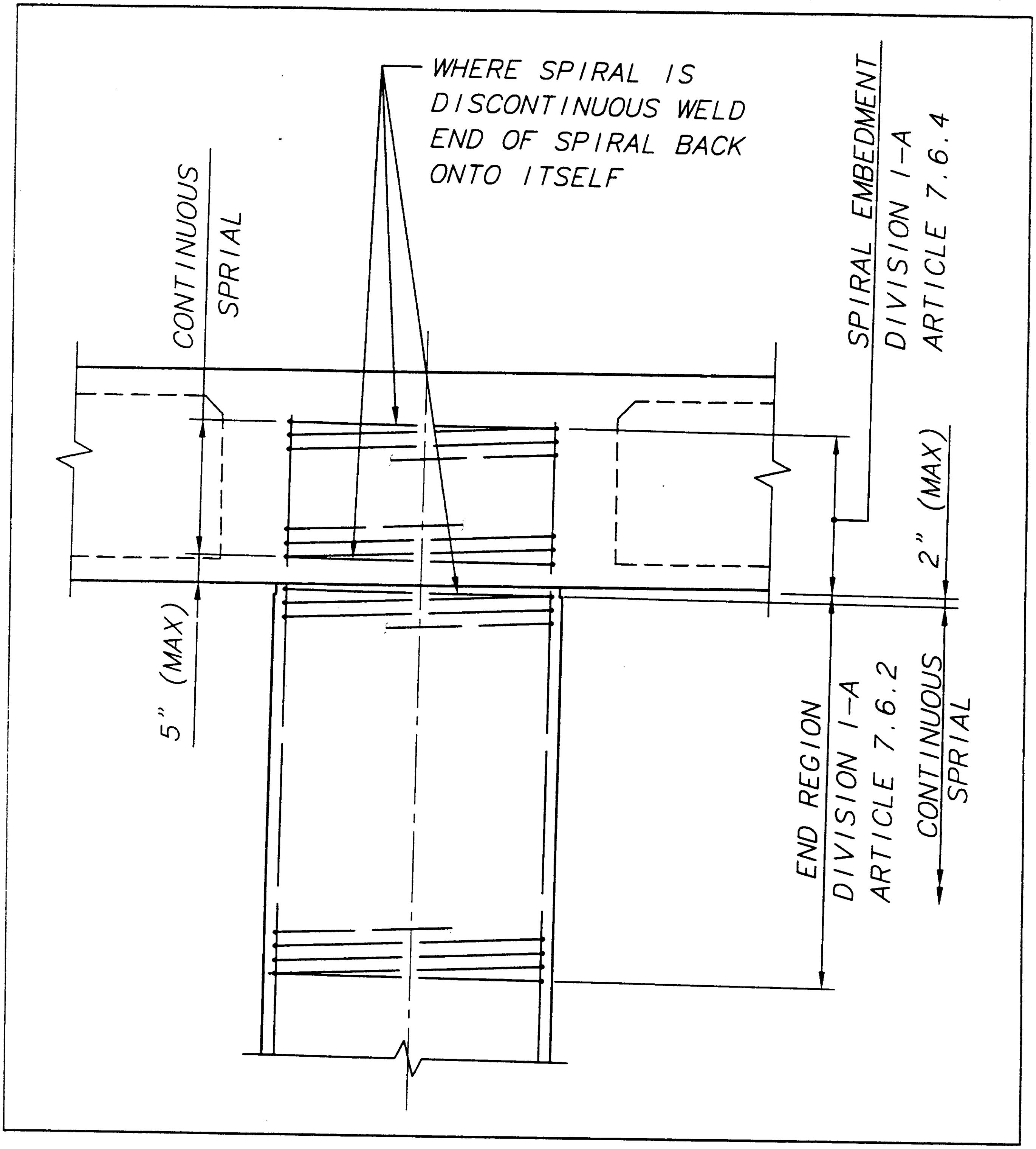


Figure 34 — Column-to-Bent Joint Detail (Discontinuous Spiral Option)

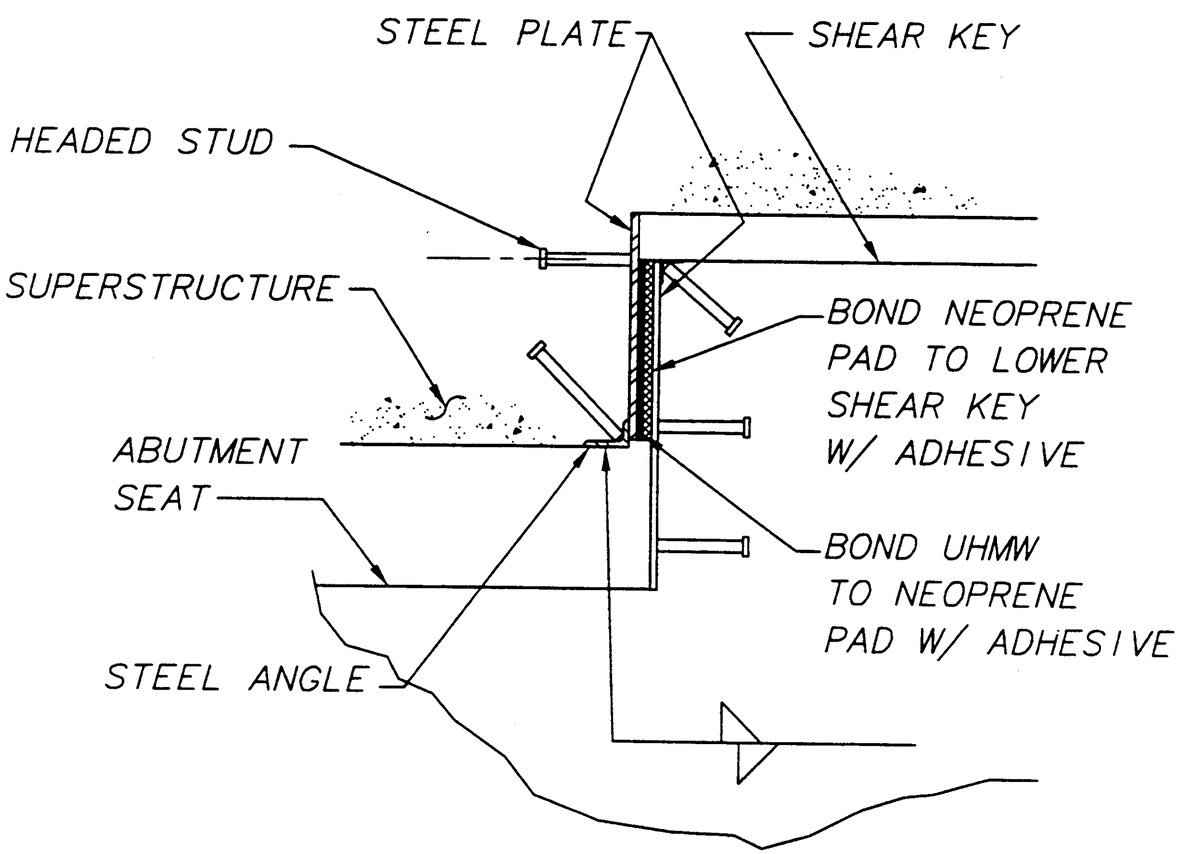


Figure 35 — Shear Key Detail

Section IV
Analysis and Design Using Single-Mode Spectral Method with Spring Supports

SECTION IV

ANALYSIS AND DESIGN USING SINGLE-MODE SPECTRAL METHOD WITH SPRING SUPPORTS

DATA

The bridge is to be built near Glacier National Park, Montana. The soil is a 250-foot-deep glacial deposit of sand and gravel.

REQUIRED

Design the bridge for seismic loading using the Proposed Revisions to the *AASHTO Standard Specifications for Highway Bridges*, Division I-A: Seismic Design, July 1994.

SOLUTION

DESIGN STEP 1

PRELIMINARY DESIGN

The preceding version of the seismic design of this bridge considered the case where all the longitudinal force was taken by the bent, and the bent foundations were fixed against translation and rotation. The abutments were fixed against transverse displacements, vertical displacements, and rotation about the bridge longitudinal axis. Additionally, the bent columns had uncracked moments of inertia.

This alternate example considers the case where the end diaphragms resist longitudinal translation, the foundations are spring supported, and a reduced effective moment of inertia is considered for the columns. In this case, the abutments have a different form than those used in the previous section. These new abutment details are shown in Figures 36a, 36b, 36c, and 36d. The backwall of the end diaphragm is configured such that the soil is bearing against the end diaphragm all the time. No gap exists between the soil and backwall. Due to this configuration, any longitudinal displacements result in longitudinal soil forces. Thus, soil springs will be used to model this condition. Likewise the wingwalls attract transverse force as soon as movement occurs. This will also be incorporated into the model using springs. Finally, at the bent foundations, rotational springs will be used to model the soil flexibility. Translational springs will not be included here, but their inclusion would be approached in the same fashion as for rotational springs.

Because much of the analysis and design of the bridge is the same as for the preceding example, only those sections of the overall analysis and design that change are discussed herein.

DESIGN STEP 1
(continued)

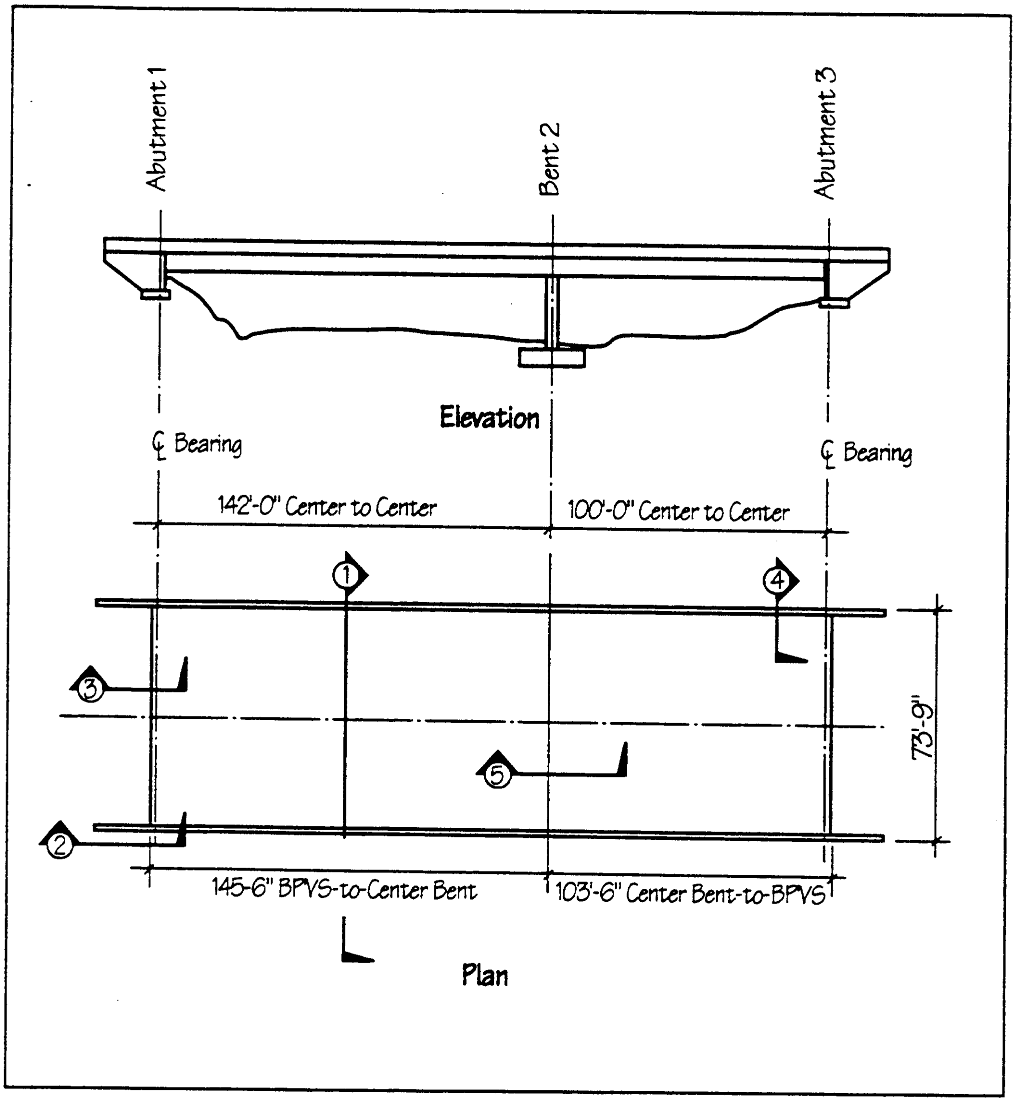


Figure 36a — Bridge Layout with Stub Abutment

DESIGN STEP 1
(continued)

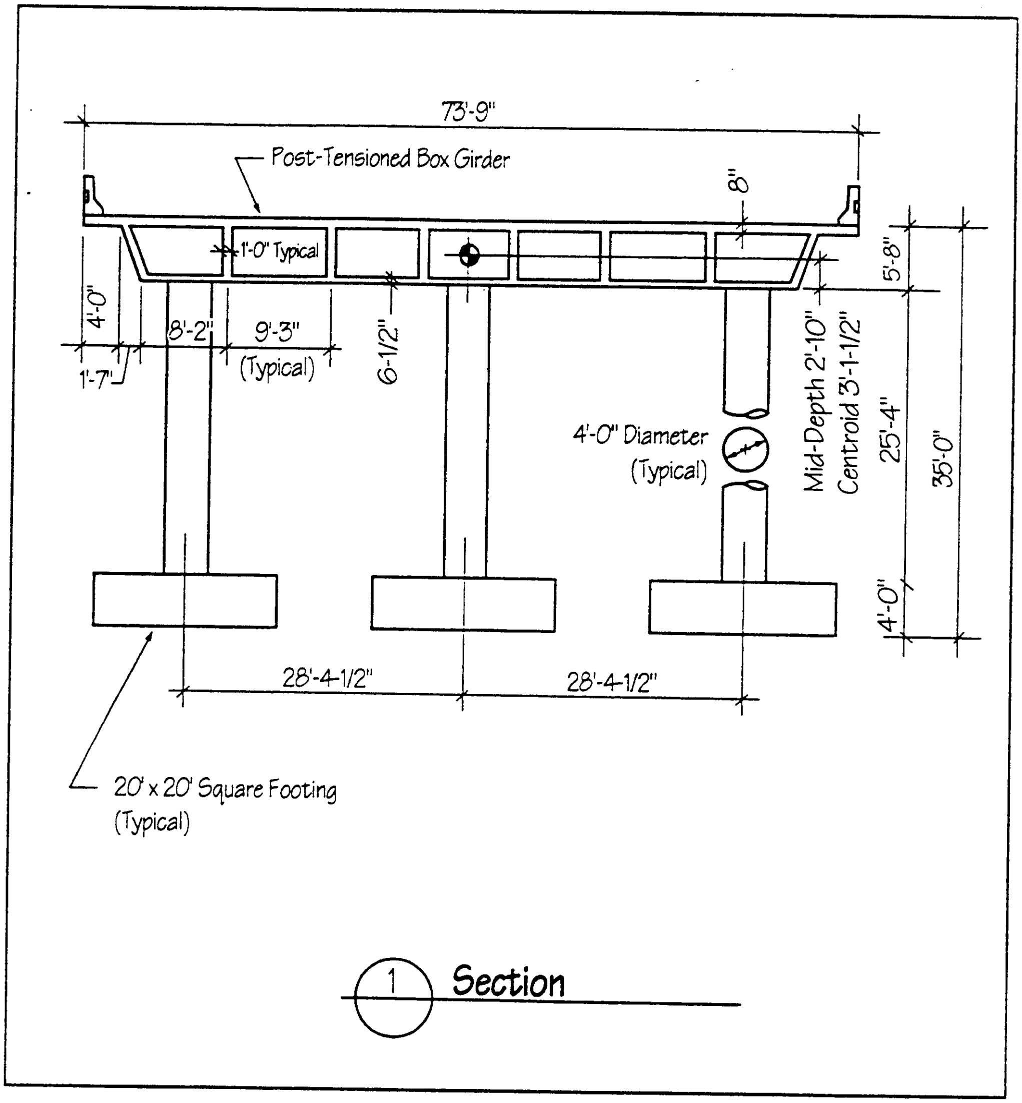


Figure 36b — Bridge Layout with Stub Abutment

DESIGN STEP 1
(continued)

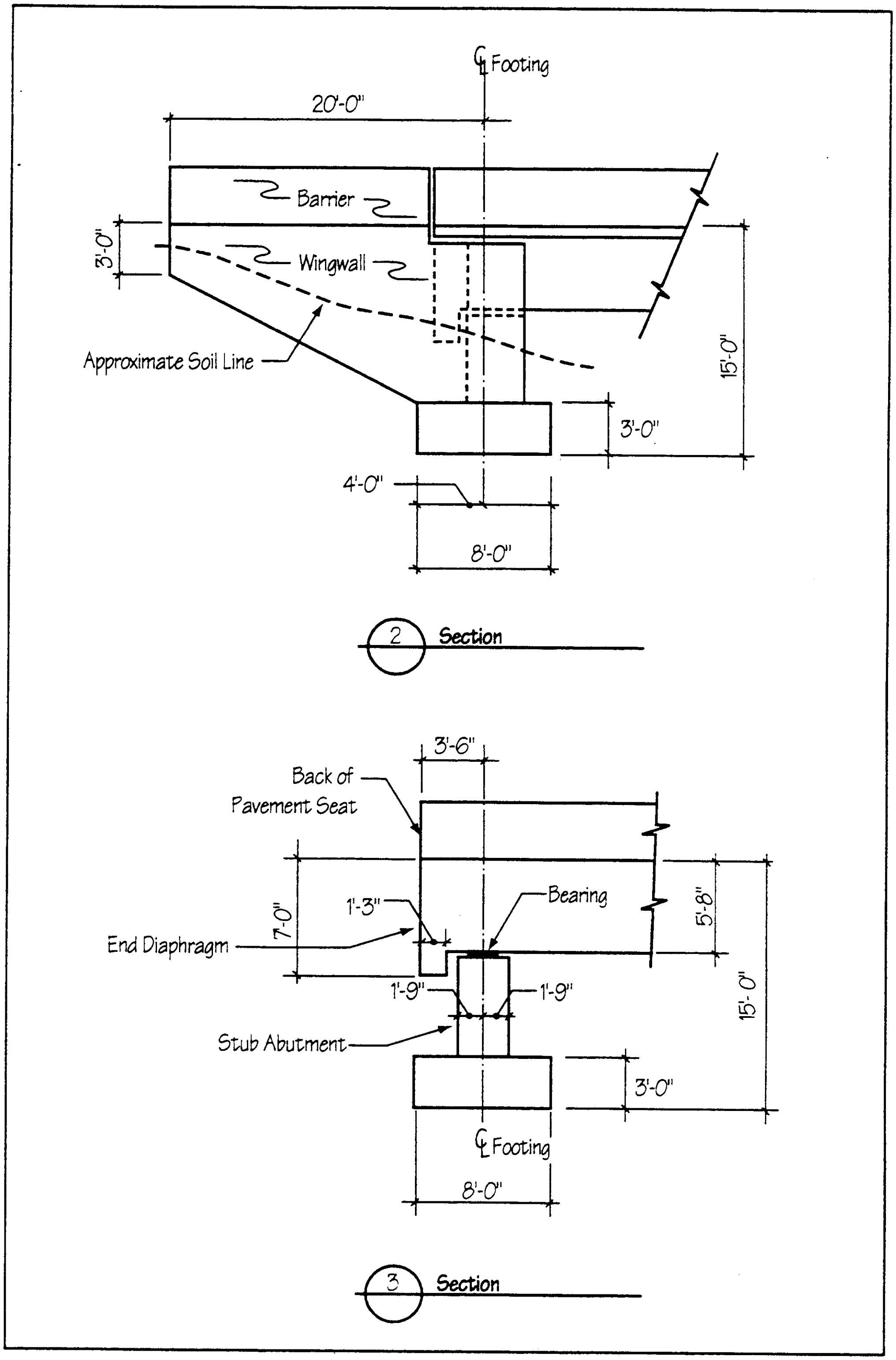


Figure 36c — Bridge Layout with Stub Abutment

DESIGN STEP 1
(continued)

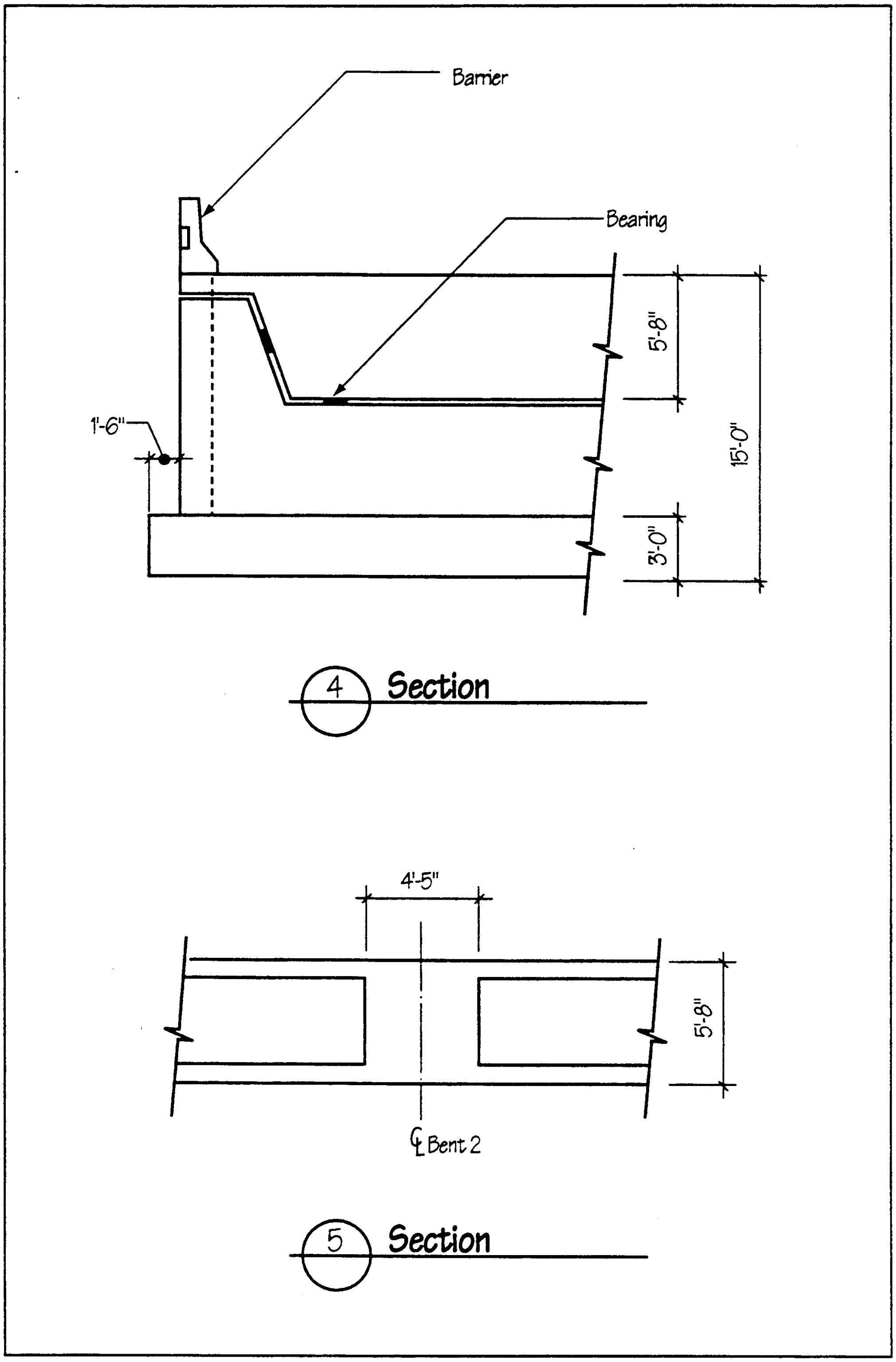


Figure 36d — Bridge Layout with Stub Abutment

DESIGN STEPS 2 THROUGH 5

No changes. The Single-Mode Spectral Method will be used for both the transverse and longitudinal analysis.

DESIGN STEP 6 — DETERMINE ELASTIC SEISMIC FORCES AND DISPLACEMENTS

Design Step 6.1 — Description of Model

The same "stick" model as was used previously is used here, except that springs are used to support the structure and the bent column stiffnesses are modified to account for realistic behavior. The determination of the foundation stiffnesses is discussed in Design Step 6.2. The column stiffness is discussed below.

Design Step 6.1.1 — Effective Column Stiffness

The use of gross or uncracked moments of inertia for reinforced concrete elements with small axial compression produces a model with a higher stiffness than the structure. In this bridge, the superstructure is post-tensioned; thus, the use of the gross moment of inertia for the superstructure is appropriate. However, for the columns, the axial load is small enough that the actual stiffness is somewhat less than that corresponding to the gross moment of inertia. The reduction in stiffness is the result of cracking along the height of the column that would occur in an earthquake.

In this portion of the example, the use of the effective moment of inertia in determining both forces and displacements will be demonstrated. It should be noted that there currently is not a consensus regarding the consideration of the effective column stiffness. This is the topic of ongoing debate within the bridge design community. Some groups advocate using the gross moment of inertia to determine internal forces and reactions and using the effective moment of inertia to determine maximum displacements. This is a bounding process and would lead to upper bounds for forces and displacements. In this example, it turns out that the AASHTO upper limit of the earthquake force coefficient controls even when the effective stiffness is used. Thus, the applied forces would not change, regardless of whether the effective or gross (uncracked) stiffnesses are used.

Note that even though the forces applied to the bridge are the same, the internal forces are distributed differently when the column moment of inertia is reduced.

Design Step 6.1.1 (continued)

Effective column moments of inertia that are related to axial load and steel content are given in Figure 37, which was developed for bridge columns by Priestley, Seible, and Chai (1992). The figures may be used to estimate the effective moment of inertia.

The axial dead load in the columns is approximately 1100 kips; thus for a 4-foot-diameter column and 4000 psi concrete the axial load is about 0.15 $f'_c A_g$. For a steel content of about 2 percent, the effective moment of inertia is roughly equal to 0.5 I_{gross}. This is used for the column moment of inertia in the spring-supported analyses.

Design Step
6.1.1
(continued)

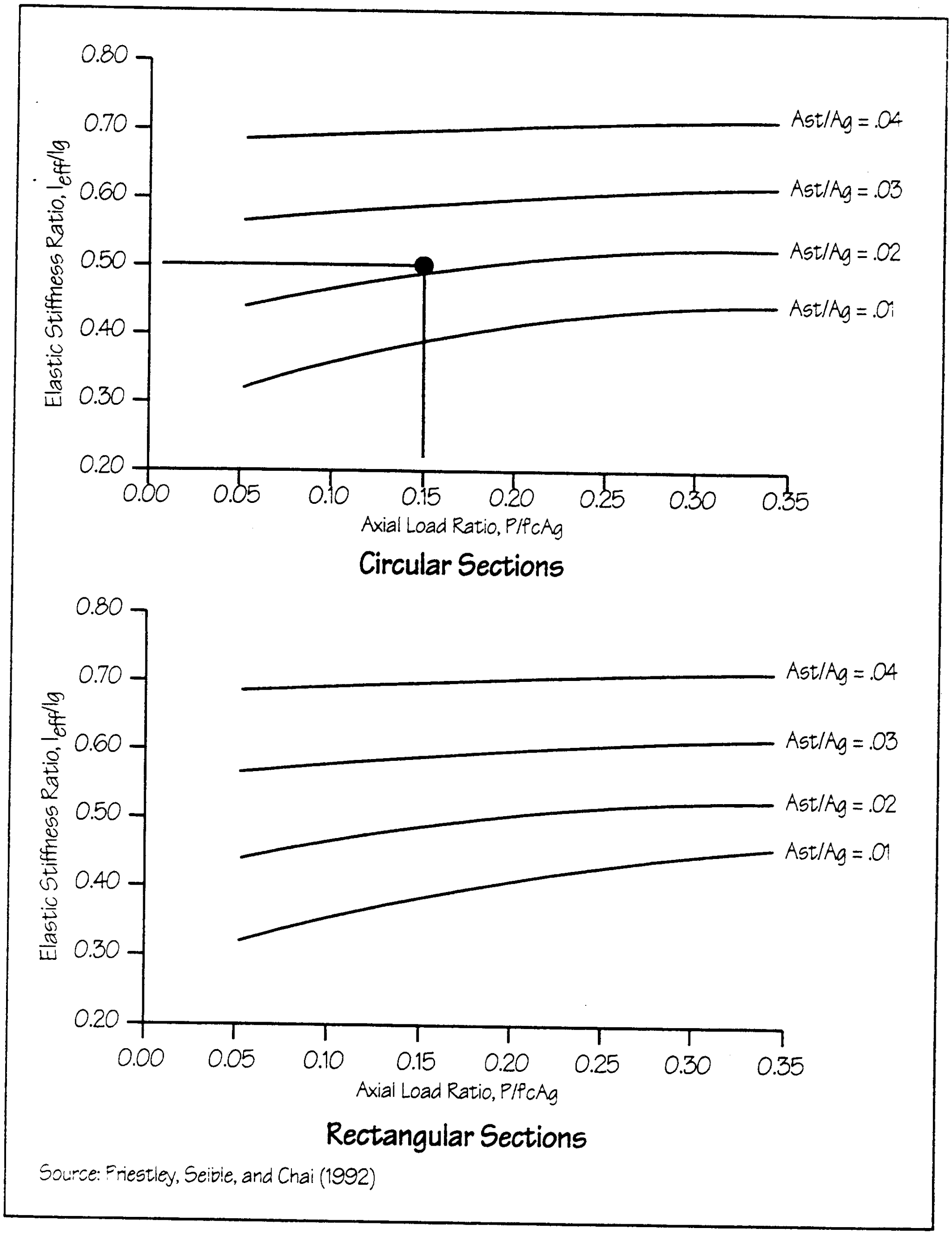


Figure 37 — Effective Stiffness of Bridge Columns

Design Step 6.2

Abutment and Bent Foundation Stiffnesses

[Division I-A, Article 7.4]

The bridge is supported with springs as shown in Figure 38.

The abutments are modeled with horizontal translational springs that act at the intersection of the superstructure work line and the centerline of the bearings. The abutments are fixed against vertical translation and rotation about the superstructure longitudinal axis (x axis). There is no restraint to rotation about the vertical (y axis) or the transverse axis (z axis).

The bent foundations are modeled with rotational springs that act about the longitudinal axis and transverse axis. Rotation about the vertical axis is fixed as is translation in each of the three directions.

Background

Establishing meaningful soil stiffnesses for bridge foundations is a complex problem. Often strain- and frequency-dependent properties exist. Furthermore, coupling may exist between adjacent degrees-of-freedom. Accurate modeling of these complex phenomena is more of an issue for time-history analyses than for static or modal analyses. Therefore such complex behavior is often simplified to linear springs that are used between fixed supports and the structure. Coupling is typically only a consideration for pile and drilled-shaft foundations.

There are several methods available for establishing spring constants for use in a seismic analysis. The complexity of the methods varies widely, as does the input information required.

One rational method for calculating foundation spring stiffnesses is to consider the foundation bonded to an elastic half-space. This is an elastic hemispherical medium that extends to infinity below the foundation, as shown in Figure 39. This in itself is an approximation, since it does not include actual ground contours, soil layering, or partial uplift. The development and application of elastic half-space constants is beyond the scope of this example. However, information about the method may be found in Bowles (1988), Richart, et al. (1970), and FHWA (1987).

Design Step 6.2 (continued)

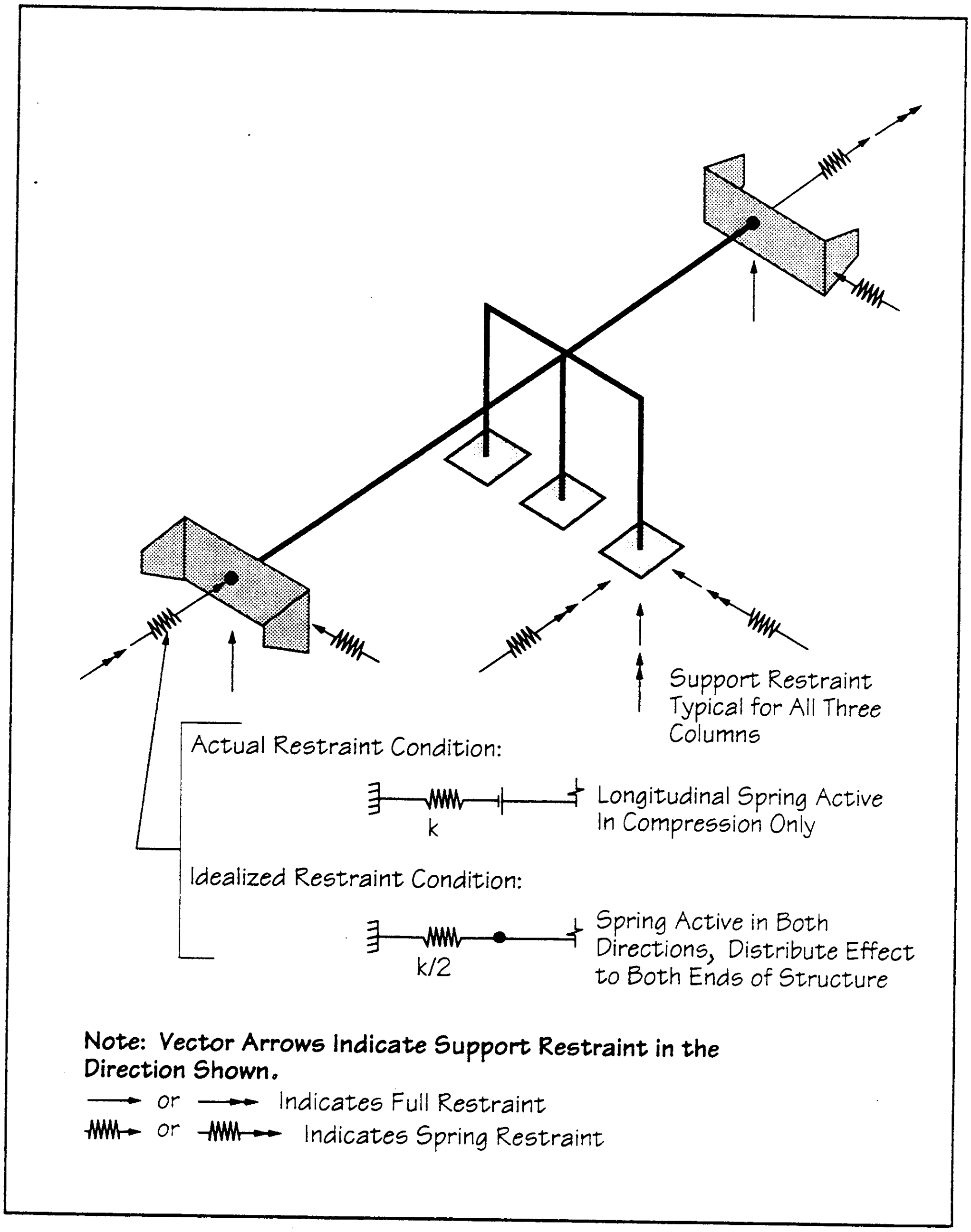


Figure 38 — Details of Supports for Spring Foundation Model

Design Step 6.2 (continued)

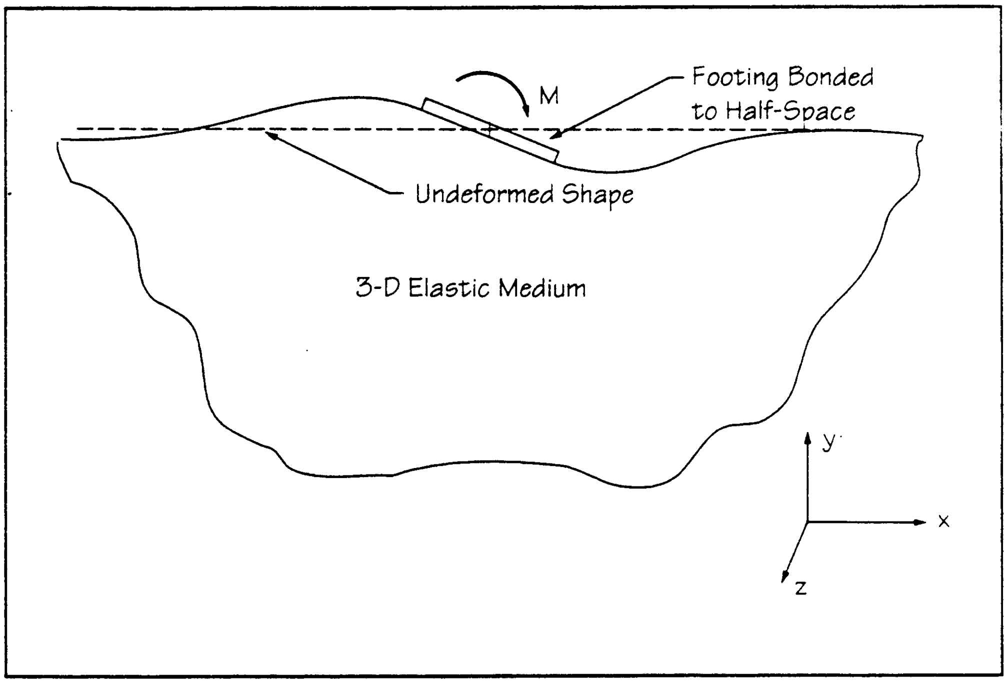


Figure 39 — Elastic Half-Space Deformation in X-Y Plane Due to an Applied Moment

Several of these references also discuss approaches based on the theory of elastic subgrade reaction. Although they are not as accurate in modeling the actual dynamic phenomenon as the half-space models, they work reasonably well for static and modal analyses. Essentially, they are beam on elastic foundation approaches, and in the simplest form, a rigid foundation is used with elastic subgrade springs. This type model is shown in Figure 40. As the name implies, the necessary input parameter is the modulus of subgrade reaction, which often is estimated from the bearing strength of the soil. The elastic subgrade approach is used in this example for the column foundations.

The elastic half-space method works reasonably well for spread footings. However, it does not lend itself to the treatment of abutments due to the irregular configuration of these elements. For this reason empirical methods have been used to predict spring constants for abutments. For instance, Caltrans uses a relatively simple empirical method for determining these constants. Due to the potentially high stresses that may occur in the soil adjacent to the abutments and due to the allowance for gaps between the superstructure and abutment, the method is iterative.

Design Step 6.2 (continued)

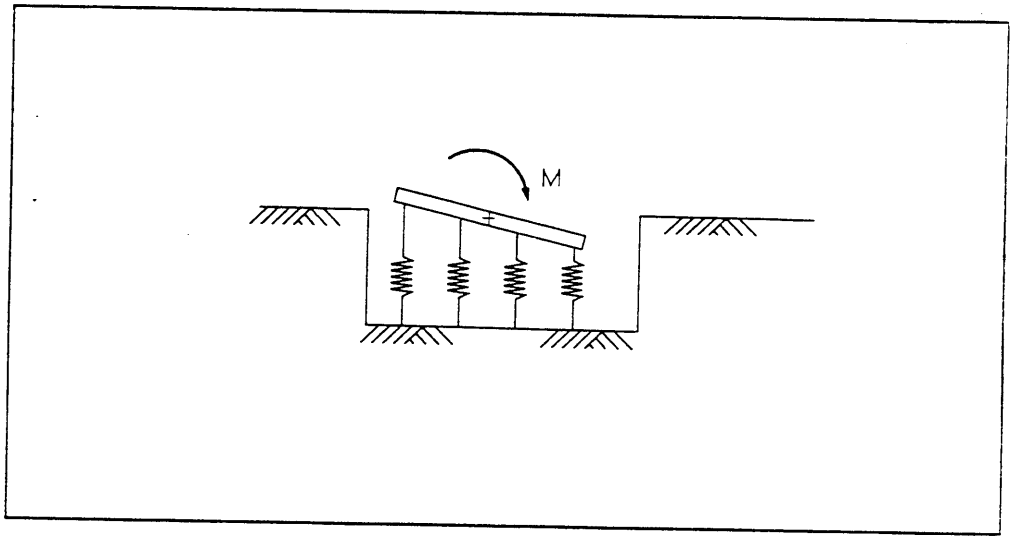


Figure 40 — Elastic Subgrade Model Deformation Due to an Applied Moment

Information on the application of the method is contained in the Caltrans' *Bridge Design Aids Manual* (1994). Discussion of the method is provided by Po Lam and Martin (1993) and recent test data is presented by Maroney, et al. (1994). The Caltrans method will be used in this example for the abutments.

The elastic subgrade method can also be used for abutments. Simplified procedures for using the method have been recently developed by Po Lam and Martin (1993).

a) Column Spread Footing Rotational Springs

Full translational restraint at the column bases is reasonable since the translational flexibility of the columns typically significantly exceeds that of the soil. Thus, the results will not be sensitive to the soil translational stiffness. This may not always be true, particularly when large diameter columns, rigid piers, or pile foundations on soft soils are used.

The elastic subgrade method for determining rotational stiffness assumes the footing is rigid relative to the soil and that the soil is an elastic foundation with a stiffness given by the modulus of subgrade reaction k_s. If the modulus is not known, it can be estimated based on the allowable bearing pressure and the factor of safety. Two methods of estimating the modulus are used; one is an equation given by Bowles (1988) and the other a figure given by PCI (1992), as shown in Figure 41.

Design Step 6.2 (continued)

Although one method alone would be sufficient for design, both will be used herein and the average used in the model.

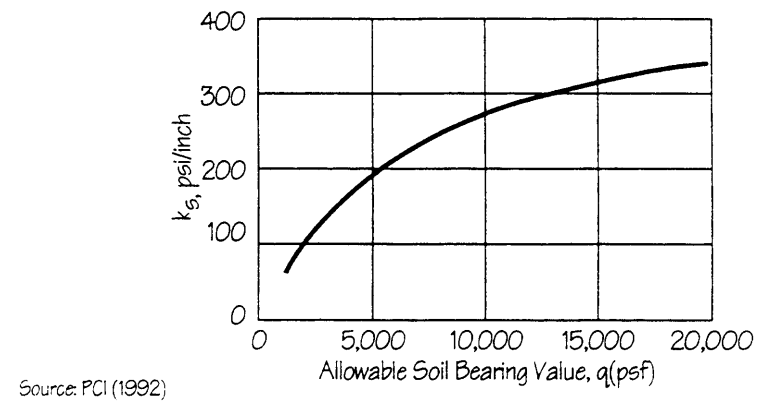


Figure 41 — Approximate Relationship Between Allowable Soil Bearing Pressure and Modulus of Subgrade Reaction

The spread footing can be characterized as shown in Figure 42. If a moment M is applied, a maximum stress σ is developed and a rotation results as shown. If the stress is assumed to vary linearly then the rotational stiffness can be determined as shown below.

Bowles Method

$B_f := 20 \cdot ft$ Width of footing

$L_f := 20 \cdot ft$ Length of footing in direction of applied moment

$q_a := 8 \cdot \frac{kip}{ft^2}$ Allowable bearing stress under footing

$FS := 3$ Factor of safety used in determining allowable bearing stress

Design Step 6.2
(continued)

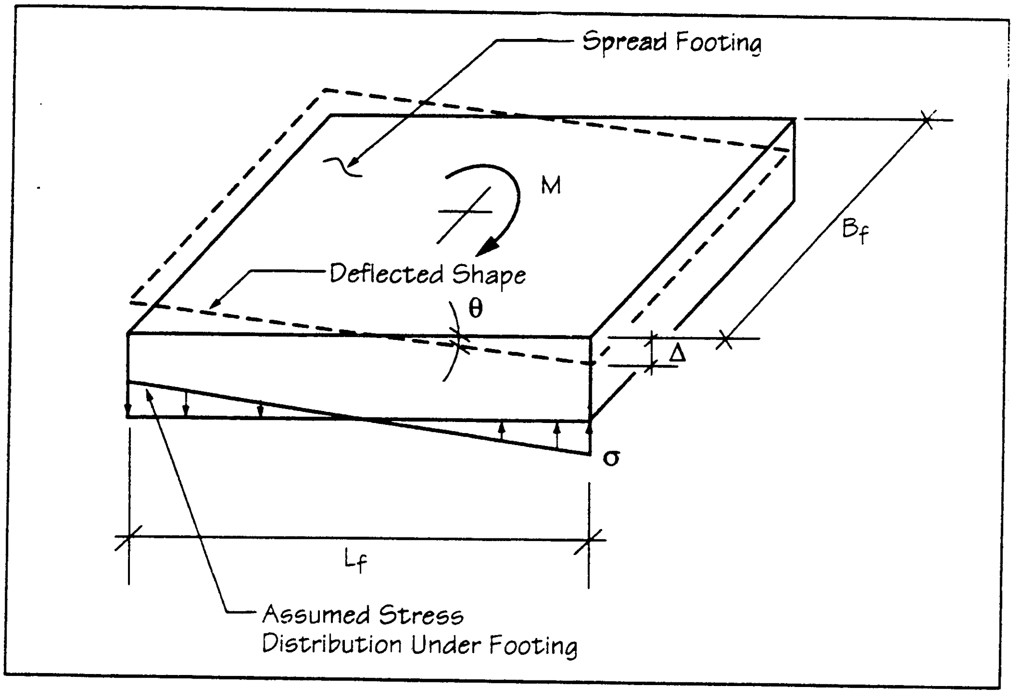


Figure 42 — Column Spread Footing Rotational Stiffness Definitions

$k_s := q_a \cdot FS \cdot 12 \cdot \frac{1}{ft}$

Modulus of subgrade reaction (empirical equation that assumes 1 inch of settlement at ultimate)

$k_s = 288 \cdot \frac{kip}{ft^3}$

It can be shown that the rotational stiffness k_r is given by the following expression.

$k_r := k_s \cdot \frac{B_f \cdot L_f^{\ 3}}{12}$ $k_r = 3840000 \cdot kip \cdot \frac{ft}{rad}$

PCI Method

$k_{r\_alt} := 5760000 \cdot kip \cdot \frac{ft}{rad}$ k_r based on Figure 41

Design Step 6.2 (continued)

The average rotational stiffness is

$$k_r := \frac{k_r + k_{r\_alt}}{2} \qquad k_r = 4800000 \cdot kip \cdot \frac{ft}{rad}$$

b) Abutment Longitudinal Springs (End Diaphragms)

The seismic behavior of soils behind abutments is nonlinear. This occurs, in part, because the soil exerts different pressures as the end diaphragm moves into the soil and as it moves away from the soil. Caltrans simplifies this problem into consideration of compression stress only, using 200 kip per inch per foot of width as an approximate stiffness that applies for an 8-foot-high wall. This is used to calculate a spring stiffness that is divided in half and distributed to either end of the bridge, as shown in Figure 38. This is necessary because an elastic dynamic model will develop both tension and compression forces in the elastic springs at abutments. The resultant forces at the abutment are then doubled for design to account for the fact that the soil is acting at one end of the bridge only at any given time.

Note that the springs at the abutments act in directions that are orthogonal to the abutment, not necessarily the superstructure. In this example, there is no difference. If skew is included, the springs at the abutments will be skewed with respect to the superstructure.

The calculations of the spring constants are given below.

Establish end diaphragm longitudinal stiffness using the Caltrans method.

$$k_p := 200 \cdot \frac{kip}{in \cdot ft}$$

Basic stiffness per foot of width based on an 8-ft-high wall and well-compacted backfill material

The 200-kip/inch/foot stiffness is assumed to be independent of wall height by some designers, and is assumed to scale linearly with height by others. In this example, the average of these two methods will be used. It is rational to expect that the actual flexibility is a function of end diaphragm height. Note also that recent large scale experiments have shown that the 200 kip/inch/foot may not be a realistic value (Maroney, et al., 1994). Therefore, this value may be refined in the future.

Design Step 6.2 (continued)

Recall the following dimensions.

$H_a := 7 \cdot ft$ Height of end diaphragm

$W_a := 74 \cdot ft$ Width of end diaphragm (could use actual width between wingwalls)

Calculate the stiffness with and without the correction.

$k_{a1} := k_p \cdot W_a$ $\quad k_{a1} = 177600 \cdot \frac{kip}{ft}$ No correction for height

$k_{a2} := k_p \cdot W_a \cdot \frac{7}{8}$ $\quad k_{a2} = 155400 \cdot \frac{kip}{ft}$ Linear correction for height

Use the average of the two stiffnesses.

$k_a := \frac{k_{a1} + k_{a2}}{2}$ $\quad k_a = 166500 \cdot \frac{kip}{ft}$ Average value of k_a

Use half of the stiffness applied at either end of the bridge.

$k_a := \frac{k_a}{2}$ $\quad k_a = 83250 \cdot \frac{kip}{ft}$

This spring constant is valid provided that the strength of the soil behind the end diaphragm is not exceeded. If the strength is exceeded, the soil will develop a dynamic passive failure wedge. There are rational methods that are used to calculate the force required to move this soil wedge, but these are somewhat complex. Therefore, a simple method used by Caltrans (1994) will be used to estimate this force in this example.

Caltrans uses a uniform pressure of 5 ksf as the static equivalent of that required to produce incipient movement of the wedge. This is divided by 0.65, which approximates the ratio of maximum static resistance stress to maximum dynamic resistance under cyclic loading. The force corresponding to the dynamic pressure is used as a check. An elastic analysis is performed to determine the forces at the abutments. If this

Design Step 6.2 (continued)

indicated value exceeds the failure load, the assumed stiffness based on 200 kip per inch per foot is reduced to approximate the yielding effect of the soil failure wedge movement. This is an iterative process that is used until a valid solution is obtained. The forces will be checked later.

Note that some designers adjust this pressure for heights other than 8 feet. This is not done here. The maximum force is calculated below.

Determine maximum force that can be delivered to end diaphragm using Caltrans approximate method.

$p_{max} := 5 \cdot \frac{kip}{ft^2}$ Assumed maximum static pressure

$p_{max} := \frac{p_{max}}{0.65}$ Adjust to account for seismic (dynamic) increase in maximum pressure (empirical)

$p_{max} = 7.7 \cdot \frac{kip}{ft^2}$

$F_{max} := p_{max} \cdot H_a \cdot W_a$ Maximum force based upon estimated dynamic strength

$F_{max} = 3985 \cdot kip$ To be used to check maximum longitudinal seismic force in Design Step 6.4

c) Abutment Transverse Springs (Wingwalls)

The same stiffness that is used behind the end diaphragm is used with the wingwalls. Because wingwalls typically taper, an effectiveness factor of two-thirds is used to calculate the spring constant. This simply means that the 200 kip per inch per foot is applied along the length of the wall, then the height correction is made as before and then the stiffness is reduced by 33 percent. Additionally, one wingwall is typically bearing against soil that is sloping away from the bridge. The value of the stiffness for one wall is then lowered an arbitrary amount to account for the reduced effectiveness. A 67 percent reduction is assumed here, although some designers prefer to ignore the contribution from that wingwall.

Design Step 6.2 (continued)

The calculation of the transverse stiffness of the abutment is given below.

Calculate the lateral spring constants for wingwalls using Caltrans approximate method.

Use basic stiffness kp to determine the spring stiffnesses. Also assume that the wingwall is two-thirds effective due to taper.

$L_w := 20 \cdot ft$ Length of wingwall in longitudinal direction

$H_w := 12 \cdot ft$ Height of wingwall at its tallest point

Calculate the stiffness of one wingwall.

$k_{w1} := \frac{2}{3} \cdot k_p \cdot L_w$ $k_{w1} = 32000 \cdot \frac{kip}{ft}$ No correction for height

$k_{w2} := \frac{2}{3} \cdot k_p \cdot L_w \cdot \frac{H_w}{8 \cdot ft}$ $k_{w2} = 48000 \cdot \frac{kip}{ft}$ Linear correction for height

Average the two stiffnesses.

$k_w := \frac{k_{w1} + k_{w2}}{2}$ $k_w = 40000 \cdot \frac{kip}{ft}$

The wingwalls are parallel to the roadway. Thus, one wingwall acts against fill that is sloping downward away from the bridge and one wingwall acts against the soil under the roadway. Assume that one wall is one-third effective and one is fully effective. Some designers may chose to ignore the contribution of the wingwall acting against the sloping fill. Note that this approach neglects any contribution due to friction along the backwall.

Increase the single wingwall stiffness to account for both walls.

$k_w := k_w \cdot 1.33$ $k_w = 53200 \cdot \frac{kip}{ft}$

Design Step 6.2 (continued)

A check on the maximum soil resistance that can be generated by the wingwall should be made in a similar fashion to that done in the longitudinal direction.

Design Step 6.3

Transverse Analysis, Single-Mode Spectral Method

[Division I-A, Article 4.4]

Because the analysis procedure is identical to that used for the transverse analysis of the basic foundation case of Section III, only the important results will be summarized.

The displacements due to a uniform transverse load of 100 kip/foot were calculated and were entered into Column 4 of Table 13. The constants α, β, and γ were calculated and used to generate an equivalent transverse static loading. The intensities of this loading are given at the superstructure quarter points as shown in Figure 43.

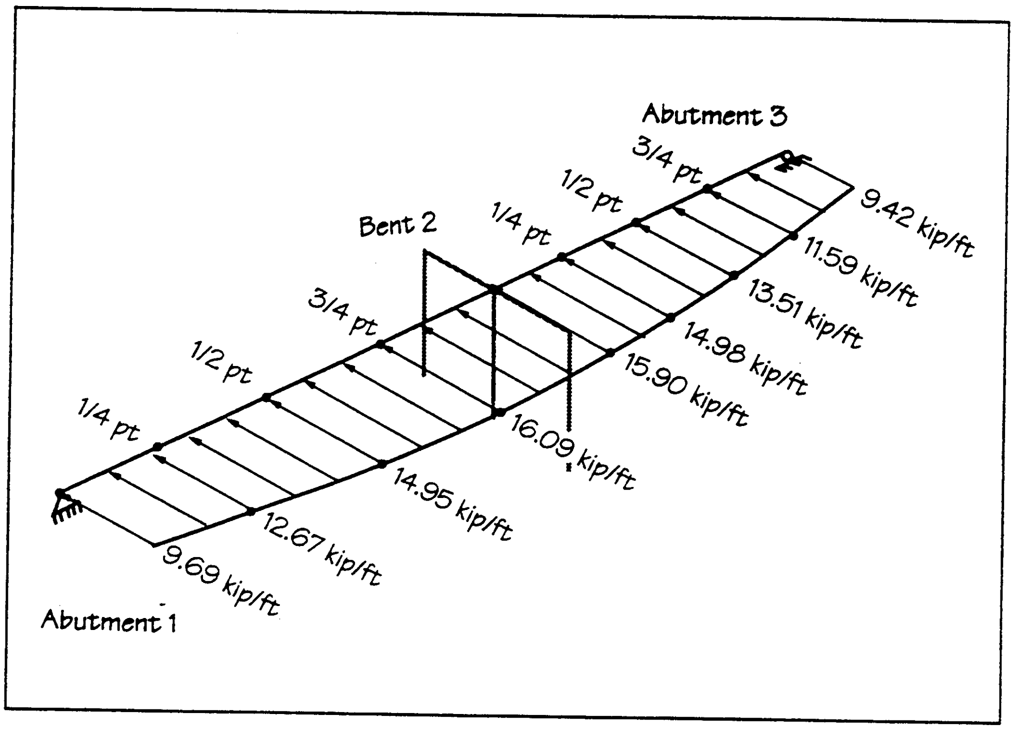


Figure 43 — Single-Mode Spectral Loads for Transverse Direction

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

Design Step 6.3 (continued)

Table 13
AASHTO Single-Mode Spectral Analysis Method for Calculation of Seismic Load

Assumptions: p_o = 100.0 k/ft; g = 32.2 ft/sec^2; w(x) = 20.1 k/ft; A = 0.28; 2.5*A = 0.70; S = 1.2

1	2	3	4	5	6	7	8
Location	Node Distance x (ft)	Tributary Length dx (ft)	Displ Due to Uniform Loading $v_s(x)$ (ft)	$\alpha(x)$ (ft^2)	$\beta(x)$ (k-ft)	$\gamma(x)$ (k-ft^2)	Equiv. Static EQ Loading $p_e(x)$ (k-ft)
Abut 1	0.0	0.0	0.2174	0.00	0.00	0.00	9.69
1/4 pt	35.5	35.5	0.2843	8.91	178.99	45.70	12.67
1/2 pt	71.0	35.5	0.3356	11.00	221.15	69.01	14.95
3/4 pt	106.5	35.5	0.3610	12.36	248.52	86.67	16.09
Bent 2	142.0	35.5	0.3567	12.74	256.06	91.89	15.90
1/4 pt	167.0	25.0	0.3362	8.66	174.09	60.37	14.98
1/2 pt	192.0	25.0	0.3031	7.99	160.62	51.48	13.51
3/4 pt	217.0	25.0	0.2600	7.04	141.48	40.07	11.59
Abut 3	242.0	25.0	0.2114	5.89	118.44	28.21	9.42
Sum =		242.0		74.59	1499.35	473.39	

(continued on next page)

	Basic Transverse	Basic Longitudinal	Spring Transverse	Spring Longitudinal
Uniform Load				
Single-Mode			▬	
Multimode				

Design Step 6.3 (continued)

Table 13 (continued)
AASHTO Single-Mode Spectral Analysis Method for Calculation of Seismic Load

Summary of Coefficients

α =	74.59 ft^2	α = sum of [$v_s(x)$ *dx]
β =	1499.35 k-ft	β = sum of [w(x) * $v_s(x)$ * dx]
γ =	473.39 $k\text{-}ft^2$	γ = sum of [w(x) * $v_s(x)^2$ * dx]

Calculate Period T, C_s and $p_e(x)$

T =	0.279 sec	$T = 2\pi * (\gamma / p_o * g * \alpha)^{1/2}$
C_s =	0.94	$C_s = 1.2*A*S / T^{2/3}$ or
C_s (min) =	0.70	$C_s = C_s$ or 2.5*A (whichever is less)
$p_e(x)$ =	44.6 *$v_s(x)$	$p_e(x) = \beta * C_s / \gamma * w(x) * v_s(x)$

The period in the transverse direction for the spring-supported bridge is T = 0.279 second.

The SAP90 input file for this analysis is F1TS4SM.

The response values of the structure, both internal forces and displacements, are given in Table 14.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode			▬	
Multimode				

Design Step 6.3 (continued)

Table 14
Response for Single-Mode Method, Transverse Direction, and Spring Foundation

			Forces and Moments				
			Longitudinal		Transverse		
			Shear (kips)	Moment (kip-ft)	Shear (kips)	Moment (kip-ft)	Axial (kips)
Abutment 1			0	0	1550	687	0
Bent 2	Center Column	Top	0	0	89.5	1253	0
		Bottom	0	0	89.5	1016	0
	Outboard Column	Top	4.1	57.2	89.2	1248	50.1
		Bottom	4.1	46.5	89.2	1012	50.1
Abutment 3			0	0	1494	975	0

Displacements (ft)								
Abut 1	1/4 pt	1/2 pt	3/4 pt	Bent 2	1/4 pt	1/2 pt	3/4 pt	Abut 3
0.0298	0.0396	0.0471	0.0509	0.0502	0.0472	0.0422	0.0359	0.0287

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

Design Step 6.4

Longitudinal Analysis, Single-Mode Spectral Method
[Division I-A, Article 4.4]

Perform a longitudinal analysis for the spring-support condition.

Design Step 6.4.1

Static Displacements
[Division I-A, Article 4.4, *(Step 1)*]

The tabular method of calculating the equivalent loading to use for the Single-Mode Spectral Method in the longitudinal direction will be used below to show the application of the table for loading in this direction.

The displacements of the bridge under a uniform longitudinal loading are calculated. In this example, these are the displacements calculated for a 100-kip/foot longitudinal load. These are shown in Table 15 in Column 4.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				▬
Multimode				

Design Step 6.4.1 (continued)

Table 15
AASHTO Single-Mode Spectral Analysis Method for Calculation of Seismic Load

Assumptions:

p_o =	100.0	k/ft	A =	0.28
g =	32.2	ft/sec^2	2.5*A =	0.70
w(x) =	20.1	k/ft	S =	1.2

1	2	3	4	5	6	7	8
Location	Node Distance x (ft)	Tributary Length dx (ft)	Displ Due to Uniform Loading $v_s(x)$ (ft)	α (x) (ft^2)	β (x) (k-ft)	γ (x) (k-ft^2)	Equiv. Static EQ Loading $p_e(x)$ (k-ft)
Abut 1	0.0	0.0	0.1414	0.00	0.00	0.00	13.37
1/4 pt	35.5	35.5	0.1471	5.12	102.93	14.85	13.91
1/2 pt	71.0	35.5	0.1508	5.29	106.28	15.83	14.26
3/4 pt	106.5	35.5	0.1524	5.38	108.17	16.40	14.41
Bent 2	142.0	35.5	0.1520	5.40	108.60	16.53	14.38
1/4 pt	167.0	25.0	0.1508	3.79	76.08	11.52	14.26
1/2 pt	192.0	25.0	0.1486	3.74	75.22	11.26	14.06
3/4 pt	217.0	25.0	0.1454	3.68	73.87	10.86	13.75
Abut 3	242.0	25.0	0.1412	3.58	72.01	10.32	13.36
Sum =		242.0		35.98	723.17	107.58	

(continued on next page)

Design Step 6.4.1 (continued)

Table 15 (continued)
AASHTO Single-Mode Spectral Analysis Method for Calculation of Seismic Load

Summary of Coefficients

$\alpha =$	35.98 ft^2	α = sum of [$v_s(x)$ *dx]
$\beta =$	723.17 k-ft	β = sum of [w(x) * $v_s(x)$ * dx]
$\gamma =$	107.58 k-ft^2	γ = sum of [w(x) * $v_s(x)^2$ * dx]

Calculate Period T, C_s and $p_e(x)$

T =	0.191 sec	$T = 2\pi * (\gamma / p_o * g * \alpha)^{1/2}$
C_s =	1.21	$C_s = 1.2*A*S / T^{2/3}$ or
C_s (min) =	0.70	$C_s = C_s$ or 2.5*A (whichever is less)
$p_e(x)$ =	94.6 *$v_s(x)$	$p_e(x) = \beta * C_s / \gamma * w(x) * v_s(x)$

Design Step 6.4.2

Calculate α, β, and γ Factors
[Division I-A, Article 4.4, *(Step 2)*]

From the last row of Columns 5, 6, and 7, the factors are

$\alpha = 35.98\ ft^2$
$\beta = 723.2$ k-ft
$\gamma = 107.7$ k-ft^2

Design Step 6.4.3

Fundamental Period
[Division I-A, Article 4.4, *(Step 3)*]

The period has also been calculated in the table and the result is given below.

T = 0.191 second

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				▬
Multimode				

Design Step 6.4.4

Equivalent Longitudinal Load
[Division I-A, Article 4.4, *(Step 4)*]

The intensities of loading are given in Column 8 of Table 15.

Design Step 6.4.5

Displacements and Member Forces
[Division I-A, Article 4.4, *(Step 5)*]

The response values of the structure, both internal forces and displacements, are given in Table 16. The SAP90 input file for this analysis is F1LS4SM.

At this point, the longitudinal force developed at the abutment must be checked. If the force exceeds that calculated in Design Step 6.2, then the longitudinal stiffness should be adjusted and the analysis repeated.

From the analysis, the maximum longitudinal force is 1636 kips at each end. This value is doubled to account for the fact that the springs were divided in half.

The resulting maximum force is 3272 kips, which is less than the maximum strength, 3985 kips, calculated in Design Step 6.2.

Thus, the analysis is adequate, and no revision is necessary.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

Design Step
6.4.5
(continued)

Table 16
Response for Single-Mode Method, Longitudinal Direction, and Spring Foundation

			Forces and Moments				
			Longitudinal		Transverse		
			Shear (kips)	Moment (kip-ft)	Shear (kips)	Moment (kip-ft)	Axial (kips)
Abutment 1			1639	0	0	0	4.9
Bent 2	Center	Top	34.6	471	0	0	1.7
		Bottom	34.6	405	0	0	1.7
	Outboard	Top	34.3	467	0	0	1.7
		Bottom	34.3	403	0	0	1.7
Abutment 3			1636	0	0	0	10

Displacements (ft)								
Abut 1	1/4 pt	1/2 pt	3/4 pt	Bent 2	1/4 pt	1/2 pt	3/4 pt	Abut 3
0.0197	0.0205	0.021	0.0213	0.0212	0.0211	0.0207	0.0203	0.0197

DESIGN STEP 7

DETERMINE DESIGN FORCES FOR SPRING SUPPORT CONDITION
[Division I-A, Article 7.2.1]

The spring support condition is very stiff both in the longitudinal direction, because of the soil spring, and in the transverse direction, because the bridge superstructure acts as a large diaphragm spanning between the abutments. Consequently, Bent No. 2 intermediate columns attract very little seismic force in either orthogonal direction. Hence, the columns will probably not hinge for this support condition. Therefore, it is not necessary to check the plastic hinging forces.

Design Step 7 of Section IV of the design example is patterned after Section III, except that it will be abbreviated to only address the modified forces. Some of the explanations are also omitted, for brevity.

Design Step 7.1 — Determine Nonseismic Forces

Design Step 7.1.1 — Determine Dead Load Forces

The dead load forces are summarized in Table 17 below. Note that there are longitudinal shear forces at the abutment, which account for the soil springs resisting the longitudinal sidesway, assuming the shoring is removed after the abutments and superstructure are complete and the backfill placed.

Table 17
Dead Load Forces with Spring Supports

		Longitudinal Force		Transverse Force		Vertical
		V_L	M_L	V_T	M_T	P
Location	Load Case	(kips)	(k-ft)	(kips)	(k-ft)	(+/- kips)
Abut 1	Dead Load	60	0	0	0	1071
Bent 2 Columns	Dead Load	41	302	0	0	1096
Abut 3	Dead Load	63	0	0	0	586

Note that column forces are per column.

Design Step 7.2

Determine Seismic Forces

Design Step 7.2.1

Summary of Elastic Seismic Forces

The Single-Mode Spectral Method results for the spring support condition are used to determine the modified design forces.

A summary of the full elastic seismic forces for an earthquake along each of the principal axis (both longitudinal and transverse) is shown in Table 18, which is a condensed version of results from Tables 14 and 16.

Table 18
Full Elastic Seismic Forces with Spring Supports

Location	Load Case	Longitudinal Force		Transverse Force		Vertical
		V_L (kips)	M_L (k-ft)	V_T (kips)	M_T (k-ft)	P (+/- kips)
Abutment 1	Longitudinal Analysis *	3275	0	0	0	5
	Transverse Analysis	0	0	1550	687	0
Bent 2	Longitudinal Analysis	35	467	0	0	2
Columns **	Transverse Analysis	4	57	89	1248	50
Abutment 3	Longitudinal Analysis *	3275	0	0	0	10
	Transverse Analysis	0	0	1494	975	0

The longitudinal earthquake force into the soil can only be at one end of the superstructure at a time. Because the spring stiffness used at each end of the model is half of the total value, the tension and compression loads at each end (see Table 16) must be added together (absolute value) to get the maximum force occurring in the soil at either of the ends.

$V_L = 1639 + 1636 = 3275$ kips

Also, the values for the Bent No. 2 columns are maximum values from center or exterior columns at both top and bottom.

Design Step 7.2.2

Combination of Orthogonal Seismic Forces [Division I-A, Article 3.9]

See Table 19 for a summary of the seismic forces for LC1 and LC2.

The definition of LC1 and LC2 is as follows.

LC1 = 100 percent of the Longitudinal Analysis Results + 30 percent of the Transverse Analysis Results

LC2 = 30 percent of the Longitudinal Analysis Results + 100 percent of the Transverse Analysis Results

For example, in Bent No. 2, the longitudinal column moment for LC1 is derived as follows.

$M = (1.0 * M_L) + (0.3 * M_T)$
$M = (1.0 * 467) + (0.3 * 57) = 484$ k-ft.

All other forces in the table are calculated similarly.

Table 19
Orthogonal Seismic Force Combinations
LC1 and LC2 with Spring Supports

Location	Load Case	Longitudinal Force V_L (kips)	Longitudinal Force M_L (k-ft)	Transverse Force V_T (kips)	Transverse Force M_T (k-ft)	Vertical P (+/- kips)
Abutment 1	Load Case 1	3275	0	465	206	5
	Load Case 2	983	0	1550	687	2
Bent 2 Columns	Load Case 1	36	484	27	374	17
	Load Case 2	15	197	89	1248	51
Abutment 3	Load Case 1	3275	0	448	293	10
	Load Case 2	983	0	1494	975	3

Design Step 7.3

Determine Modified Design Forces
[Division I-A, Article 7.2.1(A)]

Design Step 7.3.1

Modified Design Forces for Structural Members and Connections

Even though the Bent No. 2 columns will probably remain elastic for the spring support condition, the Specification allows the elastic seismic column moments to be reduced by R = 5. Use Equation (7-1) in Division I-A to calculate the modified forces for members and connections.

Group Load = 1.0 (D + B + SF + E + EQM)

Division 1-A
Eqn (7-1)

For this problem, forces B, SF, and E are assumed zero, only D and EQM forces are combined. The equation reduces to

Group Load = 1.0 (D + EQM)

Where EQM = (LC1 or LC2 forces) divided by R

a) Response Modification Reduction Factor, R
[Division I-A, Article 3.7, Table 3]

The R Factor remains the same for either support condition.

R reduces the seismic column moments, but increases the seismic lateral shear force on the connection of the superstructure to the abutment. A summary of the R values used to modify EQM is presented below.

R = 5.0 For moments in multiple column bents
R = 0.8 For shear and axial connection force of superstructure to abutment
R = 1.0 For connection of column to superstructure or foundation

b) Calculate the Modified Design Forces with EQM

Table 20 summarizes the modified design forces using EQM.

Design Step 7.3.1 (continued)

For example, in Bent No. 2, the longitudinal column moment using LC1 is derived as follows.

$M = (D + EQ/R)$
$M = (302 + 484/5) = 399$ k-ft

All other forces in the table are calculated similarly.

Table 20
Modified Design Forces for Structural Members and Connections with Spring Supports

Location	Load Case	Longitudinal Force		Transeverse Force		Vertical Force	
		V_L (kips)	M_L (k-ft)	V_T (kips)	M_T (k-ft)	P_{max} (kips)	P_{min} (kips)
Abutment 1	R Factor Used in EQM	1	-	0.8	-	0.8	0.8
	Load Case 1	3335	-	581	-	1077	1065
	Load Case 2	1043	-	1938	-	1073	1069
Bent 2 Columns	R Factor Used in EQM	1	5.0	1	5.0	1	1
	Load Case 1	77	399	27	75	1113	1079
	Load Case 2	56	341	89	250	1147	1045
Abutment 3	R Factor Used in EQM	1	-	0.8	-	0.8	0.8
	Load Case 1	3338	-	560	-	599	574
	Load Case 2	1046	-	1868	-	590	582

Design Step 7.3.2

Modified Design Forces for Foundations
[Division I-A, Article 7.2.1(B)]

Use Equation (7-2) in Division I-A to calculate the maximum forces in the bent column foundations.

Group Load = 1.0 (D + B + SF + E + EQF) Division 1-A Eqn (7-2)

Design Step
7.3.2
(continued)

For this problem, forces B, SF, and E are assumed zero; only D and EQF forces are combined. The equation reduces to

Group Load = 1.0 (D + EQF)

Where EQF = (LC1 or LC2 forces) divided by R

a) Recall the Response Modification Reduction Factor, R
[Division I-A, Article 7.2.1(B)]

R = 1 for calculating the modified design forces in the foundation

b) Calculate the Modified Design Forces with EQF

Table 21 summarizes the values of EQF modified design forces.

For example, in Bent No. 2, the longitudinal column moment using LC1 is derived as follows.

$M = (D + EQ/R)$
$M = (302 + 484/1.0) = 786$ k-ft

All other forces in the table are calculated similarly.

Design Step 7.3.2 (continued)

Table 21
Modified Design Forces for Foundations with Spring Supports

Location	Load Case	Longitudinal Force		Transverse Force		Vertical Force	
		V_L (kips)	M_L (k-ft)	V_T (kips)	M_T (k-ft)	P_{max} (kips)	P_{min} (kips)
Abutment 1	R Factor Used in EQF	1	-	1	-	1	1
	Load Case 1	3335	-	465	-	1076	1066
	Load Case 2	1043	-	1550	-	1073	1070
Bent 2 Columns	R Factor Used in EQF	1	1	1	1	1	1
	Load Case 1	76	719	27	304	1113	1079
	Load Case 2	55	470	89	1012	1147	1045
Abutment 3	R Factor Used in EQF	1	-	1	-	1	1
	Load Case 1	3338	-	448	-	596	576
	Load Case 2	1046	-	1494	-	589	583

Design Step 7.4

Plastic Hinging Forces

As was discussed previously, for the spring support condition, the modified design forces are so small that it is not necessary to calculate the plastic hinging forces in the columns. By inspection, the plastic hinging forces would all be larger; therefore, the modified design forces will be used.

Design Step 7.5

Summary of Forces

Design Step 7.5.1

Summary of Column Forces

Table 22 summarizes the forces at the top of the outboard columns of Bent No. 2. These forces were derived in Design Steps 7.1 to 7.4. Note that plastic hinging forces were not calculated. At this point, because the seismic forces are so small, it is likely that a nonseismic load case will control. However, this design example only addresses the seismic load cases.

Design Step
7.5.1
(continued)

Table 22
Summary of Forces on Bent 2 Columns with Spring Supports

Description of Force Type	Load Case	Longitudinal Force V_L (kips)	Longitudinal Force M_L (k-ft)	Transverse Force V_T (kips)	Transverse Force M_T (k-ft)	Vertical Force P_{max} or P (kips)	Vertical Force P_{min} (kips)
Dead Load	Dead Load	41	302	0	0	1096	-
Elastic Seismic Forces	Longitudinal Analysis	35	467	0	0	2	-
	Transverse Analysis	4	57	89	1248	50	-
Orthogonal Seismic Forces	Load Case 1	36	484	27	374	17	-
	Load Case 2	15	197	89	1248	51	-
Modified Forces Member/Conn	Load Case 1	77	399	27	75	1113	1079
	Load Case 2	56	341	89	250	1147	1045
Modified Forces Foundations	Load Case 1	76	719	27	304	1113	1079
	Load Case 2	55	470	89	1012	1147	1045

Plastic Hinging Forces Not Calculated for Spring Support Condition.

DESIGN STEP 8

SUMMARY OF DESIGN FORCES

This design step is similar to Design Step 8 in Section III, except that the spring support condition is used. As was discussed previously, the plastic hinging forces were not calculated in Design Step 7.

Design Step 8.1

Column or Pile Bent Design Forces

[Division I-A, Article 7.2.3]

a) Axial Forces per Division I-A, Article 7.2.3(a)

Use the modified forces (same as the elastic forces because R = 1.0) calculated in Design Step 7.3.1 and summarized in Table 22.

Elastic (Modified) Forces

$Pmin_u = 1045$ kips $Pmax_u = 1147$ kips ← Use

Hinging Forces

Not calculated.

b) Moments per Division I-A, Article 7.2.3(b)

Use the modified moments calculated in Design Step 7.3.1 and summarized in Table 22. LC2 controls

$M_L := 341 \cdot kip \cdot ft$ Longitudinal moment

$M_T := 250 \cdot kip \cdot ft$ Transverse moment

For a circular column, the modified biaxial bending moment can be converted to a moment about a single axis by

$M_u := \sqrt{M_L^2 + M_T^2}$ $M_u = 423 \cdot kip \cdot ft$

Design Step 8.1 (continued)

c) Shear Forces per Division I-A, Article 7.2.3(c)

Use the modified forces (same as the elastic forces because R = 1.0) calculated in Design Step 7.3.1 and summarized in Table 22. LC2 controls

Elastic (Modified) Forces

$V_L := 56 \cdot kip \cdot ft$ Longitudinal shear

$V_T := 89 \cdot kip \cdot ft$ Transverse shear

For a circular column, the modified biaxial shear can be converted to a shear about a single axis by

$V_u := \sqrt{V_L^2 + V_T^2}$ $V_u = 105 \cdot kip \cdot ft$

Hinging Forces Not calculated

d) Summary of Column Forces

Note that both M_u and V_u are the vector sum of the orthogonal forces.

$Pmin_u = 1045$ kips $Pmax_u = 1147$ kips
$M_u = 423$ kip-ft
$Vmax_p = 105$ kips

Design Step 8.2

Pier Design Forces

Not applicable.

Design Step 8.3

Connection Design Forces
[Division I-A, Article 7.2.5]

Design Step 8.3.1

Longitudinal Linkage Connections
[Division I-A, Article 7.2.5(A)]

There are no linkage connections; therefore, provisions are not applicable.

Design Step 8.3.2

Hold-Down Devices
[Division I-A, Article 7.2.5(B)]

The vertical seismic reaction at the abutment is negligible; therefore, no hold downs are required.

Design Step 8.3.3

Column and Pier Connection to Cap Beam
[Division I-A, Article 7.2.5(C)]

For seismic loads, the recommended connection design forces between the column and superstructure cap beam are the forces developed at the top of the column due to column hinging. The code allows the designer to use the modified forces calculated in Design Step 7.3.1. Because the column will probably remain elastic, use the modified forces, already summarized in Design Step 8.1(d).

Design Step 8.3.4

Column Connection to Foundation
[Division I-A, Article 7.2.5(C)]

The recommended connection design force between the column and foundation are the forces developed at the bottom of the column due to column hinging. Again, because the columns will probably not hinge, use the modified forces summarized in Design Step 8.1(d).

Design Step 8.4

Cap Beam Design Forces

The cap beam must be designed for the load combinations in Division I, Table 3.22.1A, except for Group VII seismic loads, Division I-A is to be used instead. As was discussed previously, use the modified design forces in the column.

Design Step 8.5

Miscellaneous Design Forces

a) Transfer of Longitudinal Seismic Force into the Soil

From Step 7.3.1 in Table 20, the maximum longitudinal force against the end diaphragm is 3338 kips at Abutment No. 3. The analysis using Basic Supports in Section III assumed rollers at the abutment. All the longitudinal seismic force is then taken by the center bent columns, and none is taken at the abutment. The analysis in Section IV using Spring Supports modeled the stiffness of the end diaphragm pushing against the soil as the superstructure tries to move longitudinally along the length of the bridge.

Design Step 8.5 (continued)

This is the load case from which the 3338 kip force was taken. See Figure 44.

This longitudinal force must be resisted entirely by the end diaphragm, which hangs off the end of the superstructure and pushes into the soil. The end diaphragm is trying to push a triangular wedge of soil, developing a passive pressure. In the analysis portion of this example, a maximum pressure of 7.7 ksf was used. This passive pressure capacity produced a maximum force of 3985 kips. Because the calculated force of 3338 kips is less, the original abutment spring stiffness did not require modification.

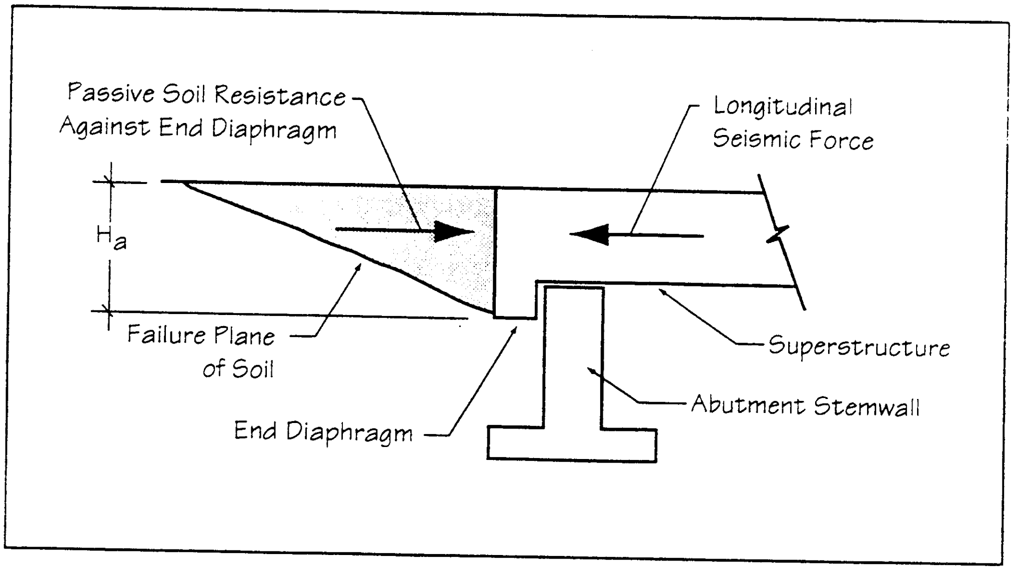


Figure 44 — Soil Resistance at End Diaphragm

Design Step 8.6

Foundation Design Forces
[Division I-A, Article 7.2.6]

The design forces for the spread footing under Bent No. 2 intermediate columns may be either a) or b) below.

a) Modified Design Forces for Foundations
[Division I-A, Article 7.2.1(B)]

These forces were calculated in Design Step 7.3.2 and are summarized in Table 22, LC2 being the controlling load case. Note that R = 1.0 represents the full elastic seismic forces.

Design Step 8.6 (continued)

$Pmin_u = 1045$ kip $\qquad Pmax_u = 1147$ kips

Resultant Moment Calculation

$M_L := 470 \cdot kip \cdot ft$ — Longitudinal moment

$M_T := 1012 \cdot kip \cdot ft$ — Transverse moment

For a circular column, the modified biaxial bending moment can be converted to a moment about a single axis by

$$M_u := \sqrt{{M_L}^2 + {M_T}^2} \qquad M_u = 1116 \cdot kip \cdot ft$$

Resultant Shear Calculation

$V_L := 55 \cdot kip \cdot ft$ — Longitudinal shear

$V_T := 89 \cdot kip \cdot ft$ — Transverse shear

For a circular column, the modified biaxial shear can be converted to a shear about a single axis by

$$V_u := \sqrt{{V_L}^2 + {V_T}^2} \qquad V_u = 105 \cdot kip \cdot ft$$

Summary of Forces

Note that both M_u and V_u are the vector sum of the orthogonal forces

$Pmin_u = 1045$ kips $\qquad Pmax_u = 1147$ kips
$M_u = 1116$ kip-ft
$V_u = 105$ kip

b) Forces from Column Plastic Hinging
[Division I-A, Article 7.2.2]

Not calculated.

Design Step 8.7

Abutment Design Forces
[Division I-A, Article 7.2.7]

a) Forces Transferred from Superstructure to Abutment Shear Key
[Division I-A, Article 7.4.2(B)]

Table 20 lists the seismic forces on the abutment at the level of the bearings. Note that R = 0.8 for connections. A summary of the forces at Abutment No. 1 are

Vertical Reaction

$P := 1077$ kip   with R = 0.8

Transverse Shear

The transverse shear is transferred from the superstructure, through the shear key, to the abutment. For the shear key design, use the EQM force with R = 0.8. Note that the transverse force for the spring support condition is larger than it was for the basic support condition because the average transverse displacement of the bridge superstructure is increased.

$V_T := 1938$ kip   with R = 0.8

b) Transverse Forces Transferred from the Abutment into the Soil

Refer to the Modified Design Forces for Foundations, summarized in Table 21 of Design Step 7.3.2. For this condition, R = 1.0 for all cases.

Vertical Reaction

$P = 1076$ kips   with R = 1.0

Transverse Shear

$V_T = 1550$ kips   with R = 1.0

Longitudinal Shear

See Design Step 8.5.

DESIGN STEP 10 — DESIGN STRUCTURAL COMPONENTS

As was discussed in the introduction to Design Step 7, this section deals with the spring support condition, which does not allow the superstructure to move freely at the abutments. This design step will discuss the results of the column design but does not include detailed calculations.

Design Step 10.1 — Final Column Design

The modified forces from Design Step 8.1 are used to design the intermediate bent columns for seismic loads. For the spring support condition, the forces are small relative to the basic support condition. Note that forces due to other loads, such as live load or stream flow, may end up controlling over these smaller load combinations with seismic loads.

Based on the design axial load, the ϕ factor used for the design of the flexure reinforcing is $\phi = 0.61$. A 36-inch-diameter column with 1.0 percent reinforcing will be sufficient to resist these loads for the spring support condition. This compares to the 48-inch-diameter column with 1.9 percent reinforcing for the basic support condition.

In the case of the basic support condition (Section III), the bent columns are designed to be ductile and form a plastic hinge. If a larger than expected earthquake occurs, the columns should perform well. In the spring support condition (Section IV), the columns may not perform as well should the seismic loading exceed the design level earthquake, and should a failure at the abutment result in larger than expected deflections at the columns.

Conclusion: The manner in which the superstructure is terminated at the abutment has a large impact on the intermediate bent columns. Allowing the superstructure to translate longitudinally (Section III), results in a larger column than if the superstructure is locked in against the soil (Section IV). Design philosophies of local jurisdictions often control which support condition is used.

DESIGN STEP 11

DESIGN FOUNDATIONS (FOOTINGS UNDER BENT NO. 2 COLUMNS)

The column requirements for the spring support condition (36-inch diameter with 1 percent reinforcement) are much smaller than for the basic support condition (48-inch diameter with 1.9 percent reinforcement). Therefore, the resulting footing size is also smaller. Instead of a 20-foot-square footing, only a 10-foot-square footing is required to resist the dead plus seismic forces. Note that this footing size is based on the fact that no plastic hinging forces are transferred from the column to the footing. Also, other load combinations may control and require a larger footing.

The maximum soil stress is larger with this 10-foot footing (21.9 ksf) than with the 20-foot footing (9.8 ksf). The reason is the footing size in the spring support condition is controlled by the soil pressure criteria, while the footing size in the basic support condition is controlled by the one-half uplift criteria.

Section V
Analysis Using Uniform Load Method for Both Basic and Spring Supports

SECTION V — ANALYSIS USING UNIFORM LOAD METHOD FOR BOTH BASIC AND SPRING SUPPORTS

INTRODUCTION — APPLICATION OF THE UNIFORM LOAD METHOD

For the purpose of illustration, this section explains the use of the Uniform Load Method for analyzing this bridge in both the transverse and longitudinal directions. Additionally, the method is demonstrated for both the basic support condition and the spring support condition.

Because the "stick" model configurations are the same as those used for the Single-Mode Method in the previous two sections, they are not discussed again here. Instead the analysis sections, Design Step 6.3 for the transverse direction and Design Step 6.4 for the longitudinal direction, are the only ones discussed. The basic support condition results for each loading direction are presented first; the spring support results are then presented for each loading direction.

ANALYSIS — BRIDGE WITH BASIC SUPPORT CONDITIONS

Design Step 6.3

Transverse Analysis, Uniform Load Method, Basic Supports
[Division I-A, Article 4.3]

The Uniform Load Method is a static loading method in which a uniform load is applied to the bridge in the direction under consideration, either transverse or longitudinal. Based on the deflection produced by the load, a stiffness in the appropriate lateral direction is calculated. From this, a period and then the seismically induced lateral force are determined. This is then applied to the structure, and the internal forces, reactions, and displacements are determined.

Design Step 6.3.1

Static Displacements
[Division I-A, Article 4.3, *(Step 1)*]

The displacements for a 100-kip-per-foot transverse loading is given in Table 23. The SAP90 input file for this analysis is F1TUL4C.

The analysis performed in this step could be performed by hand calculations. However, SAP90 was used to perform the analysis so that all the member forces could be determined directly.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

Design Step 6.3.1 (continued)

Table 23
Transverse Displacements for 100-kip/foot Uniform Load

Displacements (feet)

Abut 1	1/4 Span	1/2 Span	3/4 Span	Bent 2	1/4 Span	1/2 Span	3/4 Span	Abut 2
0.0000	0.0688	0.1218	0.1488	0.1454	0.1254	0.0921	0.0489	0.0000

The maximum displacement is 0.1488 foot at the 3/4 point of Span 1.

Design Step 6.3.2

Transverse Stiffness
[Division I-A, Article 4.3 *(Step 2)*]

Recall the following for the bridge.

$L := 242 \cdot ft$ Overall length of bridge

$W := 4876 \cdot kip$ Weight of bridge

$p_o := 100 \cdot \frac{kip}{ft}$ Unit uniform load applied to bridge

Lateral stiffness of bridge based on the calculated maximum deflection from the SAP90 model.

$K := \frac{p_o \cdot L}{0.1488 \cdot ft}$ $K = 162634 \cdot \frac{kip}{ft}$

Division I-A Eqn (4-1)

Design Step 6.3.3

Fundamental Period
[Division I-A, Article 4.3, *(Step 3)*]

The fundamental period of the bridge in the transverse direction is given below.

Note that the constant 'g', which is the acceleration due to gravity, must have consistent units.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load	▬			
Single-Mode				
Multimode				

Design Step 6.3.3 (continued)

The fundamental period, T

$$T := 2\cdot\pi\cdot\sqrt{\frac{W}{g\cdot K}} \qquad T = 0.192\cdot sec$$

Division I-A
Eqn (4-3)

Design Step 6.3.4

Equivalent Transverse Load
[Division I-A, Article 4.3 *(Step 4)*]

The equivalent static earthquake load is calculated using Equations (3-1) and (4-4) of the Specification.

The equivalent transverse load is defined in terms of the seismic response coefficient C_s which in turn is based on the acceleration coefficient and the site coefficient. Recall that the acceleration coefficient A and the soil coefficient S are

$A := 0.28 \qquad S := 1.2$

The seismic response coefficient is

$$C_s := \frac{1.2\cdot sec^{\frac{2}{3}}\cdot A\cdot S}{T^{\frac{2}{3}}} \qquad C_s = 1.212$$

Division I-A
Eqn (3-1)

Note that the constant 1.2 in Equation (3-1) carries units of seconds raised to the 2/3 power. This is the result of A and S being nondimensional and the requirement that Cs also be nondimensional.

The value of the coefficient does not need to exceed 2.5 times the acceleration coefficient.

$2.5\cdot A = 0.7 \qquad$ Thus, $\qquad C_s := 0.7$

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load	▬			
Single-Mode				
Multimode				

Design Step
6.3.4
(continued)

The equivalent lateral load is then

$$p_e := C_s \cdot \frac{W}{L} \qquad p_e = 14.10 \cdot \frac{kip}{ft}$$

Division I-A
Eqn (4-4)

Design Step
6.3.5

Displacements and Member Forces
[Division I-A, Article 4.3, *(Step 5)*]

The original displacements and forces determine by the SAP90 analysis can now simply be scaled by the ratio of the original uniform load (100 kips/foot) and the equivalent lateral load.

The results applicable to the seismic design are given in Table 24. These are scaled.

It is seen in the table that longitudinal forces are developed in the columns of the bent even though loading is only applied in the transverse direction. This is due to the unequal spans. Since the bent is not in the center of the bridge there is some rotation about a vertical axis that is induced in the bent. This then causes longitudinal forces to develop in the columns.

Design Step
6.3.5
(continued)

Table 24
Response for Uniform Load, Transverse Direction, and Basic Foundation

			Forces and Moments				
			Longitudinal		Transverse		
			Shear (kips)	Moment (kip-ft)	Shear (kips)	Moment (kip-ft)	Axial (kips)
Abutment 1			0	0	1608	586	0
Bent 2	Center	Top	0	0	78.2	1067	0
		Bottom	0	0	78.2	914	0
	Outboard	Top	7.9	107	77.5	1058	42.7
		Bottom	7.9	92.5	77.5	907	42.7
Abutment 3			0	0	1572	832	0

Displacements (ft)								
Abut 1	1/4 pt	1/2 pt	3/4 pt	Bent 2	1/4 pt	1/2 pt	3/4 pt	Abut 3
0	0.0097	0.0172	0.0210	0.0205	0.0177	0.0130	0.0069	0

Design Step 6.4

Longitudinal Analysis, Uniform Load Method, Basic Supports
[Division I-A, Article 4.3]

The response of the bridge is determined in the longitudinal direction in the same fashion as for transverse loading. However, in this case the uniform load is applied along the deck in the longitudinal direction.

Design Step 6.4.1

Static Displacements
[Division I-A, Article 4.3, *(Step 1)*]

The displacements for a 100-kip/foot longitudinal loading are given in Table 25. The SAP90 input file for this analysis is F1LSM4C.

Table 25
Longitudinal Displacements for 100-kip/foot Uniform Load

Displacements (feet)								
Abut 1	1/4 Span	1/2 Span	3/4 Span	Bent 2	1/4 Span	1/2 Span	3/4 Span	Abut 2
2.551	2.550	2.547	2.542	2.535	2.539	2.541	2.543	2.543

Because of the large axial stiffness of the superstructure the longitudinal displacements are essentially the same all along the length of the bridge.

Design Step 6.4.2

Longitudinal Stiffness
[Division I-A, Article 4.3, *(Step 2)*]

Longitudinal stiffness of bridge is based on the calculated maximum deflection from SAP90 model.

Recall the following for the bridge.

$L := 242 \cdot ft$ Overall length of bridge

$W := 4876 \cdot kip$ Weight of bridge

$p_o := 100 \cdot \frac{kip}{ft}$ Unit uniform load applied to bridge

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load		▬		
Single-Mode				
Multimode				

Design Step 6.4.2 (continued)

Lateral stiffness of bridge based on the calculated maximum deflection from the SAP90 model.

$$K := \frac{p_o \cdot L}{2.551 \cdot ft} \qquad K = 9486 \cdot \frac{kip}{ft}$$

Division I-A Eqn (4-1)

Design Step 6.4.3

Fundamental Period
[Division I-A, Article 4.3, *(Step 3)*]

Using the same weight that was calculated for the transverse loading, the fundamental period can be calculated.

The fundamental period, T

$$T := 2 \cdot \pi \cdot \sqrt{\frac{W}{g \cdot K}} \qquad T = 0.794 \cdot sec$$

Division I-A Eqn (4-3)

Design Step 6.4.4

Equivalent Longitudinal Load
[Division I-A, Article 4.3, *(Step 4)*]

The equivalent longitudinal load is defined in terms of the seismic response coefficient C_s which in turn is based on the acceleration coefficient and the site coefficient. Recall that the acceleration coefficient A and the soil coefficient S are

$$A := 0.28 \qquad S := 1.2$$

The seismic response coefficient is

$$C_s := \frac{1.2 \cdot sec^{\frac{2}{3}} \cdot A \cdot S}{T^{\frac{2}{3}}} \qquad C_s = 0.47$$

Division I-A Eqn (3-1)

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load		▬		
Single-Mode				
Multimode				

Design Step 6.4.4 (continued)

The value of the coefficient does not need to exceed 2.5 times the acceleration coefficient.

$2.5 \cdot A = 0.7$ This does not control.

The equivalent lateral load is then

$$p_e := C_s \cdot \frac{W}{L} \qquad p_e = 9.47 \cdot \frac{kip}{ft}$$

Division I-A Eqn (4-4)

Design Step 6.4.5

Displacements and Member Forces
[Division I-A, Article 4.3 *(Step 5)*]

The original displacements and forces determined by the SAP90 analysis can now be scaled by the ratio of the original uniform load and the equivalent load just calculated. The results of this scaling are given in Table 26.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load		▬		
Single-Mode				
Multimode				

Design Step
6.4.5
(continued)

Table 26
Response for Uniform Load, Longitudinal Direction, and Basic Foundation

			Forces and Moments				
			Longitudinal		Transverse		
			Shear (kips)	Moment (kip-ft)	Shear (kips)	Moment (kip-ft)	Axial (kips)
Abutment 1			0	0	0	0	105
Bent 2	Center	Top	771	9978	0	0	35.5
		Bottom	771	9566	0	0	35.5
	Outboard	Top	760	9790	0	0	35.5
		Bottom	760	9481	0	0	35.5
Abutment 3			0	0	0	0	211

Displacements (ft)								
Abut 1	1/4 pt	1/2 pt	3/4 pt	Bent 2	1/4 pt	1/2 pt	3/4 pt	Abut 3
0.242	0.242	0.241	0.241	0.240	0.240	0.241	0.241	0.242

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

ANALYSIS

BRIDGE WITH SPRING SUPPORT CONDITIONS

Design Step 6.3

Transverse Analysis, Uniform Load Method

[Division I-A, Article 4.3]

Since the process for determining the response of the system with springs included is the same as that used in the previous steps, only a summary of results is given.

The SAP90 input file for the analysis is F1TS4.
The period of the bridge in the transverse direction is T = 0.299 second.
The scaled results applicable to the seismic design are given below in Table 27.

Table 27
Response for Uniform Load, Transverse Direction, and Spring Foundation

			Forces and Moments				
			Longitudinal		Transverse		
			Shear (kips)	Moment (kip-ft)	Shear (kips)	Moment (kip-ft)	Axial (kips)
Abutment 1			0	0	1595	691	0
Bent 2	Center	Top	0	0	89.7	1255	0
		Bottom	0	0	89.7	1018	0
	Outboard	Top	3.8	53.6	89.3	1250	50.2
		Bottom	3.8	43.6	89.3	1014	50.2
Abutment 3			0	0	1550	977	0

Displacements (ft)								
Abut 1	1/4 pt	1/2 pt	3/4 pt	Bent 2	1/4 pt	1/2 pt	3/4 pt	Abut 3
0.0307	0.0401	0.0473	0.0505	0.0503	0.0474	0.0428	0.0367	0.0298

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

Design Step 6.4

Longitudinal Analysis, Uniform Load Method
[Division I-A, Article 4.3]

Recalculate the longitudinal response using spring supports and reduced column moment of inertia. The response of the bridge is determined in the same fashion as for transverse loading. Again, only a summary of the results is given.

The SAP90 input file for the analysis is F1LS4.
The period of the bridge in the longitudinal direction is T = 0.194 second.
The scaled results applicable to the seismic design are given below in Table 28.

Table 28
Response for Uniform Load, Longitudinal Direction, and Spring Foundation

			Forces and Moments				
			Longitudinal		Transverse		
			Shear (kips)	Moment (kip-ft)	Shear (kips)	Moment (kip-ft)	Axial (kips)
Abutment 1			1656	0	0	0	5
Bent 2	Center	Top	34.9	476	0	0	1.7
		Bottom	34.9	409	0	0	1.7
	Outboard	Top	34.7	472	0	0	1.7
		Bottom	34.7	407	0	0	1.7
Abutment 3			1653	0	0	0	10.1

Displacements (ft)								
Abut 1	1/4 pt	1/2 pt	3/4 pt	Bent 2	1/4 pt	1/2 pt	3/4 pt	Abut 3
0.0199	0.0207	0.0213	0.0215	0.0214	0.0213	0.021	0.0205	0.0199

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode				

Section VI
Analysis Using Multimode Spectral Method for Both Basic and Spring Supports

SECTION VI — ANALYSIS USING MULTIMODE SPECTRAL METHOD FOR BOTH BASIC AND SPRING SUPPORTS

INTRODUCTION — APPLICATION OF THE MULTIMODE SPECTRAL METHOD

For the purpose of illustration, repeat the analyses of the bridge using the Multimode Spectral Method, Procedure 3 of AASHTO I-A. As stated in Section III, this step is not required for this example. The Multimode Spectral Method is employed here to illustrate its application to both bridge examples — the basic and the spring-supported case. In fact, with many of the available analysis programs, the Multimode Spectral Method is perhaps the easiest method to apply because the programs perform the calculations directly.

ANALYSIS — BRIDGE WITH BASIC AND BRIDGE WITH SPRING SUPPORT CONDITIONS

Design Step 6.1 — Description of Models and Frequency Analysis Using Multimode Spectral Method

[Division I-A, Article 4.5]

The Multimode Spectral Method is based on the fact that the dynamic response of linear elastic structures may be characterized by the superposition of response of each of the structure's modes of vibration. In practice, the maximum response for each mode is estimated using an input design response spectra. For typical dynamic loadings, the maximum response in each mode does not occur at the same time; thus a simple summation of the individual maxima would be overly conservative. To account for this fact, various methods of combining the modal responses have been developed. A thorough discussion of modal dynamic analysis may be found in structural dynamics textbooks (e.g., Clough & Penzien, 1993).

A description of the models and the application of the Multimode Spectral Method will be discussed first along with the results that are general to both directions of loading (e.g., mode shapes, periods, and loading). Following this, the response and internal force results for both the transverse and longitudinal loading directions will be discussed. In this discussion, both the basic and spring-supported models and results are described.

Design Step 6.1.1

Mathematical Models for Multimode Response
[Division I-A, Article 4.5.2]

The models described for both the Uniform Load and Single-Mode Spectral Methods also are used for the multimodal analysis. The only difference is that mass is required for all the elements. In the SAP90 analysis, the mass is specified per unit length of the members. The program then determines the appropriate mass to assign to each node. Additionally, lumped nodal masses may also be specified for elements not modeled directly.

Design Step 6.1.2

Mode Shapes and Periods
[Division I-A, Article 4.5.3]

Typically for prismatic frame elements, half of the element mass is assigned to each end node. For the case where all masses are considered lumped at discrete points (e.g., no rotational inertia) and for which there are no nodes that have identical displacements, the number of vibration modes for the model will be equal to three times the number of nodes less the number of support translational restraints.

For the basic model there are 17 nodes and 13 translational restraints. Therefore, there are 38 modes of vibration, and each has a unique period and mode shape. This can be seen in Table 29 as the 'number of masses'. This table is taken from the SAP90 output for the basic model, input file F1TUL4C. Likewise for the spring supported model, as given in F1TS4, the abutment nodes are allowed to translate in the 'Z' direction; thus two more modes of vibration are added, and the number of masses listed in the output file is 40. See Table 30.

The program used to analyze the models will typically determine as many mode shapes and periods as the user specifies. Normally, not all are required to get an appropriate estimate of the response and internal forces.

The vibration properties are given under various names. For instance, mode shape, vibration shape, and eigenvector are all names for the same quantity. Likewise, frequency, period, and eigenvalue are all used to characterize the rate of vibration. These terms differ by the units used and by the inverse of one another (e.g., period is the inverse of frequency). These are all seen in Tables 29 and 30.

Design Step
6.1.2
(continued)

Table 29
System Equations and Frequencies for Basic Foundation Model

```
PROGRAM:SAP90/FILE:f1tul4c.EIG
C F1TUL4C 4 ft. dia. col.
E I G E N   S Y S T E M   P A R A M E T E R S

  NUMBER OF EQUATIONS                =     78
  NUMBER OF MASSES                   =     38
  NUMBER OF VALUES TO BE EVALUATED   =     15
  SIZE OF SUBSPACE                   =     19
```

MODE NUMBER	EIGENVALUE (RAD/SEC)**2	CIRCULAR FREQ (RAD/SEC)	FREQUENCY (CYCLES/SEC)	PERIOD (SEC)
1	0.606219E+02	0.778601E+01	1.239182	0.806984
2	0.199010E+03	0.141071E+02	2.245216	0.445392
3	0.818561E+03	0.286105E+02	4.553502	0.219611
4	0.128067E+04	0.357865E+02	5.695596	0.175574
5	0.244260E+04	0.494226E+02	7.865859	0.127132
6	0.840641E+04	0.916865E+02	14.592353	0.068529
7	0.957479E+04	0.978508E+02	15.573445	0.064212
8	0.174203E+05	0.131986E+03	21.006200	0.047605
9	0.191682E+05	0.138449E+03	22.034859	0.045383
10	0.269107E+05	0.164045E+03	26.108526	0.038302
11	0.437236E+05	0.209102E+03	33.279597	0.030048
12	0.622932E+05	0.249586E+03	39.722864	0.025174
13	0.932711E+05	0.305403E+03	48.606414	0.020573
14	0.130158E+06	0.360774E+03	57.419016	0.017416
15	0.139134E+06	0.373006E+03	59.365809	0.016845

Table 30
System Equations and Frequencies for Spring Foundation Model

```
PROGRAM:SAP90/FILE:f1ts4.EIG
C F1TS4 springs at abut. and col. / 4 ft. dia. col. Icol=0.5Igross
 E I G E N   S Y S T E M   P A R A M E T E R S

  NUMBER OF EQUATIONS                =     86
  NUMBER OF MASSES                   =     40
  NUMBER OF VALUES TO BE EVALUATED   =     15
  SIZE OF SUBSPACE                   =     19
```

MODE NUMBER	EIGENVALUE (RAD/SEC)**2	CIRCULAR FREQ (RAD/SEC)	FREQUENCY (CYCLES/SEC)	PERIOD (SEC)
1	0.183861E+03	0.135595E+02	2.158068	0.463377
2	0.498960E+03	0.223374E+02	3.555110	0.281285
3	0.798505E+03	0.282578E+02	4.497374	0.222352
4	0.107685E+04	0.328154E+02	5.222735	0.191471
5	0.198898E+04	0.445980E+02	7.097985	0.140885
6	0.242655E+04	0.492600E+02	7.839981	0.127551
7	0.832955E+04	0.912663E+02	14.525490	0.068844
8	0.844526E+04	0.918981E+02	14.626040	0.068371
9	0.957443E+04	0.978490E+02	15.573151	0.064213
10	0.196357E+05	0.140127E+03	22.301983	0.044839
11	0.269082E+05	0.164037E+03	26.107317	0.038303
12	0.415340E+05	0.203799E+03	32.435592	0.030830
13	0.436080E+05	0.208825E+03	33.235574	0.030088
14	0.645186E+05	0.254005E+03	40.426157	0.024736
15	0.132191E+06	0.363581E+03	57.865732	0.017281

Design Step 6.1.3

Multimode Spectral Analysis [Division I-A, Articles 4.5.4 and 3.6.2]

(a) Spectral Loading

A design response spectrum must be input to provide loading for the models. This spectrum is specified in Section 3.6.2 of Division I-A, and it applies in both the transverse and longitudinal directions. As shown in Figure 8, Equation (3-2) and the corresponding upper limit of two and a half times A effectively define the spectrum as a function of period T. Most programs will require period-spectrum data pairs to be input. Thus, the user must calculate the C_{sm} values that will define a smooth function within the analysis software. (C_{sm} is the modal analysis version of C_s.) The range used must cover the entire range of expected periods for the structure.

Some State Departments of Transportation (DOTs) and some local agencies have their own design response spectra that are used in lieu of the Division I-A spectra. These may be pre-determined spectra that have been selected to replace the AASHTO spectra. They may be acceleration amplification factors that use the AASHTO acceleration coefficient A but alter the spectral shape as a function of period T. Or they may be site-specific spectra that are developed by geotechnical engineers and incorporate the influence of soil conditions immediately adjacent to the bridge. As stated in Division I-A, Article 3.6, these alternate spectra should be 5 percent damped spectra. Regardless of the spectra chosen, the values used as input for the program will need to be expressed in the same form.

(b) Minimum Number of Modes

As mentioned above, most dynamic response can be adequately characterized without using all the vibration modes. The minimum number of modes required is specified in Division I-A, Article 4.5.4 as three times the number of spans.

Another way of assessing how many modes are sufficient to characterize response is to calculate the percentage of mass that participates in each mode, then ensure that some minimum value of participating mass is used in the analysis for each direction. Typically, at least 90 percent of the total mass should be used in each of the three directions.

Design Step
6.1.3
(continued)

However, the designer should not rely on only the percent of mass participation. The mode shapes should be inspected to determine that important masses are excited by the selected modes. If an insufficient number of modes is chosen, then both the internal structural forces and the reactions may be incorrect. The designer should ensure that enough modes have been used to accurately predict the forces and reactions.

Two scenarios regarding this issue may arise: 1) very stiff springs are used to model supports, and, due to the high stiffness of the springs, the cumulative mass participation factor converges to a number less than 90 percent for any reasonable number of modes used. This is not a problem as long as those supports are not intended to move; 2) movement of a support is suppressed due to truncation of the number of modes used, and the reaction forces at that particular support are erroneous, even though 90 percent of the mass participation has been ensured. This can be corrected simply by ensuring that the modes corresponding to movement of the masses at the support location are included in the analysis.

Example results for the basic and spring models are given in Tables 31 and 32, respectively. The first three columns show the participating mass in each direction for each mode. The next three columns show the cumulative participating mass in each direction. It can be seen that the first four modes for the two models capture most of the mass, although to obtain 90 percent in all three directions, 12 modes must be used for the basic model and 11 modes for the spring model.

Figures 45 and 46 show the first four mode shapes for the basic and spring model, respectively. When the participating masses shown in Tables 31 and 32 are compared with the primary displacement direction of the mode, the participating mass values are evident. The figures also show that the substitution of springs for rigid and sliding supports causes the ordering of the modes to change.

Design Step
6.1.3
(continued)

Table 31
Participating Mass for Basic Foundation Model

PROGRAM:SAP90/FILE:f1tul4c.EIG
C F1TUL4C 4 ft. dia. col.

P A R T I C I P A T I N G M A S S - (percent)

MODE	X-DIR	Y-DIR	Z-DIR	X-SUM	Y-SUM	Z-SUM
1	93.148	0.646	0.000	93.148	0.646	0.000
2	4.807	26.201	0.000	97.955	26.847	0.000
3	0.322	45.473	0.000	98.277	72.321	0.000
4	0.000	0.000	88.890	98.277	72.321	88.890
5	0.032	4.434	0.000	98.309	76.754	88.890
6	0.013	0.000	0.000	98.322	76.755	88.890
7	0.000	7.999	0.000	98.322	84.754	88.890
8	0.000	0.000	0.000	98.322	84.754	88.890
9	0.000	0.000	0.000	98.322	84.754	88.890
10	0.000	12.396	0.000	98.322	97.151	88.890
11	0.001	1.149	0.000	98.323	98.300	88.890
12	0.000	0.000	0.000	98.323	98.300	88.890
13	0.000	0.000	7.103	98.323	98.300	95.993
14	0.000	0.000	0.000	98.323	98.300	95.993
15	0.000	0.000	0.000	98.323	98.300	95.993

Table 32
Participating Mass for Spring Foundation Model

C F1TS4 springs at abut. and col. / 4 ft. dia. col. Icol=0.5Igross

P A R T I C I P A T I N G M A S S - (percent)

MODE	X-DIR	Y-DIR	Z-DIR	X-SUM	Y-SUM	Z-SUM
1	0.031	25.329	0.000	0.031	25.329	0.000
2	0.000	0.000	95.699	0.031	25.329	95.699
3	0.603	46.588	0.000	0.633	71.917	95.699
4	97.627	0.232	0.000	98.261	72.148	95.699
5	0.000	0.000	0.004	98.261	72.148	95.703
6	0.026	4.607	0.000	98.286	76.755	95.703
7	0.005	0.001	0.000	98.291	76.756	95.703
8	0.000	0.000	2.643	98.291	76.756	98.347
9	0.000	7.997	0.000	98.291	84.754	98.347
10	0.000	0.000	0.000	98.291	84.754	98.347
11	0.000	12.386	0.000	98.291	97.139	98.347
12	0.000	0.000	0.000	98.291	97.139	98.347
13	0.000	1.161	0.000	98.292	98.300	98.347
14	0.053	0.000	0.000	98.345	98.300	98.347
15	0.001	0.000	0.000	98.345	98.300	98.347

Design Step
6.1.3
(continued)

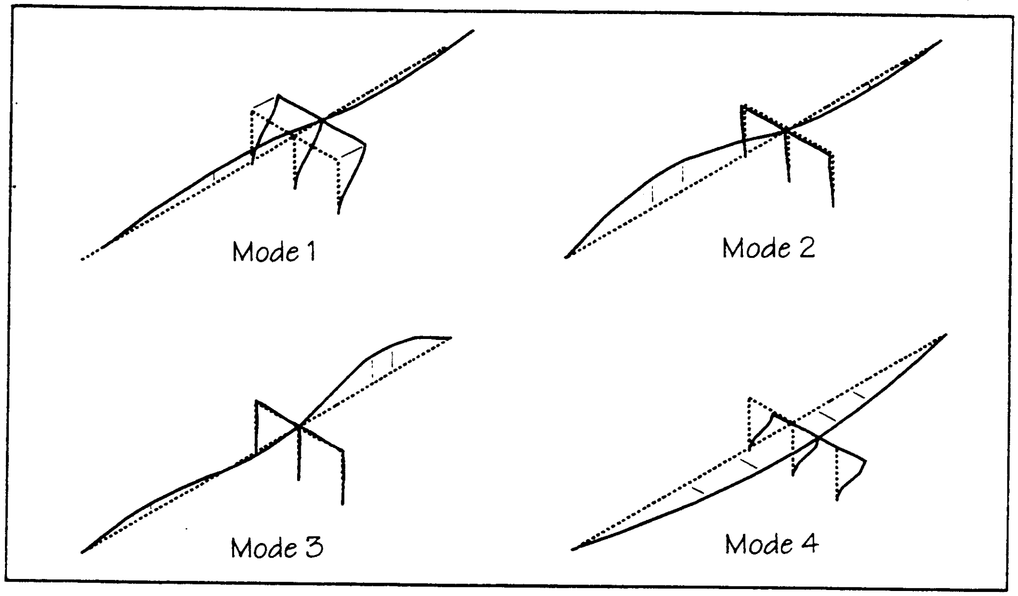


Figure 45 — Mode Shapes for Model with Basic Supports

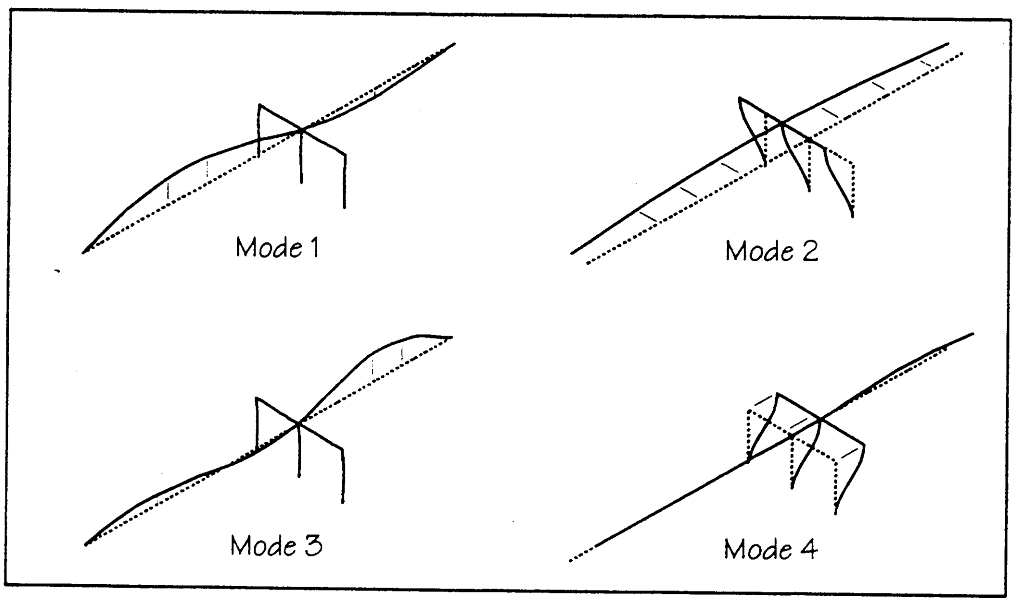


Figure 46 — Mode Shapes for Model with Spring Supports

Typically the modes are numbered sequentially from the longest period mode to the shortest.

Design Step
6.1.3
(continued)

Structures are usually described as having a transverse and a longitudinal period. It is evident that many periods actually exist. In fact, as more nodes are added, the number of periods increases. The actual structure has an infinite number of periods owing to its mass being distributed throughout rather than being lumped at discrete points. Therefore, the designations transverse and longitudinal period correspond to the modes with the longest period in each direction.

(c) Summary of Periods Obtained by Three Analysis Methods

At this point a summary of the transverse and longitudinal periods determined by the three methods of analysis is helpful in assessing the accuracy of each method. The periods calculated for the Multimode Spectral Method are taken as being the most accurate, and the periods from the other two methods may be compared to these values. Table 33 summarizes the periods in both directions. It is evident that all the methods produce similar periods. As shown, the Uniform-Load Method is the least accurate, since it produces periods that have the greatest difference from the Multimode Spectral Method. The Single-Mode Method is seen to provide periods that are very nearly equal to the multimode values.

Table 33
Summary of Periods from Various Analysis Methods and Models

Foundation	Procedure	Period (sec)	
		Transverse	Longitudinal
Basic	Uniform Load	0.192	0.794
	Single-Mode	0.172	0.791
	Multimode	0.176 (mode 4)	0.807 (mode 1)
Springs	Uniform Load	0.299	0.194
	Single-Mode	0.279	0.191
	Multimode	0.281 (mode 2)	0.191 (mode 4)

Design Step
6.1.3
(continued)

(d) Checking and Quality Assurance

The comparison of periods is important as a checking method. The Uniform Load Method is essentially a hand calculation that may be used to approximately check the results of the modal analysis, which is typically performed using a computer. Quick hand checks of this nature are always helpful.

Another check for assuring that the structural model is accurate and is, in fact, what was intended, is to closely examine the mode shapes, particularly the fundamental or longest period mode shapes. Often, errors in element connectivity, internal releases, and support conditions are readily apparent in the mode shapes. Some programs allow the mode shapes to be "animated," meaning that they are shown vibrating on the computer screen. This feature can be invaluable in detecting connectivity problems.

An example of using the mode shape plots to debug connectivity problems is shown in Figure 47. The far left-hand column was not fixed in the SAP90 input file. This was done simply by applying the incorrect last node number of a restraint generation command. The effect of not connecting the column to the foundation is readily apparent by the relative displacement between the column and the undeformed, broken-line sketch.

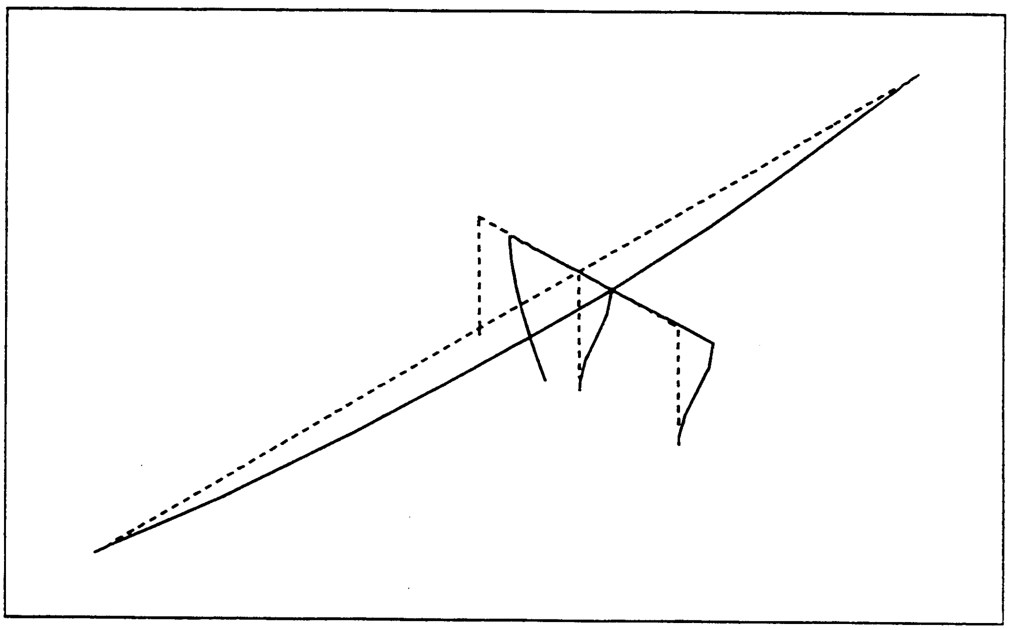


Figure 47 — Shape of Mode 4 with One Column Base Released

Design Step 6.1.3 (continued)

Hand Check ✓ 6.1.3 (3-Basic) Check No. 1 Longitudinal Period

Finally, a rough check on the longitudinal fundamental period can be made by lumping all the mass at the bent and rerunning the mode shape and frequency analysis. This is valid for the longitudinal direction since the superstructure is so stiff that all the mass displaces the same amount longitudinally. Because only one mass is used and it has three translational degrees-of-freedom, there are only three mode shapes and periods. Only the translational one is of interest.

This calculation was done for the basic foundation model. The SAP90 input file is F1LML and the result of the analysis was a longitudinal period of 0.791 second. This is very close to the multimodal 0.807 second period.

ANALYSIS

Design Step 6.3

BRIDGE WITH BASIC SUPPORT CONDITIONS

Transverse Analysis, Multimode Spectral Method, Basic Supports

[Division I-A, Article 4.5]

Using the Multimode Spectral Method, perform a transverse analysis for the basic foundation case. Transverse analysis means that the input response spectrum was assigned to the transverse direction, and in this case no longitudinal or vertical spectra were used.

The analysis program handles all the calculations, including the modal combinations. In this case, 15 modes were used to characterize the response. This number was kept constant for all the analyses.

The results are given in Table 34. The SAP90 input file for this analysis is F1TUL4C. Shown in the table are only those items that are to be used in the design process or that are useful in comparing with the results previously obtained.

Combination of Modal Forces and Displacements
[Division I-A, Article 4.5.5]

The response of the model in each of the calculated modes must be superimposed to estimate the overall response. As mentioned previously, all the modal maximum responses do not occur simultaneously; therefore a simple summation of the modal absolute values is not appropriate. Most programs use either the square root of the sum of the squares method (SRSS) or the complete quadratic combination method (CQC). The simplest is the SRSS method, and it is adequate when the modal periods are well spaced. When the periods are close, coupling between modal response can occur and thus the CQC method should be used. This method accounts for coupling between modes, preserves the signs of the cross-modal terms, and is based on random vibration fundamentals. Most programs now have the CQC method as an option. The method requires very little additional run time for most models. Therefore, it should be used exclusively. This eliminates the judgment of what constitutes closely spaced periods.

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode	—			

Design Step 6.3 (continued)

Table 34
Response for Multimode Method, Transverse Direction, and Basic Foundation

			Forces and Moments				
			Longitudinal		Transverse		
			Shear (kips)	Moment (kip-ft)	Shear (kips)	Moment (kip-ft)	Axial (kips)
Abutment 1			0	0	1237	607	0
Bent 2	Center	Top	0	0	79.9	1090	0
		Bottom	0	0	79.9	934	0
	Outboard	Top	8.1	110	79.7	1088	44.3
		Bottom	8.1	94.7	79.7	932	44.3
Abutment 3			0	0	1258	862	0

Displacements (ft)								
Abut 1	1/4 pt	1/2 pt	3/4 pt	Bent 2	1/4 pt	1/2 pt	3/4 pt	Abut 3
0	0.0097	0.0173	0.0213	0.0209	0.018	0.0132	0.0069	0

	Basic		Spring	
	Transverse	Longitudinal	Transverse	Longitudinal
Uniform Load				
Single-Mode				
Multimode	▬			

Design Step 6.4

Longitudinal Analysis, Multimode Spectral Method, Basic Supports

[Division I-A, Article 4.5]

Perform the analysis for loading in the longitudinal direction.

The results of the spectral analysis in the longitudinal direction are given in Table 35 for the basic foundation condition. The SAP90 input file for this analysis is F1LSM4C.

Table 35
Response for Multimode Method, Longitudinal Direction, and Basic Foundation

			Forces and Moments				
			Longitudinal		Transverse		
			Shear (kips)	Moment (kip-ft)	Shear (kips)	Moment (kip-ft)	Axial (kips)
Abutment 1			0	0	0	0	280
Bent 2	Center	Top	725	9280	0	0	116
		Bottom	725	9111	0	0	116
	Outboard	Top	715	9106	0	0	116
		Bottom	715	9032	0	0	116
Abutment 3			0	0	0	0	294

Displacements (ft)								
Abut 1	1/4 pt	1/2 pt	3/4 pt	Bent 2	1/4 pt	1/2 pt	3/4 pt	Abut 3
0.2352	0.2351	0.2348	0.2344	0.2338	0.2341	0.2343	0.2344	0.2345

ANALYSIS

Design Step 6.3

BRIDGE WITH SPRING SUPPORT CONDITIONS

Transverse Analysis, Multimode Spectral Method, Spring Supports

Repeat the spectral analysis for loading of the spring-supported structure in the transverse direction.

The results of the transverse spectral analysis are given in Table 36. The SAP90 input file for this analysis is F1TS4.

Table 36
Response for Multimode Method, Transverse Direction, and Spring Foundation

			Forces and Moments				
			Longitudinal		Transverse		
			Shear (kips)	Moment (kip-ft)	Shear (kips)	Moment (kip-ft)	Axial (kips)
Abutment 1			0	0	1369	703	0
Bent 2	Center	Top	0	0	91.1	1274	0
		Bottom	0	0	91.1	1033	0
	Outboard	Top	3.9	54.6	91	1272	51.2
		Bottom	3.9	44.4	91	1032	51.2
Abutment 3			0	0	1415	998	0

Displacements (ft)								
Abut 1	1/4 pt	1/2 pt	3/4 pt	Bent 2	1/4 pt	1/2 pt	3/4 pt	Abut 3
0.0292	0.0394	0.0474	0.0515	0.0511	0.0481	0.0431	0.0366	0.0293

Design Step 6.4

Longitudinal Analysis, Multimode Spectral Method, Spring Supports

Repeat the spectral analysis for loading of the spring-supported structure in the longitudinal direction.

The results of the longitudinal spectral analysis are given in Table 37. The SAP90 input file for this analysis is F1LS4.

Table 37
Response for Multimode Method, Longitudinal Direction, and Spring Foundation

			Forces and Moments				
			Longitudinal		Transverse		
			Shear (kips)	Moment (kip-ft)	Shear (kips)	Moment (kip-ft)	Axial (kips)
Abutment 1			1416	0	0	0	24
Bent 2	Center	Top	38	532	0	0	39
		Bottom	38	431	0	0	39
	Outboard	Top	37.7	527	0	0	39
		Bottom	37.7	429	0	0	39
Abutment 3			1483	0	0	0	66

Displacements (ft)								
Abut 1	1/4 pt	1/2 pt	3/4 pt	Bent 2	1/4 pt	1/2 pt	3/4 pt	Abut 3
0.0197	0.0205	0.0211	0.0213	0.0214	0.0212	0.0208	0.0203	0.0198

SUMMARY AND COMPARISON OF RESULTS

The bridge has been analyzed by three different methods for two different foundation cases in both the transverse and longitudinal directions. All three of the methods used are acceptable according to the AASHTO Specification. Therefore the results should be similar. Table 38 contains a summary of the results for selected members. Each row of the table contains the results for one member, and thus comparisons between the results may be made by examining the values in each row.

Table 38
Summary of Results from Various Analysis Methods

Foundation			Basic		
Analysis Method			Uniform Load	Single-Mode	Multimode
Maximum Abutment Force (kip)	Transverse Direction	T A	1608	1277	1258
		L A	0	0	0
	Longitudinal Direction	T A	0	0	0
		L A	0	0	0
Outboard Column	Moment (kip-ft)	T A	1058	1053	1088
		L A	9790	9790	9106
	Shear (kip)	T A	77.5	77.2	79.7
		L A	771	771	715
	Axial (kip)	T A	42.7	42.5	44.3
		L A	35.5	35.5	116
Max Displacement (ft)		T A	0.0210	0.0208	0.0213
		L A	0.2417	0.2417	0.2352

(continued on the next page)

SUMMARY
(continued)

Table 38 (continued)
Summary of Results from Various Analysis Methods

Foundation			Springs		
Analysis Method			Uniform Load	Single-Mode	Multimode
Maximum Abutment Force (kip)	Transverse Direction	T A	1595	1550	1415
		L A	0	0	0
	Longitudinal Direction	T A	0	0	0
		L A	1656	1639	1483
Outboard Column	Moment (kip-ft)	T A	1250	1248	1272
		L A	472	467	527
	Shear (kip)	T A	89.3	89.2	91
		L A	34.7	34.3	37.7
	Axial (kip)	T A	50.2	50.1	51.2
		L A	1.7	1.7	39
Max Displacement (ft)		T A	0.0505	0.0509	0.0515
		L A	0.0215	0.0213	0.0214

TA - Transverse Analysis
LA - Longitudinal Analysis

Table 38 indicates that within each of the two foundation cases — basic and springs — the results are very similar with the exception of the axial forces induced in the columns by the longitudinal loading.

Trends

Some general trends are evident. These primarily relate the differences in response produced by using the soil to resist longitudinal forces and by accounting for soil flexibility.

SUMMARY
(continued)

Forces

- Adding spring supports may increase the transverse shear at those supports. This may occur at the abutments and at intermediate supports.

- If the soil at the abutments is used to resist longitudinal force, as was the case with the spring-supported model, the forces carried by the intermediate supports may be significantly smaller than if no soil resistance is used.

The first trend is particularly counterintuitive. An explanation is given in the next two paragraphs.

Typically it is expected that increasing a structure's flexibility will elongate the period and reduce the seismic forces. In this case, the first is true. The period in the transverse direction increased from 0.176 second to 0.281 second. However, both these periods are small enough to remain in the range where the seismic amplification is considered constant. This is seen in Division I-A, Equation (3-1) and its corresponding maximum value of two and a half times A.

In this structure, the dynamic loading is the same for both the basic and spring foundation cases, although their dynamic response is quite different. The spring supported case develops relatively large displacements all along its length, while the basic structure only develops large displacements midway between the abutments. The larger displacements, overall, for the spring case cause more mass to participate in the critical fundamental modes. For instance, in Table 31, 89 percent of the mass participates in the first transverse mode for the basic model, whereas Table 32 indicates that 96 percent participates in the first transverse mode for the spring case. This means that more inertial force will be induced in the structure for the spring case, and correspondingly larger reactions will be produced.

The large differences in column forces under longitudinal loading derive from the assumptions for the soil springs. Judgment is necessary to decide to what extent the abutment soil is to be relied upon. This largely depends on how confident the designer feels about the assumed soil stiffnesses and strengths. Proper consideration of the soil effects requires close coordination between the structural and geotechnical engineers.

SUMMARY
(continued)

However, such issues of design philosophy are often resolved by the DOT in whose jurisdiction the bridge falls. Some departments allow the soil to be relied upon, and some do not.

Displacements

- Adding transverse springs may increase the transverse deflection.
- Carrying the longitudinal load into the soil at the abutments may significantly reduce the longitudinal deflection.

Discussion of Axial Forces

In this example, differences exist for the axial column forces of Bent No. 2 under longitudinal loading. For the basic-support condition, the axial force obtained by the Multimode Spectral Method is 116 kips, whereas the static methods both give 35.5 kips. For the spring-supported case the Multimode Spectral Method gives 39 kips, compared with 1.7 kips from the static methods. At first glance, the modal forces might be considered erroneous, because the static results agree with one another but not with the dynamic method. However, upon closer consideration, the dynamic results can be shown to be reasonable. Inspection of the mode shapes (Figures 45 and 46 indicate that Mode 1 for the basic-support condition and Mode 4 for the spring-supported model are the primary longitudinal modes. These are the modes that, typically, will be excited the most by longitudinal lateral loading. A close look at these mode shapes reveals that there are vertical components of the modes along the superstructure. Also, it is clear that the vertical component of Mode 1 for the basic-support condition is larger than that of the spring-supported case. Because the spans of the superstructure are not equal, a net vertical force is required at Bent No. 2 to react to the inertial forces developed during vibration in the primary longitudinal modes. This is the reason that the dynamic axial forces in the columns do not equal the static axial forces. Physically, the static axial forces are those required to equilibrate the vertical reactions developed at the abutments when a longitudinal load is applied to the bridge. The dynamic axial forces include this effect plus the inertial forces caused by vibration of the superstructure.

SUMMARY
(continued)

When dynamic results seem to conflict with static results, the designer needs to ascertain whether the results are erroneous or realistic. This can be assessed by progressively altering the dynamic model, reanalyzing, and comparing with the static results. In this example, the mass of the superstructure was first lumped at the bent, and the resulting axial force was found to be identical to that from the static analysis. Next, the multimode analysis was conducted for the basic model using only the primary longitudinal mode; all higher modes were ignored. This resulted in 78 kips of axial force in the columns. The inference is that the difference between 116 kips and 78 kips is contributed by higher modes of vibration. By progressively changing the model and considering the results, the apparent erroneous differences between the static and dynamic models were shown to be real dynamic effects that the static analyses ignored.

Quasi-Static Analysis Methods versus Modal Analysis Methods

Finally, it is important to recognize that the static and dynamic analyses may not always produce nearly identical forces and displacements. Certainly for structures with complex or irregular distributions of mass and/or stiffness, the static and dynamic methods would be expected to give different results. In such cases, dynamic analyses would be required to obtain proper estimates of the forces and displacements. However, even with regular structures, such as this example bridge, some forces or displacements may be different.

Section VII
Notations

SECTION VII NOTATIONS

a	Center to Center Distance of Bent No. 2 Outboard Columns
A	Acceleration Coefficient
A_b	Area of Individual Reinforcing Bar
A_c	Area of Column
A_{cb}	Area of Cap Beam
A_{core}	Column Core Area
A_{cv}	Concrete Area Resisting Shear
A_d	Area of Superstructure
A_g	Gross Area of Column
A_{sp}	Area of Spiral Reinforcement
A_v	Area of Shear Reinforcement
A_{vf}	Area of Shear Friction Reinforcement
B	Loads Resulting from Buoyancy
B_f	Width of Footing
b_w	Diameter of Circular Column
C_s	Elastic Seismic Response Coefficient
d	Effective Depth of Column
D	Dead Load Effect
d_b	Nominal Diameter of Reinforcing Bar
d_c	Column Diameter

NOTATIONS
(continued)

d_{core}	Column Core Diameter
d_f	Depth of Footing
DF_c	Column Distribution Factor
DF_l	Left-Hand Span Distribution Factor
DF_r	Right-Hand Span Distribution Factor
d_s	Diameter of Spiral, Depth of Soil Above Footing
E	Loads Resulting from Earth Pressure
E_c	Young's Modulus of Concrete
e_{max}	Maximum Column Eccentricity on Footing
e_{min}	Minimum Column Eccentricity on Footing
EQF	Modified Earthquake Forces - Foundations
EQM	Modified Earthquake Forces - Members and Connections
f'_c	Specified Compressive Strength of Concrete
F_{max}	Maximum Permissible Abutment Force
FS	Factor of Safety For Soil Bearing Stress
f_y	Specified Yield Strength of Reinforcement
f_{yh}	Specified Yield Strength of Transverse Reinforcement
g	Acceleration Due to Gravity
h	Column Height - Top of Footing to Superstructure c.g.
H	Height of Columns
H_a	Height of Abutment Endwall

NOTATIONS
(continued)

H_{clr}	Clear Height of Columns
H_w	Height of Wingwall
I_b	Moment of Inertia of all Bent Columns
I_c	Moment of Inertia of Columns
IC	Importance Classification
I_d	Moment of Inertia of Superstructure
I_g	Approximate Uncracked Moment of Inertia of Columns
I_{yd}	Superstructure Moment of Inertia about Vertical Axis
I_{zd}	Superstructure Moment of Inertia about Horizontal Axis
k	Soil Spring Constant
K	Lateral Stiffness of Bridge
k_a	Abutment Stiffness
k_{a1}	Uncorrected Abutment Stiffness
k_{a2}	Corrected Abutment Stiffness
k_b	Lateral Stiffness of Bent
k_c	Relative Stiffness of Columns
k_{ld}	Relative Stiffness of Left-Hand Superstructure
k_p	Caltrans Basic Stiffness of Abutment Backfill
k_r	Rotational Stiffness of Spread Footing
k_{rd}	Relative Stiffness of Right-Hand Superstructure
k_{r_alt}	PCI Rotational Stiffness of Spread Footing

NOTATIONS
(continued)

k_s	Modulus of Subgrade
k_{sb}	Stiffness of Uniformly Loaded Simple-Span Beam
k_w	Stiffness of Soil Acting Against Wingwall
k_{w1}	Uncorrected Wingwall Stiffness
k_{w2}	Corrected Wingwall Stiffness
L	Overall Length of Bridge
LC1	Load Case 1
LC2	Load Case 2
L_d	Development Length
L_{db}	Basic Development Length
L_{dh}	Development Length of Standard Hook
L_f	Length of Footing
L_{hb}	Basic Development Length of Standard Hook
L_l	Length of Left-Hand Span
L_r	Length of Right-Hand Span
L_s	Length of Shear Key
L_w	Length of Wingwall
m	Mass per Unit Length of Beam
M	Moment
M_b	Moment at Bottom of Column
M_f	Moment at Bottom of Footing

NOTATIONS
(continued)

M_L	Longitudinal Moment
$Mmax_f$	Maximum Moment at Bottom of Footing
$Mmin_f$	Minimum Moment at Bottom of Footing
$Mmax_n$	Maximum Nominal Column Moment
$Mmin_n$	Minimum Nominal Column Moment
$Mmax_p$	Maximum Column Moment due to Plastic Hinging
$Mmin_p$	Minimum Column Moment due to Plastic Hinging
M_n	Nominal Moment Strength
M_{nbot}	Nominal Moment Strength at Bottom of Column
M_{ntop}	Nominal Moment Strength at Top of Column
M_o	Moment at Centerline of Cap Beam due to Plastic Hinging of Column
M_p	Plastic Hinging Moment
M_t	Moment at Top of Column
M_T	Transverse Moment
M_u	Factored Moment due to Modified Forces
N	Minimum Support Length at Abutment
P	Vertical Force
P_d	Weight of Footing and Soil above Top of Footing
p_e	Equivalent Lateral Load
p_{max}	Caltrans Maximum Soil Pressure, Static
P_{max}	Maximum Vertical Force

NOTATIONS
(continued)

P_{min}	Minimum Vertical Force
$Pmax_f$	Maximum Axial Force at Bottom of Footing
$Pmin_f$	Minimum Axial Force at Bottom of Footing
$Pmax_p$	Maximum Column Axial Force due to Plastic Hinging
$Pmin_p$	Minimum Column Axial Force due to Plastic Hinging
$Pmax_u$	Maximum Column Axial Force due to Modified Forces
$Pmin_u$	Minimum Column Axial Force due to Modified Forces
P_n	Nominal Axial Strength
p_o	Uniform Load Applied to Bridge
q	Bearing Stress under Footing
q_a	Allowable Bearing Stress under Footing
q_{eq}	Ultimate Bearing Stress under Footing
R	Response Modification Factors
r_b	Distance of Longitudinal Reinforcement from Column Centerline
s	Spacing of Spiral and Stirrup Reinforcement
S	Soil Coefficient, Skew of Bridge
SF	Loads Resulting from Stream Flow
SPC	Seismic Performance Category
T	Fundamental Period
V	Shear Carried by Columns
V_c	Nominal Shear Strength Provided by Concrete

NOTATIONS (continued)

V_L	Longitudinal Shear
$Vmax_p$	Maximum Column Shear Force due to Plastic Hinging
$Vmin_p$	Minimum Column Shear Force due to Plastic Hinging
V_n	Nominal Shear Strength
V_o	Shear in Cap Beam due to Plastic Hinging of Column
V_p	Shear Corresponding to Plastic Hinging of Column
v_s	Displacement of Superstructure in Loaded Direction
V_s	Nominal Shear Strength Provided by Reinforcement
V_T	Transverse Shear
V_{total}	Total Shear Carried by Bent
V_u	Factored Shear due to Modified Forces
w	Weight of Superstructure per Unit Length
W	Weight of Bridge
W_a	Width of Abutment
z_{bar}	Centroid of Tension Side Reinforcement in Column
α	Coefficient used to Calculate Fundamental Period of Bridge
α_{half_pt}	Intermediate Value of α
β	Coefficient used to Calculate Fundamental Period of Bridge
β_{half_pt}	Intermediate Value of β
Δ	Deflection of Edge of Footing
ΔP	Axial Load due to Plastic Hinging

NOTATIONS
(continued)

δ	Deflection at Cap Beam
ϕ	Strength Reduction Factor
ϕ_{max}	Maximum Strength Reduction Factor
ϕ_{min}	Minimum Strength Reduction Factor
γ	Coefficient used to Calculate Fundamental Period of Bridge
γ_{half_pt}	Intermediate Value of γ
γ_{conc}	Unit Weight of Concrete
γ_{soil}	Unit Weight of Soil
μ	Coefficient of Friction
σ	Stress under Spread Footing
σ_a	Minimum Column Axial Core Stress
σ_{max}	Maximum Column Axial Stress
σ_{min}	Minimum Column Axial Stress
θ	Rotation of Footing, Resultant Direction of Acceleration
ρ_s	Volumetric Spiral Ratio

Section VIII
References

SECTION VIII REFERENCES

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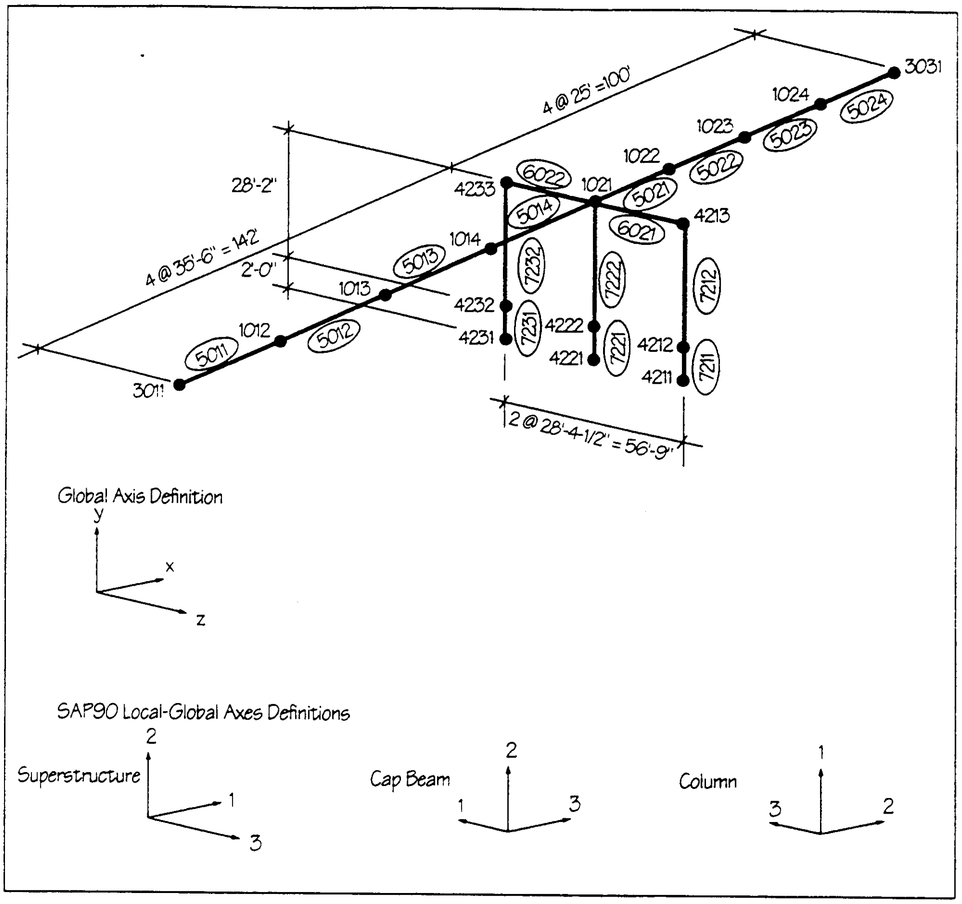
Section IX
Input for Computer Analysis



SAP90 Model of Bridge

Filename: F1TSM4C

```
C F1TSM4C 4ft. dia. col.
C Units are KIP FEET  trans. loading, single-mode spectral loads, fixed base

SYSTEM
R=0 L=2 C=0 V=15 T=0.0001 P=0.001 W=0 N=9999        :control information

JOINTS
4211  X=142    Y=0.0    Z=-28.375                   :column base
4221  X=142    Y=0.0    Z=0.0                       :column base
4231  X=142    Y=0.0    Z=28.375                    :column base
4212  X=142    Y=2.0    Z=-28.375                   :top of footing
4222  X=142    Y=2.0    Z=0.0                       :top of footing
4232  X=142    Y=2.0    Z=28.375                    :top of footing
3011  X=0.0    Y=30.17  Z=0.0                       :abutment 1
1012  X=35.5   Y=30.17  Z=0.0                       :superstructure/span 1
1014  X=106.5  Y=30.17  Z=0.0      G=1012,1014,1    :superstructure/span 1
1021  X=142    Y=30.17  Z=0.0                       :superstructure/span 2
1024  X=217    Y=30.17  Z=0.0      G=1021,1024,1    :superstructure/span 2
3031  X=242    Y=30.17  Z=0.0                       :abutment 3
4213  X=142    Y=30.17  Z=-28.375                   :capbeam
4233  X=142    Y=30.17  Z=28.375                    :capbeam

RESTRAINTS
3011 3031 20 R=0,1,1,1,0,0                          :abutment supports
4211 4231 10 R=1,1,1,1,1,1                          :column foundations

FRAME
NM=3 NL=10 NSEC=0 Y=0,-1                            :frame element specs.
C  superstructure properties
1  A=120 J=60000 I=575,51000  AS=0,0  E=518400  G=0  W=18.9 M=18.9/32.2  TC=0
C column properties
2  A=12.6 J=25  I=12.6,12.6 AS=0 E=518400  G=0 W=1.89  M=1.89/32.2   TC=0
C capbeam properties
3  A=25 J=10000  I=10E7,10E7  AS=0 E=518400  G=0 W=3.75  M=3.75/32.2   TC=0
C
C lateral static loads
C
1  WG=0,0,100                           :uniform lateral load
2  WG=0,0,0                             :
3  TRAP=0,0,0,35.5,0,-8.19              :
4  TRAP=0,0,-8.19,35.5,0,-14.51         :
5  TRAP=0,0,-14.51,35.5,0,-17.71        :
6  TRAP=0,0,-17.71,35.5,0,-17.31        :  single-mode spectral load
7  TRAP=0,0,-17.31,25,0,-14.91          :
8  TRAP=0,0,-14.91,25,0,-10.97          :
9  TRAP=0,0,-10.97,25,0,-5.83           :
10 TRAP=0,0,-5.83,25,0,0                :
C
C Note that the trapezoidal loads shown here are slightly different from those
C shown in Figure 9 of the text.  The origin of the differences is a slightly
C different numerical integration scheme that was used early in the example
C development.  The values shown in this input file and analysis results
C listed in the text are based on the earlier integration scheme.
C
C superstructure elements
5011 3011 1012 M=1 LP=4232,4212  NSL=3,2            :superstructure
5012 1012 1013 M=1 LP=4232,4212  NSL=4,2            :  "      "
5013 1013 1014 M=1 LP=4232,4212  NSL=5,2            :  "      "
5014 1014 1021 M=1 LP=4232,4212  NSL=6,2            :  "      "
5021 1021 1022 M=1 LP=4232,4212  NSL=7,2            :  "      "
5022 1022 1023 M=1 LP=4232,4212  NSL=8,2            :  "      "
5023 1023 1024 M=1 LP=4232,4212  NSL=9,2            :  "      "
5024 1024 3031 M=1 LP=4232,4212  NSL=10,2           :  "      "
```

```
C
C column elements
7211 4211 4212 M=2 LP=4211,4221 G=1,20,20,20                 :column\footing
7221 4221 4222 M=2 LP=4211,4221                              :column\footing
7212 4212 4213 M=2 LP=4211,4221 G=1,20,20,20 RE=0,2.83      :column elements
7222 4222 1021 M=2 LP=4211,4221 RE=0,2.83                    :column elements
C
C capbeam elements
6021 4213 1021 M=3 LP=1021,1022                         :capbeam elements
6022 1021 4233 M=3 LP=1021,1022                         :capbeam elements

SPEC
A=0  S=32.2*0.28  D=0.05
0.0     2.5*0    2.5*.67*0    2.5           :spectral values
0.437   2.5*0    2.5*.67*0    2.5           :spectral values
0.5     2.5*0    2.5*.67*0    2.29          :spectral values
0.6     2.5*0    2.5*.67*0    2.024         :spectral values
0.7     2.5*0    2.5*.67*0    1.827         :spectral values
0.8     2.5*0    2.5*.67*0    1.671         :spectral values
0.9     2.5*0    2.5*.67*0    1.545         :spectral values
1.0     2.5*0    2.5*.67*0    1.440         :spectral values
1.2     2.5*0    2.5*.67*0    1.275         :spectral values
1.4     2.5*0    2.5*.67*0    1.151         :spectral values
1.6     2.5*0    2.5*.67*0    1.053         :spectral values
1.8     2.5*0    2.5*.67*0    0.973         :spectral values
2.0     2.5*0    2.5*.67*0    0.907         :spectral values
2.5     2.5*0    2.5*.67*0    0.782         :spectral values
3.0     2.5*0    2.5*.67*0    0.692         :spectral values
100.    2.5*0    2.5*.67*0    0.0           :spectral values

COMBO
1 C=1,0  D=0                  :single-mode transverse earthquake loading
2 C=0,1  D=0                  :dead load only
3 C=0,0  D=1                  :multi-mode spectral transverse loading
```

Filename: F1LSM4C

```
C F1LSM4C  4 ft. dia. col.
C FHWA example 1 longitudinal loading, single-mode method, fixed base
C Units are KIP FEET

SYSTEM
R=0 L=2 C=0 V=15 T=0.0001 P=0.001 W=0 N=9999          :control information

JOINTS
4211  X=142     Y=0.0     Z=-28.375                    :column base/mid-footing
4221  X=142     Y=0.0     Z=0.0                        :column base/mid-footing
4231  X=142     Y=0.0     Z=28.375                     :column base/mid-footing
4212  X=142     Y=2.0     Z=-28.375                    :top of footing
4222  X=142     Y=2.0     Z=0.0                        :top of footing
4232  X=142     Y=2.0     Z=28.375                     :top of footing
3011  X=0.0     Y=30.17   Z=0.0                        :abutment 1
1012  X=35.5    Y=30.17   Z=0.0                        :superstructure/span 1
1014  X=106.5   Y=30.17   Z=0.0     G=1012,1014,1      :superstructure/span 1
1021  X=142     Y=30.17   Z=0.0                        :superstructure/span 2
1024  X=217     Y=30.17   Z=0.0     G=1021,1024,1      :superstructure/span 2
3031  X=242     Y=30.17   Z=0.0                        :abutment 3
4213  X=142     Y=30.17   Z=-28.375                    :capbeam
4233  X=142     Y=30.17   Z=28.375                     :capbeam

RESTRAINTS
3011 3031 20 R=0,1,1,1,0,0                             :abutment supports
4211 4231 10 R=1,1,1,1,1,1                             :column foundations

FRAME
NM=3 NL=2 NSEC=0 Y=0,-1                                :frame element specs.
C  superstructure properties
1  A=120 J=60000 I=575,51000  AS=0,0  E=518400  G=0  W=18.9 M=18.9/32.2  TC=0
C column properties
2   A=12.6 J=26   I=12.6,12.6 AS=0 E=518400  G=0 W=1.89 M=1.89/32.2 TC=0
C capbeam properties
3  A=25 J=10000  I=10E7,10E7  AS=0 E=518400  G=0 W=3.75  M=3.75/32.2   TC=0
C
C lateral static loads
1  WL=100,0,0                                          :uniform longitudinal load
2  WG=0,0,0                                            :zero lateral load
C
C superstructure elements
5011 3011 1012 M=1 LP=4232,4212  NSL=1,2                 :superstructure
5012 1012 1013 M=1 LP=4232,4212  NSL=1,2  G=1,1,1,1  :  "      "
5014 1014 1021 M=1 LP=4232,4212  NSL=1,2             :  "      "
5021 1021 1022 M=1 LP=4232,4212  NSL=1,2  G=2,1,1,1  :  "      "
5024 1024 3031 M=1 LP=4232,4212  NSL=1,2             :  "      "
C
C column elements
7211 4211 4212 M=2 LP=4211,4221 G=1,20,20,20              :column/footing
7221 4221 4222 M=2 LP=4211,4221                           :column/footing
7212 4212 4213 M=2 LP=4211,4221 G=1,20,20,20 RE=0,2.83  :column elements
7222 4222 1021 M=2 LP=4211,4221 RE=0,2.83               :column elements
C
C capbeam elements
6021 4213 1021 M=3 LP=1021,1022                        :capbeam elements
6022 1021 4233 M=3 LP=1021,1022                        :capbeam elements
```

```
SPEC
A=0   S=32.2*0.28   D=0.05
0.0      2.5    2.5*.67*0     2.5*0            :spectral values
0.437    2.5    2.5*.67*0     2.5*0            :spectral values
0.5      2.29   2.5*.67*0     2.5*0            :spectral values
0.6      2.024  2.5*.67*0     2.5*0            :spectral values
0.7      1.827  2.5*.67*0     2.5*0            :spectral values
0.8      1.671  2.5*.67*0     2.5*0            :spectral values
0.9      1.545  2.5*.67*0     2.5*0            :spectral values
1.0      1.440  2.5*.67*0     2.5*0            :spectral values
1.2      1.275  2.5*.67*0     2.5*0            :spectral values
1.4      1.151  2.5*.67*0     2.5*0            :spectral values
1.6      1.053  2.5*.67*0     2.5*0            :spectral values
1.8      0.973  2.5*.67*0     2.5*0            :spectral values
2.0      0.907  2.5*.67*0     2.5*0            :spectral values
2.5      0.782  2.5*.67*0     2.5*0            :spectral values
3.0      0.692  2.5*.67*0     2.5*0            :spectral values
100.     0.0    2.5*.67*0     2.5*0            :spectral values

COMBO
1 C=1,0   D=0                  :uniform longitudinal earthquake loading
2 C=0,1   D=0                  :dead load only
3 C=0,0   D=1                  :multimode spectral transverse loading
```

Filename: F1LSMPE

```
C F1LSMPE  4 ft. dia. col.  (loading scaled to pe)
C FHWA example 1 longitudinal loading, uniform load method, fixed base
C Units are KIP FEET

SYSTEM
R=0 L=2 C=0 V=15 T=0.0001 P=0.001 W=0 N=9999          :control information

JOINTS
4211  X=142    Y=0.0     Z=-28.375                   :column base/mid-footing
4221  X=142    Y=0.0     Z=0.0                       :column base/mid-footing
4231  X=142    Y=0.0     Z=28.375                    :column base/mid-footing
4212  X=142    Y=2.0     Z=-28.375                   :top of footing
4222  X=142    Y=2.0     Z=0.0                       :top of footing
4232  X=142    Y=2.0     Z=28.375                    :top of footing
3011  X=0.0    Y=30.17   Z=0.0                       :abutment 1
1012  X=35.5   Y=30.17   Z=0.0                       :superstructure/span 1
1014  X=106.5  Y=30.17   Z=0.0     G=1012,1014,1     :superstructure/span 1
1021  X=142    Y=30.17   Z=0.0                       :superstructure/span 2
1024  X=217    Y=30.17   Z=0.0     G=1021,1024,1     :superstructure/span 2
3031  X=242    Y=30.17   Z=0.0                       :abutment 3
4213  X=142    Y=30.17   Z=-28.375                   :capbeam
4233  X=142    Y=30.17   Z=28.375                    :capbeam

RESTRAINTS
3011 3031 20 R=0,1,1,1,0,0                           :abutment supports
4211 4231 10 R=1,1,1,1,1,1                           :column foundations


FRAME
NM=3 NL=2 NSEC=0 Y=0,-1                              :frame element specs.
C  superstructure properties
1  A=120 J=60000 I=575,51000  AS=0,0  E=518400  G=0  W=18.9 M=18.9/32.2  TC=0
C column properties
2   A=12.6 J=26    I=12.6,12.6 AS=0 E=518400  G=0 W=1.89 M=1.89/32.2 TC=0
C capbeam properties
3  A=25 J=10000  I=10E7,10E7  AS=0 E=518400  G=0 W=3.75  M=3.75/32.2   TC=0
C
C lateral static loads
1  WL=9.472,0,0                                      :uniform longitudinal load
2  WG=0,0,0                                          :zero lateral load
C
C superstructure elements
5011 3011 1012 M=1 LP=4232,4212  NSL=1,2               :superstructure
5012 1012 1013 M=1 LP=4232,4212  NSL=1,2  G=1,1,1,1   :  "       "
5014 1014 1021 M=1 LP=4232,4212  NSL=1,2              :  "       "
5021 1021 1022 M=1 LP=4232,4212  NSL=1,2  G=2,1,1,1   :  "       "
5024 1024 3031 M=1 LP=4232,4212  NSL=1,2              :  "       "
C
C column elements
7211 4211 4212 M=2 LP=4211,4221 G=1,20,20,20              :column/footing
7221 4221 4222 M=2 LP=4211,4221                           :column/footing
7212 4212 4213 M=2 LP=4211,4221 G=1,20,20,20 RE=0,2.83    :column elements
7222 4222 1021 M=2 LP=4211,4221 RE=0,2.83                 :column elements
C
C capbeam elements
6021 4213 1021 M=3 LP=1021,1022                      :capbeam elements
6022 1021 4233 M=3 LP=1021,1022                      :capbeam elements
```

Filename: F1LSMPE (continued)

```
SPEC
A=0   S=32.2*0.28   D=0.05
0.0     2.5      2.5*.67*0     2.5*0           :spectral values
0.437   2.5      2.5*.67*0     2.5*0           :spectral values
0.5     2.29     2.5*.67*0     2.5*0           :spectral values
0.6     2.024    2.5*.67*0     2.5*0           :spectral values
0.7     1.827    2.5*.67*0     2.5*0           :spectral values
0.8     1.671    2.5*.67*0     2.5*0           :spectral values
0.9     1.545    2.5*.67*0     2.5*0           :spectral values
1.0     1.440    2.5*.67*0     2.5*0           :spectral values
1.2     1.275    2.5*.67*0     2.5*0           :spectral values
1.4     1.151    2.5*.67*0     2.5*0           :spectral values
1.6     1.053    2.5*.67*0     2.5*0           :spectral values
1.8     0.973    2.5*.67*0     2.5*0           :spectral values
2.0     0.907    2.5*.67*0     2.5*0           :spectral values
2.5     0.782    2.5*.67*0     2.5*0           :spectral values
3.0     0.692    2.5*.67*0     2.5*0           :spectral values
100.    0.0      2.5*.67*0     2.5*0           :spectral values

COMBO
1 C=1,0   D=0                  :uniform longitudinal earthquake loading
2 C=0,1   D=0                  :dead load only
3 C=0,0   D=1                  :multimode spectral transverse loading
```

Filename: F1TS4SM

```
C F1TS4SM springs at abut. and col. / 4 ft. dia. col. Icol=0.5Igross
C FHWA example 1 transverse loading, single-mode spectral method
C Units are KIP FEET

SYSTEM
R=0 L=2 C=0 V=15 T=0.0001 P=0.001 W=0 N=9999         :control information

JOINTS
4211  X=142    Y=0.0    Z=-28.375                    :column base/mid-footing
4221  X=142    Y=0.0    Z=0.0                        :column base/mid-footing
4231  X=142    Y=0.0    Z=28.375                     :column base/mid-footing
4212  X=142    Y=2.0    Z=-28.375                    :top of footing
4222  X=142    Y=2.0    Z=0.0                        :top of footing
4232  X=142    Y=2.0    Z=28.375                     :top of footing
3011  X=0.0    Y=30.17  Z=0.0                        :abutment 1
1012  X=35.5   Y=30.17  Z=0.0                        :superstructure/span 1
1014  X=106.5  Y=30.17  Z=0.0     G=1012,1014,1      :superstructure/span 1
1021  X=142    Y=30.17  Z=0.0                        :superstructure/span 2
1024  X=217    Y=30.17  Z=0.0     G=1021,1024,1      :superstructure/span 2
3031  X=242    Y=30.17  Z=0.0                        :abutment 3
4213  X=142    Y=30.17  Z=-28.375                    :capbeam
4233  X=142    Y=30.17  Z=28.375                     :capbeam

RESTRAINTS
3011 3031 20 R=0,1,0,1,0,0                           :abutment supports
4211 4231 10 R=1,1,1,0,1,0                           :column foundations

SPRINGS
C translational and rotational soil springs at foundations
3011 3031 20 K=83E3,0,52E3,0,0,0                     :abutment soil springs
4211 4231 10 K=0,0,0,4.8E6,0,4.8E6                   :column base soil springs

FRAME
NM=3 NL=10 NSEC=0 Y=0,-1                             :frame element specs.
C  superstructure properties
1  A=120 J=60000 I=575,51000  AS=0,0  E=518400  G=0  W=18.9 M=18.9/32.2  TC=0
C column properties
2  A=12.6 J=13 I=6.3,6.3  AS=0 E=518400 G=0 W=1.89 M=1.89/32.2 TC=0
C capbeam properties
3  A=25 J=10000  I=10E7,10E7  AS=0 E=518400  G=0 W=3.75  M=3.75/32.2   TC=0
C
C lateral static loads
C
1  WG=0,100,0                           :uniform lateral load (not used)
2  WG=0,0,0                             :
3  TRAP=0,0,-9.65,35.5,0,-12.62         :
4  TRAP=0,0,-12.62,35.5,0,-14.90        :
5  TRAP=0,0,-14.90,35.5,0,-16.03        :
6  TRAP=0,0,-16.03,35.5,0,-15.84        :  single-mode spectral load
7  TRAP=0,0,-15.84,25,0,-14.93          :
8  TRAP=0,0,-14.93,25,0,-13.46          :
9  TRAP=0,0,-13.46,25,0,-11.54          :
10 TRAP=0,0,-11.54,25,0,-9.39           :
C
C Note that the trapezoidal loads shown here are slightly different from those
C shown in Figure 43 of the text.  The origin of the differences is a slightly
C different numerical integration scheme that was used early in the example
C development.  The values shown in this input file and analysis results
C listed in the text are based on the earlier integration scheme.
```

```
C
C superstructure elements
5011 3011 1012 M=1 LP=4232,4212  NSL=3,2              :superstructure
5012 1012 1013 M=1 LP=4232,4212  NSL=4,2              :  "      "
5013 1013 1014 M=1 LP=4232,4212  NSL=5,2              :  "      "
5014 1014 1021 M=1 LP=4232,4212  NSL=6,2              :  "      "
5021 1021 1022 M=1 LP=4232,4212  NSL=7,2              :  "      "
5022 1022 1023 M=1 LP=4232,4212  NSL=8,2              :  "      "
5023 1023 1024 M=1 LP=4232,4212  NSL=9,2              :  "      "
5024 1024 3031 M=1 LP=4232,4212  NSL=10,2             :  "      "
C
C column elements
7211 4211 4212 M=2 LP=4211,4221 G=1,20,20,20            :column/footing
7221 4221 4222 M=2 LP=4211,4221                         :column/footing
7212 4212 4213 M=2 LP=4211,4221 G=1,20,20,20 RE=0,2.83  :column elements
7222 4222 1021 M=2 LP=4211,4221 RE=0,2.83               :column elements
C
C capbeam elements
6021 4213 1021 M=3 LP=1021,1022                     :capbeam elements
6022 1021 4233 M=3 LP=1021,1022                     :capbeam elements

SPEC
A=0  S=32.2*0.28  D=0.05
0.0      2.5*0    2.5*.67*0    2.5               :spectral values
0.437    2.5*0    2.5*.67*0    2.5               :spectral values
0.5      2.5*0    2.5*.67*0    2.29              :spectral values
0.6      2.5*0    2.5*.67*0    2.024             :spectral values
0.7      2.5*0    2.5*.67*0    1.827             :spectral values
0.8      2.5*0    2.5*.67*0    1.671             :spectral values
0.9      2.5*0    2.5*.67*0    1.545             :spectral values
1.0      2.5*0    2.5*.67*0    1.440             :spectral values
1.2      2.5*0    2.5*.67*0    1.275             :spectral values
1.4      2.5*0    2.5*.67*0    1.151             :spectral values
1.6      2.5*0    2.5*.67*0    1.053             :spectral values
1.8      2.5*0    2.5*.67*0    0.973             :spectral values
2.0      2.5*0    2.5*.67*0    0.907             :spectral values
2.5      2.5*0    2.5*.67*0    0.782             :spectral values
3.0      2.5*0    2.5*.67*0    0.692             :spectral values
100.     2.5*0    2.5*.67*0    0.0               :spectral values

COMBO
1 C=1,0  D=0                   :single-mode transverse earthquake loading
2 C=0,1  D=0                   :dead load only
3 C=0,0  D=1                   :multimode spectral transverse loading
```

Filename: F1LS4SM

```
C F1LS4SM  springs at abut. and col. / 4 ft. dia. col. Icol=0.5Igross
C FHWA example 1 longitudinal loading, single-mode spectral method
C Units are KIP FEET

SYSTEM
R=0 L=2 C=0 V=15 T=0.0001 P=0.001 W=0 N=9999          :control information

JOINTS
4211  X=142    Y=0.0    Z=-28.375                     :column base/mid-footing
4221  X=142    Y=0.0    Z=0.0                         :column base/mid-footing
4231  X=142    Y=0.0    Z=28.375                      :column base/mid-footing
4212  X=142    Y=2.0    Z=-28.375                     :top of footing
4222  X=142    Y=2.0    Z=0.0                         :top of footing
4232  X=142    Y=2.0    Z=28.375                      :top of footing
3011  X=0.0    Y=30.17  Z=0.0                         :abutment 1
1012  X=35.5   Y=30.17  Z=0.0                         :superstructure/span 1
1014  X=106.5  Y=30.17  Z=0.0     G=1012,1014,1       :superstructure/span 1
1021  X=142    Y=30.17  Z=0.0                         :superstructure/span 2
1024  X=217    Y=30.17  Z=0.0     G=1021,1024,1       :superstructure/span 2
3031  X=242    Y=30.17  Z=0.0                         :abutment 3
4213  X=142    Y=30.17  Z=-28.375                     :capbeam
4233  X=142    Y=30.17  Z=28.375                      :capbeam

RESTRAINTS
3011 3031 20 R=0,1,0,1,0,0                            :abutment supports
4211 4231 10 R=1,1,1,0,1,0                            :column foundations

SPRINGS
C translational and rotational soil springs at foundations
3011 3031 20 K=83E3,0,52E3,0,0,0                      :abutment soil springs
4211 4231 10 K=0,0,0,4.8E6,0,4.8E6                    :column base soil springs

FRAME
NM=3 NL=2 NSEC=0 Y=0,-1                               :frame element specs.
C  superstructure properties
1  A=120 J=60000 I=575,51000  AS=0,0  E=518400  G=0  W=18.9 M=18.9/32.2  TC=0
C column properties
2  A=12.6 J=13 I=6.3,6.3  AS=0 E=518400 G=0 W=1.89 M=1.89/32.2 TC=0
C capbeam properties
3  A=25 J=10000  I=10E7,10E7  AS=0 E=518400  G=0 W=3.75  M=3.75/32.2   TC=0
C
C lateral static loads
1  WL=13.96,0,0                                       :uniform lateral load
2  WG=0,0,0                                           :zero lateral load
C
C superstructure elements
5011 3011 1012 M=1 LP=4232,4212  NSL=1,2              :superstructure
5012 1012 1013 M=1 LP=4232,4212  NSL=1,2  G=1,1,1,1   :  "     "
5014 1014 1021 M=1 LP=4232,4212  NSL=1,2              :  "     "
5021 1021 1022 M=1 LP=4232,4212  NSL=1,2  G=2,1,1,1   :  "     "
5024 1024 3031 M=1 LP=4232,4212  NSL=1,2              :  "     "
C
C column elements
7211 4211 4212 M=2 LP=4211,4221 G=1,20,20,20              :column/footing
7221 4221 4222 M=2 LP=4211,4221                           :column/footing
7212 4212 4213 M=2 LP=4211,4221 G=1,20,20,20 RE=0,2.83    :column elements
7222 4222 1021 M=2 LP=4211,4221 RE=0,2.83                 :column elements
C
C capbeam elements
6021 4213 1021 M=3 LP=1021,1022                       :capbeam elements
6022 1021 4233 M=3 LP=1021,1022                       :capbeam elements
```

Filename: F1LS4SM (continued)

```
SPEC
A=0  S=32.2*0.28  D=0.05
0.0      2.5     2.5*.67*0     2.5*0          :spectral values
0.437    2.5     2.5*.67*0     2.5*0          :spectral values
0.5      2.29    2.5*.67*0     2.5*0          :spectral values
0.6      2.024   2.5*.67*0     2.5*0          :spectral values
0.7      1.827   2.5*.67*0     2.5*0          :spectral values
0.8      1.671   2.5*.67*0     2.5*0          :spectral values
0.9      1.545   2.5*.67*0     2.5*0          :spectral values
1.0      1.440  .2.5*.67*0     2.5*0          :spectral values
1.2      1.275   2.5*.67*0     2.5*0          :spectral values
1.4      1.151   2.5*.67*0     2.5*0          :spectral values
1.6      1.053   2.5*.67*0     2.5*0          :spectral values
1.8      0.973   2.5*.67*0     2.5*0          :spectral values
2.0      0.907   2.5*.67*0     2.5*0          :spectral values
2.5      0.782   2.5*.67*0     2.5*0          :spectral values
3.0      0.692   2.5*.67*0     2.5*0          :spectral values
100.     0.0     2.5*.67*0     2.5*0          :spectral values

COMBO
1 C=1,0  D=0                   :uniform transverse earthquake loading
2 C=0,1  D=0                   :dead load only
3 C=0,0  D=1                   :multimode spectral transverse loading
```

Filename: F1TUL4C

```
C F1TUL4C 4 ft. dia. col.
C FHWA example 1 transverse loading, uniform load method, fixed base
C Units are KIP FEET

SYSTEM
R=0 L=2 C=0 V=15 T=0.0001 P=0.001 W=0 N=9999          :control information

JOINTS
4211  X=142    Y=0.0    Z=-28.375                     :column base/mid-footing
4221  X=142    Y=0.0    Z=0.0                         :column base/mid-footing
4231  X=142    Y=0.0    Z=28.375                      :column base/mid-footing
4212  X=142    Y=2.0    Z=-28.375                     :top of footing
4222  X=142    Y=2.0    Z=0.0                         :top of footing
4232  X=142    Y=2.0    Z=28.375                      :top of footing
3011  X=0.0    Y=30.17  Z=0.0                         :abutment 1
1012  X=35.5   Y=30.17  Z=0.0                         :superstructure/span 1
1014  X=106.5  Y=30.17  Z=0.0     G=1012,1014,1       :superstructure/span 1
1021  X=142    Y=30.17  Z=0.0                         :superstructure/span 2
1024  X=217    Y=30.17  Z=0.0     G=1021,1024,1       :superstructure/span 2
3031  X=242    Y=30.17  Z=0.0                         :abutment 3
4213  X=142    Y=30.17  Z=-28.375                     :capbeam
4233  X=142    Y=30.17  Z=28.375                      :capbeam

RESTRAINTS
3011 3031 20 R=0,1,1,1,0,0                            :abutment supports
4211 4231 10 R=1,1.1,1,1,1                            :column foundations

FRAME
NM=3 NL=2 NSEC=0 Y=0,-1                               :frame element specs.
C  superstructure properties
1  A=120 J=60000 I=575,51000  AS=0,0  E=518400  G=0  W=18.9 M=18.9/32.2  TC=0
C column properties
2  A=12.6 J=25 I=12.6,12.6 AS=0 E=518400 G=0 W=1.89 M=1.89/32.2 TC=0
C 2  A=26.2 J=110  I=55,55 AS=0 E=518400  G=0 W=3.93  M=3.93/32.2  TC=0
C capbeam properties
3  A=25 J=10000  I=10E7,10E7  AS=0 E=518400  G=0 W=3.75  M=3.75/32.2  TC=0
C
C lateral static loads
1  WG=0,0,100                                         :uniform lateral load
2  WG=0,0,0                                           :zero lateral load
C
C superstructure elements
5011 3011 1012 M=1 LP=4232,4212  NSL=1,2              :superstructure
5012 1012 1013 M=1 LP=4232,4212  NSL=1,2  G=1,1,1,1   :  "     "
5014 1014 1021 M=1 LP=4232,4212  NSL=1,2              :  "     "
5021 1021 1022 M=1 LP=4232,4212  NSL=1,2  G=2,1,1,1   :  "     "
5024 1024 3031 M=1 LP=4232,4212  NSL=1,2              :  "     "
C
C column elements
7211 4211 4212 M=2 LP=4211,4221 G=1,20,20,20               :column/footing
7221 4221 4222 M=2 LP=4211,4221                            :column/footing
7212 4212 4213 M=2 LP=4211,4221 G=1,20,20,20 RE=0,2.83     :column elements
7222 4222 1021 M=2 LP=4211,4221 RE=0,2.83                  :column elements
C
C capbeam elements
6021 4213 1021 M=3 LP=1021,1022                       :capbeam elements
6022 1021 4233 M=3 LP=1021,1022                       :capbeam elements
```

```
SPEC
A=0  S=32.2*0.28  D=0.05
0.0      2.5*0      2.5*.67*0     2.5            :spectral values
0.437    2.5*0      2.5*.67*0     2.5            :spectral values
0.5      2.5*0      2.5*.67*0     2.29           :spectral values
0.6      2.5*0      2.5*.67*0     2.024          :spectral values
0.7      2.5*0      2.5*.67*0     1.827          :spectral values
0.8      2.5*0      2.5*.67*0     1.671          :spectral values
0.9      2.5*0      2.5*.67*0     1.545          :spectral values
1.0      2.5*0      2.5*.67*0     1.440          :spectral values
1.2      2.5*0      2.5*.67*0     1.275          :spectral values
1.4      2.5*0      2.5*.67*0     1.151          :spectral values
1.6      2.5*0      2.5*.67*0     1.053          :spectral values
1.8      2.5*0      2.5*.67*0     0.973          :spectral values
2.0      2.5*0      2.5*.67*0     0.907          :spectral values
2.5      2.5*0      2.5*.67*0     0.782          :spectral values
3.0      2.5*0      2.5*.67*0     0.692          :spectral values
100.     2.5*0      2.5*.67*0     0.0            :spectral values

COMBO
1 C=1,0  D=0                       :uniform transverse earthquake loading
2 C=0,1  D=0                       :dead load only
3 C=0,0  D=1                       :multimode spectral transverse loading
```

Filename: F1TS4

```
C F1TS4 springs at abut. and col. / 4 ft. dia. col. Icol=0.5Igross
C FHWA example 1 transverse loading, uniform load method
C Units are KIP FEET

SYSTEM
R=0 L=2 C=0 V=15 T=0.0001 P=0.001 W=0 N=9999          :control information

JOINTS
4211  X=142    Y=0.0    Z=-28.375                     :column base/mid-footing
4221  X=142    Y=0.0    Z=0.0                         :column base/mid-footing
4231  X=142    Y=0.0    Z=28.375                      :column base/mid-footing
4212  X=142    Y=2.0    Z=-28.375                     :top of footing
4222  X=142    Y=2.0    Z=0.0                         :top of footing
4232  X=142    Y=2.0    Z=28.375                      :top of footing
3011  X=0.0    Y=30.17  Z=0.0                         :abutment 1
1012  X=35.5   Y=30.17  Z=0.0                         :superstructure/span 1
1014  X=106.5  Y=30.17  Z=0.0     G=1012,1014,1       :superstructure/span 1
1021  X=142    Y=30.17  Z=0.0                         :superstructure/span 2
1024  X=217    Y=30.17  Z=0.0     G=1021,1024,1       :superstructure/span 2
3031  X=242    Y=30.17  Z=0.0                         :abutment 3
4213  X=142    Y=30.17  Z=-28.375                     :capbeam
4233  X=142    Y=30.17  Z=28.375                      :capbeam

RESTRAINTS
3011 3031 20 R=0,1,0,1,0,0                            :abutment supports
4211 4231 10 R=1,1,1,0,1,0                            :column foundations

SPRINGS
C translational and rotational soil springs at foundations
3011 3031 20 K=83E3,0,52E3,0,0,0                      :abutment soil springs
4211 4231 10 K=0,0,0,4.8E6,0,4.8E6                    :column base soil springs

FRAME
NM=3 NL=2 NSEC=0 Y=0,-1                               :frame element specs.
C  superstructure properties
1  A=120 J=60000 I=575,51000  AS=0,0  E=518400  G=0  W=18.9 M=18.9/32.2  TC=0
C column properties
2  A=12.6 J=13 I=6.3,6.3  AS=0 E=518400 G=0 W=1.89 M=1.89/32.2 TC=0
C capbeam properties
3  A=25 J=10000  I=10E7,10E7  AS=0 E=518400  G=0 W=3.75  M=3.75/32.2   TC=0
C
C lateral static loads
1  WG=0,0,100                                         :uniform lateral load
2  WG=0,0,0                                           :zero lateral load
C
C superstructure elements
5011 3011 1012 M=1 LP=4232,4212  NSL=1,2              :superstructure
5012 1012 1013 M=1 LP=4232,4212  NSL=1,2  G=1,1,1,1   :  "     "
5014 1014 1021 M=1 LP=4232,4212  NSL=1,2              :  "     "
5021 1021 1022 M=1 LP=4232,4212  NSL=1,2  G=2,1,1,1   :  "     "
5024 1024 3031 M=1 LP=4232,4212  NSL=1,2              :  "     "
C
C column elements
7211 4211 4212 M=2 LP=4211,4221 G=1,20,20,20             :column/footing
7221 4221 4222 M=2 LP=4211,4221                          :column/footing
7212 4212 4213 M=2 LP=4211,4221 G=1,20,20,20 RE=0,2.83   :column elements
7222 4222 1021 M=2 LP=4211,4221 RE=0,2.83                :column elements
C
C capbeam elements
6021 4213 1021 M=3 LP=1021,1022                       :capbeam elements
6022 1021 4233 M=3 LP=1021,1022                       :capbeam elements
```

Filename: F1TS4 (continued)

```
SPEC
A=0  S=32.2*0.28  D=0.05
0.0     2.5*0     2.5*.67*0    2.5              :spectral values
0.437   2.5*0     2.5*.67*0    2.5              :spectral values
0.5     2.5*0     2.5*.67*0    2.29             :spectral values
0.6     2.5*0     2.5*.67*0    2.024            :spectral values
0.7     2.5*0     2.5*.67*0    1.827            :spectral values
0.8     2.5*0     2.5*.67*0    1.671            :spectral values
0.9     2.5*0     2.5*.67*0    1.545            :spectral values
1.0     2.5*0     2.5*.67*0    1.440            :spectral values
1.2     2.5*0     2.5*.67*0    1.275            :spectral values
1.4     2.5*0     2.5*.67*0    1.151            :spectral values
1.6     2.5*0     2.5*.67*0    1.053            :spectral values
1.8     2.5*0     2.5*.67*0    0.973            :spectral values
2.0     2.5*0     2.5*.67*0    0.907            :spectral values
2.5     2.5*0     2.5*.67*0    0.782            :spectral values
3.0     2.5*0     2.5*.67*0    0.692            :spectral values
100.    2.5*0     2.5*.67*0    0.0              :spectral values

COMBO
1 C=1,0  D=0                   :uniform transverse earthquake loading
2 C=0,1  D=0                   :dead load only
3 C=0,0  D=1                   :multimode spectral transverse loading
```

Filename: F1LML

```
C F1LML all mass lumped at bent (check of main analysis)
C FHWA example 1 longitudinal loading, uniform load method, fixed base
C Units are KIP FEET

SYSTEM
R=0 L=2 C=0 V=3 T=0.0001 P=0.001 W=0 N=9999          :control information

JOINTS
4211  X=142    Y=0.0     Z=-28.375                    :column base/mid-footing
4221  X=142    Y=0.0     Z=0.0                        :column base/mid-footing
4231  X=142    Y=0.0     Z=28.375                     :column base/mid-footing
4212  X=142    Y=2.0     Z=-28.375                    :top of footing
4222  X=142    Y=2.0     Z=0.0                        :top of footing
4232  X=142    Y=2.0     Z=28.375                     :top of footing
3011  X=0.0    Y=30.17   Z=0.0                        :abutment 1
1012  X=35.5   Y=30.17   Z=0.0                        :superstructure/span 1
1014  X=106.5  Y=30.17   Z=0.0      G=1012,1014,1     :superstructure/span 1
1021  X=142    Y=30.17   Z=0.0                        :superstructure/span 2
1024  X=217    Y=30.17   Z=0.0      G=1021,1024,1     :superstructure/span 2
3031  X=242    Y=30.17   Z=0.0                        :abutment 3
4213  X=142    Y=30.17   Z=-28.375                    :capbeam
4233  X=142    Y=30.17   Z=28.375                     :capbeam

RESTRAINTS
3011 3031 20 R=0,1,1,1,0,0                            :abutment supports
4211 4231 10 R=1,1,1,1,1,1                            :column foundations

MASSES
1021 M=4872/32.2,4872/32.2,4872/32.2,0,0,0            :all mass lumped at bent

FRAME
NM=3 NL=2 NSEC=0 Y=0,-1                               :frame element specs.
C  superstructure properties
1  A=120 J=60000 I=575,51000  AS=0,0  E=518400  G=0  W=18.9 M=0/32.2  TC=0
C column properties
2  A=12.6 J=110  I=12.6,12.6 AS=0 E=518400  G=0 W=1.89  M=0/32.2   TC=0
C capbeam properties
3  A=25 J=10000  I=10E7,10E7  AS=0 E=518400  G=0 W=3.75  M=0/32.2   TC=0
C
C lateral static loads
1  WL=100,0,0                                         :uniform longitudinal load
2  WG=0,0,0                                           :zero lateral load
C
C superstructure elements
5011 3011 1012 M=1 LP=4232,4212  NSL=1,2              :superstructure
5012 1012 1013 M=1 LP=4232,4212  NSL=1,2  G=1,1,1,1   :  "     "
5014 1014 1021 M=1 LP=4232,4212  NSL=1,2              :  "     "
5021 1021 1022 M=1 LP=4232,4212  NSL=1,2  G=2,1,1,1   :  "     "
5024 1024 3031 M=1 LP=4232,4212  NSL=1,2              :  "     "
C
C column elements
7211 4211 4212 M=2 LP=4211,4221 G=1,20,20,20          :column/footing
7221 4221 4222 M=2 LP=4211,4221                       :column/footing
7212 4212 4213 M=2 LP=4211,4221 G=1,20,20,20 RE=0,2.83 :column elements
7222 4222 1021 M=2 LP=4211,4221 RE=0,2.83             :column elements
C
C capbeam elements
6021 4213 1021 M=3 LP=1021,1022                       :capbeam elements
6022 1021 4233 M=3 LP=1021,1022                       :capbeam elements
```

Filename: F1LML (continued)

```
SPEC
A=0  S=32.2*0.28  D=0.05
0.0     2.5     2.5*.67*0    2.5*0          :spectral values
0.437   2.5     2.5*.67*0    2.5*0          :spectral values
0.5     2.29    2.5*.67*0    2.5*0          :spectral values
0.6     2.024   2.5*.67*0    2.5*0          :spectral values
0.7     1.827   2.5*.67*0    2.5*0          :spectral values
0.8     1.671   2.5*.67*0    2.5*0          :spectral values
0.9     1.545   2.5*.67*0    2.5*0          :spectral values
1.0     1.440   2.5*.67*0    2.5*0          :spectral values
1.2     1.275   2.5*.67*0    2.5*0          :spectral values
1.4     1.151   2.5*.67*0    2.5*0          :spectral values
1.6     1.053   2.5*.67*0    2.5*0          :spectral values
1.8     0.973   2.5*.67*0    2.5*0          :spectral values
2.0     0.907   2.5*.67*0    2.5*0          :spectral values
2.5     0.782   2.5*.67*0    2.5*0          :spectral values
3.0     0.692   2.5*.67*0    2.5*0          :spectral values
100.    0.0     2.5*.67*0    2.5*0          :spectral values

COMBO
1 C=1,0  D=0                :uniform longitudinal earthquake loading
2 C=0,1  D=0                :dead load only
3 C=0,0  D=1                :multimode spectral transverse loading
```

Filename: F1LS4

```
C F1LS4 springs at abut. and col. / 4 ft. dia. col. Icol=0.5Igross
C FHWA example 1 longitudinal loading, uniform load method
C Units are KIP FEET

SYSTEM
R=0 L=2 C=0 V=15 T=0.0001 P=0.001 W=0 N=9999        :control information

JOINTS
4211  X=142    Y=0.0    Z=-28.375                  :column base/mid-footing
4221  X=142    Y=0.0    Z=0.0                      :column base/mid-footing
4231  X=142    Y=0.0    Z=28.375                   :column base/mid-footing
4212  X=142    Y=2.0    Z=-28.375                  :top of footing
4222  X=142    Y=2.0    Z=0.0                      :top of footing
4232  X=142    Y=2.0    Z=28.375                   :top of footing
3011  X=0.0    Y=30.17  Z=0.0                      :abutment 1
1012  X=35.5   Y=30.17  Z=0.0                      :superstructure/span 1
1014  X=106.5  Y=30.17  Z=0.0     G=1012,1014,1    :superstructure/span 1
1021  X=142    Y=30.17  Z=0.0                      :superstructure/span 2
1024  X=217    Y=30.17  Z=0.0     G=1021,1024,1    :superstructure/span 2
3031  X=242    Y=30.17  Z=0.0                      :abutment 3
4213  X=142    Y=30.17  Z=-28.375                  :capbeam
4233  X=142    Y=30.17  Z=28.375                   :capbeam

RESTRAINTS
3011 3031 20 R=0,1,0 1,0,0                         :abutment supports
4211 4231 10 R=1,1,1,0,1,0                         :column foundations

SPRINGS
C translational and rotational soil springs at foundations
3011 3031 20 K=83E3,0,52E3,0,0,0                :abutment soil springs
4211 4231 10 K=0,0,0,4.8E6,0,4.8E6              :column base soil springs

FRAME
NM=3 NL=2 NSEC=0 Y=0,-1                            :frame element specs.
C  superstructure properties
1  A=120 J=60000 I=575,51000  AS=0,0  E=518400  G=0  W=18.9 M=18.9/32.2  TC=0
C column properties
2  A=12.6 J=13 I=6.3,6.3  AS=0 E=518400 G=0 W=1.89 M=1.89/32.2 TC=0
C capbeam properties
3  A=25 J=10000  I=10E7,10E7  AS=0 E=518400  G=0 W=3.75  M=3.75/32.2   TC=0
C
C lateral static loads
1  WL=100,0,0                                      :uniform lateral load
2  WG=0,0,0                                        :zero lateral load
C
C superstructure elements
5011 3011 1012 M=1 LP=4232,4212  NSL=1,2              :superstructure
5012 1012 1013 M=1 LP=4232,4212  NSL=1,2  G=1,1,1,1  :  "      "
5014 1014 1021 M=1 LP=4232,4212  NSL=1,2             :  "      "
5021 1021 1022 M=1 LP=4232,4212  NSL=1,2  G=2,1,1,1  :  "      "
5024 1024 3031 M=1 LP=4232,4212  NSL=1,2             :  "      "
C
C column elements
7211 4211 4212 M=2 LP=4211,4221 G=1,20,20,20              :column/footing
7221 4221 4222 M=2 LP=4211,4221                           :column/footing
7212 4212 4213 M=2 LP=4211,4221 G=1,20,20,20 RE=0,2.83    :column elements
7222 4222 1021 M=2 LP=4211,4221 RE=0,2.83                 :column elements
C
C capbeam elements
6021 4213 1021 M=3 LP=1021,1022                     :capbeam elements
6022 1021 4233 M=3 LP=1021,1022                     :capbeam elements
```

```
SPEC
A=0  S=32.2*0.28  D=0.05
0.0       2.5     2.5*.67*0     2.5*0                :spectral values
0.437     2.5     2.5*.67*0     2.5*0                :spectral values
0.5       2.29    2.5*.67*0     2.5*0                :spectral values
0.6       2.024   2.5*.67*0     2.5*0                :spectral values
0.7       1.827   2.5*.67*0     2.5*0                :spectral values
0.8       1.671   2.5*.67*0     2.5*0                :spectral values
0.9       1.545   2.5*.67*0     2.5*0                :spectral values
1.0       1.440   2.5*.67*0     2.5*0                :spectral values
1.2       1.275   2.5*.67*0     2.5*0                :spectral values
1.4       1.151   2.5*.67*0     2.5*0                :spectral values
1.6       1.053   2.5*.67*0     2.5*0                :spectral values
1.8       0.973   2.5*.67*0     2.5*0                :spectral values
2.0       0.907   2.5*.67*0     2.5*0                :spectral values
2.5       0.782   2.5*.67*0     2.5*0                :spectral values
3.0       0.69    2.5*.67*0     2.5*0                :spectral values
100.      0.0     2.5*.67*0     2.5*0                :spectral values

COMBO
1 C=1,0   D=0                   :uniform transverse earthquake loading
2 C=0,1   D=0                   :dead load only
3 C=0,0   D=1                   :multimode spectral transverse loading
```

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