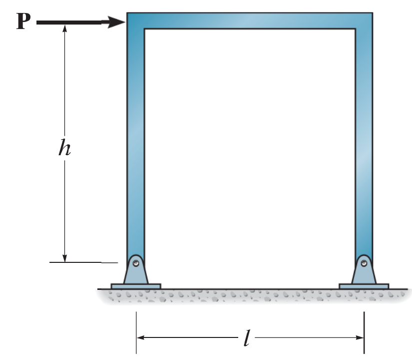
7.4 Portal Frames and Trusses

Frames. Portal frames are frequently used over the entrance of a bridge* and as a main stiffening element in building design in order to transfer horizontal forces applied at the top of the frame to the foundation. On bridges, these frames resist the forces caused by wind, earthquake, and unbalanced traffic loading on the bridge deck. Portals can be pin supported, fixed supported, or supported by partial fixity. The approximate analysis of each case will now be discussed for a simple three-member portal.

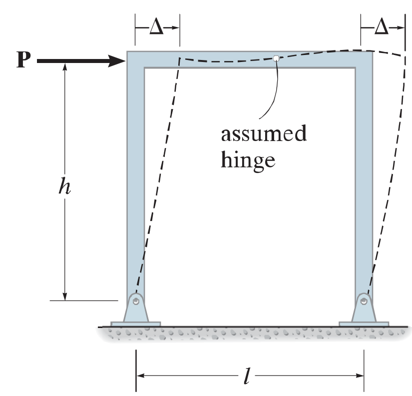


(a)

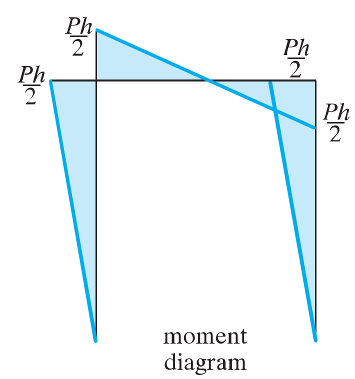
Fig. 7–7

Pin Supported. A typical pin-supported portal frame is shown in Fig. 7–7*a*. Since four unknowns exist at the supports but only three equilibrium equations are available for solution, this structure is statically indeterminate to the first degree. Consequently, only one assumption must be made to reduce the frame to one that is statically determinate.

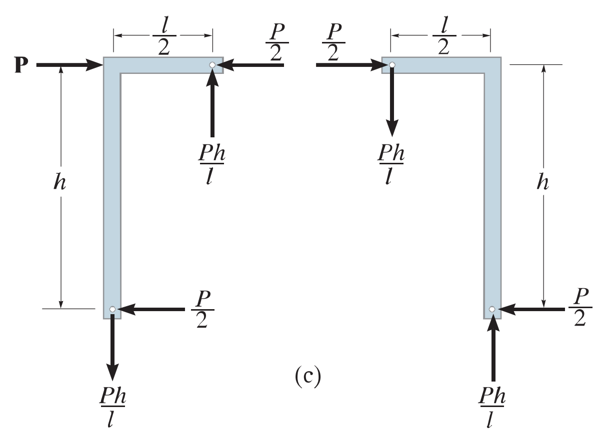
The elastic deflection of the portal is shown in Fig. 7–7*b*. This diagram indicates that a point of inflection, that is, where the moment changes from positive bending to negative bending, is located *approximately* at the girder's midpoint. Since the moment in the girder is zero at this point, we can *assume* a hinge exists there and then proceed to determine the reactions at the supports using statics. If this is done, it is found that the horizontal reactions (shear) at the base of each column are *equal* and the other reactions are those indicated in Fig. 7–7*c*. Furthermore, the moment diagrams for this frame are indicated in Fig. 7–7*d*.



(b)



(d)



(c)

*See Fig. 3–4.

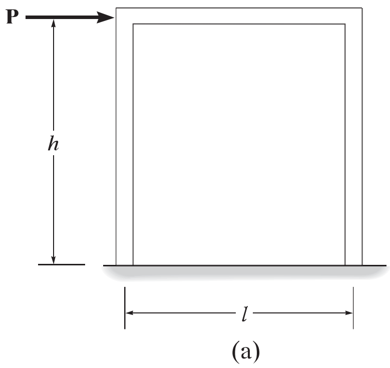
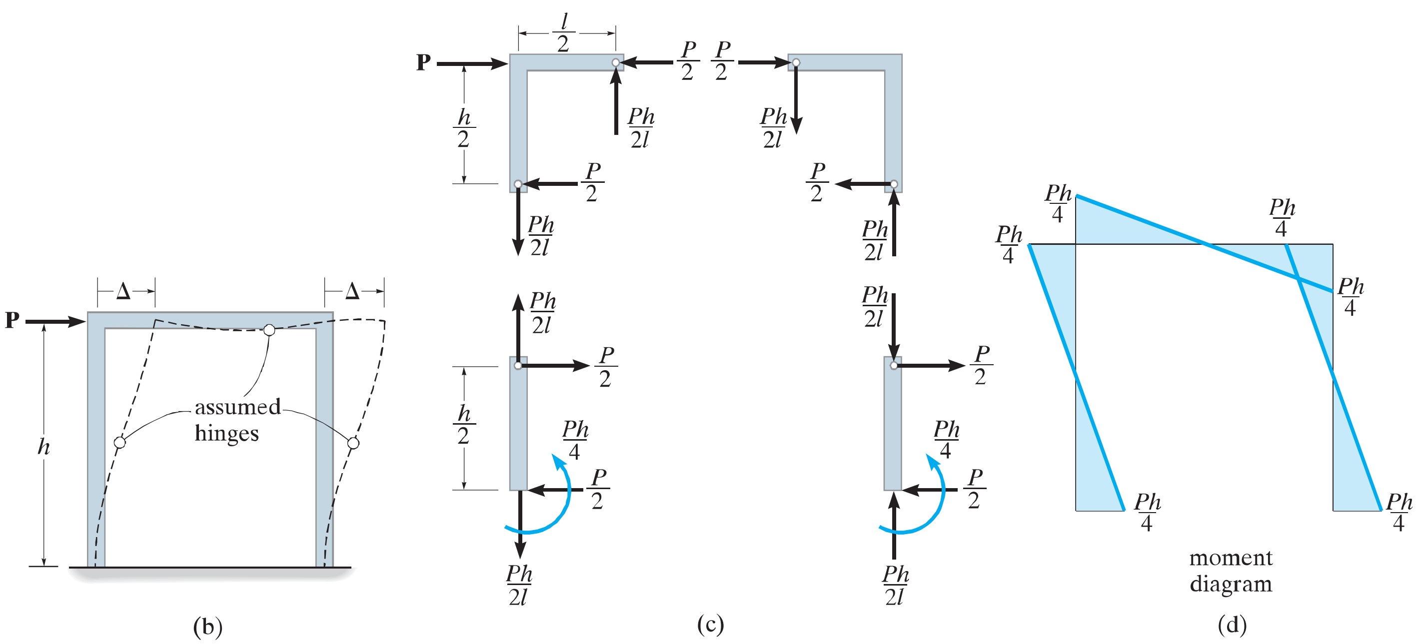


Fig. 7–8

Fixed Supported. Portals with two fixed supports, Fig. 7–8*a*, are statically indeterminate to the third degree since there are a total of six unknowns at the supports. If the vertical members have equal lengths and cross-sectional areas, the frame will deflect as shown in Fig. 7–8*b*. For this case we will *assume* points of inflection occur at the midpoints of all three members, and therefore hinges are placed at these points. The reactions and moment diagrams for each member can therefore be determined by dismembering the frame at the hinges and applying the equations of equilibrium to each of the four parts. The results are shown in Fig. 7–8*c*. Note that, as in the case of the pin-connected portal, the horizontal reactions (shear) at the base of each column are *equal*. The moment diagram for this frame is indicated in Fig. 7–8*d*.



Partial Fixity. Since it is both difficult and costly to construct a perfectly fixed support or foundation for a portal frame, it is conservative and somewhat realistic to assume a slight rotation occurs at the supports, Fig. 7–9*a*. As a result, the points of inflection on the columns lie somewhere between the case of having a pin-supported portal, Fig. 7–7*a*, where the "inflection points" are at the supports (base of columns), and a fixed-supported portal, Fig. 7–8*a*, where the inflection points are at the center of the columns. Many engineers arbitrarily define the location at $h/3$, Fig. 7–9*b*, and therefore place hinges at these points, and also at the center of the girder.

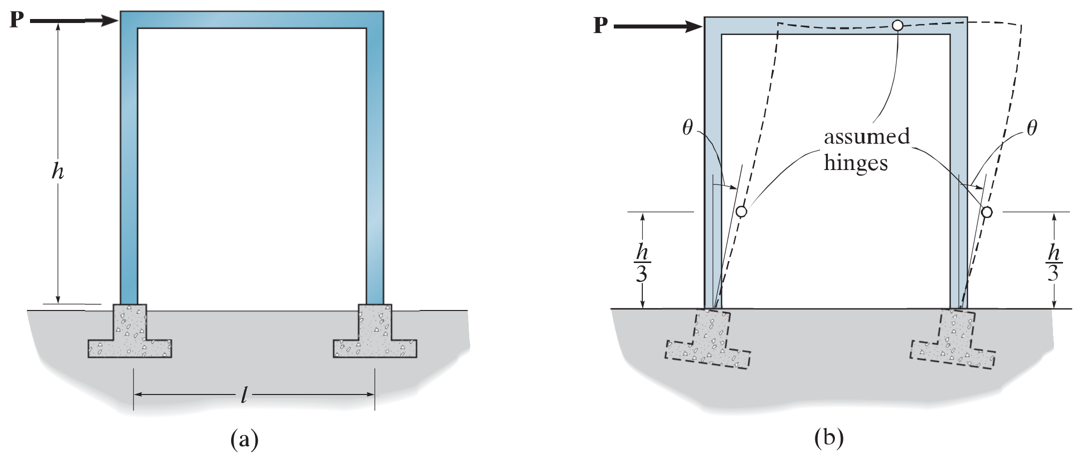


Fig. 7–9

Trusses. When a portal is used to span large distances, a truss may be used in place of the horizontal girder. Such a structure is used on large bridges and as transverse bents for large auditoriums and mill buildings. A typical example is shown in Fig. 7–10*a*. In all cases, the suspended truss is assumed to be pin connected at its points of attachment to the columns. Furthermore, the truss keeps the columns straight within the region of attachment when the portal is subjected to the sidesway Δ, Fig. 7–10*b*. Consequently, we can analyze trussed portals using the same assumptions as those used for simple portal frames. For pin-supported columns, assume the horizontal reactions (shear) are equal, as in Fig. 7–7*c*. For fixed-supported columns, assume the horizontal reactions are equal and an inflection point (or hinge) occurs on each column, measured midway between the base of the column and the *lowest point* of truss member connection to the column. See Fig. 7–8*c* and Fig. 7–10*b*.

The following example illustrates how to determine the forces in the members of a trussed portal using the approximate method of analysis described above.

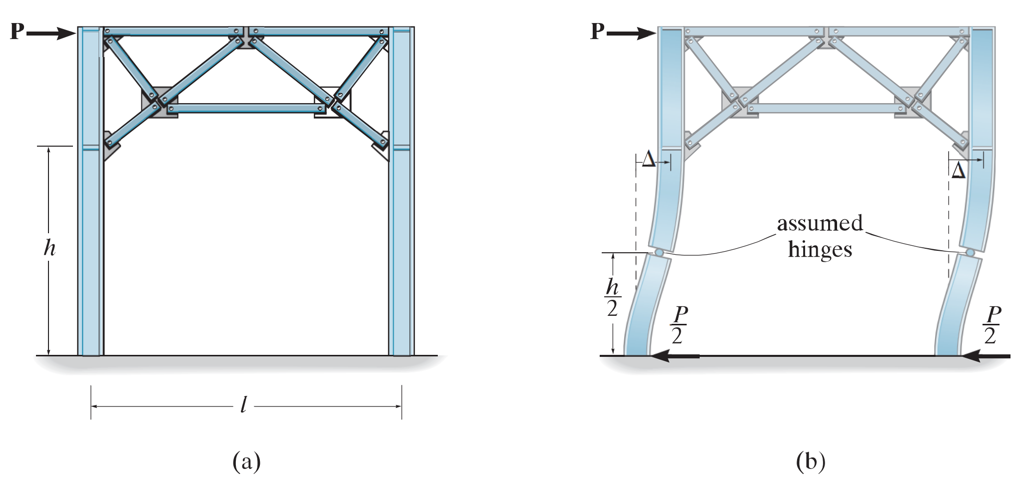


Fig. 7–10

EXAMPLE 7.4

Determine by approximate methods the forces acting in the members of the Warren portal shown in Fig. 7–11*a*.

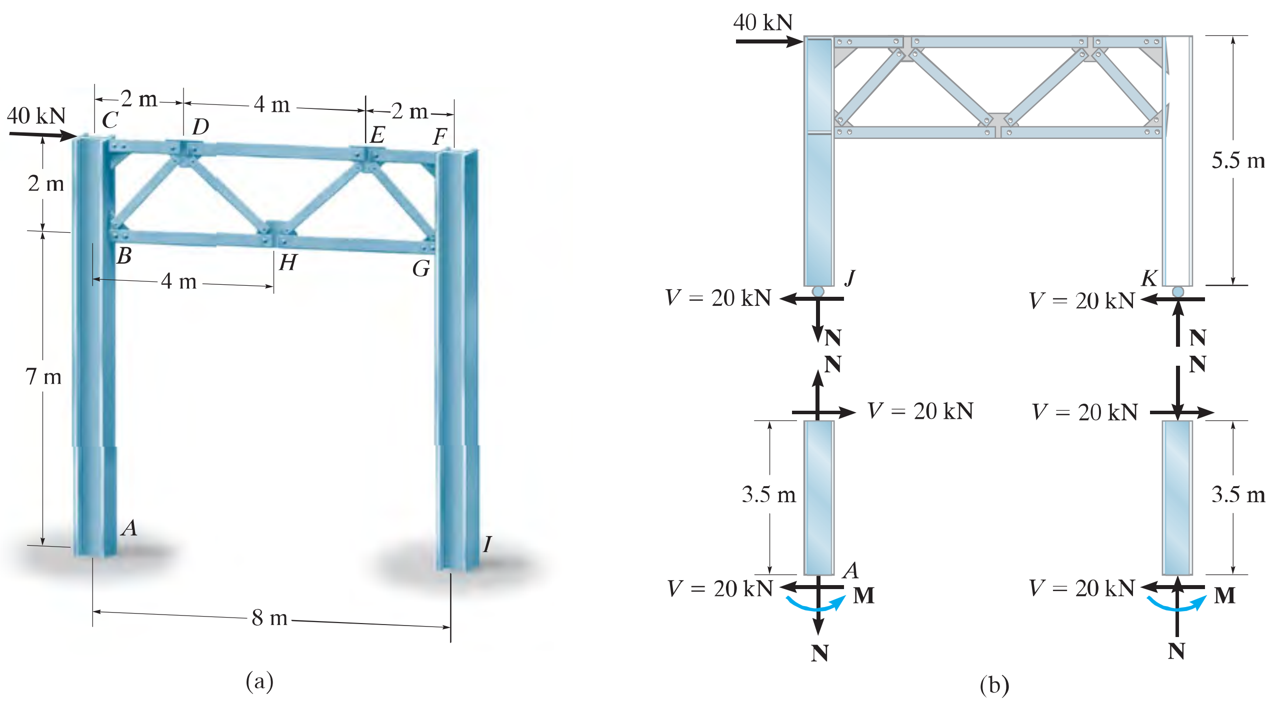


Fig. 7–11

SOLUTION

The truss portion *B*, *C*, *F*, *G* acts as a rigid unit. Since the supports are fixed, a point of inflection is assumed to exist $7\text{ m}/2 = 3.5\text{ m}$ above *A* and *I*, and equal horizontal reactions or shear act at the base of the columns, i.e., $\Sigma F_x = 0$; $V = 40\text{ kN}/2 = 20\text{ kN}$. With these assumptions, we can separate the structure at the hinges *J* and *K*, Fig. 7–11*b*, and determine the reactions on the columns as follows:

Lower Half of Column

$$\curvearrowright + \Sigma M_A = 0; \qquad M - 3.5(20) = 0 \qquad M = 70\text{ kN}\cdot\text{m}$$

Upper Portion of Column

$$\curvearrowright + \Sigma M_J = 0; \qquad -40(5.5) + N(8) = 0 \qquad N = 27.5\text{ kN}$$

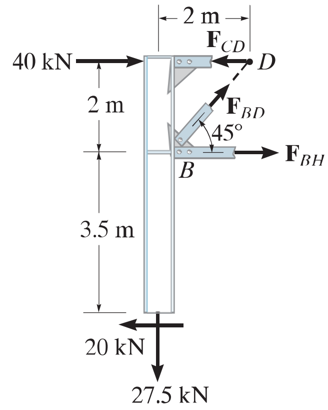
Using the method of sections, Fig. 7–11*c*, we can now proceed to obtain the forces in members CD, BD, and BH.

$$+\uparrow \Sigma F_y = 0; \quad -27.5 + F_{BD} \sin 45^\circ = 0 \qquad F_{BD} = 38.9 \text{ kN (T)} \quad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_B = 0; \quad -20(3.5) - 40(2) + F_{CD}(2) = 0 \quad F_{CD} = 75 \text{ kN (C)} \quad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_D = 0; \quad F_{BH}(2) - 20(5.5) + 27.5(2) = 0 \quad F_{BH} = 27.5 \text{ kN (T)} \quad \textit{Ans.}$$

In a similar manner, show that one obtains the results on the free-body diagram of column FGI in Fig. 7–11*d*. Using these results, we can now find the force in each of the other truss members of the portal using the method of joints.



(c)

Joint *D*, Fig. 7–11*e*

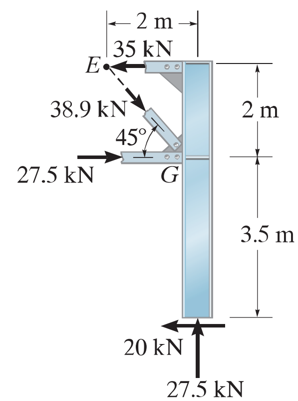
$$+\uparrow \Sigma F_y = 0; \quad F_{DH} \sin 45^\circ - 38.9 \sin 45^\circ = 0 \quad F_{DH} = 38.9 \text{ kN (C)} \quad \textit{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 75 - 2(38.9 \cos 45^\circ) - F_{DE} = 0 \quad F_{DE} = 20 \text{ kN (C)} \quad \textit{Ans.}$$

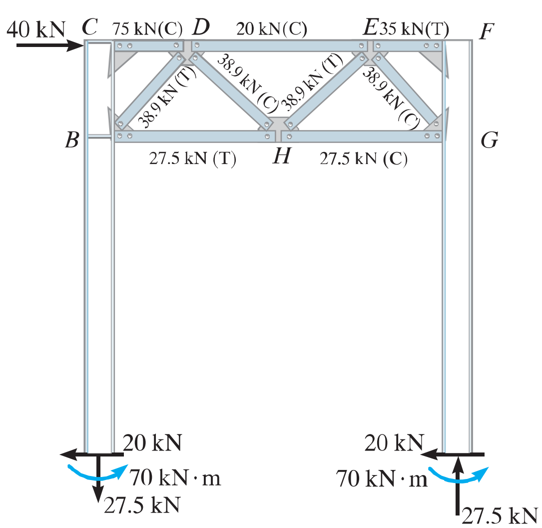
Joint *H*, Fig. 7–11*f*

$$+\uparrow \Sigma F_y = 0; \quad F_{HE} \sin 45^\circ - 38.9 \sin 45^\circ = 0 \quad F_{HE} = 38.9 \text{ kN (T)} \quad \textit{Ans.}$$

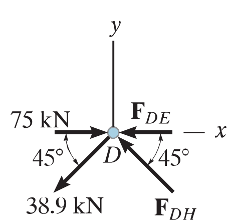
These results are summarized in Fig. 7–11*g*.



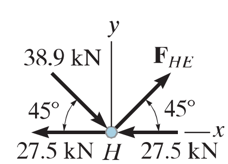
(d)



(g)



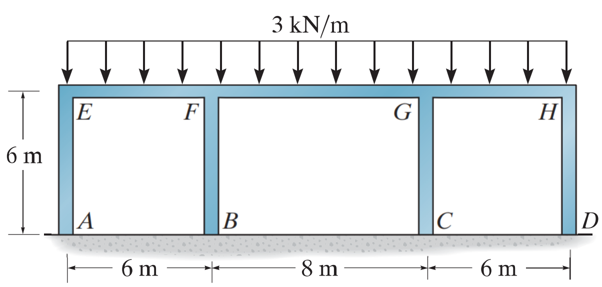
(e)



(f)

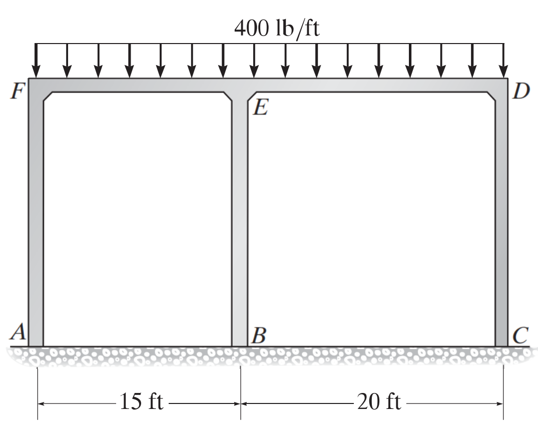
PROBLEMS

7–13. Determine (approximately) the internal moments at joints *A* and *B* of the frame.



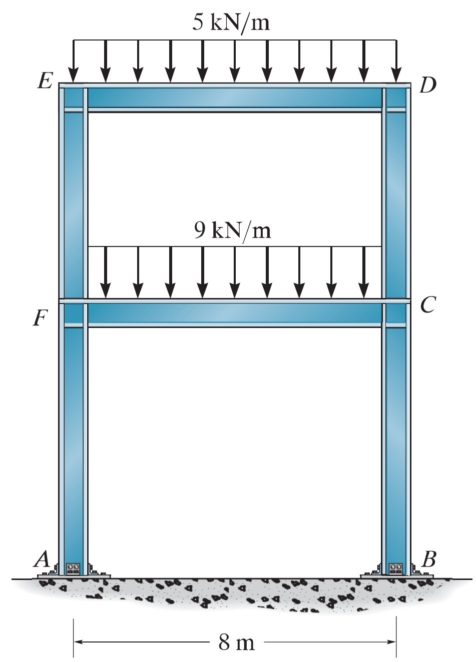
Prob. 7–13

7–14. Determine (approximately) the internal moments at joints *F* and *D* of the frame.



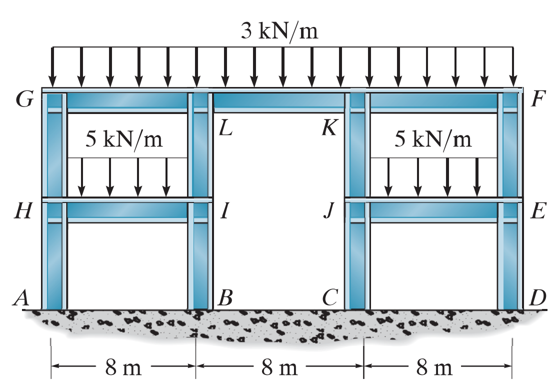
Prob. 7–14

7–15. Determine (approximately) the internal moment at *A* caused by the vertical loading.



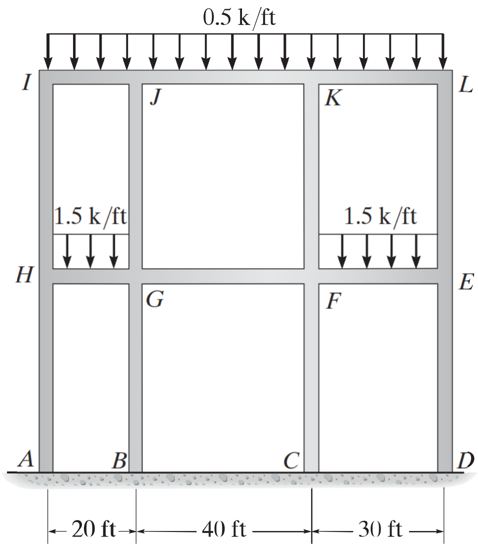
Prob. 7–15

***7–16.** Determine (approximately) the internal moments at *A* and *B* caused by the vertical loading.



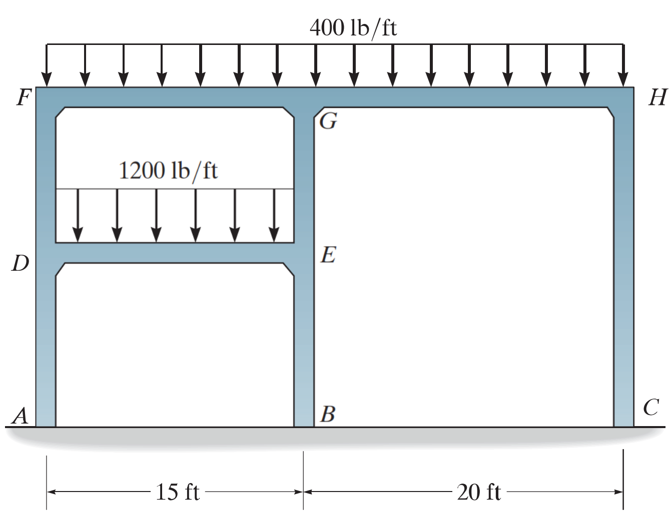
Prob. 7–16

7–17. Determine (approximately) the internal moments at joints I and L. Also, what is the internal moment at joint H caused by member HG?



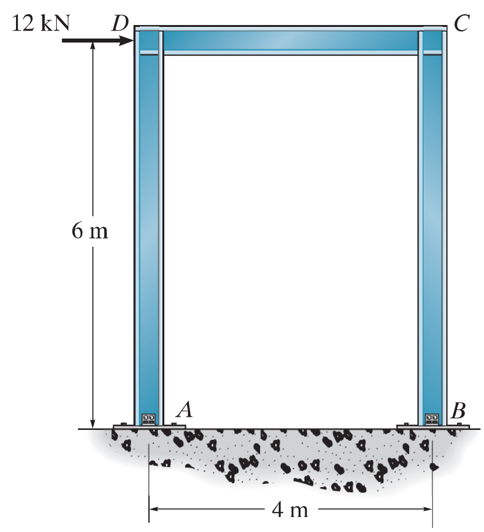
Prob. 7–17

7–18. Determine (approximately) the support actions at A, B, and C of the frame.



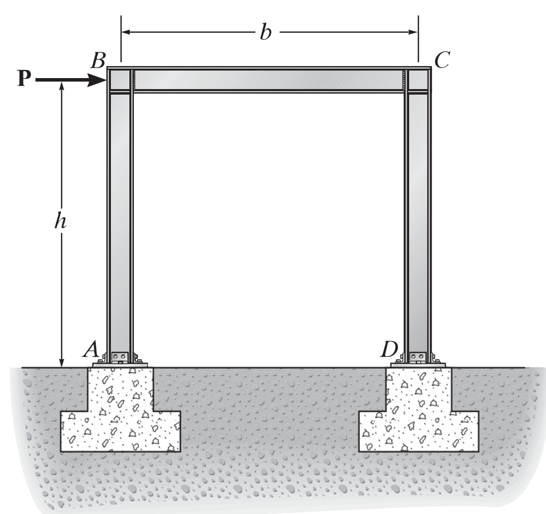
Prob. 7–18

7–19. Determine (approximately) the support reactions at A and B of the portal frame. Assume the supports are (a) pinned, and (b) fixed.



Prob. 7–19

***7–20.** Determine (approximately) the internal moment and shear at the ends of each member of the portal frame. Assume the supports at A and D are partially fixed, such that an inflection point is located at $h/3$ from the bottom of each column.

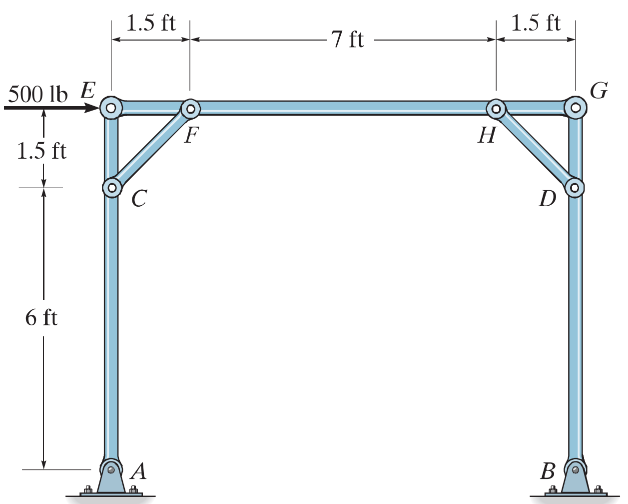


Prob. 7–20

7

7–21. Draw (approximately) the moment diagram for member ACE of the portal constructed with a *rigid* member EG and knee braces CF and DH. Assume that all points of connection are pins. Also determine the force in the knee brace CF.

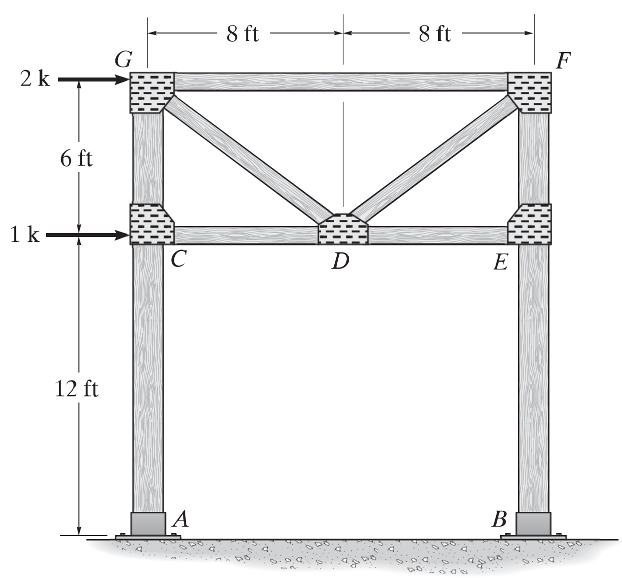
7–22. Solve Prob. 7–21 if the supports at A and B are fixed instead of pinned.



Probs. 7–21/7–22

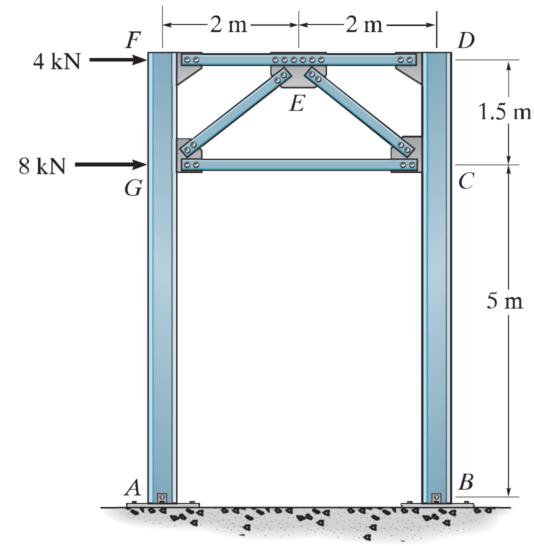
7–23. Determine (approximately) the force in each truss member of the portal frame. Also find the reactions at the fixed column supports A and B. Assume all members of the truss to be pin connected at their ends.

***7–24.** Solve Prob. 7–23 if the supports at A and B are pinned instead of fixed.



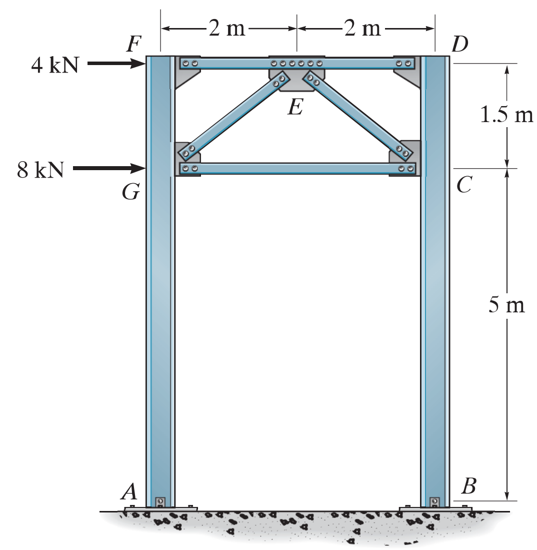
Probs. 7–23/7–24

7–25. Draw (approximately) the moment diagram for column AGF of the portal. Assume all truss members and the columns to be pin connected at their ends. Also determine the force in all the truss members.



Prob. 7–25

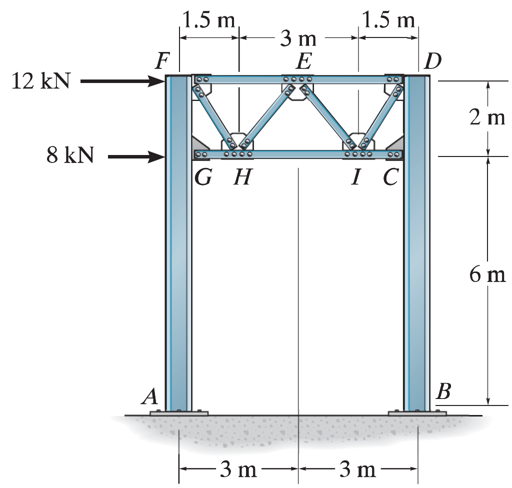
7–26. Draw (approximately) the moment diagram for column AGF of the portal. Assume all the members of the truss to be pin connected at their ends. The columns are fixed at A and B. Also determine the force in all the truss members.



Prob. 7–26

7–27. Determine (approximately) the force in each truss member of the portal frame. Also find the reactions at the fixed column supports A and B. Assume all members of the truss to be pin connected at their ends.

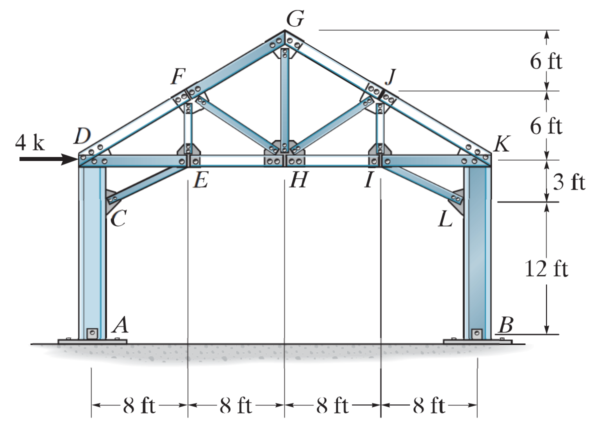
***7–28.** Solve Prob. 7–27 if the supports at A and B are pinned instead of fixed.



Probs. 7–27/7–28

7–31. Draw (approximately) the moment diagram for column ACD of the portal. Assume all truss members and the columns to be pin connected at their ends. Also determine the force in members FG, FH, and EH.

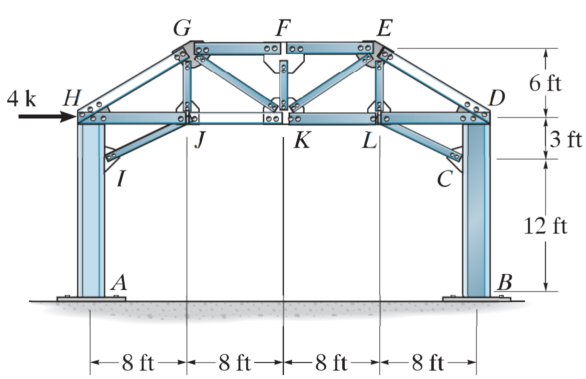
***7–32.** Solve Prob. 7–31 if the supports at A and B are fixed instead of pinned.



Probs. 7–31/7–32

7–29. Determine (approximately) the force in members GF, GK, and JK of the portal frame. Also find the reactions at the fixed column supports A and B. Assume all members of the truss to be connected at their ends.

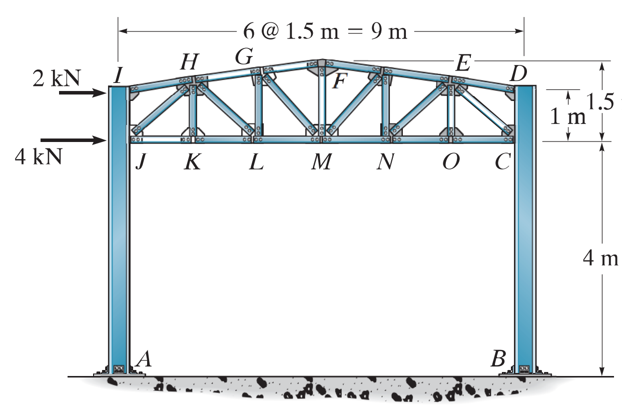
7–30. Solve Prob. 7–29 if the supports at A and B are pin connected instead of fixed.



Probs. 7–29/7–30

7–33. Draw (approximately) the moment diagram for column AJI of the portal. Assume all truss members and the columns to be pin connected at their ends. Also determine the force in members HG, HL, and KL.

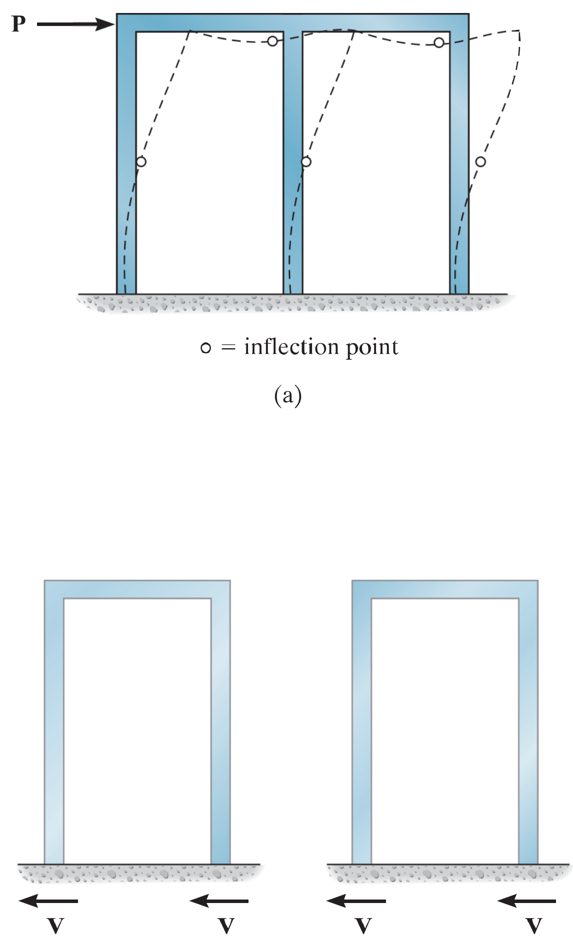
7–34. Solve Prob. 7–33 if the supports at A and B are fixed instead of pinned.



Probs. 7–33/7–34

7.5 Lateral Loads on Building Frames: Portal Method

In Sec. 7–4 we discussed the action of lateral loads on portal frames and found that for a frame fixed supported at its base, points of inflection occur at approximately the center of each girder and column and the columns carry equal shear loads, Fig. 7–8. A building bent deflects in the same way as a portal frame, Fig. 7–12*a*, and therefore it would be appropriate to assume inflection points occur at the center of the columns and girders. If we consider each bent of the frame to be composed of a series of portals, Fig. 7–12*b*, then as a further assumption, the *interior columns* would represent the effect of *two portal columns* and would therefore carry twice the shear V as the two exterior columns.



(a)

(b)

Fig. 7–12

7

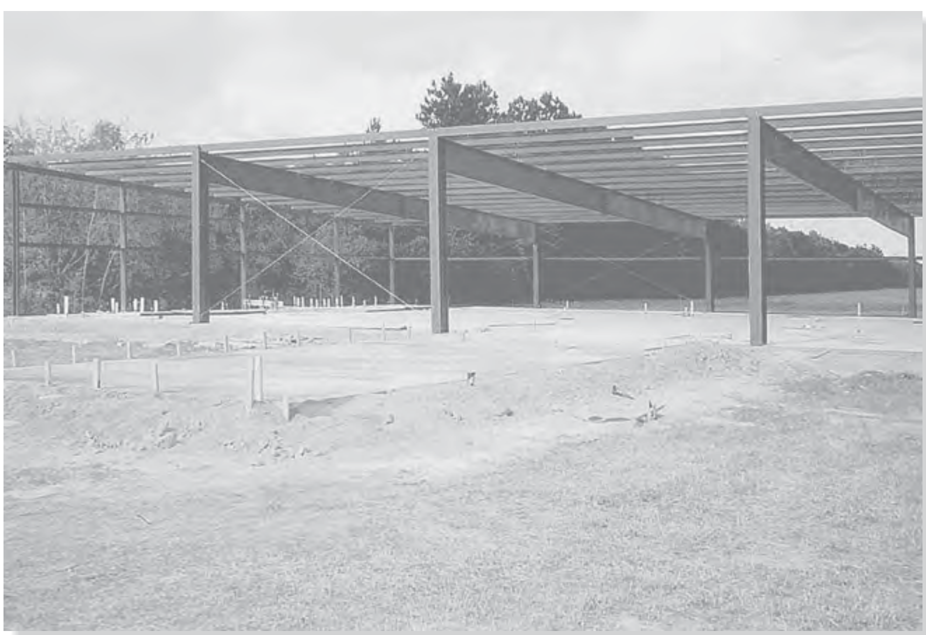
In summary, the portal method for analyzing fixed-supported building frames requires the following assumptions:

1. A hinge is placed at the center of each girder, since this is assumed to be a point of zero moment.
2. A hinge is placed at the center of each column, since this is assumed to be a point of zero moment.
3. At a given floor level the shear at the interior column hinges is twice that at the exterior column hinges, since the frame is considered to be a superposition of portals.

These assumptions provide an adequate reduction of the frame to one that is statically determinate yet stable under loading.

By comparison with the more exact statically indeterminate analysis, *the portal method is most suitable for buildings having low elevation* and *uniform framing*. The reason for this has to do with the structure's action under load. In this regard, *consider the frame as acting like a cantilevered beam* that is fixed to the ground. Recall from mechanics of materials that *shear resistance* becomes more important in the design of *short* beams, whereas *bending* is more important if the beam is *long*. (See Sec. 7–6.) The portal method is based on the assumption related to shear as stated in item 3 above.

The following examples illustrate how to apply the portal method to analyze a building bent.



The portal method of analysis can be used to (approximately) perform a lateral-load analysis of this single-story frame.

EXAMPLE 7.5

Determine (approximately) the reactions at the base of the columns of the frame shown in Fig. 7–13*a*. Use the portal method of analysis.

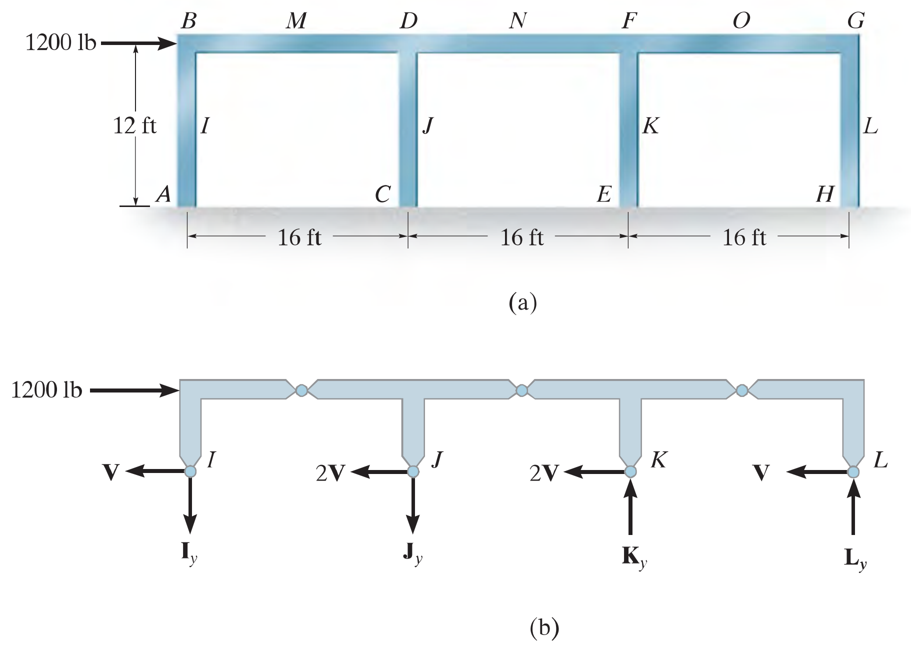


Fig. 7–13

7

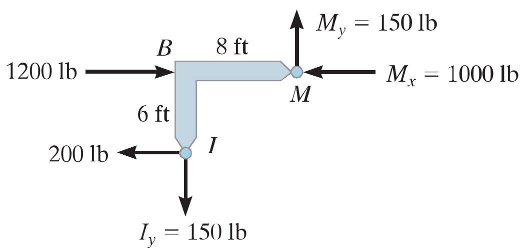
SOLUTION

Applying the first two assumptions of the portal method, we place hinges at the centers of the girders and columns of the frame, Fig. 7–13*a*. A section through the column hinges at *I*, *J*, *K*, *L* yields the free-body diagram shown in Fig. 7–13*b*. Here the third assumption regarding the column shears applies. We require

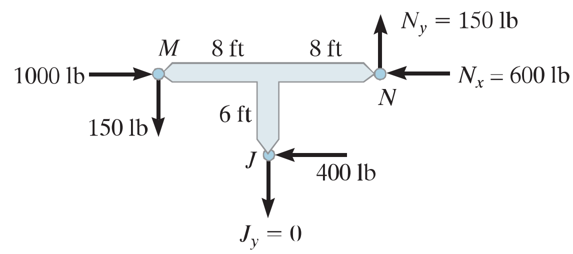
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 1200 - 6V = 0 \qquad V = 200 \text{ lb}$$

Using this result, we can now proceed to dismember the frame at the hinges and determine their reactions. *As a general rule, always start this analysis at the corner or joint where the horizontal load is applied*. Hence, the free-body diagram of segment *IBM* is shown in Fig. 7–13*c*. The three reaction components at the hinges I_y, M_x, and M_y are determined by applying $\Sigma M_M = 0$, $\Sigma F_x = 0$, $\Sigma F_y = 0$, respectively. The adjacent segment *MJN* is analyzed next, Fig. 7–13*d*, followed by segment *NKO*, Fig. 7–13*e*, and finally segment *OGL*, Fig. 7–13*f*. Using these results, the free-body diagrams of the columns with their support reactions are shown in Fig. 7–13*g*.

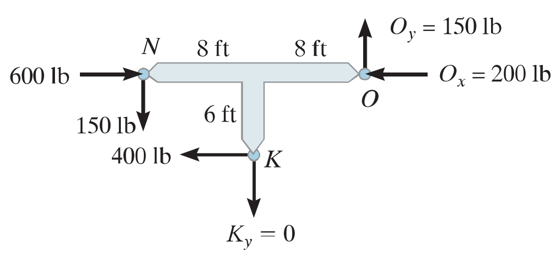
If the horizontal segments of the girders in Figs. 7–13*c*, *d*, *e* and *f* are considered, show that the moment diagram for the girder looks like that shown in Fig. 7–13*h*.



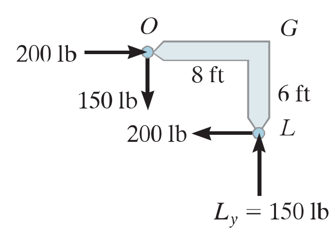
(c)



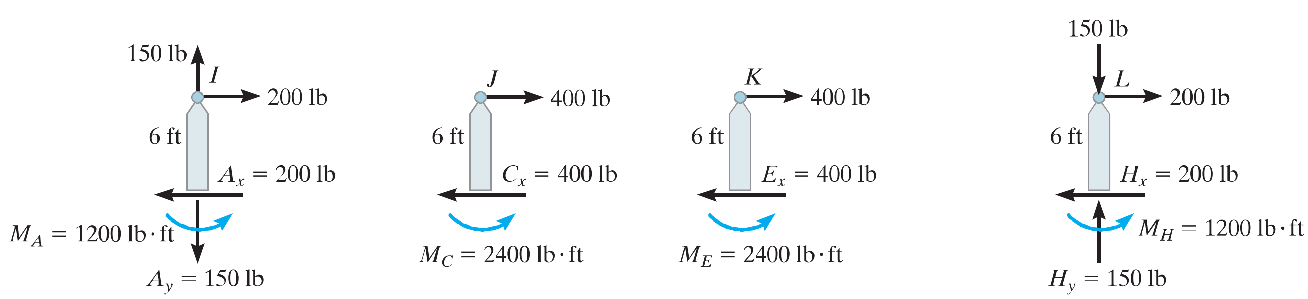
(d)



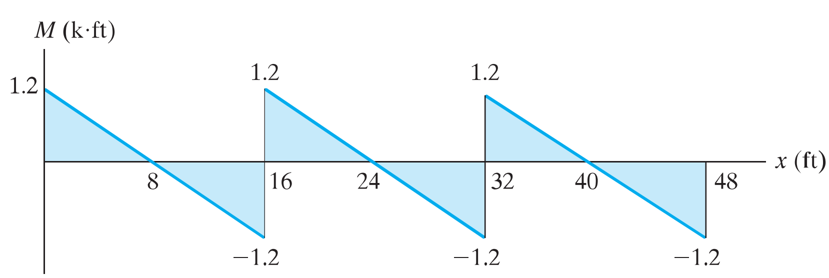
(e)



(f)



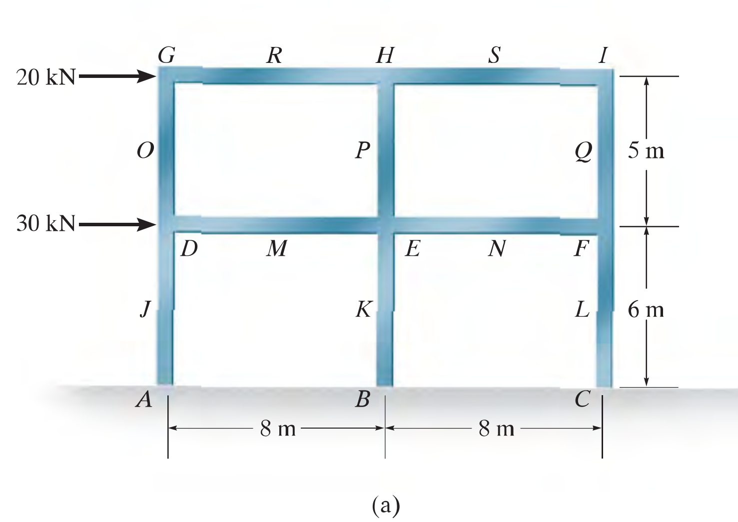
(g)



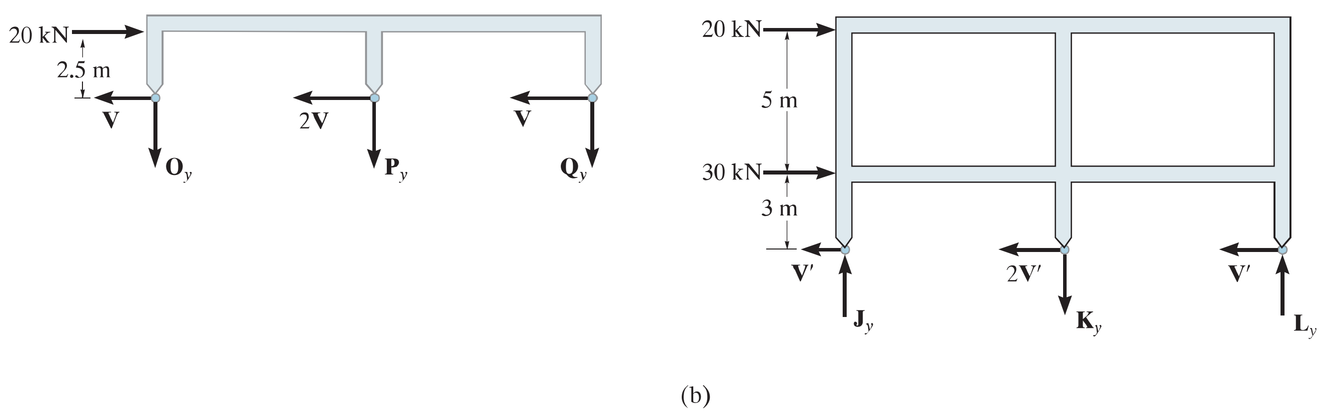
(h)

EXAMPLE 7.6

Determine (approximately) the reactions at the base of the columns of the frame shown in Fig. 7–14*a*. Use the portal method of analysis.



(a)



(b)

Fig. 7–14

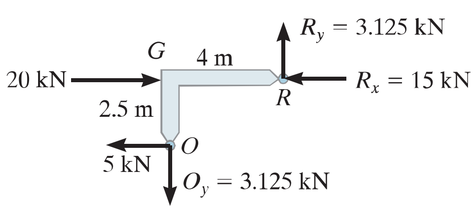
SOLUTION

First hinges are placed at the *centers* of the girders and columns of the frame, Fig. 7–14*a*. A section through the hinges at O, P, Q and J, K, L yields the free-body diagrams shown in Fig. 7–14*b*. The column shears are calculated as follows:

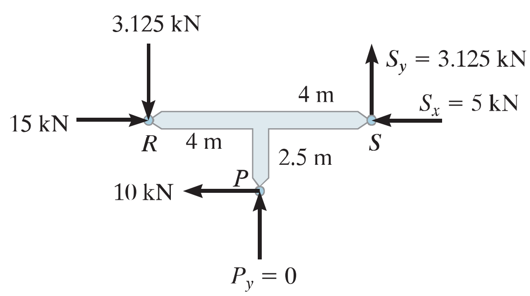
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 20 - 4V = 0 \qquad V = 5 \text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 20 + 30 - 4V' = 0 \qquad V' = 12.5 \text{ kN}$$

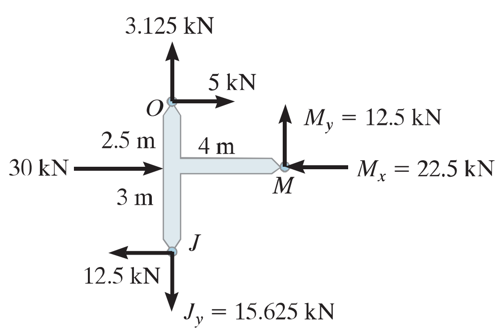
Using these results, we can now proceed to analyze each part of the frame. The analysis starts with the *corner* segment OGR, Fig. 7–14*c*. The three unknowns O_y, R_x, and R_y have been calculated using the equations of equilibrium. With these results segment OJM is analyzed next, Fig. 7–14*d*; then segment JA, Fig. 7–14*e*; RPS, Fig. 7–14*f*; $PMKN$, Fig. 7–14*g*; and KB, Fig. 7–14*h*. Complete this example and analyze segments SIQ, then QNL, and finally LC, and show that $C_x = 12.5$ kN, $C_y = 15.625$ kN, and $M_C = 37.5\text{ kN}\cdot\text{m}$. Also, use the results and show that the moment diagram for $DMENF$ is given in Fig. 7–14*i*.



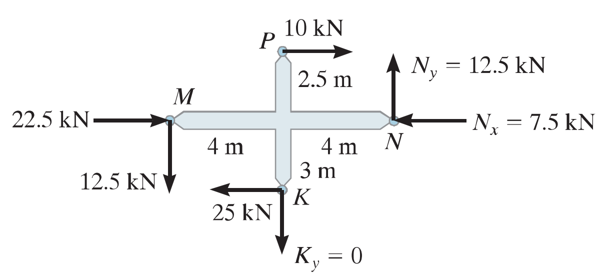
(c)



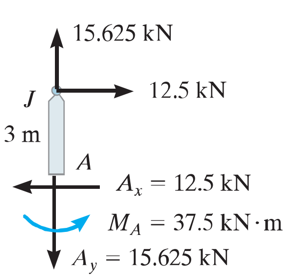
(f)



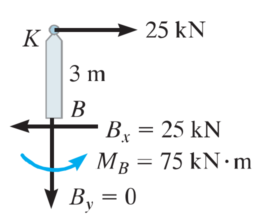
(d)



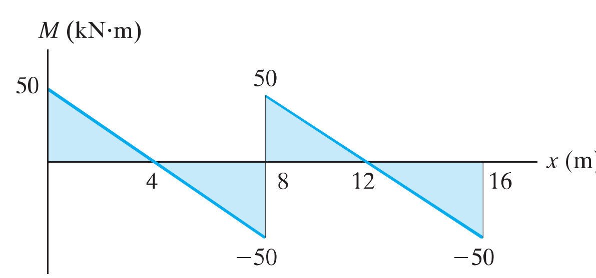
(g)



(e)



(h)



(i)