

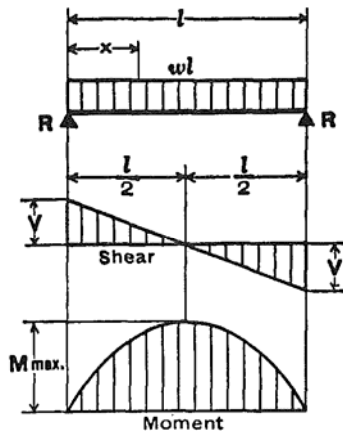
# BEAM DIAGRAMS AND FORMULAS

## For various static loading conditions

For meaning of symbols, see page 2 - 293

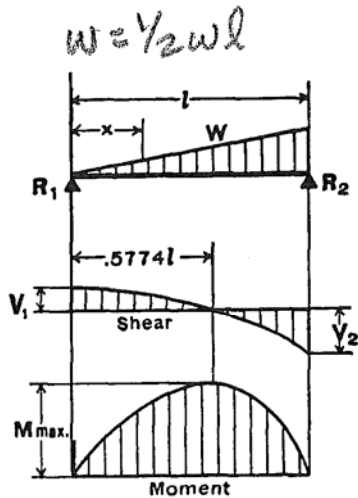
# AISC Beam Tables

### 1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



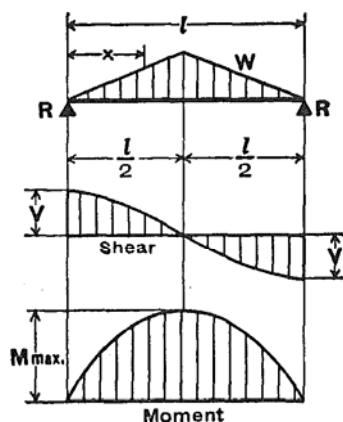
$$\begin{aligned} \text{Total Equiv. Uniform Load} &= wl \\ R = V &= \frac{wl}{2} \\ V_x &= w \left( \frac{l}{2} - x \right) \\ M_{\text{max. (at center)}} &= \frac{wl^2}{8} \\ M_x &= \frac{wx}{2} (l - x) \\ \Delta_{\text{max. (at center)}} &= \frac{5wl^4}{384EI} \\ \Delta_x &= \frac{wx}{24EI} (l^3 - 2lx^2 + x^3) \end{aligned}$$

### 2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END



$$\begin{aligned} \text{Total Equiv. Uniform Load} &= \frac{16W}{9\sqrt{3}} = 1.0264W \\ R_1 = V_1 &= \frac{W}{3} \\ R_2 = V_2 \text{ max.} &= \frac{2W}{3} \\ V_x &= \frac{W}{3} - \frac{Wx^2}{l^2} \\ M_{\text{max. (at } x = \frac{l}{\sqrt{3}} = .5774l)} &= \frac{2Wl}{9\sqrt{3}} = .1283 Wl \\ M_x &= \frac{Wx}{3l^2} (l^2 - x^2) \\ \Delta_{\text{max. (at } x = l\sqrt{1 - \sqrt{\frac{8}{15}}} = .5193l)} &= .01304 \frac{Wl^3}{EI} \\ \Delta_x &= \frac{Wx}{180EI l^2} (3x^4 - 10l^2x^2 + 7l^4) \end{aligned}$$

### 3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER



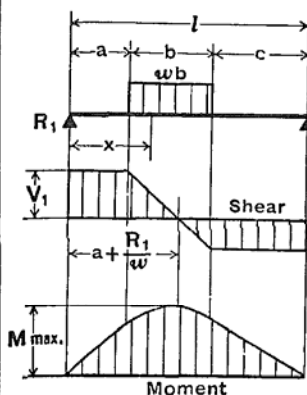
$$\begin{aligned} \text{Total Equiv. Uniform Load} &= \frac{4W}{3} \\ R = V &= \frac{W}{2} \\ V_x \text{ (when } x < \frac{l}{2}) &= \frac{W}{2l^2} (l^2 - 4x^2) \\ M_{\text{max. (at center)}} &= \frac{Wl}{6} \\ M_x \text{ (when } x < \frac{l}{2}) &= Wx \left( \frac{1}{2} - \frac{2x^2}{3l^2} \right) \\ \Delta_{\text{max. (at center)}} &= \frac{Wl^3}{60EI} \\ \Delta_x \text{ (when } x < \frac{l}{2}) &= \frac{Wx}{480EI l^2} (5l^2 - 4x^2)^2 \end{aligned}$$

# BEAM DIAGRAMS AND FORMULAS

## For various static loading conditions

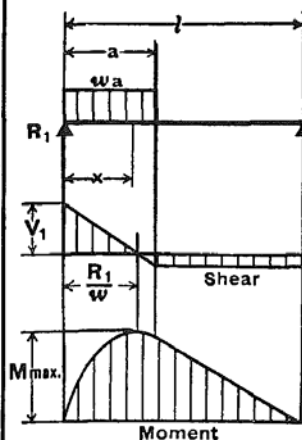
For meaning of symbols, see page 2 - 293

### 4. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED



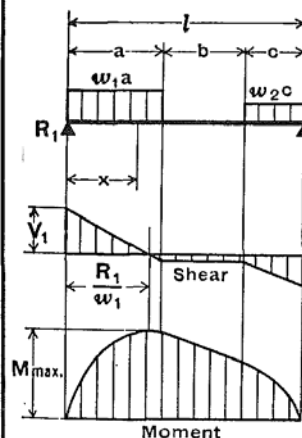
$$\begin{aligned}
 R_1 &= V_1 \quad \left( \text{max. when } a < c \right) \quad \dots = \frac{wb}{2l} (2c + b) \\
 R_2 &= V_2 \quad \left( \text{max. when } a > c \right) \quad \dots = \frac{wb}{2l} (2a + b) \\
 V_x &\quad \left( \text{when } x > a \text{ and } < (a + b) \right) \quad \dots = R_1 - w(x - a) \\
 M_{\text{max.}} &\quad \left( \text{at } x = a + \frac{R_1}{w} \right) \quad \dots = R_1 \left( a + \frac{R_1}{2w} \right) \\
 M_x &\quad \left( \text{when } x < a \right) \quad \dots = R_1 x \\
 M_x &\quad \left( \text{when } x > a \text{ and } < (a + b) \right) \quad \dots = R_1 x - \frac{w}{2} (x - a)^2 \\
 M_x &\quad \left( \text{when } x > (a + b) \right) \quad \dots = R_2 (l - x)
 \end{aligned}$$

### 5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



$$\begin{aligned}
 R_1 &= V_1 \text{ max.} \quad \dots = \frac{wa}{2l} (2l - a) \\
 R_2 &= V_2 \quad \dots = \frac{wa^2}{2l} \\
 V_x &\quad \left( \text{when } x < a \right) \quad \dots = R_1 - wx \\
 M_{\text{max.}} &\quad \left( \text{at } x = \frac{R_1}{w} \right) \quad \dots = \frac{R_1^2}{2w} \\
 M_x &\quad \left( \text{when } x < a \right) \quad \dots = R_1 x - \frac{wx^2}{2} \\
 M_x &\quad \left( \text{when } x > a \right) \quad \dots = R_2 (l - x) \\
 \Delta x &\quad \left( \text{when } x < a \right) \quad \dots = \frac{wx}{24EI} (a^2(2l-a)^2 - 2ax^2(2l-a) + lx^3) \\
 \Delta x &\quad \left( \text{when } x > a \right) \quad \dots = \frac{wa^2(l-x)}{24EI} (4xl - 2x^2 - a^2)
 \end{aligned}$$

### 6. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END



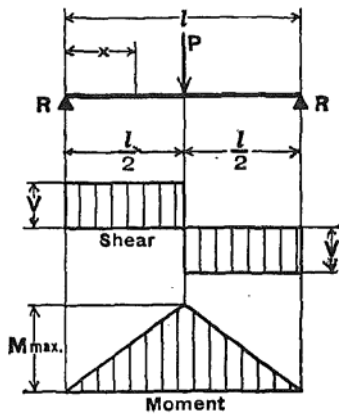
$$\begin{aligned}
 R_1 &= V_1 \quad \dots = \frac{w_1 a (2l - a) + w_2 c^2}{2l} \\
 R_2 &= V_2 \quad \dots = \frac{w_2 c (2l - c) + w_1 a^2}{2l} \\
 V_x &\quad \left( \text{when } x < a \right) \quad \dots = R_1 - w_1 x \\
 V_x &\quad \left( \text{when } x > a \text{ and } < (a + b) \right) \quad \dots = R_1 - w_1 a \\
 V_x &\quad \left( \text{when } x > (a + b) \right) \quad \dots = R_2 - w_2 (l - x) \\
 M_{\text{max.}} &\quad \left( \text{at } x = \frac{R_1}{w_1} \text{ when } R_1 < w_1 a \right) \quad \dots = \frac{R_1^2}{2w_1} \\
 M_{\text{max.}} &\quad \left( \text{at } x = l - \frac{R_2}{w_2} \text{ when } R_2 < w_2 c \right) \quad \dots = \frac{R_2^2}{2w_2} \\
 M_x &\quad \left( \text{when } x < a \right) \quad \dots = R_1 x - \frac{w_1 x^2}{2} \\
 M_x &\quad \left( \text{when } x > a \text{ and } < (a + b) \right) \quad \dots = R_1 x - \frac{w_1 a}{2} (2x - a) \\
 M_x &\quad \left( \text{when } x > (a + b) \right) \quad \dots = R_2 (l - x) - \frac{w_2 (l - x)^2}{2}
 \end{aligned}$$

# BEAM DIAGRAMS AND FORMULAS

## For various static loading conditions

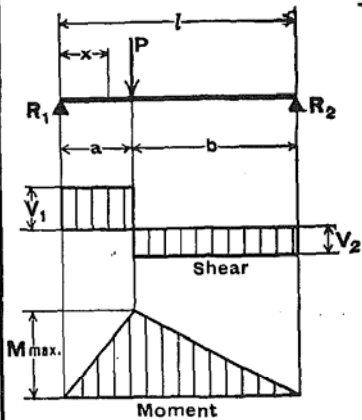
For meaning of symbols, see page 2 - 293

### 7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER



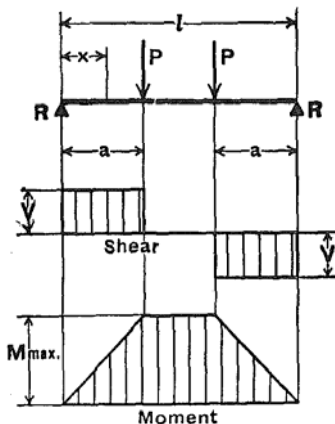
$$\begin{aligned}
 \text{Total Equiv. Uniform Load} &= 2P \\
 R = V &= \frac{P}{2} \\
 M_{\text{max.}} \text{ (at point of load)} &= \frac{Pl}{4} \\
 M_x \text{ (when } x < \frac{l}{2} \text{)} &= \frac{Px}{2} \\
 \Delta_{\text{max.}} \text{ (at point of load)} &= \frac{Pl^3}{48EI} \\
 \Delta_x \text{ (when } x < \frac{l}{2} \text{)} &= \frac{Px}{48EI} (3l^2 - 4x^2)
 \end{aligned}$$

### 8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT



$$\begin{aligned}
 \text{Total Equiv. Uniform Load} &= \frac{8 Pab}{l^2} \\
 R_1 = V_1 \text{ (max. when } a < b \text{)} &= \frac{Pb}{l} \\
 R_2 = V_2 \text{ (max. when } a > b \text{)} &= \frac{Pa}{l} \\
 M_{\text{max.}} \text{ (at point of load)} &= \frac{Pab}{l} \\
 M_x \text{ (when } x < a \text{)} &= \frac{Pbx}{l} \\
 \Delta_{\text{max.}} \text{ (at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b \text{)} &= \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI l} \\
 \Delta_a \text{ (at point of load)} &= \frac{Pa^2b^2}{3EI l} \\
 \Delta_x \text{ (when } x < a \text{)} &= \frac{Pbx}{6EI l} (l^2 - b^2 - x^2)
 \end{aligned}$$

### 9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



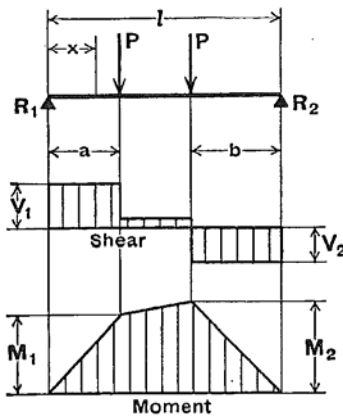
$$\begin{aligned}
 \text{Total Equiv. Uniform Load} &= \frac{8 Pa}{l} \\
 R = V &= P \\
 M_{\text{max.}} \text{ (between loads)} &= Pa \\
 M_x \text{ (when } x < a \text{)} &= Px \\
 \Delta_{\text{max.}} \text{ (at center)} &= \frac{Pa}{24EI} (3l^2 - 4a^2) \\
 \Delta_x \text{ (when } x < a \text{)} &= \frac{Px}{6EI} (3la - 3a^2 - x^2) \\
 \Delta_x \text{ (when } x > a \text{ and } < (l-a) \text{)} &= \frac{Pa}{6EI} (3lx - 3x^2 - a^2)
 \end{aligned}$$

# BEAM DIAGRAMS AND FORMULAS

## For various static loading conditions

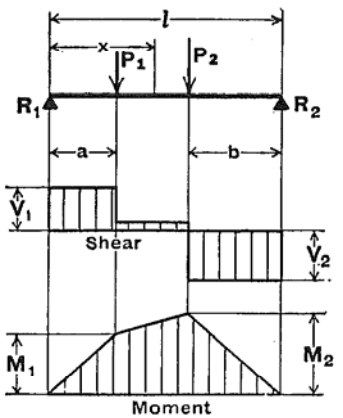
For meaning of symbols, see page 2 - 293

### 10. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



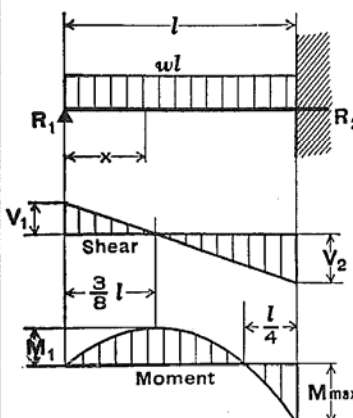
$$\begin{aligned}
 R_1 &= V_1 \left( \text{max. when } a < b \right) \dots\dots\dots = \frac{P}{l} (l - a + b) \\
 R_2 &= V_2 \left( \text{max. when } a > b \right) \dots\dots\dots = \frac{P}{l} (l - b + a) \\
 V_x &\left( \text{when } x > a \text{ and } < (l - b) \right) \dots\dots\dots = \frac{P}{l} (b - a) \\
 M_1 &\left( \text{max. when } a > b \right) \dots\dots\dots = R_1 a \\
 M_2 &\left( \text{max. when } a < b \right) \dots\dots\dots = R_2 b \\
 M_x &\left( \text{when } x < a \right) \dots\dots\dots = R_1 x \\
 M_x &\left( \text{when } x > a \text{ and } < (l - b) \right) \dots\dots\dots = R_1 x - P (x - a)
 \end{aligned}$$

### 11. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$$\begin{aligned}
 R_1 &= V_1 \dots\dots\dots = \frac{P_1 (l - a) + P_2 b}{l} \\
 R_2 &= V_2 \dots\dots\dots = \frac{P_1 a + P_2 (l - b)}{l} \\
 V_x &\left( \text{when } x > a \text{ and } < (l - b) \right) \dots\dots\dots = R_1 - P_1 \\
 M_1 &\left( \text{max. when } R_1 < P_1 \right) \dots\dots\dots = R_1 a \\
 M_2 &\left( \text{max. when } R_2 < P_2 \right) \dots\dots\dots = R_2 b \\
 M_x &\left( \text{when } x < a \right) \dots\dots\dots = R_1 x \\
 M_x &\left( \text{when } x > a \text{ and } < (l - b) \right) \dots\dots\dots = R_1 x - P_1 (x - a)
 \end{aligned}$$

### 12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER— UNIFORMLY DISTRIBUTED LOAD



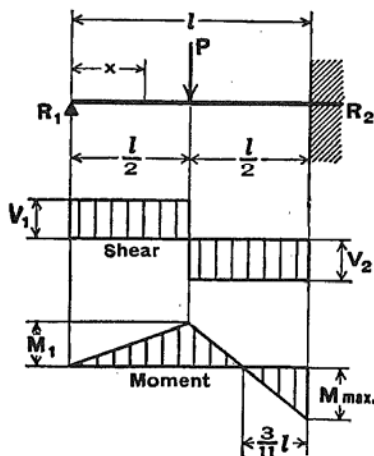
$$\begin{aligned}
 \text{Total Equiv. Uniform Load} &\dots\dots\dots = wl \\
 R_1 &= V_1 \dots\dots\dots = \frac{3wl}{8} \\
 R_2 &= V_2 \text{ max.} \dots\dots\dots = \frac{5wl}{8} \\
 V_x &\dots\dots\dots = R_1 - wx \\
 M \text{ max.} &\dots\dots\dots = \frac{wl^2}{8} \\
 M_1 &\left( \text{at } x = \frac{3}{8} l \right) \dots\dots\dots = \frac{9}{128} wl^2 \\
 M_x &\dots\dots\dots = R_1 x - \frac{wx^2}{2} \\
 \Delta \text{ max.} &\left( \text{at } x = \frac{l}{16} (1 + \sqrt{33}) = .4215l \right) \dots\dots\dots = \frac{wl^4}{185EI} \\
 \Delta_x &\dots\dots\dots = \frac{wx}{48EI} (l^3 - 3lx^2 + 2x^3)
 \end{aligned}$$

# BEAM DIAGRAMS AND FORMULAS

## For various static loading conditions

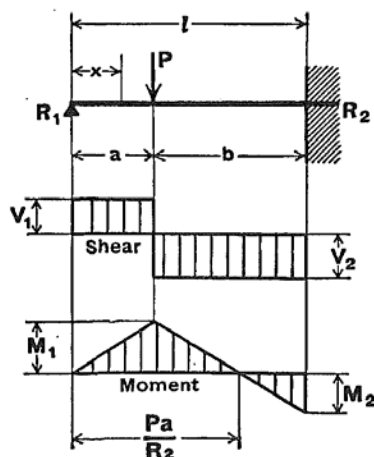
For meaning of symbols, see page 2 - 293

### 13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER— CONCENTRATED LOAD AT CENTER



$$\begin{aligned}
 \text{Total Equiv. Uniform Load} &= \frac{3P}{2} \\
 R_1 = V_1 &= \frac{5P}{16} \\
 R_2 = V_2 \text{ max.} &= \frac{11P}{16} \\
 M \text{ max. (at fixed end)} &= \frac{3Pl}{16} \\
 M_1 \text{ (at point of load)} &= \frac{5Pl}{32} \\
 M_x \text{ (when } x < \frac{l}{2}) &= \frac{5Px}{16} \\
 M_x \text{ (when } x > \frac{l}{2}) &= P \left( \frac{l}{2} - \frac{11x}{16} \right) \\
 \Delta \text{ max. (at } x = l \sqrt{\frac{1}{5}} = .4472l) &= \frac{Pl^3}{48EI\sqrt{5}} = .009317 \frac{Pl^3}{EI} \\
 \Delta x \text{ (at point of load)} &= \frac{7Pl^3}{768EI} \\
 \Delta x \text{ (when } x < \frac{l}{2}) &= \frac{Px}{96EI} (3l^2 - 5x^2) \\
 \Delta x \text{ (when } x > \frac{l}{2}) &= \frac{P}{96EI} (x-l)^2 (11x-2l)
 \end{aligned}$$

### 14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER— CONCENTRATED LOAD AT ANY POINT



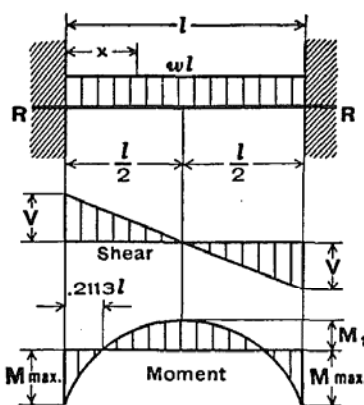
$$\begin{aligned}
 R_1 = V_1 &= \frac{Pb^2}{2l^3} (a+2l) \\
 R_2 = V_2 &= \frac{Pa}{2l^3} (3l^2 - a^2) \\
 M_1 \text{ (at point of load)} &= R_1 a \\
 M_2 \text{ (at fixed end)} &= \frac{Pab}{2l^2} (a+l) \\
 M_x \text{ (when } x < a) &= R_1 x \\
 M_x \text{ (when } x > a) &= R_1 x - P(x-a) \\
 \Delta \text{ max. (when } a < .414l \text{ at } x = l \frac{l^2+a^2}{3l^2-a^2}) &= \frac{Pa}{3EI} \frac{(l^2-a^2)^3}{(3l^2-a^2)^2} \\
 \Delta \text{ max. (when } a > .414l \text{ at } x = l \sqrt{\frac{a}{2l+a}}) &= \frac{Pab^2}{6EI} \sqrt{\frac{a}{2l+a}} \\
 \Delta a \text{ (at point of load)} &= \frac{Pa^2b^3}{12EI l^3} (3l+a) \\
 \Delta x \text{ (when } x < a) &= \frac{Pb^2x}{12EI l^3} (3al^2 - 2lx^2 - ax^2) \\
 \Delta x \text{ (when } x > a) &= \frac{Pa}{12EI l^3} (l-x)^2 (3l^2x - a^2x - 2a^2l)
 \end{aligned}$$

# BEAM DIAGRAMS AND FORMULAS

## For various static loading conditions

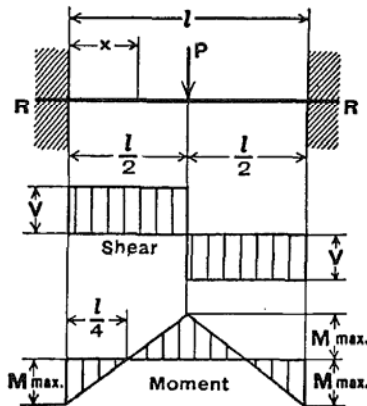
For meaning of symbols, see page 2 - 293

### 15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS



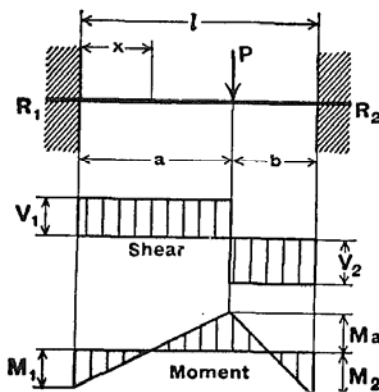
$$\begin{aligned}
 \text{Total Equiv. Uniform Load} &= \frac{2wl}{3} \\
 R = V &= \frac{wl}{2} \\
 V_x &= w\left(\frac{l}{2} - x\right) \\
 M_{\text{max. (at ends)}} &= \frac{wl^2}{12} \\
 M_1 \text{ (at center)} &= \frac{wl^2}{24} \\
 M_x &= \frac{w}{12} (6lx - l^2 - 6x^2) \\
 \Delta_{\text{max. (at center)}} &= \frac{wl^4}{384EI} \\
 \Delta_x &= \frac{wx^2}{24EI} (l - x)^2
 \end{aligned}$$

### 16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER



$$\begin{aligned}
 \text{Total Equiv. Uniform Load} &= P \\
 R = V &= \frac{P}{2} \\
 M_{\text{max. (at center and ends)}} &= \frac{Pl}{8} \\
 M_x \text{ (when } x < \frac{l}{2}) &= \frac{P}{8} (4x - l) \\
 \Delta_{\text{max. (at center)}} &= \frac{Pl^3}{192EI} \\
 \Delta_x \text{ (when } x < \frac{l}{2}) &= \frac{Px^2}{48EI} (3l - 4x)
 \end{aligned}$$

### 17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT ANY POINT



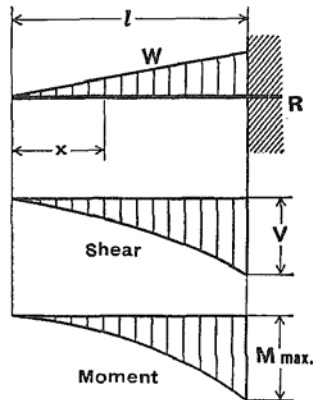
$$\begin{aligned}
 R_1 = V_1 \text{ (max. when } a < b) &= \frac{Pb^2}{l^3} (3a + b) \\
 R_2 = V_2 \text{ (max. when } a > b) &= \frac{Pa^2}{l^3} (a + 3b) \\
 M_1 \text{ (max. when } a < b) &= \frac{Pab^2}{l^2} \\
 M_2 \text{ (max. when } a > b) &= \frac{Pa^2b}{l^2} \\
 M_a \text{ (at point of load)} &= \frac{2Pa^2b^2}{l^3} \\
 M_x \text{ (when } x < a) &= R_1x - \frac{Pab^2}{l^2} \\
 \Delta_{\text{max. (when } a > b \text{ at } x = \frac{2a}{3a+b})} &= \frac{2Pa^3b^2}{3EI (3a + b)^2} \\
 \Delta_a \text{ (at point of load)} &= \frac{Pa^3b^3}{3EI l^3} \\
 \Delta_x \text{ (when } x < a) &= \frac{Pb^2x^2}{6EI l^3} (3a - 3ax - bx)
 \end{aligned}$$

# BEAM DIAGRAMS AND FORMULAS

## For various static loading conditions

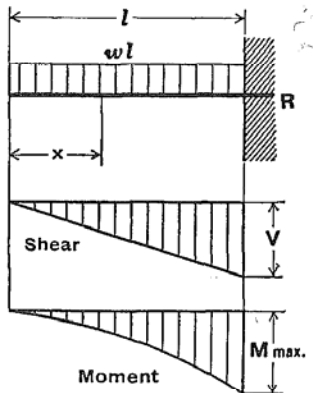
For meaning of symbols, see page 2 - 293

### 18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END



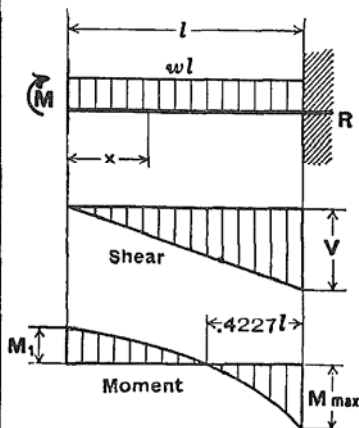
Total Equiv. Uniform Load . . . . .	$= \frac{8}{3} W$
$R = V$ . . . . .	$= W$
$V_x$ . . . . .	$= W \frac{x^2}{l^2}$
$M \text{ max. (at fixed end)}$ . . . . .	$= \frac{Wl}{3}$
$M_x$ . . . . .	$= \frac{Wx^3}{3l^2}$
$\Delta \text{ max. (at free end)}$ . . . . .	$= \frac{Wl^3}{15EI}$
$\Delta_x$ . . . . .	$= \frac{W}{60EI l^2} (x^5 - 5l^4x + 4l^5)$

### 19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . .	$= 4wl$
$R = V$ . . . . .	$= wl$
$V_x$ . . . . .	$= wx$
$M \text{ max. (at fixed end)}$ . . . . .	$= \frac{wl^2}{2}$
$M_x$ . . . . .	$= \frac{wx^2}{2}$
$\Delta \text{ max. (at free end)}$ . . . . .	$= \frac{wl^4}{8EI}$
$\Delta_x$ . . . . .	$= \frac{w}{24EI} (x^4 - 4l^3x + 3l^4)$

### 20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—UNIFORMLY DISTRIBUTED LOAD



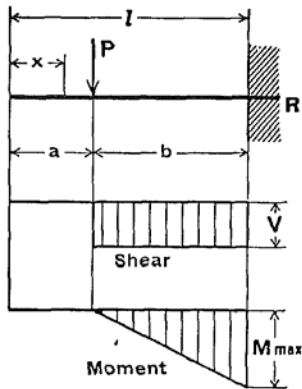
Total Equiv. Uniform Load . . . . .	$= \frac{8}{3} wl$
$R = V$ . . . . .	$= wl$
$V_x$ . . . . .	$= wx$
$M \text{ max. (at fixed end)}$ . . . . .	$= \frac{wl^2}{3}$
$M_1$ (at deflected end) . . . . .	$= \frac{wl^2}{6}$
$M_x$ . . . . .	$= \frac{w}{6} (l^2 - 3x^2)$
$\Delta \text{ max. (at deflected end)}$ . . . . .	$= \frac{wl^4}{24EI}$
$\Delta_x$ . . . . .	$= \frac{w}{24EI} (l^2 - x^2)^2$

# BEAM DIAGRAMS AND FORMULAS

## For various static loading conditions

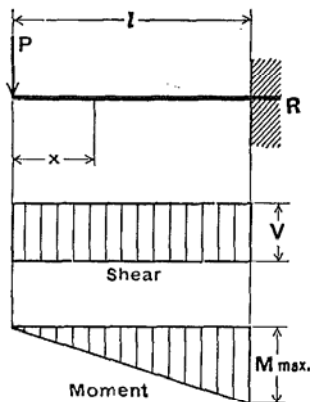
For meaning of symbols, see page 2 - 293

### 21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT



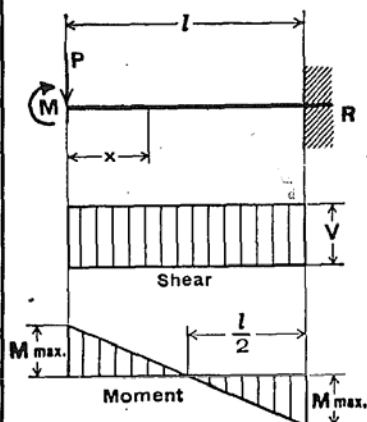
$$\begin{aligned}
 \text{Total Equiv. Uniform Load} & \dots = \frac{8Pb}{l} \\
 R = V & \dots = P \\
 M_{\text{max. (at fixed end)}} & \dots = Pb \\
 M_x \text{ (when } x > a) & \dots = P(x - a) \\
 \Delta_{\text{max. (at free end)}} & \dots = \frac{Pb^2}{6EI} (3l - b) \\
 \Delta_a \text{ (at point of load)} & \dots = \frac{Pb^3}{3EI} \\
 \Delta_x \text{ (when } x < a) & \dots = \frac{Pb^2}{6EI} (3l - 3x - b) \\
 \Delta_x \text{ (when } x > a) & \dots = \frac{P(l - x)^2}{6EI} (3b - l + x)
 \end{aligned}$$

### 22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END



$$\begin{aligned}
 \text{Total Equiv. Uniform Load} & \dots = 8P \\
 R = V & \dots = P \\
 M_{\text{max. (at fixed end)}} & \dots = Pl \\
 M_x & \dots = Px \\
 \Delta_{\text{max. (at free end)}} & \dots = \frac{Pl^3}{3EI} \\
 \Delta_x & \dots = \frac{P}{6EI} (2l^3 - 3l^2x + x^3)
 \end{aligned}$$

### 23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—CONCENTRATED LOAD AT DEFLECTED END



$$\begin{aligned}
 \text{Total Equiv. Uniform Load} & \dots = 4P \\
 R = V & \dots = P \\
 M_{\text{max. (at both ends)}} & \dots = \frac{Pl}{2} \\
 M_x & \dots = P\left(\frac{l}{2} - x\right) \\
 \Delta_{\text{max. (at deflected end)}} & \dots = \frac{Pl^3}{12EI} \\
 \Delta_x & \dots = \frac{P(l - x)^2}{12EI} (l + 2x)
 \end{aligned}$$

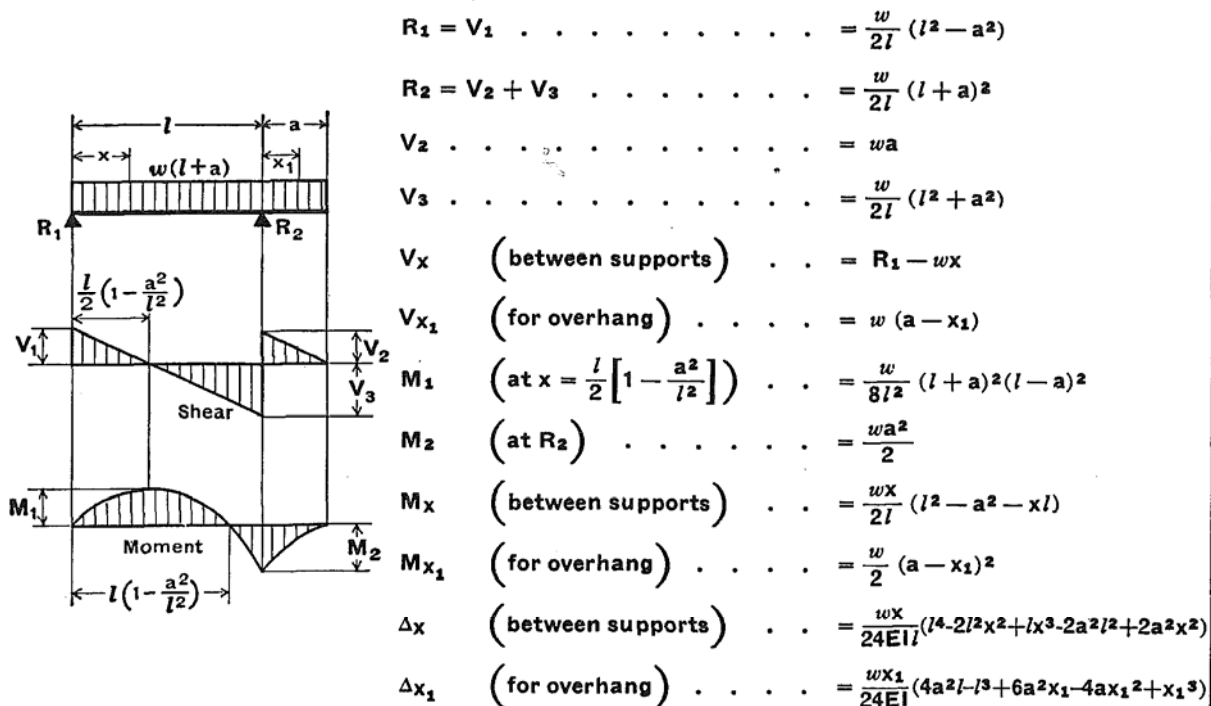


# BEAM DIAGRAMS AND FORMULAS

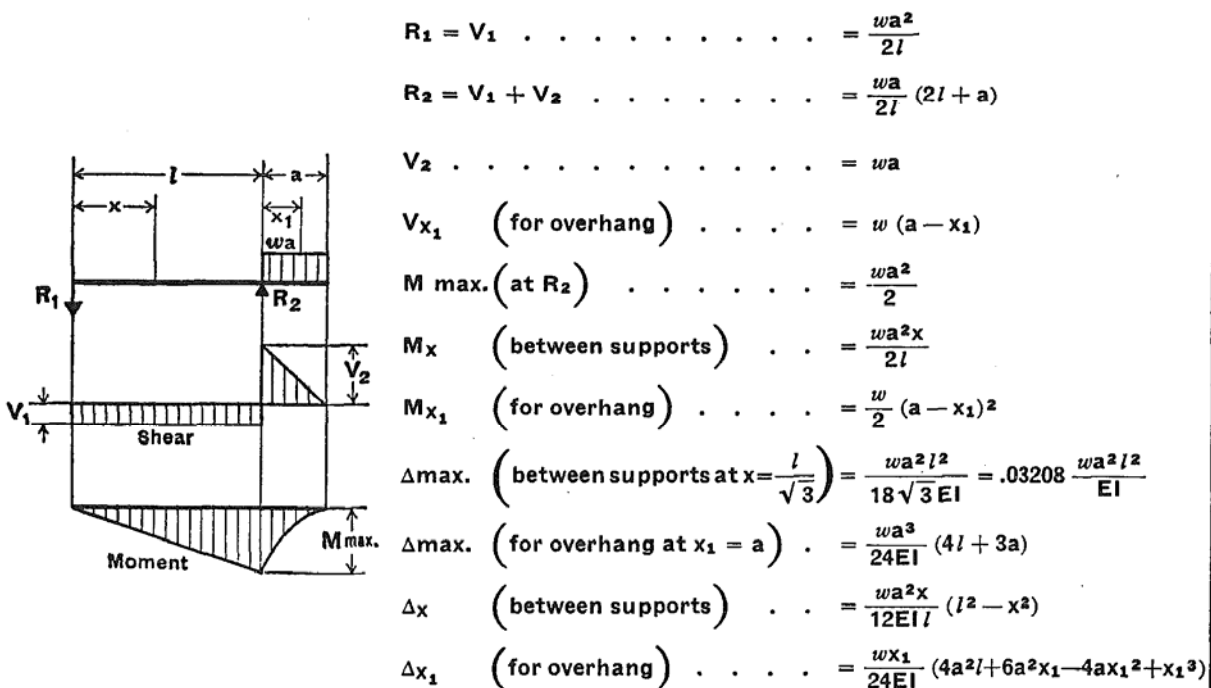
## For various static loading conditions

For meaning of symbols, see page 2 - 293

### 24. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD



### 25. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD ON OVERHANG

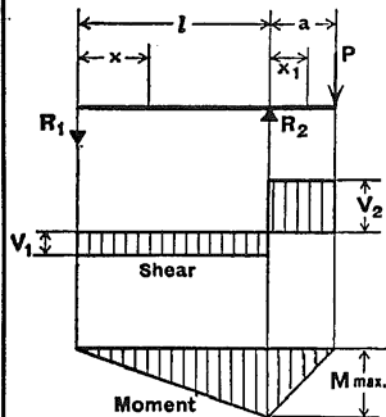


# BEAM DIAGRAMS AND FORMULAS

## For various static loading conditions

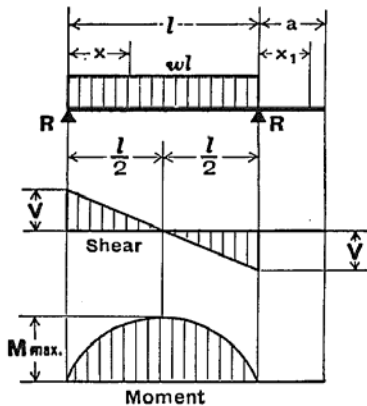
For meaning of symbols, see page 2 - 293

### 26. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT END OF OVERHANG



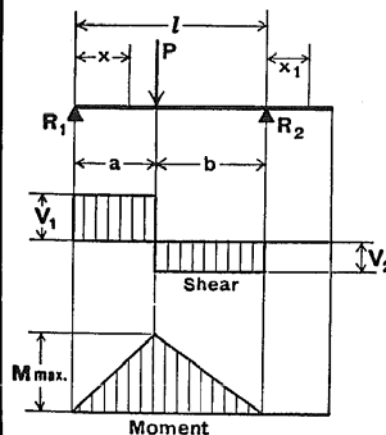
$$\begin{aligned}
 R_1 &= V_1 \dots\dots\dots = \frac{Pa}{l} \\
 R_2 &= V_1 + V_2 \dots\dots\dots = \frac{P}{l} (l + a) \\
 V_2 &\dots\dots\dots = P \\
 M_{\text{max.}} & \left( \text{at } R_2 \right) \dots\dots\dots = Pa \\
 M_x & \left( \text{between supports} \right) \dots\dots\dots = \frac{Pax}{l} \\
 M_{x_1} & \left( \text{for overhang} \right) \dots\dots\dots = P(a - x_1) \\
 \Delta_{\text{max.}} & \left( \text{between supports at } x = \frac{l}{\sqrt{3}} \right) = \frac{Pa l^2}{9\sqrt{3}EI} = .06415 \frac{Pa l^2}{EI} \\
 \Delta_{\text{max.}} & \left( \text{for overhang at } x_1 = a \right) \dots\dots\dots = \frac{Pa^2}{3EI} (l + a) \\
 \Delta x & \left( \text{between supports} \right) \dots\dots\dots = \frac{Pax}{6EI} (l^2 - x^2) \\
 \Delta x_1 & \left( \text{for overhang} \right) \dots\dots\dots = \frac{Px_1}{6EI} (2al + 3ax_1 - x_1^2)
 \end{aligned}$$

### 27. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD BETWEEN SUPPORTS



$$\begin{aligned}
 \text{Total Equiv. Uniform Load} &\dots\dots\dots = wl \\
 R &= V \dots\dots\dots = \frac{wl}{2} \\
 V_x &\dots\dots\dots = w \left( \frac{l}{2} - x \right) \\
 M_{\text{max.}} & \left( \text{at center} \right) \dots\dots\dots = \frac{wl^2}{8} \\
 M_x &\dots\dots\dots = \frac{wx}{2} (l - x) \\
 \Delta_{\text{max.}} & \left( \text{at center} \right) \dots\dots\dots = \frac{5wl^4}{384EI} \\
 \Delta x &\dots\dots\dots = \frac{wx}{24EI} (l^3 - 2lx^2 + x^3) \\
 \Delta x_1 &\dots\dots\dots = \frac{wl^3 x_1}{24EI}
 \end{aligned}$$

### 28. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT ANY POINT BETWEEN SUPPORTS



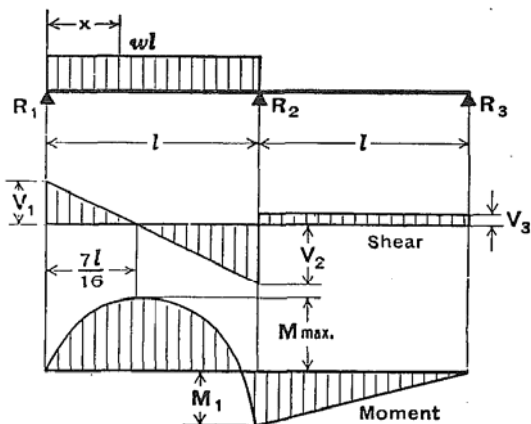
$$\begin{aligned}
 \text{Total Equiv. Uniform Load} &\dots\dots\dots = \frac{8Pab}{l^2} \\
 R_1 &= V_1 \left( \text{max. when } a < b \right) \dots\dots\dots = \frac{Pb}{l} \\
 R_2 &= V_2 \left( \text{max. when } a > b \right) \dots\dots\dots = \frac{Pa}{l} \\
 M_{\text{max.}} & \left( \text{at point of load} \right) \dots\dots\dots = \frac{Pab}{l} \\
 M_x & \left( \text{when } x < a \right) \dots\dots\dots = \frac{Pbx}{l} \\
 \Delta_{\text{max.}} & \left( \text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b \right) = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI} \\
 \Delta a & \left( \text{at point of load} \right) \dots\dots\dots = \frac{Pa^2 b^2}{3EI} \\
 \Delta x & \left( \text{when } x < a \right) \dots\dots\dots = \frac{Pbx}{6EI} (l^2 - b^2 - x^2) \\
 \Delta x & \left( \text{when } x > a \right) \dots\dots\dots = \frac{Pa(l-x)}{6EI} (2lx - x^2 - a^2) \\
 \Delta x_1 &\dots\dots\dots = \frac{Pabx_1}{6EI} (l + a)
 \end{aligned}$$

# BEAM DIAGRAMS AND FORMULAS

## For various static loading conditions

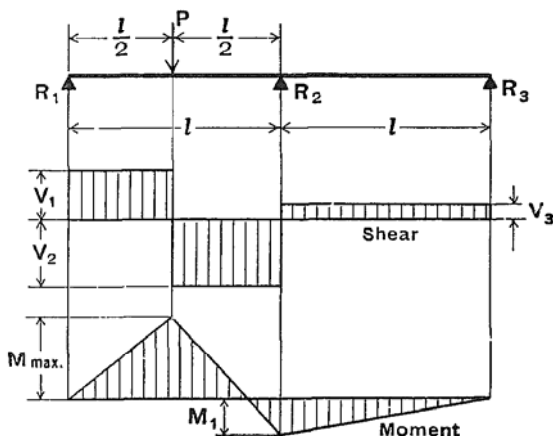
For meaning of symbols, see page 2 - 293

### 29. CONTINUOUS BEAM—TWO EQUAL SPANS—UNIFORM LOAD ON ONE SPAN



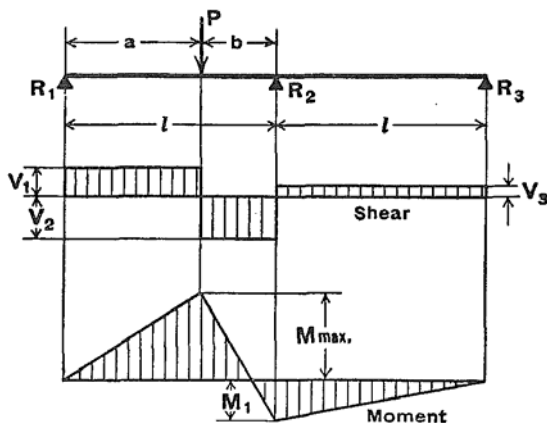
$$\begin{aligned}
 \text{Total Equiv. Uniform Load} &= \frac{49}{64} wl \\
 R_1 = V_1 &= \frac{7}{16} wl \\
 R_2 = V_2 + V_3 &= \frac{5}{8} wl \\
 R_3 = V_3 &= -\frac{1}{16} wl \\
 V_2 &= \frac{9}{16} wl \\
 M_{\text{max.}} \left( \text{at } x = \frac{7}{16} l \right) &= \frac{49}{512} wl^2 \\
 M_1 \left( \text{at support } R_2 \right) &= \frac{1}{16} wl^2 \\
 M_x \left( \text{when } x < l \right) &= \frac{wx}{16} (7l - 8x) \\
 \Delta \text{ Max. (0.472 } l \text{ from } R_1) &= 0.0092 \frac{wl^4}{EI}
 \end{aligned}$$

### 30. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT CENTER OF ONE SPAN



$$\begin{aligned}
 \text{Total Equiv. Uniform Load} &= \frac{13}{8} P \\
 R_1 = V_1 &= \frac{13}{32} P \\
 R_2 = V_2 + V_3 &= \frac{11}{16} P \\
 R_3 = V_3 &= -\frac{3}{32} P \\
 V_2 &= \frac{19}{32} P \\
 M_{\text{max.}} \left( \text{at point of load} \right) &= \frac{13}{64} Pl \\
 M_1 \left( \text{at support } R_2 \right) &= \frac{3}{32} Pl \\
 \Delta \text{ Max. (0.480 } l \text{ from } R_1) &= 0.015 \frac{Pl^3}{EI}
 \end{aligned}$$

### 31. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT ANY POINT



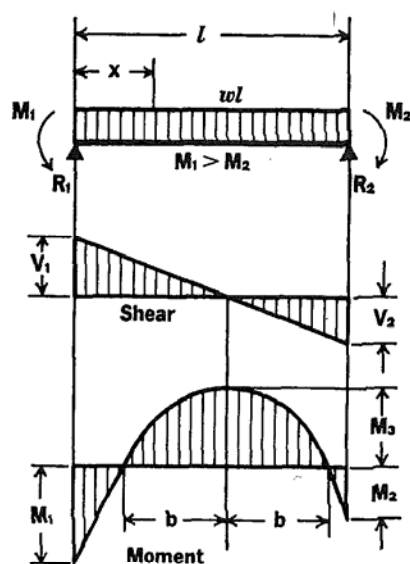
$$\begin{aligned}
 R_1 = V_1 &= \frac{Pb}{4l^3} (4l^2 - a(l+a)) \\
 R_2 = V_2 + V_3 &= \frac{Pa}{2l^3} (2l^2 + b(l+a)) \\
 R_3 = V_3 &= -\frac{Pab}{4l^3} (l+a) \\
 V_2 &= \frac{Pa}{4l^3} (4l^2 + b(l+a)) \\
 M_{\text{max.}} \left( \text{at point of load} \right) &= \frac{Pab}{4l^3} (4l^2 - a(l+a)) \\
 M_1 \left( \text{at support } R_2 \right) &= \frac{Pab}{4l^2} (l+a)
 \end{aligned}$$

# BEAM DIAGRAMS AND FORMULAS

## For various static loading conditions

For meaning of symbols, see page 2 - 293

### 32. BEAM—UNIFORMLY DISTRIBUTED LOAD AND VARIABLE END MOMENTS



$$R_1 = V_1 = \frac{wl}{2} + \frac{M_1 - M_2}{l}$$

$$R_2 = V_2 = \frac{wl}{2} - \frac{M_1 - M_2}{l}$$

$$V_x = w \left( \frac{l}{2} - x \right) + \frac{M_1 - M_2}{l}$$

$$M_3 \left( \text{at } x = \frac{l}{2} + \frac{M_1 - M_2}{wl} \right)$$

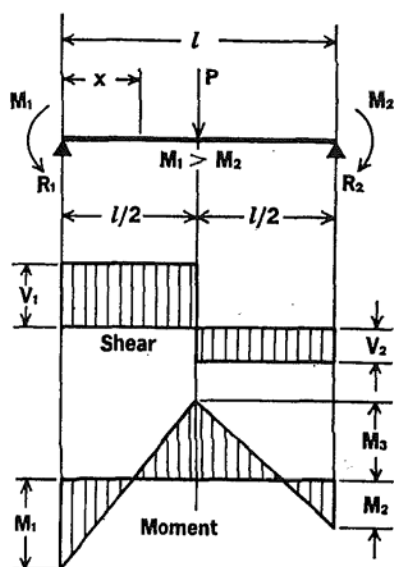
$$= \frac{wl^2}{8} - \frac{M_1 + M_2}{2} + \frac{(M_1 - M_2)^2}{2wl^2}$$

$$M_x = \frac{wx}{2} (l - x) + \left( \frac{M_1 - M_2}{l} \right) x - M_1$$

$$b \left( \text{To locate inflection points} \right) = \sqrt{\frac{l^2}{4} - \left( \frac{M_1 + M_2}{w} \right) + \left( \frac{M_1 - M_2}{wl} \right)^2}$$

$$\Delta_x = \frac{wx}{24EI} \left[ x^3 - \left( 2l + \frac{4M_1}{wl} - \frac{4M_2}{wl} \right) x^2 + \frac{12M_1}{w} x + l^3 - \frac{8M_1 l}{w} - \frac{4M_2 l}{w} \right]$$

### 33. BEAM—CONCENTRATED LOAD AT CENTER AND VARIABLE END MOMENTS



$$R_1 = V_1 = \frac{P}{2} + \frac{M_1 - M_2}{l}$$

$$R_2 = V_2 = \frac{P}{2} - \frac{M_1 - M_2}{l}$$

$$M_3 \left( \text{At center} \right) = \frac{Pl}{4} - \frac{M_1 + M_2}{2}$$

$$M_x \left( \text{When } x < \frac{l}{2} \right) = \left( \frac{P}{2} + \frac{M_1 - M_2}{l} \right) x - M_1$$

$$M_x \left( \text{When } x > \frac{l}{2} \right) = \frac{P}{2} (l - x) + \frac{(M_1 - M_2)x}{l} - M_1$$

$$\Delta_x \left( \text{When } x < \frac{l}{2} \right) = \frac{Px}{48EI} \left( 3l^2 - 4x^2 - \frac{8(l-x)}{Pl} [M_1(2l-x) + M_2(l+x)] \right)$$